Exercise 4

(b) $\langle v \rangle = \int_0^\infty v p(v) dv = \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 4\pi \int_0^\infty \underbrace{v^2}_{:=g(v)} \underbrace{v \cdot \exp\left(-\frac{mv^2}{2k_BT}\right)}_{:=f'(v)} dv$

$$= \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 4\pi \left[\underbrace{\left[v^2 \frac{k_BT}{m} \cdot \exp\left(-\frac{mv^2}{2k_BT}\right)\right]_0^\infty}_{=0} + \int_0^\infty 2 v \frac{k_BT}{m} \exp\left(-\frac{mv^2}{2k_BT}\right) dv \right]$$

$$= \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 4\pi \cdot 2 \int_0^\infty v \exp\left(-\frac{mv^2}{2k_BT}\right) \cdot \frac{k_BT}{m} dv$$

$$= \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 8\pi \left[-\frac{k_B^2T^2}{m^2} \exp\left(-\frac{mv^2}{2k_BT}\right)\right]_0^\infty$$

$$= \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 8\pi \frac{k_B^2T^2}{m^2} = \sqrt{\frac{8k_BT}{m\pi}} \quad \square$$

(c) $p = m \cdot v = m \cdot \sqrt{\frac{8k_BT}{m\pi}}$

$$\lambda = h \cdot \frac{\sqrt{m \cdot \pi}}{m\sqrt{8k_BT}} = \frac{\sqrt{\pi}}{2} \cdot \lambda_T$$

(d) $T_c = 1450K$

~~$\lambda_c = 0{,}0929 Å \hat{=} 5{,}806\%$ of $1{,}6 Å$~~

$\lambda_{T_c} = 0{,}059 Å \hat{=} 3{,}7\%$ of $1{,}6 Å$

Classical treatment may be sufficient.