Excercise Sheet 2

Ex 4:

b) $\rho(v) = \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 4\pi v^2 \cdot \exp\left(-\frac{mv^2}{2k_B T}\right)$

$\rho(v) = A\, v^2 \cdot \exp(-B v^2)$ with $A := \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 4\pi$

and $B := \frac{m}{2k_B T}$

$$\langle v \rangle = \int_0^\infty v\, \rho(v)\, dv = A \int_0^\infty v^3 \exp(-Bv^2)\, dv$$

substitute $x = v^2 \Rightarrow \frac{dx}{dv} = 2v \Leftrightarrow dv = \frac{dx}{2v}$

$$= A \int_0^\infty x\, v\, \exp(-Bx) \frac{dx}{2v} = \frac{A}{2} \int_0^\infty x \exp(-Bx)\, dx$$

partielle Integration: $f(x) = x \Rightarrow f'(x) = 1$

$g'(x) = \exp(-Bx) \Rightarrow g(x) = -\frac{1}{B}\exp(-Bx)$

$$= \frac{A}{2}\left(\left[-x\frac{1}{B}\exp(-Bx)\right]_0^\infty - \int_0^\infty -\frac{1}{B}\exp(-Bx)\, dx\right)$$

$$= \frac{A}{2}\left(0 + \frac{1}{B}\left[-\frac{1}{B}\exp(-Bx)\right]_0^\infty\right)$$

$$= \frac{A}{2}\left(0 + \frac{1}{B^2}\right) = \frac{A}{2B^2} = \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 2\pi \left(\frac{m}{2k_B T}\right)^{-2}$$

$$= \frac{2\pi}{\pi^{3/2}}\left(\frac{m}{2k_B T}\right)^{-\frac{1}{2}} = \left(\frac{2k_B T}{m\pi}\right)^{\frac{1}{2}} \cdot 2 = \sqrt{\frac{8k_B T}{m\pi}}$$

c) $\lambda = \frac{h}{p}$, $\lambda_T = \frac{h}{\sqrt{2\pi m k_B T}}$

$$\langle p \rangle = m \langle v \rangle = \sqrt{\frac{8 m k_B T}{\pi}}$$

$$\lambda = \frac{h}{\langle p \rangle} = \frac{h\sqrt{\pi}}{2\sqrt{2mk_BT}} = \lambda_T \cdot \frac{\pi}{2}$$

d) $m_{SiO_2} \approx 9,9798 \cdot 10^{-26}\ kg$

$$\lambda_{T,SiO_2} \approx 5,914 \cdot 10^{-12}\ m \approx 0,05914\ Å$$

$$\frac{0,059}{1,6} \approx 3,7\%$$