

Project 3: Riemann Sums of a Double Integral

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My objective in this project is to explore double integrals as a limit of Riemann prisms. In particular, I will be examining a function over a region defined as:

$f(x,y) = 4 - x^2 + y$

$R = [-2, 2] \times [0, 2]$

These are expressible as the limit of Riemann prisms:

$$\lim_{m,n \rightarrow \infty} \sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*) \Delta x \Delta y$$

(1)

where $\Delta x = \frac{b-a}{m}$ and $\Delta y = \frac{d-c}{n}$.
Additionally, $x \in [a, b]$ and $y \in [c, d]$.
Lookie here:

1 Approximation with Rectangular Prisms

...text ...

<div><div>y</div><div>x</div></div>	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{7}{4}$
-1	2.9375	2.4375	1.4375	-0.0625
1	4.9375	4.4375	3.4375	1.9375

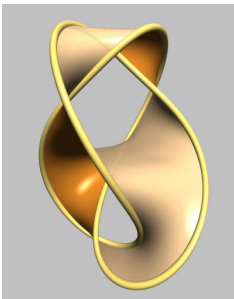


Figure 1: Awesome Image

2 Volume as a Double Integral

Because our particular function $f(x, y) \geq 0$ over the region of integration $R = [-2, 2] \times [0, 2]$, we can interpret the double integral over this region as the volume under the surface $z = f(x, y)$. Therefore,

$$\begin{aligned} V &= \int_{-2}^2 \int_0^2 (4 - x^2 + y) \, dx \, dy = \int_{-2}^2 \left[4x - \frac{x^3}{3} + xy \right]_{x=0}^{x=2} dy \\ &= \int_{-2}^2 \left(\frac{16}{3} + 2y \right) dy = \left. \frac{16}{3}y + y^2 \right|_{-2}^2 = \left[\left(\frac{16}{3}(2) + 4 \right) - \left(\frac{16}{3}(-2) + 4 \right) \right] \\ &= \frac{64}{3} = 21.\bar{3} \end{aligned}$$

3 Varying Approximations of Volume

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