

Problem Set 2

Due Wednesday November 1<sup>st</sup>, 2017 at the start of class. Please bring a hard copy of your solutions (including a print-out of your code) to class.

**Problem 1: Carbon pools**

Consider the attached simple carbon cycle model. Such a model structure could work at either the local or the global scale (although the magnitude of the NPP would obviously differ depending on the scale represented). For each pool,  $r_H$  represents the decay fraction going to heterotrophic respiration. Given this structure, do the following.

a) plot the trajectory of all pools, starting with all pools empty and going to equilibrium. What is the equilibrium carbon stock in each of the pools? What is the trajectory of NEP?

b) Starting with all pools in equilibrium (from a), assess the timing and implications for all pools of increasing NPP by 20%, as one might expect in an experiment where ecosystems were shifted to doubled CO<sub>2</sub>.

c) Do the same exercise as in b, but with NPP increasing by 0.2% per year, for 100 years.

d) Starting with all pools in equilibrium (from a), assess the timing and implications for all pools of increasing all decomposition rates by 0.2% per year for 100 years, as one might expect in a warming climate.

e) Finally, simulate an experiment in which both NPP and all decomposition rates increase by 0.2% per year, for 100 years. Again, discuss the implications for all pools compared to the previous simulations.

**Problem 2: Penman-Monteith equation**

You are moving and want to do some gardening in your new house. You can't decide between two different plant types, but know that because you don't have an easy hook-up, providing water for the plants will take some care. You decided to base your decision solely on the water needs of each plant. The characteristics corresponding to each crop type are listed below:

	Stomatal resistance (s/m)	Vegetation height (cm)
Option 1	30	30
Option 2	60	100

Fortunately, the former tenant of your new place is interested in land surface processes, and has given you some typical average measurements from a meteorological station at 2-m :  $T_a = 20^\circ\text{C}$ ,  $u = 1 \text{ m s}^{-1}$ , relative humidity is 65%,  $\rho_{\text{air}} = 1.2 \text{ kg m}^{-3}$ ,  $\rho_{\text{water}} = 1000 \text{ kg m}^{-3}$ , and  $P_s = 1000 \text{ mb}$ . The net radiation at the surface is  $280 \text{ W m}^{-2}$  and the ground heat flux is  $\text{W m}^{-2}$ .

- What would the evapo-transpiration (in mm day<sup>-1</sup>) be for each of these plant choices? Use the Penman-Monteith equation.
- An alternative way to determine the evapo-transpiration rate is to iteratively solve the system. This can be done by solving, at each iteration both the energy balance and flux-resistance equations for H, LE, and TL. Code up an iterative process to do this. What are the T<sub>L</sub> and LE if T<sub>L</sub> is solved to within 0.01°C?
- How does your answer for part b compare to that of part a? What explains any differences you might observe?

*Hint: You may want to re-arrange the Magnus-Roche equation to derive an analytical solution for T as a function of saturated vapor pressure.*

### **Problem 3: Water use efficiency**

Water use efficiency can be defined as the ratio of photosynthesis to transpiration

$$WUE = \frac{A}{T} = \frac{g_{sc}(c_i - c_a)}{g_w(e_i - e_a)}$$

- Consider a leaf with an internal CO<sub>2</sub> concentration of 230 ppm (= 230 μmol mol<sup>-1</sup>) and a leaf temperature of 30°C. The ambient CO<sub>2</sub> concentration is 380 ppm, and the air temperature is 28°C, calculate and plot water use efficiency as a function of relative humidity for humidities from 20 to 70%. Assume the stomatal conductance with respect to CO<sub>2</sub> is greater than that with respect to water by a factor of 1.6.
- Now, consider the same situation, but across a range of leaf temperatures (24 to 32°C), and with a fixed relative humidity of 40%. How does the sensitivity of WUE to leaf temperature compare to its sensitivity to relative humidity?

### **Problem 4: Limiting factors in photosynthesis**

Consider a C<sub>3</sub> leaf characterized by the parameter values below. Assume that all parameter values are at a reference temperature of 25°C, which is the temperature of the leaf.

- Plot the response of J<sub>E</sub>, J<sub>C</sub>, and J<sub>S</sub> to leaf internal CO<sub>2</sub> concentration, over the range from 0 to 80 Pa (0 to 800 ppm or 0 to 800 μmol/mol). For this exercise, the incident PAR is 1500 μmol m<sup>-2</sup> s<sup>-1</sup>. Hold on to this code. You will need it in Problem 5 and future problem sets.
- Plot the response of net photosynthesis (A<sub>n</sub> = A – R<sub>d</sub>) to the same range of CO<sub>2</sub> concentrations. Identify the region of the curve where the rate is limited by J<sub>E</sub>, J<sub>C</sub>, and J<sub>S</sub>
- Plot the response of net photosynthesis (A<sub>n</sub> = A – R<sub>d</sub>) to ambient PAR, over the range from 0 to 2000 μmol m<sup>-2</sup> s<sup>-1</sup>, at internal CO<sub>2</sub> concentrations of 20, 30, 50, and 70 Pa.

Symbol	Value	Units	Description
$\epsilon_m$	0.08	mol/mol	Maximum quantum efficiency
$K_m$	300	$\mu\text{mol/mol}$	Michaelis constant for $\text{CO}_2$
$K_i$	300	mmol/mol	Inhibition constant for $\text{O}_2$
$R_d$	1.5	$\mu\text{mol m}^{-2} \text{s}^{-1}$	Day respiration
$V_m$	100	$\mu\text{mol m}^{-2} \text{s}^{-1}$	Rubisco capacity
$C_{ca}$	38	Pa	Ambient $\text{CO}_2$ mole fraction
$[\text{O}_2]$	210	mmol/mol	Oxygen mole fraction
$\alpha_p$	0.8		Leaf absorptivity for PAR
$\beta$	0.98		Co-limitation factor
$\tau$	2.6	mmol/ $\mu\text{mol}$	$\text{CO}_2/\text{O}_2$ specificity ratio
$\theta$	0.95		Co-limitation factor

#### **Problem 5: $V_{\max}$ estimation from observations**

The file “Aspen\_CO2\_curve.csv” contains measurements of an aspen leaf. Import these data into R, and plot an A (assimilation) vs.  $p_i$  (internal  $\text{CO}_2$  concentration) curve. What is the  $V_m$  (also referred to as  $V_{\max}$ ) of this leaf? Assume all parameters except  $V_m$  are as in problem 4, and temperature-induced parameter value variations over the range of temperatures measured are negligible.

*Hint: one approach would be to optimize  $V_{\max}$  so that the predicted A vs.  $C_c$  curve most closely matches the observed values*

$$\text{NPP} = 500 \text{ g C m}^{-2} \text{ y}^{-1}$$

