Неделя 7

№19.3

$l = l(t)$

$q = \{\varphi\}$

$T = \frac{m}{2}(\dot{l}^2 + l^2\dot{\varphi}^2)$

$\Pi = mg(l_0 - l\cos\varphi)$

$$L = T - \Pi = \frac{m}{2}(\dot{l}^2 + l^2\dot{\varphi}^2) - mg(l_0 - l\cos\varphi)$$

$$p = \frac{\partial L}{\partial\dot{\varphi}} = ml^2\dot{\varphi} \Rightarrow \dot{\varphi} = \frac{p_\varphi}{ml^2}$$

$$H = p^T\tilde{\dot{q}} - L(q, \tilde{\dot{q}}, t) = \frac{p_\varphi^2}{ml^2} - \frac{m}{2}\left(\dot{l}^2 + \frac{p_\varphi^2}{ml^2}\right) + mg(l_0 - l\cos\varphi) =$$

$$= \frac{\tilde{p}_\varphi^2}{2ml^2} - \frac{m}{2}\dot{l}^2 + mg(l_0 - l\cos\varphi) \quad // \to \text{отв.}$$

№19.6 $L = \frac{3\dot{q}_1^2}{2} + \frac{\dot{q}_2^2}{2} - q_1^2 - \frac{q_2^2}{2} - q_1 q_2$

$$L = \frac{1}{2}\dot{q}^T A(q,t)\dot{q} + b^T(q,t)\dot{q} + l(q,t) = L_2 + L_1 + L_0$$

$$p = A\dot{q} + b \Rightarrow \tilde{\dot{q}} = A^{-1}(p - b)$$

$$H = \sum p_i\tilde{\dot{q}}_i - \tilde{L}$$

$$H = 2\tilde{L}_2 + L_1 - L_2 - L_1 - L_0 = \tilde{L}_2 - L_0 = \frac{1}{2}(p-b)^T A^{-1}(p-b) - L_0$$

$$L_2 = \frac{1}{2}[\dot{q}_1, \dot{q}_2]\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} + (q_1, q_2)\begin{pmatrix} -1 & -1/2 \\ -1/2 & -1/2 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow A^{-1}: \left(\begin{array}{cc|cc} 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \to \begin{pmatrix} 1/3 & 0 \\ 0 & 1 \end{pmatrix} = A^{-1}$$

$$\Rightarrow H = \frac{1}{2}\left(\frac{p_1^2}{3} + p_2^2\right) + q_1^2 + \frac{q_2^2}{2} + q_1 q_2 \quad // \to \text{отв}$$

№19.13 $H = q_1 p_2 - q_2 p_1 + a(p_1^2 + p_2^2)$

$L - ?$ $\quad \dot{q}_i = \frac{\partial H}{\partial p_i} \Rightarrow \tilde{p} = p(q, \dot{q}, t)$ $\quad p_i = \frac{\partial L}{\partial \dot{q}_i}$

$$L = \sum \tilde{p}_i \dot{q}_i - \tilde{H}$$

$$H = \underbrace{\frac{1}{2}p^T\hat{A}p}_{H_2} + \underbrace{\hat{b}^T p}_{H_1} + \underbrace{\hat{l}}_{H_0} \Rightarrow L = \tilde{H}_2 - H_0$$

$$\dot{q} = \hat{A}p + \hat{b} \Rightarrow \tilde{p} = \hat{A}^{-1}(\dot{q} - \hat{b}); \quad L = \frac{1}{2}(\dot{q} - \hat{b})^T\hat{A}^{-1}(\dot{q} - \hat{b}) \cdot H_0$$

$$\hat{A} = \begin{pmatrix} 2a & 0 \\ 0 & 2a \end{pmatrix} \Rightarrow \hat{A}^{-1} = \begin{pmatrix} 1/2a & 0 \\ 0 & 1/2a \end{pmatrix}$$

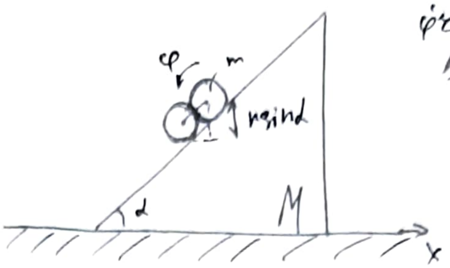
$$L = \frac{1}{2} (\dot q_1 + q_2 \, , \, \dot q_2 - q_1) \begin{pmatrix} 1/2a & 0 \\ 0 & 1/2a \end{pmatrix} \cdot \begin{pmatrix} \dot q_1 + q_2 \\ \dot q_2 - q_1 \end{pmatrix} =$$

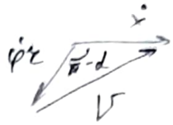
$$= \frac{\dot q_1^2 + \dot q_2^2}{4a} + \frac{\dot q_1 q_2 - \dot q_2 q_1}{2a} + \frac{q_1^2 + q_2^2}{4a}$$

$\Lambda \to$ обоб

№19.31

M, r, m, α





Введем обобщенные координаты:

$$q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} x \\ \varphi \end{pmatrix}$$

$$\Rightarrow \Pi = -mgr\sin\alpha$$

$$T = \frac{m(\dot\varphi r)^2}{2} + \frac{mr^2(\dot\varphi)^2}{2\cdot 2} + \frac{M}{2}(\dot x)^2 + \frac{m\dot x^2}{2} + mr\dot\varphi \dot x \cos\alpha$$

Система консервативна $\Rightarrow H = T + \Pi$

$$\Rightarrow H = \frac{3}{4} m(\dot\varphi r)^2 + \frac{M}{2}(\dot x)^2 - mgr\varphi\sin\alpha + \frac{m\dot x^2}{2} + mr\dot\varphi\dot x\cos\alpha$$

$$\dot p_x = -\frac{\partial H}{\partial x} = 0 \Rightarrow p_x = C_x$$

$$\dot p_\varphi = -\frac{\partial H}{\partial \varphi} \Rightarrow p_\varphi = -mgrt\sin\alpha + C_\varphi$$

$$L = \frac{3}{4} m(\dot\varphi r)^2 + \frac{M}{2}(\dot x)^2 + mgr\varphi\sin\alpha + \frac{m(\dot x)^2}{2} + mr\dot\varphi\dot x\cos\alpha$$

$$\Rightarrow p_x = \frac{\partial L}{\partial \dot x} = M\dot x + m\dot x + mr\dot\varphi\cos\alpha \Rightarrow \dot x = \frac{p_x - mr\dot\varphi\cos\alpha}{M+m} = \frac{C_x - mr\dot\varphi\cos\alpha}{M+m}$$

$$p_\varphi = \frac{\partial L}{\partial \dot\varphi} = \frac{3}{2} mr^2\dot\varphi + mr\dot x\cos\alpha \Rightarrow \dot\varphi = \frac{p_\varphi - mr\dot x\cos\alpha}{\frac{3}{2}mr^2} = \frac{-mgrt\sin\alpha - mr\dot x\cos\alpha + C_\varphi}{\frac{3}{2}mr^2}$$

$$\Rightarrow \begin{cases} \dot x = \dfrac{C_\varphi - mr\dot\varphi\cos\alpha}{M+m} \\ \dot\varphi = \dfrac{-mgrt\sin\alpha - mr\dot x\cos\alpha + C_\varphi}{\frac{3}{2}mr^2} \end{cases}$$

$$\Rightarrow \dot x = \frac{C_x}{M+m} + \frac{mr\cos\alpha}{M+m}\left(\frac{mgrt\sin\alpha + mr\dot x\cos\alpha + C_\varphi}{\frac{3}{2}mr^2}\right) =$$

$$= \frac{3/2\, mr^2 C_x + mr\cos\alpha\,(mgrt\sin\alpha + C_\varphi + mr\dot x\cos\alpha)}{\frac{3}{2}mr^2(M+m)}$$

$$\Rightarrow \dot x\left(1 - \frac{2}{3}\frac{m\dot x\cos^2\alpha}{M+m}\right) = \frac{3mr^2 C_x + 2m^2 g r^2 t\sin\alpha\cos\alpha}{3mr^2(M+m)}$$

$$\dot x = \frac{3mr^2 C_x + mr\cos\alpha\,(2mgrt\sin\alpha + 2C_\varphi)}{3mr^2(M+m) - 2m^2r^2\cos^2\alpha}$$

$$\Rightarrow x - x_0 = \frac{3mr^2 C_\varphi + mr\cos\alpha\,(mgrt\sin\alpha + 2C_\varphi)}{3mr^2(M+m) - 2m^2r^2\cos^2\alpha}\cdot t$$

$$\dot\varphi = -\frac{gt\sin\alpha}{\frac{3}{2}r} - \frac{\cos\alpha}{\frac{3}{2}r}\cdot\left(\frac{3mr^2 C_x + mr\cos\alpha\,(mgrt\sin\alpha + 2C_\varphi)}{3mr^2(M+m) - 2m^2r^2\cos^2\alpha}\right) + \frac{C_\varphi}{\frac{3}{2}mr^2}$$

$$\Rightarrow \varphi - \varphi_0 = \frac{2mr\,C_x\cos\alpha + (m+M)(mgrt\sin\alpha + 2C_\varphi)}{3mr^2(M+m) - 2m^2r^2\cos^2\alpha}\cdot t$$

№19.35

$$\begin{cases}\dot q = Aq + Bp\\ \dot p = Cq + Dp\end{cases}$$

Для гамильтоновой:

$\exists H(q,p,t)$:

$$\begin{cases}\dot q = \frac{\partial H}{\partial p}\\ \dot p = -\frac{\partial H}{\partial q}\end{cases}$$

$$\Rightarrow \begin{cases}\frac{\partial^2 H}{\partial p_i\partial q_j} = \frac{\partial^2 H}{\partial q_j\partial p_i}\\ \frac{\partial^2 H}{\partial p_i\partial p_j} = \frac{\partial^2 H}{\partial p_j\partial p_i}\\ \frac{\partial^2 H}{\partial q_i\partial q_j} = \frac{\partial^2 H}{\partial q_j\partial q_i}\end{cases}$$

$$\Rightarrow \dot q_i = a_{ik}q^k + b_{ik}p^k = \frac{\partial H}{\partial p_i}$$

$$\frac{\partial}{\partial p_j}(a_{ik}q^k + b_{ik}p^k) = \frac{\partial^2 H}{\partial p_j\partial p_i} = \frac{\partial^2 H}{\partial p_i\partial p_j} = \frac{\partial}{\partial p_i}(a_{jk}q^k + b_{jk}p^k)$$

$$\Rightarrow \underline{b_{ij} = b_{ji}} \Rightarrow \underline{B = B^T}$$

$$\dot p_i = c_{ik}q^k + d_{ik}p^k = -\frac{\partial H}{\partial q_i}$$

$$\frac{\partial}{\partial q_j}(c_{ik}q^k + d_{ik}p^k) = -\frac{\partial^2 H}{\partial q_j\partial q_i} = -\frac{\partial^2 H}{\partial q_i\partial q_j} = \frac{\partial}{\partial q_i}(c_{jk}q^k + d_{jk}p^k)$$

$$\Rightarrow \underline{c_{ij} = c_{ji}} \Rightarrow \underline{C = C^T}$$

$$\dot q_i = a_{ik}q^k + b_{ik}p^k = \frac{\partial H}{\partial p_i}$$

$$\frac{\partial}{\partial q_j}(a_{ik}q^k + b_{ik}p^k) = \frac{\partial^2 H}{\partial q_j\partial p_i} = \frac{\partial^2 H}{\partial p_i\partial q_j} = -\frac{\partial}{\partial p_i}(c_{jk}q^k + d_{jk}p^k)$$

$$\underline{a_{ij} = -d_{ji}} \Rightarrow \underline{A = -D^T}$$