NASA project

Kees Kroep 4246373

August 8, 2016

Abstract

Type an abstract

Contents

Introduction

This document is a feasability study of a lidar sensor designed for the Europa Clipper. The main focus of this document will lie on the three most challenging aspects of the sensor design. How to achieve the target precision goals of the two modus of operation within the power budget, and the effect of radiation on the system.

First the requirements will be stated in section 1.1, next the assumptions used for this study will be listed in ??, then the feasability of the Altimetry Mode and Hazard Detection Mode will be investigated in ?? and ?? respectively. Section ?? will zoom into the effect of radiation on the system, and finally ?? and ?? will conclude the system with a spec sheet etc...

1 Requirements and Specifications

1.1 High level specifications

The radiation and mass, power and volume budget are listed in table 1 and table 2.

Table 1: Overview of radiation limitations

Radiation	
Total Integrated Dose in Silicon behind 100 mil Al spherical shell	500 krad

Table 2: Requirements for mass, power and volume

Mass, Power & Volume Budget	
Optical Head	<5 kg
Inside vault	<1 kg
Max cable length between optical head and electronics	2 m
Power	<50 W

The Lidar sensor features two operation modes: the Altimetry Mode, and the Hazard Detection Mode.

1.2 Altimetry Mode

Altimetry Mode is the first phase of the landing. During the Altimetry Mode, the Lidar sensor has to provide the altitude of the sensor in relationship to the surface op Europa. The requirements for the Altimetry Mode are listed in table 3

Table 3: Requirements for Altimetry Mode

Altimetry Mode	Threshold	Goal
Max acquisition Slant Range	5 km	8 km
Range Accuracy (3-sigma)	1%	0.10%
Update Rate	0.1 Hz	1 Hz

1.3 Hazard Detection Mode

During the Hazard Detection Mode a 3d map of the surface of Europa must be created around the landing place. The Hazard Mode is operational when the sensor is close enough to the surface of Europa. The requirements for the Hazard Detection Mode can be found in table 4

Table 4: Requirements for Hazard Detection Mode

Hazard Detection Mode	Threshold	Goal
Max range (altitude)	400 m	500 m
Min Operational Range (altitude)	5 m	1 m
Range Accurcay (3-sigma on final 3D map)	10 cm	5 cm
Ground Sample Distance (per pixel on 3D map)	10 cm	5 cm
Ground Area Coverage at max altitude	100 m x 100 m	125 m x 125 m
Time for 3D map creation	1 s	1 s

1.4 Overview

A schematic overview of the sensor is shown in fig. 1.

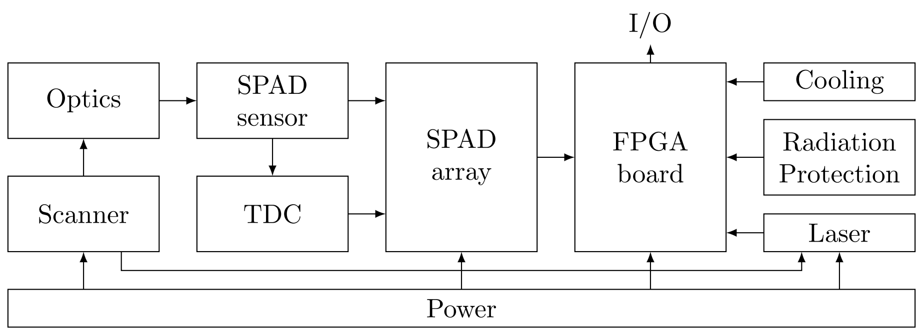

Figure 1: Schematic overview

Laser: The laser must send short pulses but powerful pulses at a predefined frequency, to transmit photons that can be detected by the SPAD Sensor. A critical requirement for the laser is the amount of photons it is able to transmit as a function of time, within the budget and technical limitations.

SPAD Sensor: The Single Photon Avalange Diode (SPAD) is responsible for generating a digital pulse when hit by a photon. A circuit build around the SPAD must ensure that the sPAD is quenched as quickly as possible to minimize the deadline. A critical requirement for the SPAD is a small jitter to maximize the accuracy of the sensor.

TDC: The Time Interval to Digital Converter (TDC) is connected to the laser and the SPAD Sensor. The TDC must measure the time difference between the transmission of teh laser and the receiving at the SPAD sensor. This measurement requires an accuracy of tens of picoseconds to meet the accuracy requirements.

Scanner: The scanner handles the scanning motion that is needed to accumulate the entire picture. Different scanning motions can introduce undesired jitter and heavily influency the amount of SPAD Sensors that are needed on teh SPAD Array.

SPAD Array: The SPAD Array integrates the SPAD Sensors on a chip and connects them to the TDC's. The layout of the SPAD Array is closely rtelated to the scanning motion that is used in the scanner.

Optics: The optics must tranfer as much of the incomming photons as possible to the sensitive area on the SPAD Sensors. The Optics part also implements a bandpass filter around the target frequency.

FPGA Board: The FPGA Board controls the sensor. It is responsible for accumulating and interpreting the measurements from the TDC's, and controlling the Scanner and Laser.

Cooling: The cooling has to keep the temperature of the FPGA chip under a threshold temperature.

Radiation Protection: The Radiation Protection shields sensitive parts of the sensor from radiation. The most sensitive part of the system is expected to be the FPGA Board, based on experience.

Power: The power block has to supply the scanner, SPAD Array, FPGA Board, and the laser with the required power. The power block has to operate within the specified power budget.

1.5 Assumptions

This section will list the assumption that are made throughout the design.

Reflectivity of surface Europa: It is assumed that out of all light that hit's Europa, 35% is reflected in a perfectly diffuse manner. The rest of the light is either reflected in a specular way that will not hit the sensor, or absorbed by the ice on the surface of Europa. This assumption is made in order to avoid the added complexity when using a more elaborate model.

Wavelength laser: The wavelength of light emitted by the laser is assumed to be $850\,nm$. This wavelength is a typcial choice for Silicon SPAD's.

Bandpass filter:A bandpass filter will be used o filter background noise. This filter is assumed to be a bandpass filter with a center frequency of $850\,nm$ and a FWHM of $10\,nm$.The minimum transmission of the filter is 50%. These values are directly taken from the "*850nm, 10nm FWHM, 12.5mm Mounted Diameter*" product made by Edmund Optics and sold for 75 \$.

Jitter of SPAD Sensor: The jitter of the SPAD Sensor, or Full Width Half Max (FWHM) is assumed to be $100\,ps$. This value is a typical FWHM for state of the art Silicon SPAD's.

Jitter of TDC: The jitter of the TDC's is assumed to be insignificant when compared to the jitter of $100\,ps$ caused by the SPAD Sensors. Experience with previous designs show that TDC jitter is generally a very small contributor to overall jitter.

Accuracy of TDC: The accuracy of the TDC is assumed to be $50\,ps$, and therefore well within the accuracy requirements. The accuracy should be $5\,cm$. Using eq. (1), the desired accuracy is $333\,ps$.

$$t = \frac{2x}{c} \tag{1}$$

where x is the accuracy in distance, and $c \approx 3 \cdot 10^8$ the speed of light. The factor two is present because the light travels the distance twice.

Sunlight: It is assumed that the light from the sun that is reflected of Europa is the only signifant contributor to background noise. No other sources of light will be considered.

Photon Detection Probability: It is assumed that the PDP of the SPADs is 35%.

Effective area on chip: It is assumed that 5% of the SPAD Array can be used to receive photons. This assumption is based on current gen Silicon SPAD's.

Laser light hitting target: It is assumed that all the light that leaves the laser is hitting the target area on Europa.

Signal to Noise Background Ratio: It is assumed that a Signal to Noise Background Ratio (SNBR) of 0 dB will be sufficient for the sensor. This value is based on previous research on Silicon SPADs.

Laser efficiency: It is assumed that the laser has an efficiency of 10%.

Surface of SPAD: It is assumed that the surface of a single SPAD is $20 \times 20\,\mu m^2$.

2 Altimetry Mode

The Altimetry Mode is the mode in which only the altitude of the device in relationship to Europa is required. The main challenges for the altimetry mode are acquiring the required resolution, acquiring the required speed, staying within the power budget, and being sufficiently resilient against the accumulated radiation for the entire trip.

The first step will be to create a model of the noise that will be present at Europa. Then the required amount of signal will be calculated, resulting in the required signal power.

2.1 Optics

The receiver optics are a good place to start of with, beacuse the optics are not very dependent on results attachieved in other areas. The optics have to transport as many desired photons, and as little unwanted photons to the active area on the SPADs as possible. All while having an acceptable depth of field.

The most basic solution is a single lens with an aperture. The opacity of the lens can be calculated with the absorption of the lens material and f-number of the lens using eq. (2).

$$\text{opacity} = \frac{1 - \text{absorption}}{\text{f-number}^2} \tag{2}$$

The performance of a possible configuration is shown in table 5

Table 5: Performance of basic optics solution

Basic Optics	
f-number	2.00
absorption	5.00 %
opacity	23.75 %

Improvements

There are a couple of additions that can improve the performance of the optics. The first and very necessary one, is the use of a bandpass filter. The transmitted signal will have a very specific bandwidth of $850\,nm$. Using a narrow bandpass filter one can filter out an enormous part of the background noise. The filter will have an opacity of 50 % for the target wavelength.

The captured photons that hit the lens need to be guided to the active area of the SPADs. If the active area on the chip is very small, one can use microlenses to improve the effectiveness of the optics. A Microlens focusses light on a single SPAD on the chip. Two types of microlenses will be considered: a spherical lens, and a square shaped lens. The presence of microlenses poses a limitation of the main lens. The f-number must be relatively large. A higher f-number means a smaller aperture and therefore more loss of photons. A way of dealing with this problem is to use a second lens instead. An overview of the available options is shown in fig. 2

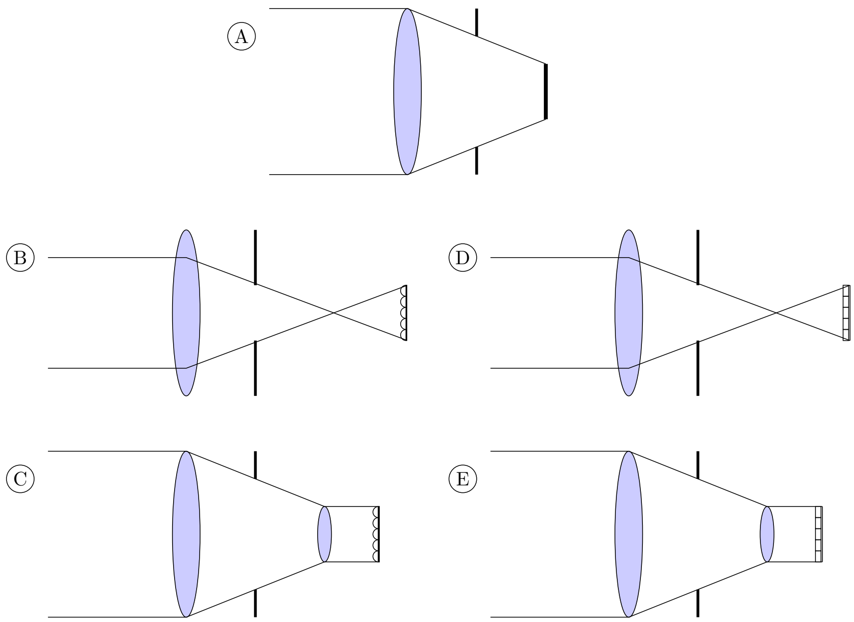

Figure 2: Overview of possible receiver optics implementations

$$\text{opacity} = (1 - \text{absorption}_1)(1 - \text{absorption}_2) \cdot \text{opacity filter} \cdot \frac{X}{\text{f-number}^2} \tag{3}$$

Where X is the active area on the chip. A comparison between the different options is shown in table 6

Table 6: comparison of different optics solutions

Type	**A**	**B**	**C**	**D**	**E**
absorption 1^{st} lens	0.05	0.05	0.05	0.05	0.05
f-number	2	8	2	8	2
absorption 2^{nd} lens	0	0	0.05	0	0.05
opacity bandpass filter	0.5	0.5	0.5	0.5	0.5
active area on chip	X	0.55	0.55	0.65	0.65
effective opacity	$X \cdot 11.75\,\%$	$0.4082\,\%$	$6.204\,\%$	$0.4924\,\%$	$14.6656\,\%$

The comparison in table 6 shows some good alternatives to the basic lens, if there is a need for it due to a small active area on the chip. However, most of the future calculations will focus on the basic model A.

2.2 Noise caused by the sun

The sun is the most dominant source of unwanted photons at Europa. This section will investigate how much energy is hitting the surface of Europa.

To calculate that the sun will be modelled as an ideal black body. The spectral irradiance of the sun can be calculated using eq. (4).

$$I_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{4}$$

where $I_\lambda(v, t)$ is spectral irradiance with unit W/m^3.
h is the planck constant
c is the speed of light in vacuum
k is the Boltzman constant
λ is the wavelength of the electromagnetic radiation
T is the absolute temperature of the body

The spectral irradiance of the sun is calculated in table 7.

Table 7: Calculation of sun irradiation

Sun irradiation	
h	$6.63 \cdot 10^{-34}\, Js$
c	$3.00 \cdot 10^{8}\, m/s$
k	$1.38 \cdot 10^{-23}\, j/K$
λ	$850.00\, nm$
T	$5.78\, kK$
I_λ	$1.51 \cdot 10^{4}\, W/m^2/nm$

The next step is to calculate the power emitted by the sun in the specified bandwidth, at the location of Europa. The specified bandwidth is in this case the assumed bandpass filter used on the lens. The emitted power is calculated by modelling the sun as a point source, and then spreading that power over a sphere with a radius equal to the distance between the sun and Europa, as is done in eq. (5).

$$P_{sun} = I_{\text{sun}} B_\lambda S \frac{r_{\text{sun}}^2}{r_{\text{europa}}^2} \tag{5}$$

where I_{sun} is the spectral irradiance of the sun at the center frequency of the filter, B_λ is the bandwidth of the filter in meters, S the surface area of the target area on Europa, r_{sun} the of radius of the sun, and r_{europa} the distance between Europa and the sun. The effective radiance of the background noise at Europa is calculated in table 8 using eq. (5).

Table 8: Calculation of background power on target area on Europa

Background power	
I_λ	$1.51 \cdot 10^{13}\, W/M^3$
B_λ	$10.00\, nm$
Surface area	$15625.00\, m^2$
r_{sun}	$6.96 \cdot 10^{5}\, km$
r_{europa}	$7.79 \cdot 10^{8}\, km$
P_B	$1.89\, kW$

The next step is to calculate the percentage of energy that hits the device when hovering over Europa. The focal length and aperture ofthe lens will be configured in such a way that the target surface on Europa fills the entire view at the maximum altitude of the Hazard Detection Mode, so that altitude will be chosen to calculate the received noise power. The amount of power received at the lens of the device can be calculated using eq. (6).

$$P'_B = \frac{P_B \cdot R_{Europa} \cdot D_l \cdot \text{opacity}}{2r^2} \tag{6}$$

where P'_B is the power hitting the lens,
P_B the noise power on the target area of Europa,
R_{Europa} the reflectivity of Europa,
D_l the diameter of the lens,
opacity the opacity of the lens,
and r the altitude of the device. The calculations are performed in table 9.

Table 9: Amount of noise power that hits the SPAD array

noise power at SPADs	
P_B	$1.89\,kW$
r	$500.00\,m$
R_{europa}	$35.00\,\%$
Diameter lens (D_l)	$50.00\,mm$
opacity optics	$11.75\,\%$
P'_B	$7.76\,\mu W$

Finally the power needs to be converted to number of photons. To calculate how many photons bounce from the surface of Europa and actually hit the light. To calculate the amount of photons one needs to know the amount of energy per photon. This can be calculated using eq. (7). The calculation is performed in table 10.

$$E_{photon} = \frac{hc}{\lambda} \tag{7}$$

Table 10: Pulse frequency for both modes of operation

energy of photon	
h	$6.63 \cdot 10^{-34}\,Js$
c	$3.00 \cdot 10^{8}\,m/s$
λ	$850.00\,nm$
E_{photon}	$2.34 \cdot 10^{-19}\,J$

The amout of photons per second can then be calculated using eq. (8). The calculation is shown in

$$\text{photon}/s = \frac{P}{E_{photon}} \tag{8}$$

Table 11: Pulse frequency for both modes of operation

photons hitting SPADs	
P'_B	$7.76 \cdot 10^{-6}\,W$
E_{photon}	$2.34 \cdot 10^{-19}\,J$
photons at SPADs	$3.31 \cdot 10^{13}\,\text{photon}/s$

2.3 Detected photon characterisation

This section tackles the problem of calculating the resolution of the set-up. To do this, the different types of detection by the SPADs, due to sunlight, signal source and dark counts, must be characterized first.

To avoid pileup a couple of temporary assumptions about the performance of the SPADs will be made. Firstly, it is temporarily assumed that the percentage of the surface that is sensitive to photons is 50 %. Secondly it is temporarily assumed that the dark count rate (DCR), of the entire array is $PSS_N = 1 \cdot 10^9 \text{photon}/s$.

The amount of detected sunlight photons detected by the SPADs per second can now be calculated using eq. (9). The calculation are shown in table 12. Both the sunlight detections and the DCR detections are uniformly distributed.

$$PPS_B = \text{photons at SPADs} \cdot \text{PDP} \cdot \text{effective area} \tag{9}$$

Table 12: Amount of detected sunlight photons detected per second

PPS for background photons	
Photons at SPADs	$3.31 \cdot 10^{13}$ photon/s
PDP	35.00 %
effective area	50.00 %
PPS_B	$5.80 \cdot 10^{12}$ photon/s

The next step is to characterize the relationship between the laser power and the received photons. It is assumed that all light emitted by the laser is hitting Europa. It is also assumed that the efficiency of the laser is 10 %. Next, similar calculation as performed with P_B can be used to calculate the amount of detected signal photons per second PPS_S. There is one important difference however: where for sunlight an altitude of $500\,m$ was used, for the signal photons an altitude of $8\,km$ must be considered. The calculation are performed in table 13. Note that even with the largest power budget of $50\,W$, the SNR will be well below $0\,dB$.

Table 13: Amount of detected signal photons detected per second

PPS for background photons	
P_S	$1.00\,W$
Altitude	$8.00\,km$
P'_S	$1.61 \cdot 10^{-11}\,W$
PPS_S	$1.20 \cdot 10^{7}$ photon/s

The signal photons are not uniformly distributed like the noise and background photons. The signal photons are assumed to be distributed in a normal distribution with a $FWHM = 100\,ps$. This distribution is convoluted with the jitter on the SPADs, which is assumed to be a normal distribution with a $FWHM = 100\,ps$. Using eq. (10) and eq. (12), the resulting distribution has a $\sigma_S = 60\,ps$. The mean μ_s is equal to the time of flight of the photon.

$$FWHM = 2\sqrt{2\ln 2}\sigma \tag{10}$$

$$\sigma = \frac{FWHM}{2\sqrt{2\ln 2}} \tag{11}$$

$$\sigma_{f\otimes g} = \sqrt{\sigma_f^2 + \sigma_g^2} \tag{12}$$

The maximum allowable $FWHM$ of the measurement is $333\,ps$ as shown in section 1.5. Using eq. (11) the maximum standard deviation is $\sigma = 141\,ps$.

2.4 Resolution

The requirement for the altimetry mode changes based on the height. The target is a resolution of at least $0.1\,\%$. To ensure that this requirement is met, both the largest altitude of $8\,km$, and the smallest altitude of $500\,m$ will be investigated.

The minimum resolution for the largest and shortest altitude are $8\,m$ and $0.5\,m$ respectively. The maximum allowable FWHM can be calculated using eq. (13). Using eq. (11) the maximum standard deviation can be calculated. The calculations are performed

$$FWHM_{max} = \frac{2x}{c} \tag{13}$$

Table 14: Pulse frequency for both modes of operation

AM requirements	short	long
altitude	$500\,m$	$8\,km$
resolution	$50\,cm$	$8\,m$
FWHM	$3.33\,ns$	$53.33\,ns$
σ	$1.42\,ns$	$22.65\,ns$

2.4.1 standard deviation shortcut

To calculate the standard deviation of the sum of multiple random variables that are independent and uncorrelated, one can use eq. (26)

$$\mathrm{Var}[aX + bY + cZ] = a^2\mathrm{Var}[X] + b^2\mathrm{Var}[Y] + c^2\mathrm{Var}[Z] \tag{14}$$

Next consider a situation where there are two random variable distributions. The first distribution is S, which is a normal distribution with $\mu_s = ToF$, where $ToF \in \{0, 50\mu\}$, and $\sigma_s = 100p$. The second distribution is N, which is a uniform distribution with $\mu_n = \frac{50\mu}{2}$ and $\sigma_n = \frac{50\mu}{\sqrt{12}}$.

$$\begin{aligned}
\mu_{mean} &= \frac{1}{110}\Big(\sum_{k=1}^{100}\mu_s + \sum_{l=1}^{10}\mu_n\Big) && (15)\\
&= \frac{100ToF + 10 * 25\mu}{110} && (16)\\
&= \frac{10}{11}ToF + 227.3n && (17)
\end{aligned}$$

$$\begin{aligned}
\mathrm{Var}_{mean} &= \mathrm{Var}\Big[\frac{1}{110}\Big(\sum_{k=1}^{100} S_k + \sum_{l=1}^{10} N_l\Big)\Big] && (18)\\
&= \frac{1}{110^2}\Big(\sum_{k=1}^{100}\mathrm{Var}[S] + \sum_{l=1}^{10}\mathrm{Var}[N]\Big) && (19)\\
&= \frac{1}{110^2}(100\sigma_s^2 + 10\sigma_n^2) && (20)\\
&= \frac{10^{-18} + 2.0833\cdot 10^{-9}}{110^2} && (21)\\
&= 1.7218\cdot 10^{-13} && (22)\\
\sigma_{mean} &= \sqrt{\mathrm{Var}_{mean}} && (23)\\
&= 414.94n && (24)
\end{aligned}$$

2.5 Sampling Method

The sampling method used has a massive impact on the performance of the system. This section investigates the options that can be chosen from.

The time that the sensor should listen to a response is determined by the maximum round trip time of a photon, and can be calculated using eq. (25).

$$t_{round} = \frac{2r}{c} \tag{25}$$

where $c \approx 3\cdot 10^8$ is the speed of light, and r the altitude of the sensor.

The next step is to calculate the relationship between oberved number of good and bad photons for a given listening period, and the resulting standard deviation of the measurement. To calculate the standard deviation of the sum of multiple random variables that are independent and uncorrelated, one can use eq. (26)

$$\mathrm{Var}[aX + bY + cZ] = a^2\mathrm{Var}[X] + b^2\mathrm{Var}[Y] + c^2\mathrm{Var}[Z] \tag{26}$$

Because the DCR and sunlight photons have the same uniform distribution they are taken from the same uniform random variable N, where n is the amount of detected sunlight and dark photons, $\mu_N = \frac{1}{2}t_{round}$, and $\sigma_N = \frac{1}{\sqrt{12}}t_{round}$. The signal photons are taken from the random variable S, where s is the amount of detected signal photons, $\mu_S = ToF$, and $\sigma_S = 42.5\,ps$. The resulting mean μ_{tot} can be calculated using eq. (28). The error with the desired μ can be calculated using eq. (30).

$$\begin{aligned}
\mu_{tot} &= \frac{1}{s+n}\Big(\sum_{k=1}^{s}\mu_S + \sum_{l=1}^{n}\mu_N\Big) && (27)\\
&= \frac{s\cdot Tof}{s+n} + \frac{n\cdot\frac{1}{2}t_{round}}{s+n} && (28)
\end{aligned}$$

$$\begin{aligned}
\mu_{\text{error}} &= |ToF - \mu_{tot}| && (29)\\
&= \Big|\frac{n(ToF - \frac{1}{2}t_{round})}{s+n}\Big| && (30)
\end{aligned}$$

The resulting standard deviation σ_{tot} can be calculated using eq. (36).

$$\mathrm{Var}_{tot} = \mathrm{Var}\Big[\frac{1}{s+n}\Big(\sum_{k=1}^{s}\mathrm{Var}_S + \sum_{l=1}^{n}\mathrm{Var}_N\Big)\Big] \tag{31}$$

$$= \frac{1}{(s+n)^2}(s\sigma_s^2 + n\sigma_n^2) \tag{32}$$

$$\sigma_{tot} = \sqrt{\mathrm{Var}_{tot}} \tag{33}$$

$$= \frac{\sqrt{s\sigma_s^2 + n\sigma_n^2}}{s+n} \tag{34}$$

$$= \frac{\sqrt{s \cdot (42.5 \cdot 10^{-12})^2 + n \cdot (\frac{1}{\sqrt{12}} t_{round})^2}}{s+n} \tag{35}$$

$$= \frac{\sqrt{s \cdot 1.8063 \cdot 10^{-21} + \frac{n}{12} t_{round}}}{s+n} \tag{36}$$

A very important restriction to the sampling method is the highest achievable peak power of the laser. If there would be no limit on the peak power, one could send out one extremely powerful pulse, listen for an extremely short period of time, and then reconstruct the altitude. The key advantage is the listening time. Shortening the listening time directly reduces the amount of background and noise that can be observed.

The maximum pulse frequency can then be calculated using eq. (37)

$$f_{pulse} = \frac{1}{t_{round}} = \frac{c}{2r} \tag{37}$$

The maximum altitude are different for the Altimetry and Hazard Detection mode. The maximum pulse frequency of both modes is shown in **??**.

In order to increase the frequency, it is intersting to see what happens if the frequency is increased. The TDC measures the time between the last outgoing pulse and the incomming pulse. This means that the performed measurement t_{TDC} can be calculated using eq. (38).

$$t_{TDC} = ToF \mod T_{pulse} \tag{38}$$

$$ToF = t_{TDC} + k \cdot T_{pulse} \qquad k \in \mathbb{N} \tag{39}$$

where t_{TDC} is the measurement of the TDC, ToF the time of flight, and T_{pulse} the time period of the laser pulses. The returned answer is related to ToF as shown in eq. (39). The precision of the measurement is maintained, but on a larger scale the information is lost.

To solve this problem one can make measurements in two different frequencies. The idea is also applied in TDCs (Name of technique?), where two ring oscillators with different frequencies are used to amplify the range of the TDC. A similar idea can be applied here, but instead of using two oscillators, the time is devided in half, and in the first half the laser sends pulses in frequency f_1, and in the second half pulses with frequncy f_2. These two different measurements can first be used to determine the large scale ToF, and then the measurements can be combined to get a high accuracy to accompany that.

As a proof of concept. This idea is applied to the Altimetry Mode. It is assumed that a difference of $5\,ns$ between λ_1 and λ_2 is sufficient. Now using a maximum ToF one can calculate that optimal two frequencies, such that $f1_{>}f_2$ and f_2 as high as possible.

$$\frac{50\mu}{5n} = 10660 \tag{40}$$

$$\sqrt{\frac{50\mu}{5n}} \approx 103 \tag{41}$$

Now two coprime numbers in $\mathbb{N}$ around 103 are 103 and 104. The resulting frequencies are

$$T_1 = 103 \cdot 5n = 515n \tag{42}$$

$$T_2 = 104 \cdot 5n = 520n \tag{43}$$

$$f_1 = \frac{1}{T_1} = 1.9417\, MHz \tag{44}$$

$$f_2 = \frac{1}{T_1} = 1.9231\, MHz \tag{45}$$

$$\tag{46}$$

The pulse frequency increases by a factor 100. A matlab plot of the resulting system is shown in fig. 3, where the measurement values of the TDC are plotted against the ToF.

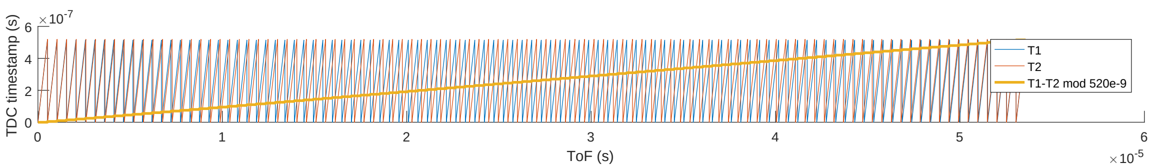

Figure 3: Matlab plot of TDC measurements vs ToF

The ToF can be calculated using eq. (48).

$$t_{1-2} = (t_1 - t_2) \mod T_2 \tag{47}$$

$$ToF = \frac{t_{1-2}}{T_2 - T_1} T_1 + \frac{t_1 + t_2 - t_{1-2}}{2} \tag{48}$$

where t_1 and t_2 are the timestamp measurements for f_1 and f_2 respectively. t_{1-2} is the modulus of the time difference between t_1 and t_2.