The stiffness matrix for each element is $\boldsymbol{k}_e = \int \boldsymbol{B}^T EA \boldsymbol{B}\, dx$,

$$\boldsymbol{k}_e = EA \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

and the right-hand side vector is $\boldsymbol{b}_e = \int \boldsymbol{N}^T f\, dx$,

$$\boldsymbol{b}_e = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

Assembling three elements into the global matrix and right-hand side vector,

$$EA \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1 \\ 1/2 \end{bmatrix}.$$

We need to apply the boundary condition $u(0) = 0$, which requires that $a_1 = 0$. The simplest way to impose this condition is to delete the first row and column of the stiffness matrix,

$$EA \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1/2 \end{bmatrix}.$$

Solving this system we have $\boldsymbol{a}^T = (1/EA) \begin{bmatrix} 2.5 & 4 & 4.5 \end{bmatrix}$. The exact solution is $u = (1/EA)\left(-x^2/2 + 3x\right)$. The is derived by integrating $-EAd^2u/dx^2 = 1$ and using the boundary conditions $u(0) = 0$ and $du(3)/dx = 0$.

What you notice is that the finite element solution is exact at the nodes, but it is not exact between the nodes. This 'exactness' is a feature of finite element methods in one dimension, but it does not carry over to higher dimensions.

This nodal 'exactness' means that looking at error in the displacement at the nodes does not tell us about the error. To quantify the error we could compute a norm of the error, e.g.

$$e_u = \left(\int_0^L (u - u_h)^2 \, dx \right)^{1/2}$$

for the displacement error, or

$$e_{\text{energy}} = \left(\int_0^L EA \left(\frac{du}{dx} - \frac{du_h}{dx} \right)^2 dx \right)^{1/2}$$

for the error in the strain energy.

Since the exact solution is quadratic, the finite element solution would be exact if you use elements which are quadratic or higher.