Project 3: Riemann Sums of a Double Integral

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My objective in this project is to explore double integrals as a limit of Riemann prisms. The function of interest is:

$$f(x,y) = 4 - x^2 + y \qquad R = [-2,2] \times [0,2]$$

These are expressible as the limit of Riemann prisms:

$$\lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*) \Delta x \Delta y \tag{1}$$

where $\Delta x = \frac{b-a}{m}$ and $\Delta y = \frac{d-c}{n}$.
Additionally, $x \in [a,b]$ and $y \in [c,d]$.
Lookie here:

1 Approximation with Rectangular Prisms

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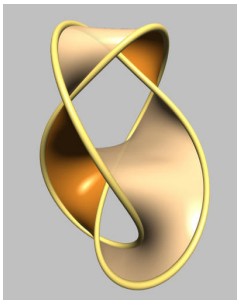

Figure 1: Awesome Image

2 Volume as a Double Integral

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$$\int_{-2}^{2}\int_{0}^{2}(4-x^2+y)\,\mathrm{d}x\,\mathrm{d}y = \int_{-2}^{2}\Big[4x-\frac{x^3}{3}+xy\Big]_{x=0}^{x=2}\,\mathrm{d}y$$

$$= \int_{-2}^{2}(\frac{16}{3}+2y)\,\mathrm{d}y = \frac{16}{3}y+y^2\Big|_{-2}^{2} = \left[\Big(\frac{16}{3}(2)+4\Big)-\Big(\frac{16}{3}(-2)+4\Big)\right]$$

$$= \frac{64}{3} = 21.\bar{3}$$

3 Varying Approximations of Volume

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