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Izvod pravila učenja za sve parametre mreže:

Za k-ti uzorak: funkcija pogreške $E_k = \frac{1}{2}(y_k - o_k)^2$

↳ izlaz mreže

Izlaz sustava neizrazitog upravljanja za k-ti uzorak: ↳ očekivani izlaz

$$O_k = \frac{\sum_{i=1}^{m} \alpha_i \beta_i z_i}{\sum_{i=1}^{m} \alpha_i \beta_i}$$

α_i, β_i - funkcije pripadnosti (jakosti paljenja i-tog pravila)

z_i - konsekvent pravila (konstante, realni brojevi)

$$\alpha_i = \frac{1}{1+e^{b_i(x-a_i)}} \qquad \beta_i = \frac{1}{1+e^{d_i(y-c_i)}} \qquad z_i = p_i x + q_i y + r_i$$

Parametre ažuriramo u skladu s algoritmom gradijentnog spusta:

$$\psi(t+1) = \psi(t) - \eta \frac{\partial E_k}{\partial \psi}$$

Određujemo parcijalne derivacije funkcije E_k po parametrima: $p_i, q_i, r_i, a_i, b_i, c_i, d_i$

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial z_i} \cdot \frac{\partial z_i}{\partial p_i}$$

$$\frac{\partial E_k}{\partial o_k} = \frac{\partial}{\partial o_k}\left(\frac{1}{2}(y_k - o_k)^2\right) = -(y_k - o_k); \quad \frac{\partial o_k}{\partial z_i} = \frac{\partial}{\partial z_i}\left(\frac{\sum_{j=1}^{m} \alpha_j \beta_j z_j}{\sum_{j=1}^{m} \alpha_j \beta_j}\right) = \frac{\alpha_i \beta_i}{\sum_{j=1}^{m} \alpha_j \beta_j};$$

$$\frac{\partial z_i}{\partial p_i} = \frac{\partial}{\partial p_i}(x p_i + y q_i + r_i) = x$$

$$\boxed{\frac{\partial E_k}{\partial p_i} = -(y_k - o_k) \cdot \frac{\alpha_i \beta_i}{\sum_{j=1}^{m} \alpha_j \beta_j} \cdot x}$$

$$\frac{\partial E_k}{\partial q_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial z_i} \cdot \frac{\partial z_i}{\partial q_i}$$

$$\frac{\partial z_i}{\partial q_i} = \frac{\partial}{\partial q_i}(p_i x + q_i y + r_i) = y$$

$$\boxed{\frac{\partial E_k}{\partial q_i} = -(y_k - o_k) \cdot \frac{\alpha_i \beta_i}{\sum_{j=1}^{m} \alpha_j \beta_j} \cdot y}$$

$$\frac{\partial E_k}{\partial r_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial z_i} \cdot \frac{\partial z_i}{\partial r_i}$$

$$\frac{\partial z_i}{\partial r_i} = \frac{\partial}{\partial r_i}(p_i x + q_i y + r_i) = 1$$

$$\boxed{\frac{\partial E_k}{\partial r_i} = -(y_k - o_k) \cdot \frac{\alpha_i \beta_i}{\sum_{j=1}^{m} \alpha_j \beta_j}}$$

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$$\frac{\partial E_k}{\partial a_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial a_i} = \frac{\partial E_k}{\partial a_i}$$

$$\frac{\partial o_k}{\partial \alpha_i} = \frac{\partial}{\partial \alpha_i}\left(\frac{\sum_{j=1}^{m} \alpha_j \beta_j z_j}{\sum_{j=1}^{m} \alpha_j \beta_j}\right) = \left(\frac{\partial}{\partial \alpha_i}\left(\sum_{j=1}^{m} \alpha_j \beta_j z_j\right)\cdot\left(\sum_{j=1}^{m} \alpha_j \beta_j\right) - \left(\sum_{j=1}^{m} \alpha_j \beta_j z_j\right)\cdot\left(\frac{\partial}{\partial \alpha_i}\left(\sum_{j=1}^{m} \alpha_j \beta_j\right)\right)\right)\cdot\frac{1}{\left(\sum_{j=1}^{m} \alpha_j \beta_j\right)^2}$$

$$= \frac{1}{\left(\sum_{j=1}^{m} \alpha_j \beta_j\right)^2}\cdot\left(\beta_i z_i \cdot \sum_{j=1}^{m} \alpha_j \beta_j - \beta_i \cdot \sum_{j=1}^{m} \alpha_j \beta_j z_j\right)$$

$$\frac{\partial \alpha_i}{\partial a_i} = \frac{\partial}{\partial a_i}\left(\frac{1}{1+e^{b_i(x-a_i)}}\right) = -\frac{1}{(1+e^{b_i(x-a_i)})^2}\cdot e^{b_i(x-a_i)}\cdot(-b_i) = b_i \cdot \underbrace{\frac{1}{1+e^{b_i(x-a_i)}}}_{\alpha_i}\cdot\underbrace{\left(\frac{e^{b_i(x-a_i)}}{1+e^{b_i(x-a_i)}}\right)}_{1-\alpha_i}$$

$$= \alpha_i(1-\alpha_i)\cdot b_i$$

$$\boxed{\frac{\partial E_k}{\partial a_i} = -(y_k - o_k)\cdot\frac{z_i \sum_{j=1}^{m} \alpha_j \beta_j - \sum_{j=1}^{m} \alpha_j \beta_j z_j}{(\sum_{j=1}^{m} \alpha_j \beta_j)^2}\cdot \beta_i \alpha_i(1-\alpha_i)\cdot b_i}$$

$$\frac{\partial E_k}{\partial b_i} = \frac{\partial E_k}{\partial o_k}\cdot\frac{\partial o_k}{\partial \alpha_i}\cdot\frac{\partial \alpha_i}{\partial b_i}$$

$$\frac{\partial \alpha_i}{\partial b_i} = \frac{\partial}{\partial b_i}\left(\frac{1}{1+e^{b_i(x-a_i)}}\right) = \frac{-1}{(1+e^{b_i(x-a_i)})^2}\cdot e^{b_i(x-a_i)}\cdot(x-a_i) = \frac{e^{b_i(x-a_i)}}{(1+e^{b_i(x-a_i)})^2}\cdot(a_i - x) =$$

$$= \alpha_i(1-\alpha_i)\cdot(a_i - x)$$

$$\boxed{\frac{\partial E_k}{\partial b_i} = -(y_k - o_k)\cdot\frac{z_i \sum_{j=1}^{m} \alpha_j \beta_j - \sum_{j=1}^{m} \alpha_j \beta_j z_j}{(\sum_{j=1}^{m} \alpha_j \beta_j)^2}\cdot \beta_i \cdot \alpha_i(1-\alpha_i)(a_i - x)}$$

$$\frac{\partial E_k}{\partial c_i} = \frac{\partial E_k}{\partial o_k}\cdot\frac{\partial o_k}{\partial \beta_i}\cdot\frac{\partial \beta_i}{\partial c_i}$$

$$\frac{\partial o_k}{\partial \beta_i} = \frac{\partial}{\partial \beta_i}\left(\frac{\sum_{j=1}^{m} \alpha_j \beta_j z_j}{\sum_{j=1}^{m} \alpha_j \beta_j}\right) = \frac{1}{(\sum_{j=1}^{m} \alpha_j \beta_j)^2}\cdot\left(\left(\frac{\partial}{\partial \beta_i}\sum_{j=1}^{m} \alpha_j \beta_j z_j\right)\cdot\left(\sum_{j=1}^{m} \alpha_j \beta_j\right) - \left(\sum_{j=1}^{m} \alpha_j \beta_j z_j\right)\cdot\left(\frac{\partial}{\partial \beta_i}\sum_{j=1}^{m} \alpha_j \beta_j\right)\right)$$

$$= \frac{1}{(\sum_{j=1}^{m} \alpha_j \beta_j)^2}\cdot\left(\alpha_i z_i \cdot \sum_{j=1}^{m} \alpha_j \beta_j - \alpha_i \cdot \sum_{j=1}^{m} \alpha_j \beta_j z_j\right)$$

$$\frac{\partial \beta_i}{\partial c_i} = \frac{\partial}{\partial c_i}\left(\frac{1}{1+e^{d_i(x-c_i)}}\right) = \beta_i(1-\beta_i)\cdot d_i$$

$$\boxed{\frac{\partial E_k}{\partial c_i} = -(y_k - o_k)\cdot\frac{z_i \sum_{j=1}^{m} \alpha_j \beta_j - \sum_{j=1}^{m} \alpha_j \beta_j z_j}{(\sum_{j=1}^{m} \alpha_j \beta_j)^2}\cdot \alpha_i \cdot \beta_i(1-\beta_i)\cdot d_i}$$

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$$\frac{\partial E_k}{\partial \alpha_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \beta_i} \cdot \frac{\partial \beta_i}{\partial \alpha_i}$$

$$\frac{\partial \beta_i}{\partial \alpha_i} = \frac{\partial}{\partial \alpha_i}\left(\frac{1}{1+e^{\alpha_i(x-c_i)}}\right) = \underbrace{\frac{-e^{\alpha_i(x-c_i)}}{\left(1+e^{\alpha_i(x-c_i)}\right)^2}}_{-\beta(1-\beta)} \cdot (x-c_i) = \beta(1-\beta)(c_i-x)$$

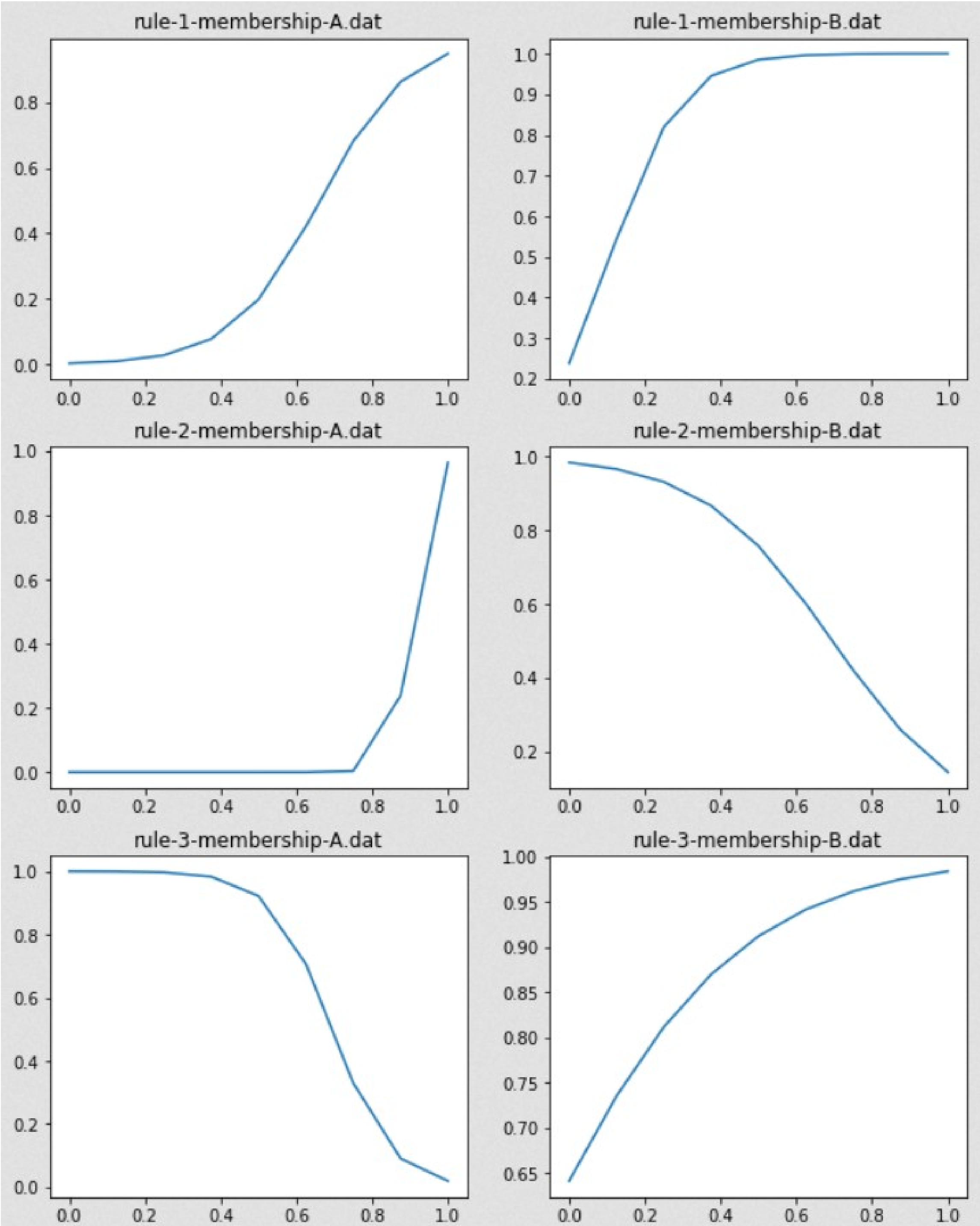
$$\boxed{\frac{\partial E_k}{\partial \alpha_i} = -(y_k - o_k)\,\frac{z_i \sum_{j=1}^{m} \alpha_j \beta_j - \sum_{j=1}^{m} \alpha_j \beta_j z_j}{\left(\sum_{j=1}^{m} \alpha_j \beta_j\right)^2} \cdot \alpha_i \cdot \beta_i \cdot (1-\beta_i) \cdot (c_i - x)}$$

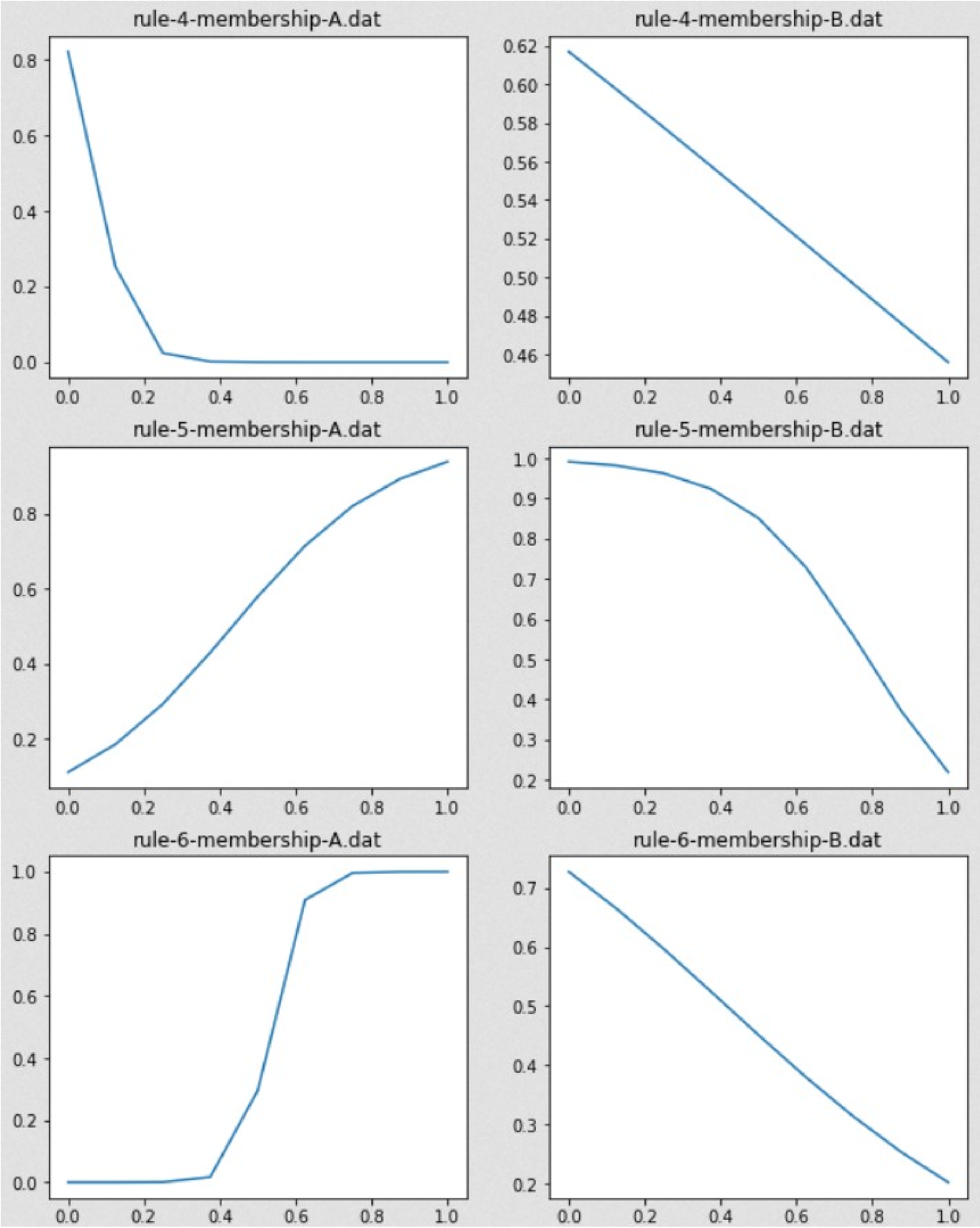
Kod pravog gradijentnog spusta ažuriranje parametara provodi se tako da se akumulira suma parcijalnih derivacija za svaki od N ulaznih primjera, pa se tek onda provede ažuriranje. Kod stohastičkog algoritma učenja parametri se ažuriraju [na]kon učenja svakog pojedinog primjera.

stohastički: $\psi(t+1) = \psi(t) - \eta \frac{\partial E_k}{\partial \psi}$ (on-line)

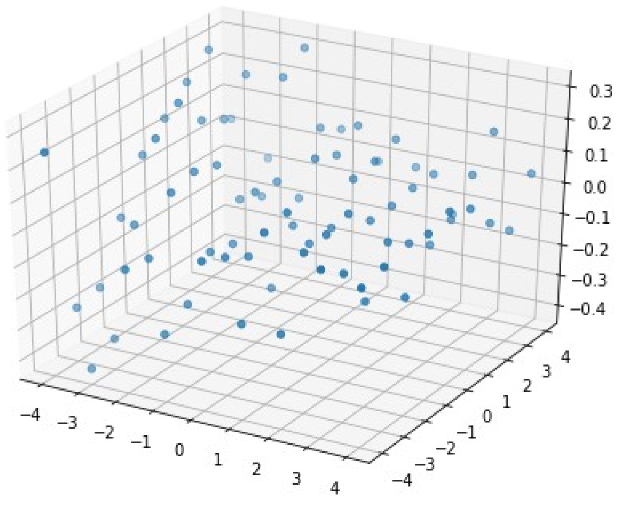
pravi: $\psi(t+1) = \psi(t) - \eta \sum_{k=1}^{N} \frac{\partial E_k}{\partial \psi}$ (batch)

Prikaz funkcija pripadnosti mreže naučene korištenjem 6 pravila. Prvi stupac označava funkcije pripadnosti varijable *x1*, a drugi varijable *x2*.



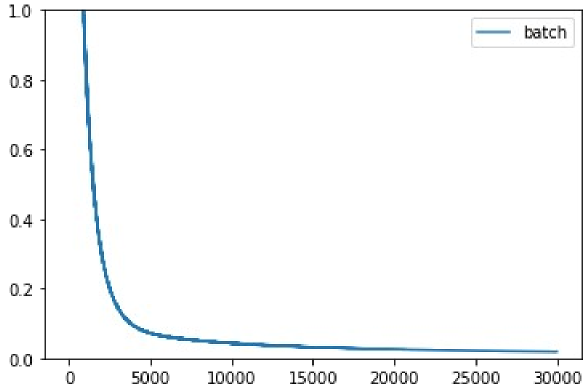

rule-4-membership-A.dat
rule-4-membership-B.dat
rule-5-membership-A.dat
rule-5-membership-B.dat
rule-6-membership-A.dat
rule-6-membership-B.dat

Prikaz odstupanja očekivanih izlaza i izlaza naučenog modela.

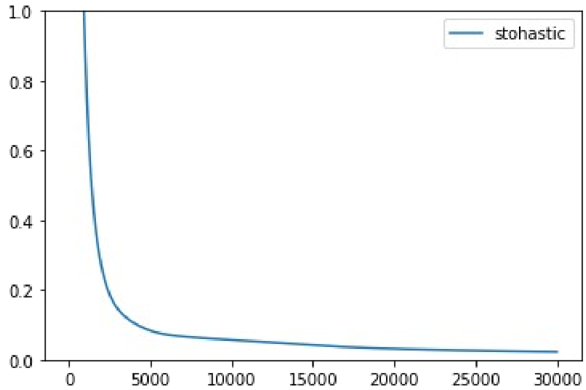


Maksimalno odstupanje: 0.295519022636

Prikaz postupka učenja za algoritam potpunog gradijenta:



Prikaz postupka učenja za stohatički gradijentni spust:



Prikaz ponašanja algoritama učenja za različite stope učenja. Za preveliku stopu učenja algoritam divergira, dok za premalu stopu učenja algoritam sporije uči.