Ecophysiology-HW2

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```
  knitr::opts_chunk$set(fig.width=4, fig.height=4,
                        fig.align="center", fig.path='Figs/', warning=FALSE,
                        message=FALSE, fig.pos = 'h',
                        tidy.opts=list(width.cutoff=60),tidy=TRUE)
```

```
rm(list = ls())
setwd("D:/Krishna/Acads/Q4/Ecophysiology/HW")
library(ggplot2)
library(reshape2)
library(latex2exp)
library(knitr)
theme = theme_light() + theme(plot.title = element_text(hjust = 0.5),
    text = element_text(size = 7))
```

Problem 1

Assumptions:

1. Simulation run for 5000 years in each case. It is assumed that this is enough time for an equilibrium to establish. However, it can be seen that not all pools reach steady state completely. eg. NEP ≈ 0 but not exactly $= 0$.

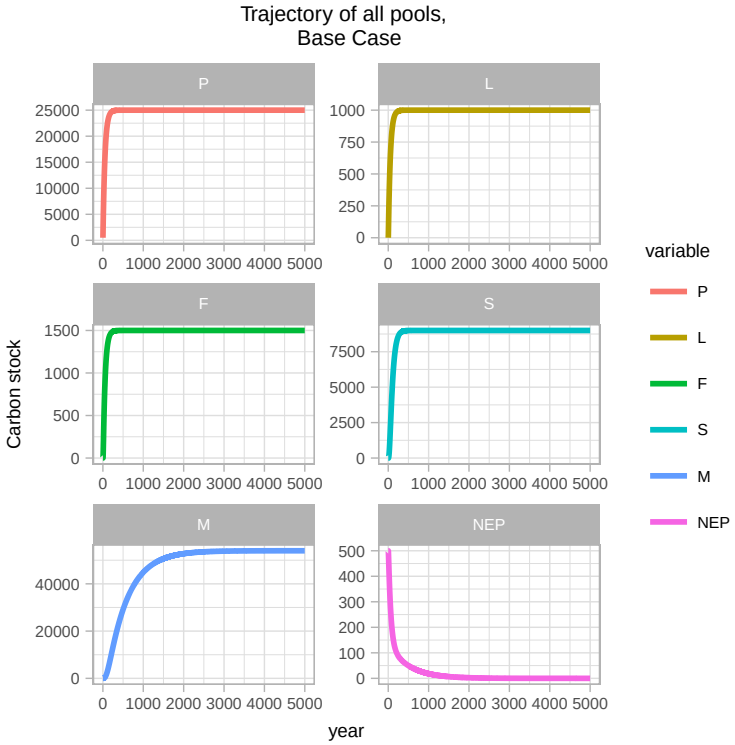
Part a)

To calculate NEP, we know that NEP=GPP-$R_h - R_a$
$\therefore NEP = NPP - R_h$

```
simulate = function(df, NPP, dec, title = "") {
    Rh = 0.4
    for (t in seq(2, dim(df)[1], 1)) {
        df$P[t] = df$P[t - 1] + NPP[t - 1] - df$P[t - 1] * 0.02 *
            dec[t - 1]
        df$L[t] = df$L[t - 1] + df$P[t - 1] * 0.02 * dec[t -
            1] - df$L[t - 1] * 0.5 * dec[t - 1]
        df$F[t] = df$F[t - 1] + df$L[t - 1] * 0.5 * (1 - Rh) *
            dec[t - 1] - df$F[t - 1] * 0.2 * dec[t - 1]
        df$S[t] = df$S[t - 1] + df$F[t - 1] * 0.2 * (1 - Rh) *
            dec[t - 1] - df$S[t - 1] * 0.02 * dec[t - 1]
        df$M[t] = df$M[t - 1] + df$S[t - 1] * 0.02 * (1 - Rh) *
            dec[t - 1] - df$M[t - 1] * 0.002 * dec[t - 1]
        df$NEP[t] = NPP[t - 1] - df$L[t - 1] * 0.5 * Rh * dec[t -
            1] - df$F[t - 1] * 0.2 * Rh * dec[t - 1] - df$S[t -
            1] * 0.02 * Rh * dec[t - 1] - df$M[t - 1] * 0.002 *
            dec[t - 1]
        df$year[t] = df$year[t - 1] + 1
```

```
    }
    df.m <- melt(df[df$year >= min(df$year) + 1, ], "year")
    g = ggplot(df.m, aes(year, value, colour = variable)) + geom_line(size = 1) +
        facet_wrap(~variable, ncol = 2, scales = "free") + labs(y = "Carbon stock",
        title = paste("Trajectory of all pools, \n", title)) +
        theme
    return(list(df, g))
}
```

```
year.end = 5000
df = data.frame(matrix(ncol = 7, nrow = year.end))
colnames(df) = c("year", "P", "L", "F", "S", "M", "NEP")
df$year = seq(0, year.end - 1)
df[df$year == 0, ] = rep(0, dim(df)[2])
df$NEP[1] = 500
NPP = rep(500, dim(df)[1])
dec = rep(1, dim(df)[1])
ret = simulate(df, NPP, dec, "Base Case")
df = ret[[1]]
ret[[2]]
```




```
kable(tail(df, 1), caption = "$Equilibrium\\ carbon\\ concentration\\ (g\\ C\\ m^{-2}\\ y^{-1})$")
```

Table 1: *Equilibrium carbon concentration* ($g\ C\ m^{-2}\ y^{-1}$)

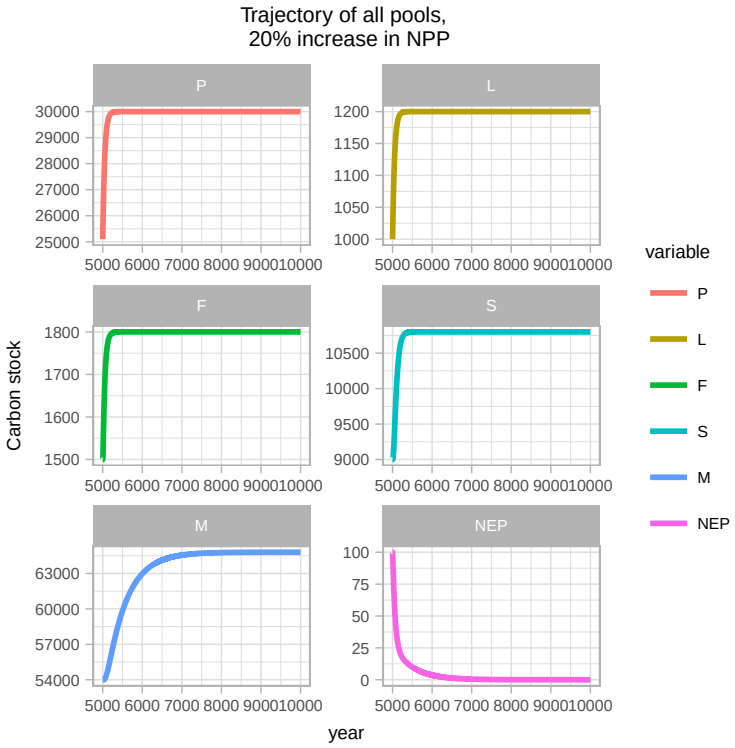
	year	P	L	F	S	M	NEP
5000	4999	25000	1000	1500	9000	53996.95	0.0061022

```
df.master = df
```

All graphs here onwards are presented from and onwards the equilibrium state, which as mentioned before is 5000 years.

Part b)

```
df[1, ] = tail(df.master, 1)
NPP = 1.2 * NPP
ret = simulate(df, NPP, dec, "20% increase in NPP")
df = ret[[1]]
ret[[2]]
```




```
kable(tail(df, 1), caption = "$Equilibrium\\ carbon\\ concentration\\ (g\\ C\\ m^{-2}\\ y^{-1})$")
```

Table 2: *Equilibrium carbon concentration* ($g\ C\ m^{-2}\ y^{-1}$)

	year	P	L	F	S	M	NEP
5000	9998	30000	1200	1800	10800	64799.39	0.0012207

1. Increasing NPP by 20% shifts the equilibrium state of each pool up by 20%. This is because the shift in NPP introduces a chain reaction untill each pool finds its respective equilibrium state. NPP is the primary input supply and thus controls the state of all other pools downstream.

2. NEP on the other hand, jumps to 100g $gCm^{-2}y{-}1$ as a result of it being directly proportional to NPP. Once the new equilibrium is established and heterogenous respiration balances out NPP, NEP dies down to zero.

3. The time required to reach equilibrium in each case appears to be the same as that of initial base case. This means that the time required to establish steady state is independent of quantum of change introduced in the system.

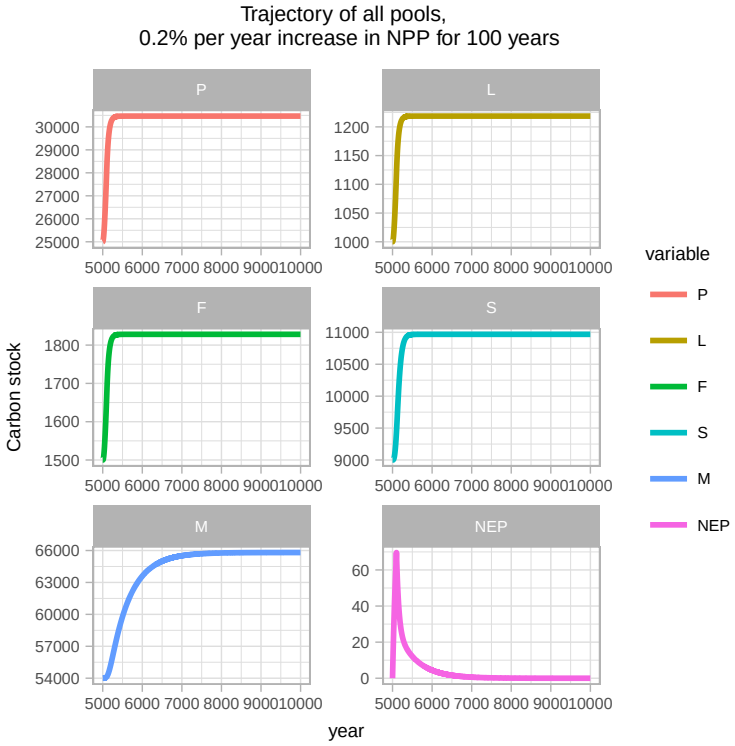
4. The passive pool takes the longest to reach steady state because its decay rate is the slowest.

Part c)

Assumption:

1. Rate of increase of NPP is assumed to be 0.2% **compounded** annually.

```
NPP.inc = 500
for (t in seq(2, dim(df)[1])) {
    if (t <= 100) {
        NPP.inc[t] = NPP.inc[t - 1] * 1.002
    } else {
        NPP.inc[t] = NPP.inc[t - 1]
    }
}
ret = simulate(df, NPP.inc, dec, "0.2% per year increase in NPP for 100 years")
df = ret[[1]]
ret[[2]]
```




```
kable(tail(df, 1), caption = "$Equilibrium\\ carbon\\ concentration\\ (g\\ C\\ m^{-2}\\ y^{-1})$")
```

Table 3: *Equilibrium carbon concentration* ($g\ C\ m^{-2}\ y^{-1}$)

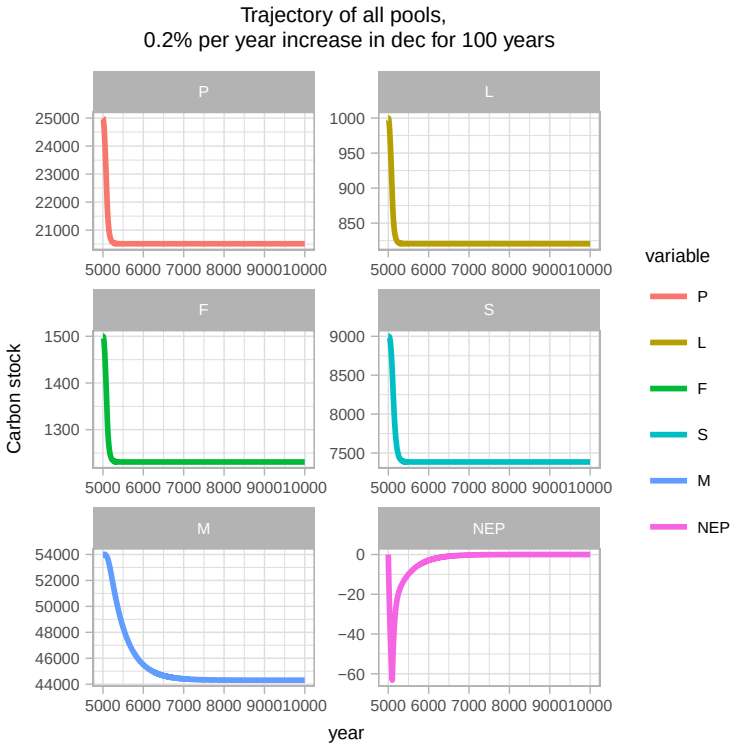
	year	P	L	F	S	M	NEP
5000	9998	30468.03	1218.721	1828.082	10968.49	65810.21	0.0014827

year	P	L	F	S	M	NEP

1. All pools again take the same time to reach their new steady states. The magnitude of the shift is directly proportional to the total final increase in NPP which is roughly 1.22 times are 22% ($1.002^{1}00 = 1.22$).
2. NEP increases proportional to NPP for the first 100 years and then exponentially decays untill R_h increases to cancel out NPP.

Part d)

```
dec.inc = dec
for (t in seq(2, dim(df)[1])) {
    if (t <= 100) {
        dec.inc[t] = dec.inc[t - 1] * 1.002
    } else {
        dec.inc[t] = dec.inc[t - 1]
    }
}
NPP = rep(500, dim(df)[1])
ret = simulate(df, NPP, dec.inc, "0.2% per year increase in dec for 100 years")
df = ret[[1]]
ret[[2]]
```




```
kable(tail(df, 1), caption = "$Equilibrium\\ carbon\\ concentration\\ (g\\ C\\ m^{-2}\\ y^{-1})$")
```

Table 4: *Equilibrium carbon concentration* ($g\ C\ m^{-2}\ y^{-1}$)

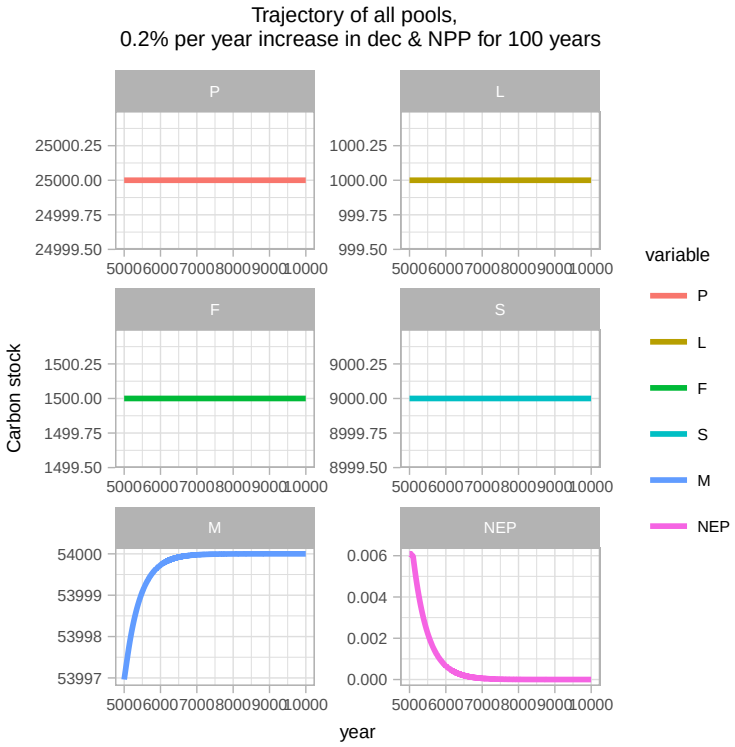
	year	P	L	F	S	M	NEP
5000	9998	20513.3	820.5321	1230.798	7384.789	44308.8	-0.0001695

1. As the decomposition rate increases,the pools shift downward untill equilibrium is established.
2. The net downward shift is by about 22% which is the total increase in decomposition rate.

3. The time taken to establish equilibrium is similar to that Part c.

Part e)

```
ret = simulate(df, NPP.inc, dec.inc, "0.2% per year increase in dec & NPP for 100 years")
df = ret[[1]]
ret[[2]]
```




```
kable(tail(df, 1), caption = "$Equilibrium\\ carbon\\ concentration\\ (g\\ C\\ m^{-2}\\ y^{-1})$")
```

Table 5: *Equilibrium carbon concentration* ($g\ C\ m^{-2}\ y^{-1}$)

	year	P	L	F	S	M	NEP
5000	9998	25000	1000	1500	9000	54000	0

*Note: Scales for M and NEP are extremely zoomed intentially to shows that 5000 years was not enough for it to reach equilibrium state. Essentially, for inference purposes, all lines are flat.

1. The decomposition rate increase is balanced by NPP increase resulting in no net change of pools.

Problem 2

Part a)

Penman Monteith equation for evapotranspiration is given by

$$
\begin{aligned}
ET_o &= \frac{\Delta(R_n - G) + \rho_a c_p(\delta e) g_a}{(\Delta + \gamma(1 + g_a/g_s))L_v} \\
where \quad g_a &= \frac{\kappa^2 u_a}{ln^2 \frac{z}{z_o}} \\
&= \frac{0.41^2 \times 1}{ln^2 \frac{2}{0.1 \times h_c}} \\
\& \quad \delta_e &= VPD = e^*(T) - e(T) \\
&= e^*(T) \times (1 - R.H.) \\
But \quad e^*(T) = e^*(20) &= 6.1094 e^{\frac{17.625 \times 20}{20 + 243.04}} \\
&= 23.33\ hPa = 2333\ Pa \\
\therefore \delta_e &= 23.33 \times (1 - 0.65) = 8.17\ hPa \\
\Delta &= \frac{\partial e^*}{\partial T} = \frac{\lambda \times e^*}{R_v \times T^2} \\
&= \frac{40680\ J.mol^{-1} \times 23.33\ hPa}{8.314\ J.mol^{-1}.K^{-1} \times 293.01^2\ K^2} \\
&= 1.33\ hPa.K^{-1} = 133\ Pa.K^{-1} \\
\therefore ET_o &= \frac{133\ Pa.K^{-1} \times (280 - 25)\ W.m^{-2} + 1.2\ Kg.m^{-3} \times 1000\ J.Kg^{-1}.K^{-1} \times 817\ Pa \times g_a}{(133\ Pa.K^{-1} + 66\ Pa.K^{-1}(1 + g_a/gs))\ 2.453 \times 10^9\ J.m^{-3}} \\
&= \frac{(33915\ + 980400 \times g_a) Pa.K^{-1}.W.m^{-2}}{(133 + 66(1 + g_a/g_s)) \times 2.453 \times 10^9\ Pa.K^{-1}.J.m^{-3}} \\
g_{s1} &= \frac{1}{30} = 0.033\ m.s^{-1} \\
g_{a1} &= 9.53 \times 10^{-3}\ m.s^{-1} \\
\& & \\
g_{s2} &= \frac{1}{60} = 0.017\ m.s^{-1} \\
g_{a2} &= 18.7 \times 10^{-3}\ m.s^{-1} \\
\therefore ET_{o1} &= 6.99 mm.day^{-1} \\
\therefore ET_{o2} &= 6.74 mm.day^{-1}
\end{aligned}
$$

```
ET = function(ga, gs) {
    et = (33915 + 980400 * ga)/((133 + 66 * (1 + ga/gs)) * 2.453 *
        10^9)
    et = 1000 * et * 60 * 60 * 24
    return(et)
}
gs1 = 1/30
```

```
ga1 = 0.00953
gs2 = 1/60
ga2 = 0.0187
# ET(ga1,gs1)
```

Part b)

```
mag.roch = function(T) {
    e = 6.1094 * exp(1)^((17.625 * T)/(T + 243.04))
    return(100 * e)
}
mag.roch.inv = function(e) {
    e.corr = log(e/6.1094)
    T = e.corr * 243.04/(17.625 - e.corr)
    return(T)
}
rho.air = 1.2
Cp = 1000
Ta = 20
# ra=1/ga
Rn = 280
G = 25
RH = 0.65
eps = 0.622
P = 1e+05
lambda = 2260000
Lv = 2.458e+09
e.star.air = mag.roch(Ta)
e.air = RH * e.star.air
qa = eps * e.air/(P - 0.378 * e.air)
option = 1
for (ra in c(1/ga1, 1/ga2)) {
    Tl = 0
    Tl.new = 10
    while (abs(Tl.new - Tl) >= 1e-05) {
        Tl = Tl.new
        H = rho.air * Cp * (Tl - Ta)/ra
        LE = Rn - H - G
        ET = LE/lambda
        qs = ET * ra/rho.air + qa
        e.leaf = qs * P/(eps + 0.378 * qs)
        Tl.new = mag.roch.inv(e.leaf/100)
    }
    ET = LE/Lv * 60 * 60 * 24 * 1000
    print(paste("Option ", option, ":"))
    print(paste("Leaf temperature = ", round(Tl.new, 2), " C"))
    print(paste("LE = ", round(LE), " W/m2"))
    print(paste("ET = ", round(ET, 2), " mm/day"))
    option = option + 1
}
```

```
## [1] "Option  1 :"
```

```
## [1] "Leaf temperature =  23.26  C"
## [1] "LE =  218  W/m2"
## [1] "ET =  7.65  mm/day"
## [1] "Option  2 :"
## [1] "Leaf temperature =  19.91  C"
## [1] "LE =  257  W/m2"
## [1] "ET =  9.04  mm/day"
```

Part c)

1. In both cases, the Penman Monteith equation is found to underestimate the evpotranspiration.

2. The difference is due to the assumption in part a that leaf temperature is same as air temperature which is clearly not the case as seen in part b.
3. This assumption of equal temperature causes us to ignore sensible heat, thus resulting in an underestimation of ET in part a.

Problem 3

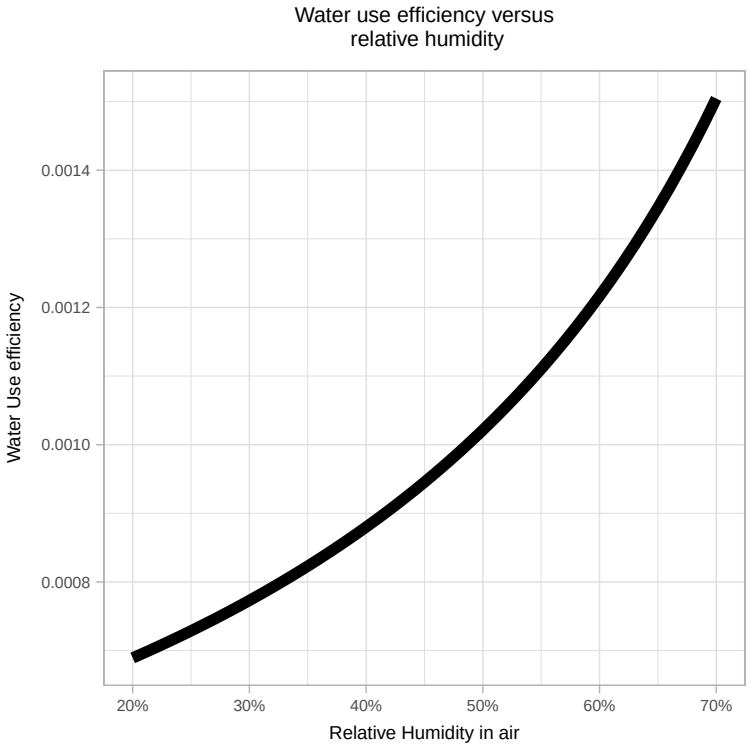
Part a)

Assumptions:

1. Leaf is saturated with water.

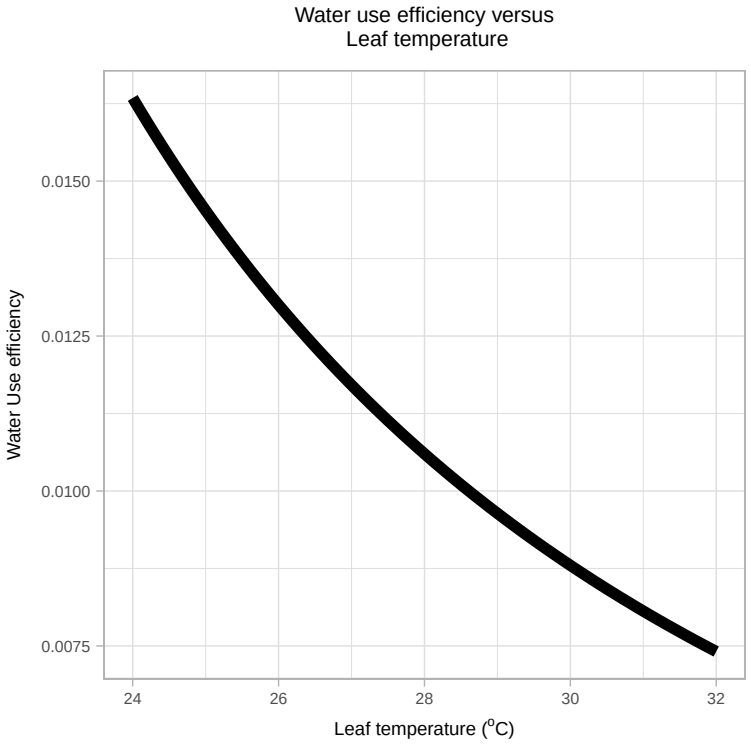
2. The stomatal conductance with respect to water is greater than that with respect to CO2 (by a factor of 1.6)

```
RH = seq(0.2, 0.7, 0.01)
P = 1e+05
ci = 0.00023 * P
ca = 0.00038 * P
ei = mag.roch(30)  # RH = 1
ea = mag.roch(28) * RH
WUE = 0.16 * abs(ci - ca)/abs(ei - ea)
df = data.frame(cbind(RH, WUE))
colnames(df) = c("RH", "WUE")
g = ggplot(df, aes(RH, WUE))
g + geom_line(size = 2) + labs(x = "Relative Humidity in air",
    y = "Water Use efficiency", title = "Water use efficiency versus\n relative humidity") +
    theme + scale_x_continuous(labels = scales::percent)
```



Part b)

```
Tl = seq(24, 32, 0.1)
RH = 0.4
ei = mag.roch(Tl)  # RH = 1
ea = mag.roch(28) * RH
WUE = 1.6 * abs(ci - ca)/abs(ei - ea)
df = data.frame(cbind(Tl, WUE))
colnames(df) = c("Tl", "WUE")
g = ggplot(df, aes(Tl, WUE))
g + geom_line(size = 2) + labs(x = TeX("Leaf temperature (^oC)"),
    y = "Water Use efficiency", title = "Water use efficiency versus\n Leaf temperature") +
    theme
```



1. WUE increases with increase in relative humidity because for the same assimilation of carbon, there is lesser transpiration occuring due to decrease in gradient.

2. WUE decreases with increase in leaf temperature because of increased transpiration.

3. However, WUE is roughly 3 times more sensitive to per degree change of leaf temperature as compared to per % change in relative humidity.

Problem 4

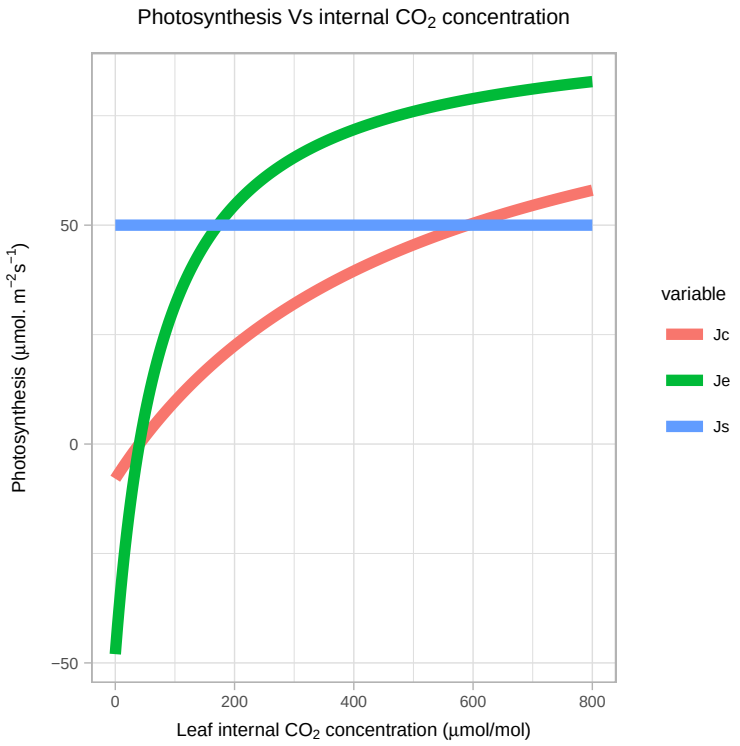
Assumptions:

1. Net rate of photosynthesis is piecewise linear.

Part a)

```
Qp = 1500
Epsm = 0.08
Km = 300
Ki = 3e+05
Rd = 1.5e-06
Vm = 100
CCa = 38
O = 210000
alpha = 0.8
beta = 0.98
tau = 2600
```

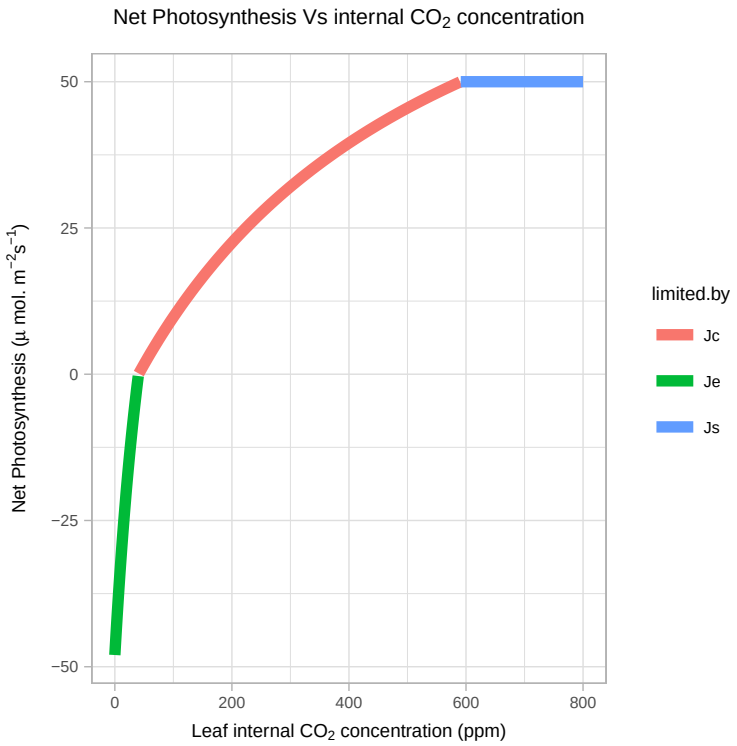
```
theta = 0.95
pi = seq(0, 800, 1)
Tau.star = O/2/tau
Kapp = Km * (1 + O/Ki)

pht = function(a = alpha, E = Epsm, p = pi, T = Tau.star, V = Vm,
    K = Kapp, Q = Qp) {
    Je = a * E * Q * (p - T)/(p + 2 * T)
    Jc = V * (p - T)/(p + K)
    Js = V/2
    df = data.frame(p, Jc, Je, Js)
    colnames(df) = c("cCO2", "Jc", "Je", "Js")
    return(df)
}

df = pht()
df2 = melt(df, id.vars = "cCO2")
g = ggplot(df2, aes(cCO2, value, color = variable))
g + geom_line(size = 2) + labs(x = TeX("Leaf internal CO_2 concentration ($\\mu$mol/mol)"),
    y = TeX("Photosynthesis ($\\mu$mol. m^{-2}s^{-1})"), title = TeX("Photosynthesis Vs internal CO_2 c
    theme
```



Part b)

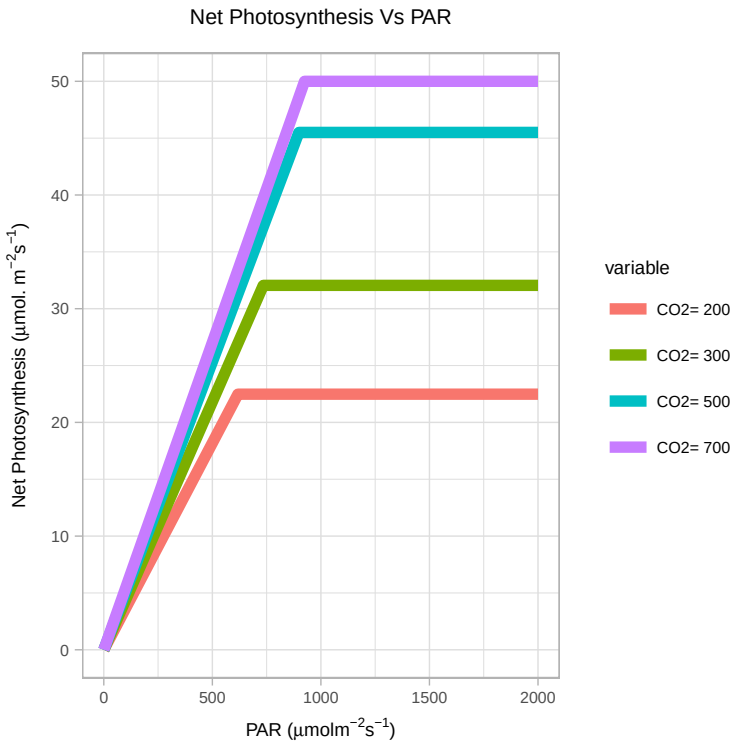
```
net.pht = function(df) {
    df$An <- apply(df[, 2:4], 1, min) - Rd
    df$limited.by = colnames(df[, 2:4])[apply(df[, 2:4], 1, which.min)]
```

```
    return(df)
}
df = net.pht(df)
g = ggplot(df, aes(cCO2, An, color = limited.by))
g + geom_line(size = 2) + labs(x = TeX("Leaf internal CO_2 concentration (ppm)"),
    y = TeX("Net Photosynthesis  ($\\mu$ mol. m^{-2}s^{-1})"),
    title = TeX("Net Photosynthesis Vs internal CO_2 concentration")) +
    theme
```



Part c)

```
Qp = seq(0, 2000, 1)
pi.range = c(20, 30, 50, 70) * 10
df.master = data.frame(Qp)
colnames(df.master) = "Qp"
for (pi.s in pi.range) {
    df = pht(p = pi.s)
    df = net.pht(df)
    df.master[paste("CO2=", pi.s)] = df$An
}

df2 = melt(df.master, id.vars = "Qp")
g = ggplot(df2, aes(Qp, value, color = variable))
g + geom_line(size = 2) + labs(x = TeX("PAR ($\\mu$molm^{-2}s^{-1})"),
    y = TeX("Net Photosynthesis ($\\mu$mol. m^{-2}s^{-1})"),
    title = TeX("Net Photosynthesis Vs PAR")) + theme
```

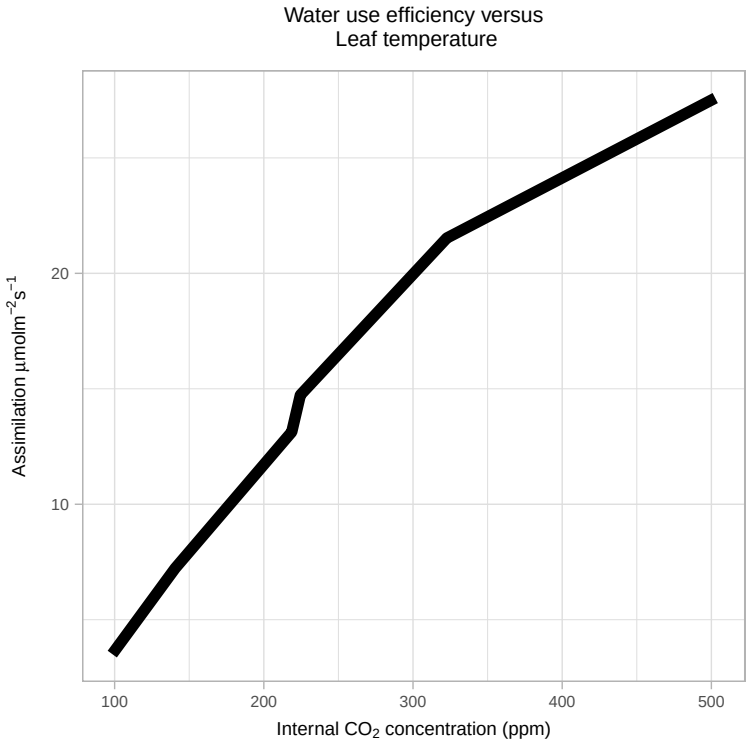


Problem 5

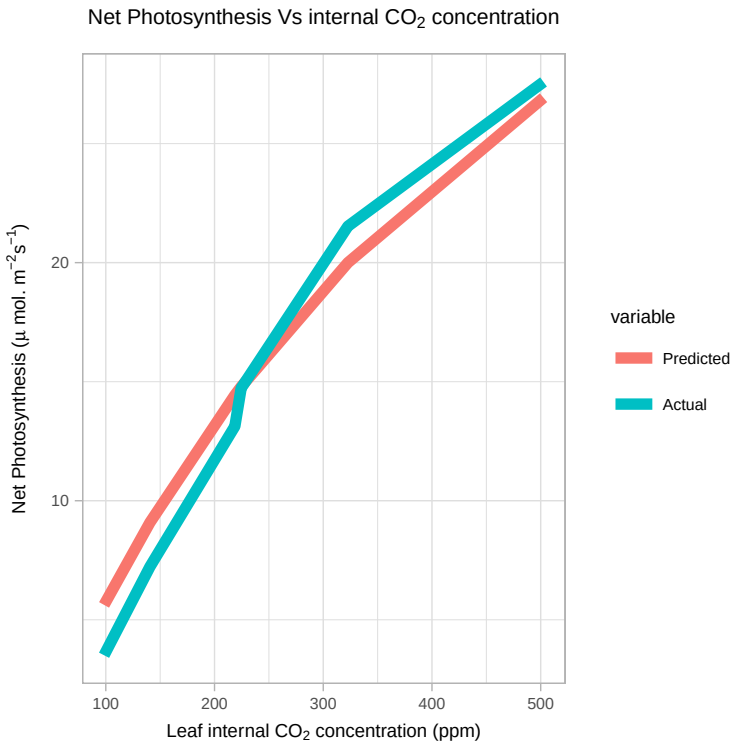
Assumptions:

1. Temperature induced parameter fluctuations are negligible.
2. PAR = 1500 $\mu molm^{-2}s^{-1}$

```
df = read.csv("Aspen_CO2_curve.csv")

g = ggplot(df, aes(Cc, A))
g + geom_line(size = 2) + labs(x = TeX("Internal CO_2 concentration (ppm)"),
    y = TeX("Assimilation $\\mu$molm^{-2}s^{-1}"), title = "Water use efficiency versus\n Leaf temperatu
    theme
```




```
Qp = 1500
norm <- function(x) (sum(x^2))
rsq = function(Vm, df) {
    df.pred = pht(p = df$Cc, V = Vm)
    df.pred = net.pht(df.pred)
    An.pred = df.pred$An
    err = norm(df$A - An.pred)
    return(err)
}
Vm.int = c(1, 400)
fit = optimize(rsq, Vm.int, df = df)
Vmax = fit$minimum
df.pred = pht(p = df$Cc, V = Vmax)
df.pred = net.pht(df.pred)
df.pred = cbind.data.frame(df.pred[, c("cCO2", "An")], df$A)
colnames(df.pred) = c("cCO2", "Predicted", "Actual")
df2 = melt(df.pred, id.vars = "cCO2")
g = ggplot(df2, aes(cCO2, value, color = variable))
g + geom_line(size = 2) + labs(x = TeX("Leaf internal CO_2 concentration (ppm)"),
    y = TeX("Net Photosynthesis  ($\\mu$ mol. m^{-2}s^{-1})"),
    title = TeX("Net Photosynthesis Vs internal CO_2 concentration")) +
    theme
```



$V_m = 59\ \mu.mol.m^{-2}.s^{-1}$