Line Assignment

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I. Question

Class 11, Exercise 10.1, Q(9): **Without using distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (–3, 2)are the vertices of a parallelogram.**

II. Solution

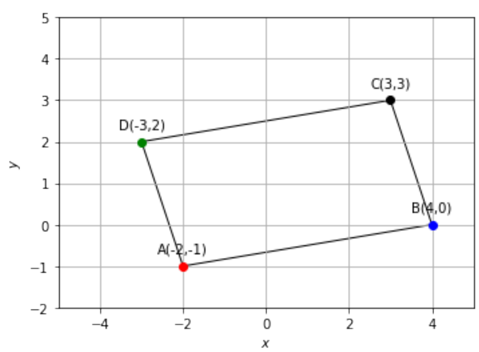

Figure 1: paralellogram ABCD

We can prove that the points are the vertices of a parallelogram if $\vec{AB} \parallel \vec{DC}$, $\vec{BC} \parallel \vec{AD}$ and $\vec{AB}$ $=\vec{DC}$, $\vec{BC} = \vec{AD}$

Theorm:if θ is the angle between $\vec{a}$ and $\vec{b}$,then

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|sin\theta$$

corollary: The two non-zero vectors $\vec{a}$ and $\vec{b}$ are

parallel to each other,if their product is a zero vector

Consider figure I,where

A $=\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ $\quad \boldsymbol{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

C$=\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $\quad \boldsymbol{D} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

let

$$\boldsymbol{P} = \boldsymbol{B} - \boldsymbol{A} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} \quad \boldsymbol{Q} = \boldsymbol{C} - \boldsymbol{D} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$$\boldsymbol{R} = \boldsymbol{A} - \boldsymbol{C} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad \boldsymbol{S} = \boldsymbol{A} - \boldsymbol{D} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

A. *proof for $\boldsymbol{P} \parallel \boldsymbol{Q}$ and $\boldsymbol{P} = \boldsymbol{Q}$*

$$\boldsymbol{P} \times \boldsymbol{Q} = |(\boldsymbol{B} - \boldsymbol{A}) \times (\boldsymbol{C} - \boldsymbol{D}))| = \begin{vmatrix} \hat{i} & \hat{j} \\ 6 & 1 \\ 6 & 1 \end{vmatrix} = 0\hat{k}$$

so,according to corollary **P||Q**

$$||\boldsymbol{P}|| = \sqrt{\begin{pmatrix} 1 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix}} = \sqrt{37} \tag{1}$$

$$||\boldsymbol{Q}|| = \sqrt{\begin{pmatrix} 1 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix}} = \sqrt{37} \tag{2}$$

form equation (1) and (2) **P=Q**

B. *proof for $\boldsymbol{R} \parallel \boldsymbol{S}$ and $\boldsymbol{R} = \boldsymbol{S}$*

$$\boldsymbol{R} \times \boldsymbol{S} = |(\boldsymbol{B} - \boldsymbol{C}) \times (\boldsymbol{A} - \boldsymbol{D})| = \begin{vmatrix} \hat{i} & \hat{j} \\ 1 & -3 \\ 1 & -3 \end{vmatrix} = 0\hat{k}$$

so,according to corollary **R||S**

$$||\boldsymbol{R}|| = \sqrt{\begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix}} = \sqrt{10} \tag{3}$$

$$||\boldsymbol{S}|| = \sqrt{\begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix}} = \sqrt{10} \tag{4}$$

form equation (3) and (4) **R=S**

since the opposite lines are parallel and equal the points (– 2, – 1), (4, 0), (3, 3) and (–3, 2) forms the vertices of a parallelogram

CONSTRUCTION

Symbol	**Value**	**Description**
A	$\begin{pmatrix} -2 \\ -1 \end{pmatrix}$	Vertex A
B	$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	Vertex B
C	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	Vertex C
D	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$	Vertex D
P	$\begin{pmatrix} 1 \\ 6 \end{pmatrix}$	vector AB
Q	$\begin{pmatrix} 1 \\ 6 \end{pmatrix}$	vector DC
R	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector BC
S	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	vector AD

Get the python code of the figures from

https://github.com/kkousar/KOUSAR_FWC/blob/main/matrices/line/code/line.py