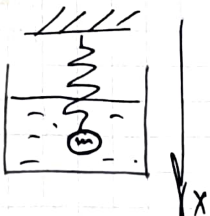
Неделя 6

№18.5

m, c

$-\beta v$



$$m\ddot{x} + \beta\dot{x} + cx = mg$$

$$\hat{q} = \hbar \cdot e^{i\omega t} \Rightarrow (-m\omega^2 + i\omega\beta + c)\hbar = mg e^{i\omega t}$$

$$\left(-\omega^2 + i\omega\frac{\beta}{m} + \frac{c}{m}\right)\hbar = g e^{i\omega t}$$

$$D(i\omega) = -\omega^2 + i\omega\frac{\beta}{m} + \frac{c}{m}$$

$$W(i\omega) = \frac{1}{D(i\omega)} = \frac{1}{-\omega^2 + i\omega\frac{\beta}{m} + \frac{c}{m}} \quad \text{– АФХ}$$

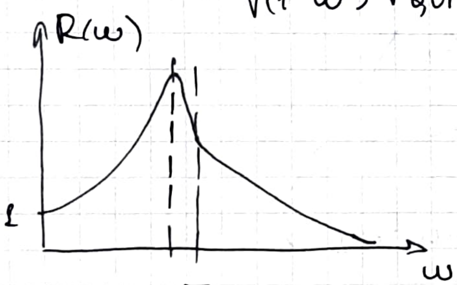
$$R(i\omega) = \frac{1}{\sqrt{\left(\frac{c}{m} - \omega^2\right)^2 + \left(\omega\frac{\beta}{m}\right)^2}} \quad \text{– АЧХ}$$

$$\varphi = \arg W(i\omega)$$

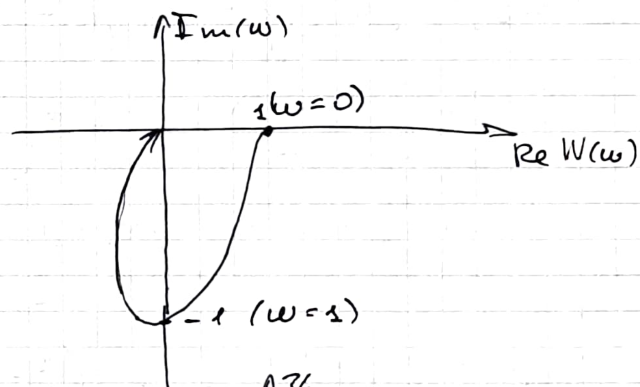
а) $\frac{c}{m} = 1;\ \frac{\beta}{m} = 0{,}1$

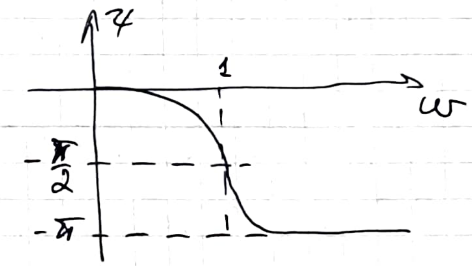
$$D(i\omega) = -\omega^2 + i\cdot 0{,}1\,\omega + 1 \Rightarrow W(i\omega) = \frac{1}{1 - \omega^2 + i\cdot 0{,}1\,\omega}$$

$$R(i\omega) = \frac{1}{\sqrt{(1-\omega^2)^2 + 0{,}01\,\omega^2}}$$



$\sqrt{\frac{c}{m}} = 1$



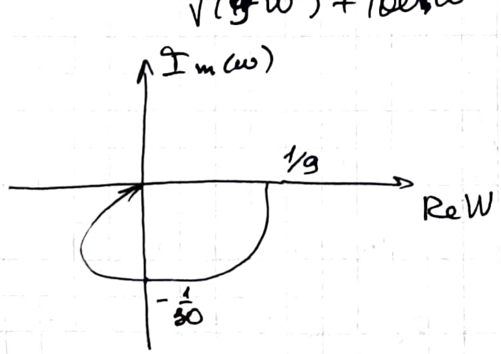


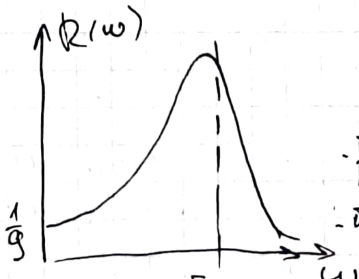
б) $\frac{c}{m} = 9;\ \frac{\beta}{m} = 10$

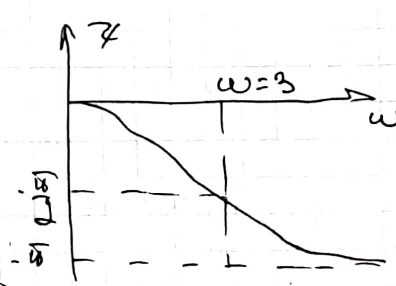
$$\Rightarrow D(i\omega) = -\omega^2 + 10i\omega + 9$$

$$W(i\omega) = \frac{1}{(9-\omega^2) + 10i\omega}$$

$$R(i\omega) = \frac{1}{\sqrt{(9-\omega^2)^2 + 100\,\omega^2}}$$

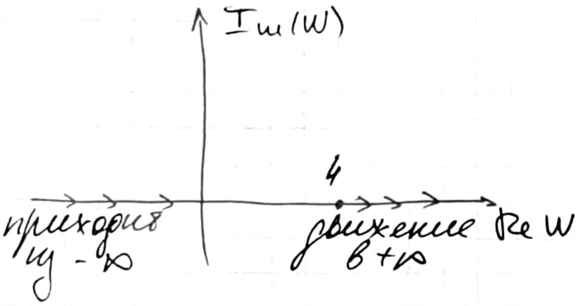


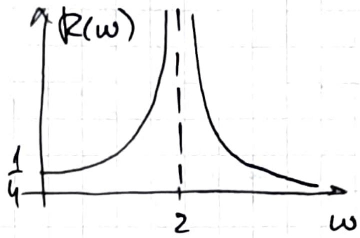


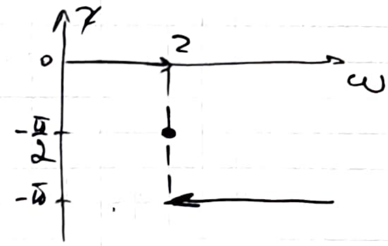


в) $\frac{c}{m} = 4$, $\beta = 0$ $\Rightarrow$ $D(\omega) = -\omega^2 + 4$ $R(\omega) = \frac{1}{|4-\omega^2|}$

$W(\omega) = \frac{1}{4-\omega^2}$

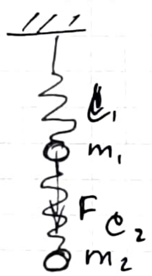






(N18.25)

m_1, m_2
c_1, c_2
$F(t) = a\sin$
$A_{m_1} = 0$



$q = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$A\ddot{q} + B\dot{q} + Cq = \begin{pmatrix} a \\ 0 \end{pmatrix}\sin pt - g\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$

$T = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2}$

$\Pi = \frac{c_1 x_1^2}{2} + \frac{c_2(x_1-x_2)^2}{2} = \frac{(c_1+c_2)x_1^2}{2} -$

$- c_2 x_1 x_2 + \frac{c_2 x_2^2}{2} = \frac{1}{2}\left((c_1+c_2)x_1^2 - 2c_2 x_1 x_2 + c_2 x_2^2\right)$

$\Rightarrow A = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$ $C = \begin{pmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{pmatrix}$

$D(\omega) = C - \omega^2 A = \begin{pmatrix} (c_1+c_2)-\omega^2 m_1 & -c_2 \\ -c_2 & c_2-\omega^2 m_2 \end{pmatrix}$

$W = D^{-1}(\omega) = \frac{1}{\det D(\omega)}\begin{pmatrix} c_2-\omega^2 m_2 & c_2 \\ c_2 & (c_1+c_2)-\omega^2 m_1 \end{pmatrix} =$

$= \frac{1}{(c_2-\omega^2 m_2)(c_1+c_2-\omega^2 m_1) - c_2^2}\left(-//- \right)$

$q = h\sin pt - gC^{-1}\begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$

$D(i\omega)\cdot h\cdot\sin pt = \begin{pmatrix} a \\ 0 \end{pmatrix}\sin pt \Rightarrow h = W(i\omega)\cdot\begin{pmatrix} a \\ 0 \end{pmatrix}$

$\Rightarrow h = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}\begin{pmatrix} a \\ 0 \end{pmatrix}$

$\Rightarrow$ решение в виде: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}\begin{pmatrix} a \\ 0 \end{pmatrix}\sin pt$

$\Rightarrow$ чтобы амплитуда колеб. m_1 была $= 0$, нужно, чтобы $W_{11} = 0$

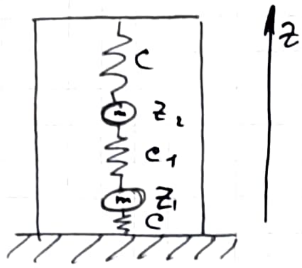
$\Rightarrow \frac{c_2 - \omega^2 m_2}{(c_2 - m_2\omega^2)(c_1+c_2-\omega^2 m_1) - c_2^2} = 0$ $\Rightarrow c_2 - m_2\omega^2 = 0$ $\omega = p$

$\Rightarrow \boxed{c_2 - m_2 p^2 = 0} \to$ Отв

№18.29

$m,\ c_1,\ c$

$z = A_0 \sin \omega t$



$$q = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$A\ddot{q} + Cq = Q(t)$$

Перейдём в НеИСО. Примем z_1 и z_2 за смещение.

$$\Rightarrow \Pi = c\frac{z_2^2}{2} + c\frac{z_1^2}{2} + \frac{c_1}{2}(z_1 - z_2)^2 + mg(z_1 + z_2)$$

$$T = \frac{m\dot{z}_1^2}{2} + \frac{m\dot{z}_2^2}{2} \Rightarrow A = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \quad C = \begin{pmatrix} c+c_1 & -c_1 \\ -c_1 & c+c_1 \end{pmatrix}$$

$$A\ddot{q} + Cq = -gm\begin{pmatrix} 1 \\ 1 \end{pmatrix} + m\ddot{z}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$ Подставим $z = A_0 \sin \omega t$

$$\Rightarrow A\ddot{q} + Cq = -gm\begin{pmatrix} 1 \\ 1 \end{pmatrix} - A_0\omega^2 m \sin\omega t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Найдём решение в виде: $q = \frac{gm}{c}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + h\sin\omega t$

$$\Rightarrow (-\omega^2 A + C)h\cancel{\sin\omega t} = -A_0 m\omega^2 \cancel{\sin\omega t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow h = -m\omega^2 A_0 (C - \omega^2 A)^{-1}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$(C - \omega^2 A) = D(\omega) = \begin{pmatrix} c+c_1-\omega^2 m & -c_1 \\ -c_1 & c+c_1-\omega^2 m \end{pmatrix}$$

$$W = D^{-1}(\omega) = \frac{1}{(c+c_1-\omega^2 m)^2 - c_1^2}\begin{pmatrix} c+c_1-\omega^2 m & c_1 \\ c_1 & c+c_1-\omega^2 m \end{pmatrix}$$

$$\Rightarrow h = \frac{-m\omega^2 A_0}{(c+c_1-\omega^2 m)^2 - c_1^2}\begin{pmatrix} c+c_1-\omega^2 m + c_1 \\ c+c_1-\omega^2 m + c_1 \end{pmatrix} = \ (=)$$

$$\cancel{c^2 + cc_1 - c\omega^2 m + c_1 c + c_1^2 - c_1\omega^2 m + c\omega^2 m + c_1\omega^2 m + \omega^4 m^2 - c_1^2}$$

$$= \cancel{\frac{c^2 + 2cc_1 + \omega^4 m^2}{c^2 + 2cc_1 + \omega^2 mc}}\ \cancel{\frac{c + 2c_1 - \omega^2 m}{c}}$$

$$\cancel{\omega^2 m(\omega^2 m - c)}$$

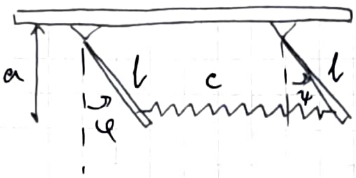
$$c^2 + cc_1 - \omega^2 mc + c_1 c + \cancel{c_1^2} - c_1\omega^2 m - c\omega^2 m - c_1\omega^2 m + \omega^4 m^2 - \cancel{c_1^2} =$$
$$= c^2 + 2cc_1 - 2\omega^2 mc - 2c_1\omega^2 m + \omega^4 m^2$$

$$(=)\ \frac{-m\omega^2 A_0\ (c + 2c_1 - \omega^2 m)}{(c+c_1-\omega^2 m)^2 - c_1^2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{-m\omega^2 A_0}{(c - \omega^2 m)}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{c}{m}}} \to \text{Отв}$$

№18.37

l, m, c, a
$A_0 \sin pt$



$A\ddot q + Cq = Q(t)$

$q = u\Theta$

$u^T A u \ddot\Theta + C u \Theta = Q(t)$

$\ddot\Theta + \Lambda\Theta = u^T Q(t)$

$T = \frac{\frac{ml^2}{3}\dot\varphi^2}{2} + \frac{\frac{ml^2}{3}\dot\psi^2}{2}$

$\Pi = \frac{ca^2}{2}(\sin\varphi - \sin\psi)^2 + \frac{mgl}{2}(1-\cos\varphi + 1-\cos\psi) \approx$
$\approx \frac{ca^2}{2}(\varphi-\psi)^2 + \frac{mgl}{2}\left(\frac{\varphi^2}{2} + \frac{\psi^2}{2}\right)$

$A = \begin{pmatrix} \frac{ml^2}{3} & 0 \\ 0 & \frac{ml^2}{3} \end{pmatrix}$ $\quad C = \begin{pmatrix} \frac{mgl}{2} + ca^2 & -ca^2 \\ -ca^2 & \frac{mgl^2}{2} + ca^2 \end{pmatrix}$

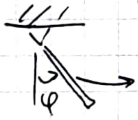
$|C - \lambda A| = 0$ Присутствует симметрия $\Rightarrow \bar u_1 = \binom{1}{1};\ \bar u_2 = \binom{1}{-1}$

$\Rightarrow \omega_1 = \sqrt{\frac{3g}{2l}} - \lambda_1 = \frac{3g}{2l}$; $\lambda_2 = \frac{3g}{2l} + \frac{6ca^2}{ml^2} \Rightarrow \omega_2 = \sqrt{\lambda_2}$

$u_1^T A u_1 = \frac{2}{3} ml^2 = u_2^T A u_2$

$u = \sqrt{\frac{3}{2}} \frac{1}{\sqrt{ml^2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Рассмотрим силу инерции, действ. на один стержень:



$y^e = -m\ddot x = mA_0\omega^2 \sin\omega t \Rightarrow$ Углы малы $\Rightarrow Q_1^e = l y^e = mlA_0\omega^2 \sin\omega t$
$Q_2^e =$

$Q = mlA_0\omega^2 \binom{1}{1} \sin\omega t$

$u^T Q = \sqrt{\frac{3}{2}} \frac{mlA_0\omega^2}{\sqrt{ml^2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \binom{1}{1} \sin\omega t = \begin{pmatrix} \sqrt{\frac{3}{2}} A_0\omega^2 \sqrt{m} \sin\omega t \\ 0 \end{pmatrix}$

$\Rightarrow$ Получаем ур-е на Θ_1: $\ddot\Theta_1 + \frac{3g}{2l}\Theta_1 = \sqrt{\frac{3m}{2}} A_0\omega^2 \sin\omega t$

Ищем реш-е в виде: $\Theta_1 = d\sin\omega t \Rightarrow -d\omega^2 + \frac{3g}{2l}d = \sqrt{\frac{3m}{2}} A_0\omega^2$

$\Rightarrow d = \sqrt{\frac{3m}{2}} \cdot \frac{2lA_0\omega^2}{3g - 2l\omega^2} \Rightarrow \vec\Theta^{\cancel{\text{частное}}} = \binom{1}{0} \sqrt{\frac{3m}{2}} \frac{2lA_0\omega^2}{3g - 2l\omega^2} \sin\omega t$

Проделаем обратный переход: $\binom{\varphi}{\psi} = u\Theta = \sqrt{\frac{3}{2m}} \cdot \frac{1}{l} \sqrt{\frac{3m}{2}} \frac{2lA_0\omega^2}{3g - 2l\omega^2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \binom{1}{0} =$

$= \frac{3A_0\omega^2}{3g - 2l\omega^2} \binom{1}{1} \sin\omega t$

$\omega = p \Rightarrow \underbrace{\binom{\varphi}{\psi}^{\cancel{\text{частн}}} = \binom{1}{1} \frac{3A_0p^2}{3g - 2lp^2} \sin pt}$

$\Rightarrow$ Общее решение имеет вид:

$$\begin{pmatrix} \varphi \\ \psi \end{pmatrix}^{\text{общ.}} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} c_1 \sin(\omega_1 t + \alpha_1) \\ c_2 \sin(\omega_2 t + \alpha_2) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \varphi \\ \psi \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} c_1 \sin(\omega_1 t + \alpha_1) \\ c_2 \sin(\omega_2 t + \alpha_2) \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \frac{3A_0 p^2}{3g - 2lp^2} \sin pt$$

$$\omega_1^2 = \frac{3g}{2l}; \quad \omega_2^2 = \frac{3g}{2l} + \frac{6ca^2}{ml^2}$$