Project 3: Riemann Sums of a Double Integral

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My objective in this project is to explore double integrals as a limit of Riemann prisms. In particular, I will be examining a function over a region defined as:

$f(x, y) = 4 - x^2 + y$ $\qquad$ $R = [-2, 2] \times [0, 2]$

These are expressible as the limit of Riemann prisms:

$$\lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta x \Delta y \tag{1}$$

where $\Delta x = \frac{b-a}{m}$ and $\Delta y = \frac{d-c}{n}$.
Additionally, $x \in [a, b]$ and $y \in [c, d]$.
Lookie here:

1 Approximation with Rectangular Prisms

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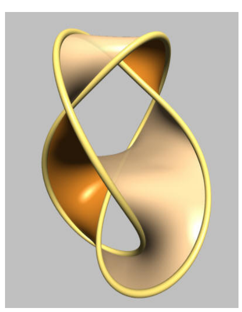

Figure 1: Awesome Image

2 Volume as a Double Integral

Because our particular function $f(x,y) \geq 0$ over the region of integration $R = [-2,2] \times [0,2]$, we can interpret the double integral over this region as the volume under the surface $z = f(x,y)$. Therefore,

$$V = \int_{-2}^{2} \int_{0}^{2} (4 - x^2 + y)\,\mathrm{d}x\,\mathrm{d}y = \int_{-2}^{2} \left[4x - \frac{x^3}{3} + xy \right]_{x=0}^{x=2} \mathrm{d}y$$
$$= \int_{-2}^{2} (\frac{16}{3} + 2y)\,\mathrm{d}y = \frac{16}{3}y + y^2 \Big|_{-2}^{2} = \left[\left(\frac{16}{3}(2) + 4 \right) - \left(\frac{16}{3}(-2) + 4 \right) \right]$$
$$= \frac{64}{3} = 21.\bar{3}$$

3 Varying Approximations of Volume

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