CE394M: Stress paths and invariants

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Overview

1. Stresses / strains in typical geotechnical lab tests
2. Friction
3. Stress invariants
4. Soil engineering properties
5. Total vs Effective stress

Stresses / strains

1D consolidation / simple shear

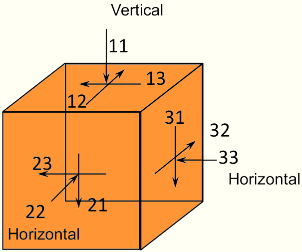
- Zero lateral strain ($\varepsilon_{22} = \varepsilon_{33} = 0$)
- Stresses: σ and τ
- Strains: $\varepsilon_{11} = \varepsilon_v$ and γ

2D plane strain

- Zero lateral strain ($\varepsilon_{22} = \gamma_{12} = \gamma_{23} = 0$)
- Stresses: s and t
- Strains: ε_v and ε_γ

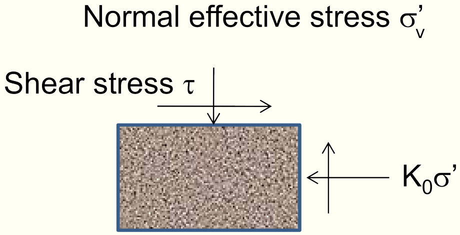
3D general (axi-symmetric as a special case)

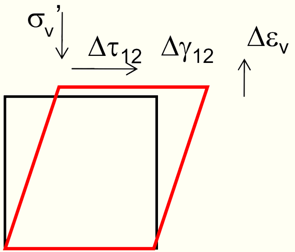
- Stresses: p and q
- Strains: ε_v and ε_s



1D simple shear

1. No lateral strain
2. Constant normal effective stress σ'_v
3. Increasing shear strain γ
4. Measure shear resistance τ
5. Measure volumetric strain ε_v or void ratio $e = e_0 - (1 + e_0)\varepsilon_v$
6. **No information for the lateral direction**





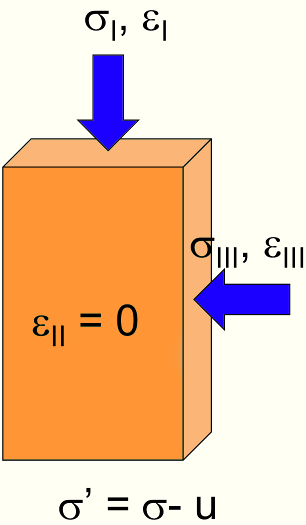
2D plane strain / Mohr-Coulomb model

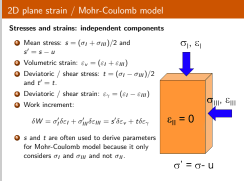
Stresses and strains: independent components

1. Mean stress: $s = (\sigma_I + \sigma_{III})/2$ and $s' = s - u$
2. Volumetric strain: $\varepsilon_v = (\varepsilon_I + \varepsilon_{III})$
3. Deviatoric / shear stress: $t = (\sigma_I - \sigma_{III})/2$ and $t' = t$.
4. Deviatoric / shear strain: $\varepsilon_\gamma = (\varepsilon_I - \varepsilon_{III})$
5. Work increment:

$$\delta W = \sigma'_I \delta\varepsilon_I + \sigma'_{III}\delta\varepsilon_{III} = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$$

6. s and t are often used to derive parameters for Mohr-Coulomb model because it only considers σ_I and σ_{III} and not σ_{II}.

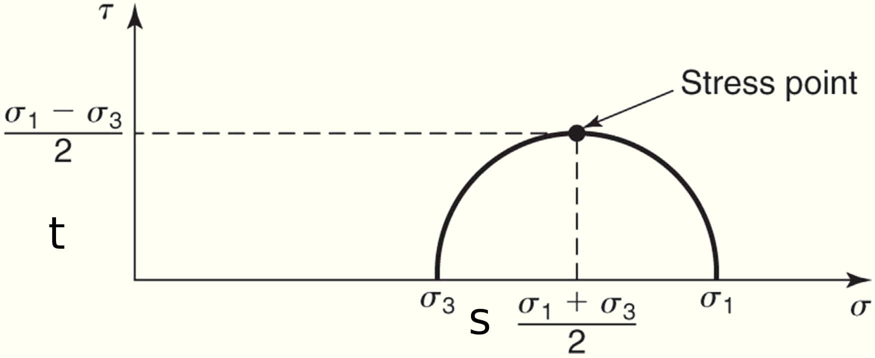




Principal stresses $\sigma_I > \sigma_{II} > \sigma_{III}$. This equation holds good and is the definition of σ in principal stress notations, i.e., $I > II > III$.

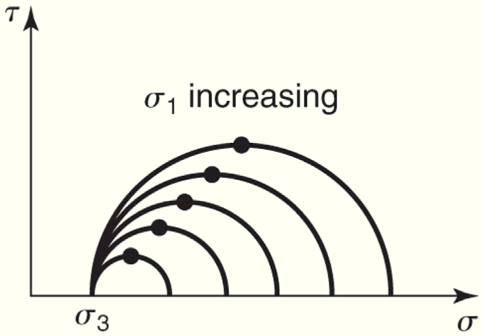
No shear stresses on these planes.

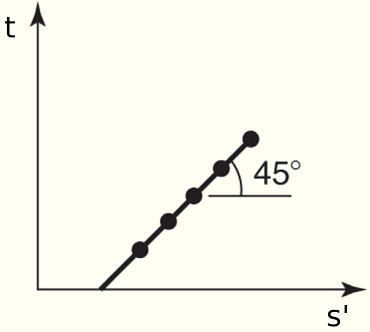
2D Mohr circle



Mohr circle to 2D stress components

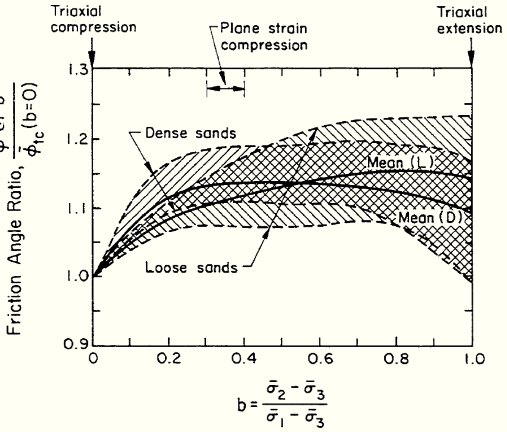
Successive Mohr circles and stress path for constant σ_3 and increasing σ_1:





Effect of σ_{II}

Bishop (1966) defined **b-value**: $b = (\sigma_{II} - \sigma_{III})/(\sigma_I - \sigma_{III})$, where $\sigma_I > \sigma_{II} > \sigma_{III}$. Triaxial compression $b = 0$, triaxial extension $b = 1$. Typically $\phi'_{ps} = (1.05 - 1.15)\phi'_{tx}$.



Effect of σ_{II} on friction angle (Kulhawy and Mayne, 1990)

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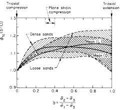

Effect of σ_{II} on friction angle (Kulhawy and Mayne, 1990)

σ_{II} do have an effect on soil behavior. For example, the friction angle depends on the loading condition: triaxial compression, plane-strain, triaxial extension and others ...., The effect of σ_{II} can be measured using true triaxial apparatus or hollow cylinder torsional shear apparatus. In general, the peak friction angle increases 10 to 15 percent from b = 0 (triaxial compression) to b = 0.3 to 0.4 (plane strain), and it stays constant or slightly decreases as b reaches 1 (tri-axial extension).

The variation of measured friction angle with changes in σ_{II} can be attributed to the effects of different mean stress and stress anisotropy on the dilatancy and particle rearrangement contributions to the total strength. For given maximum and minimum principal stresses, the TXE conditions have the largest mean effective stress, whereas the triaxial compression conditions have the smallest mean effective stress. The higher confinement for triaxial extension and plane strain conditions contributes to the increasing friction angle for these conditions. (Mitchell and Soga., 2005)

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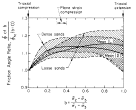
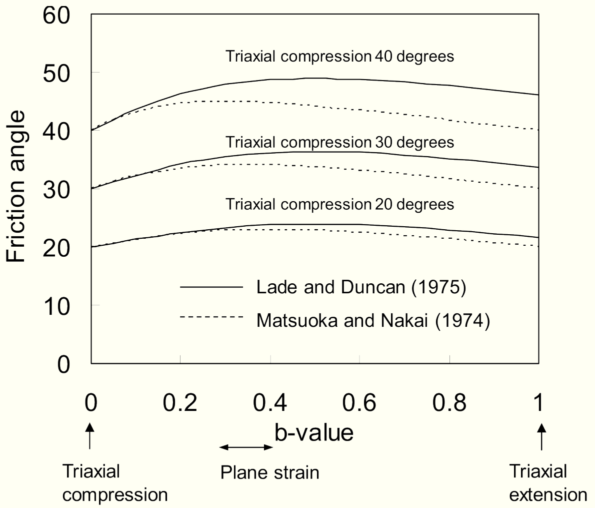


Effect of σ_{II} on friction angle (Kulhawy and Mayne, 1990)

In TXC major principal stress, σ_1 acts vertically, compared to triaxial extension mode of loading, where σ_1 acts horizontally. The effect of the intermediate principal stress is comparatively smaller, and is often assessed using the parameter $b = (\sigma_{II} - \sigma_{III})/(\sigma_I - \sigma_{III})$, which is zero in triaxial compression, and one in triaxial extension. In TXE $\sigma_I = \sigma_{II}$ (radial), while σ_{III} is the vertical stress.

Effect of σ_{II} on friction



Chapter 11., Mitchell and Soga, 2005

Effect of σ_{II} on friction

Various models fit the experimental data showing the intermediate stress effect. Lade and Duncan (1975)

$$\frac{I_1^3}{I_3} = const$$

Matsuoka and Nakai (1974)

$$\frac{I_1 I_2}{I_3} = const$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$
$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$
$$I_3 = \sigma_1\sigma_2\sigma_3$$

Matsuoka and Nakai's model gives the same friction angle for compression and extension, whereas Lade and Duncan's model gives the ratio of the TXE friction angle (ϕ'_{TXE}) to the TXC friction angle (ϕ'_{TXC}) to be 1.08 at $\phi'_{TXC} = 20^o$ to 1.15 at $\phi'_{TXC} = 40^o$.

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$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$
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Given the large scatter in the published experimental data, it is not possible to conclude that one model is better than the ohter.

Triaxial stresses and strains: independent components

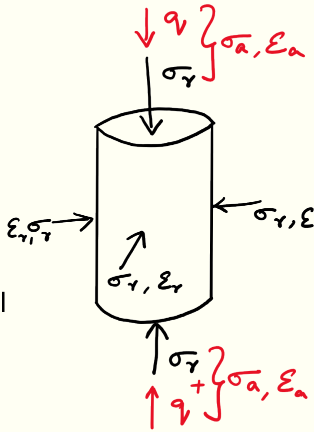
- We split the stress system into:
  - purely volumetric deformation 'p'
  - purely distortional deformation 'q'
- Mean stress:

$$p = (\sigma_a + 2\sigma_r)/3 \quad p' = p - u$$

- Volumetric strain: $\varepsilon_v = (\varepsilon_a + 2\varepsilon_r)$
- Deviatoric / shear stress (purely distortional deformation):

$$q = (\sigma_a - \sigma_r) \quad q' = q$$

- Deviatoric / shear strain: $\varepsilon_s = 2/3(\varepsilon_a - \varepsilon_r)$



Triaxial deviatoric strain

- Deviatoric / shear strain: $\varepsilon_s = 2/3(\varepsilon_a - \varepsilon_r)$
- Why 2/3?:
- Work equation: $\delta W = p'\delta\varepsilon_v + q'\delta\varepsilon_s$

$$
\begin{aligned}
\delta W &= \sigma'_1\delta\varepsilon_1 + \sigma'_2\delta\varepsilon_2 + \sigma'_3\delta\varepsilon_3 = \sigma'_a\delta\varepsilon_a + 2\sigma'_r\delta\varepsilon_r \\
\delta W &= (1/3\sigma'_a + 2/3\sigma'_r)(\delta\varepsilon_a + 2\delta\varepsilon_r) + (\sigma'_a - \sigma'_r)(2/3)(\delta\varepsilon_a - \delta\varepsilon_r) \\
&= \sigma'_a\delta\varepsilon_a + 2\sigma'_r\delta\varepsilon_r
\end{aligned}
$$

Triaxial stress - strain relationship

Relationship in terms of axial and radial direction:

$$\begin{bmatrix} \varepsilon_a \\ \varepsilon_r \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -2\nu \\ -\nu & (1-\nu) \end{bmatrix} \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \tag{1}$$

Relationship in terms of 'p' and 'q':

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \quad \text{invert} \rightarrow \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \begin{bmatrix} 1 & 2/3 \\ 1 & -1/3 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \tag{2}$$

Strain:

$$\begin{bmatrix} \varepsilon_p \\ \varepsilon_q \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2/3 & -2/3 \end{bmatrix} \begin{bmatrix} \varepsilon_a \\ \varepsilon_r \end{bmatrix} \tag{3}$$

Volume - shear coupling

Equation (1) and eq. (2) in eq. (3) to relate $\varepsilon_p, \varepsilon_q$ to p, q:

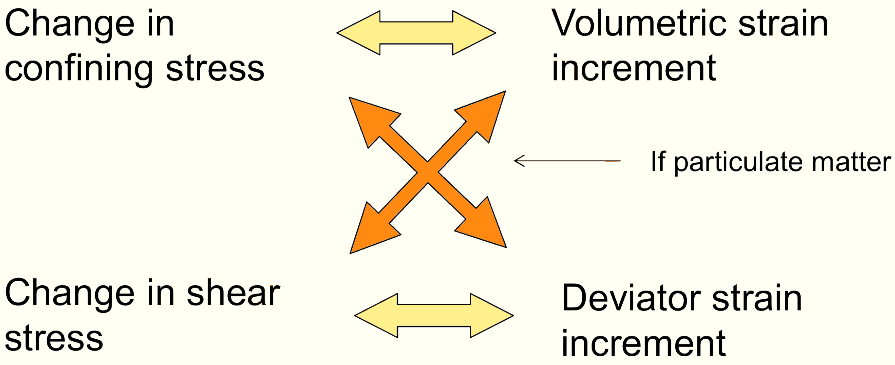
$$\begin{bmatrix} \varepsilon_p \\ \varepsilon_q \end{bmatrix} = \begin{bmatrix} 3(1-2\nu)/E & 0 \\ 0 & 2(1+\nu)/3E \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_p \\ \varepsilon_q \end{bmatrix} = \begin{bmatrix} 1/K & 0 \\ 0 & 1/3G \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}$$

1. Off-diagonal zeros indicate no coupling between volumetric and distortional effects.
2. Applying pure shear cannot induce volume changes - **not true for soils!**
3. Plasticity theory will deal with this.

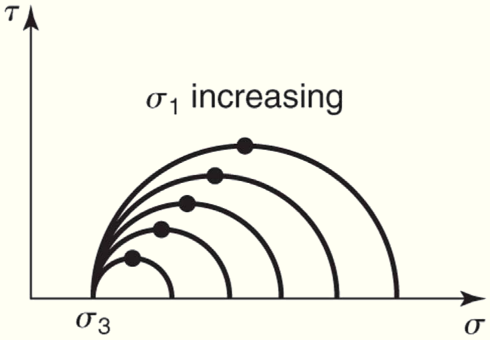
Volume shear coupling

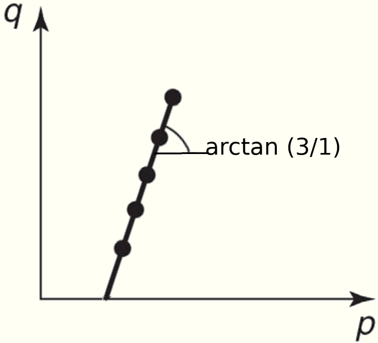
Usually volume change and shear deformation are decoupled (i.e. bulk modulus and shear modulus), but in soils they are coupled:



Mohr circle to 3D stress components

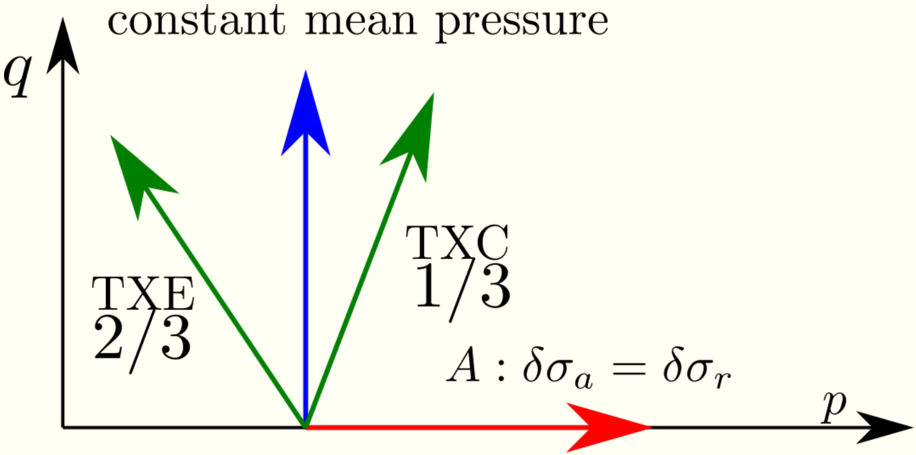
Successive Mohr circles and stress path for constant σ_3 and increasing σ_1:





Stress paths p-q

Different stress paths for initially hydrostatic stress conditions:



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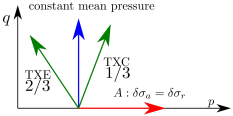


A mean total stress $p = \frac{\sigma_a + 2\sigma_r}{3}$ and deviatoric stress $q = \sigma_a - \sigma_r$. In triaxial compression the cell pressure is held constant, $\delta\sigma_r = 0$ and axial load is increased.

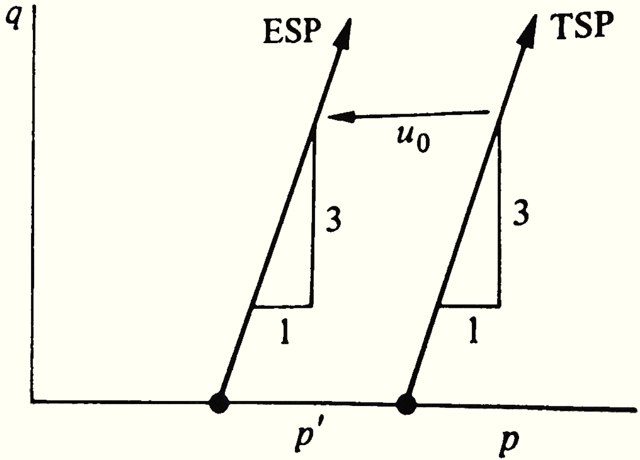
$$\delta p = \frac{\delta q}{3}$$

In triaxial extension, to keep the axial stress constant while changing the cell pressure requires that the ram force (or deviatoric stress) and cell pressure must be changed simultaneously. For $\delta\sigma_a = 0, \delta\sigma_r = -\delta q$. The the differential form is:

$$\delta p = \frac{2\delta\sigma_r}{3} = \frac{-2\delta q}{3}$$

Triaxial compression

TXC with constant back pressure u_o (TSP: Total stress path; ESP: Effective stress path)



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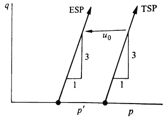
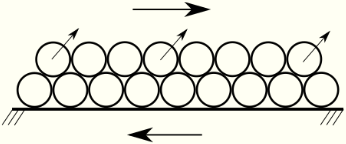
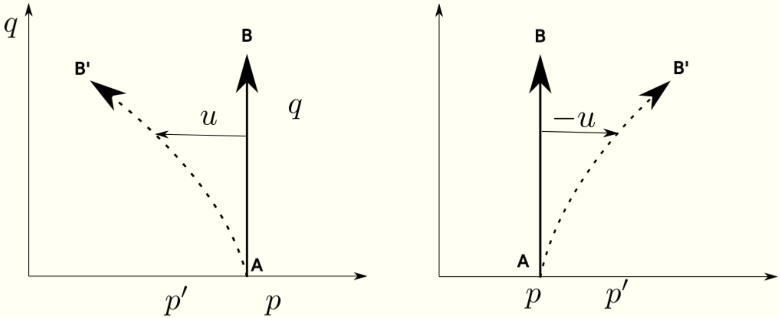


If drainage can occur freely from the soil sample to atmospheric pressure, then the pore pressure will be zero, and total and effective stresses will be identical. The measurement of volume changes of triaxial samples require the measurement of volume of pore fluid flowing into or out of the sample. If part of this pore fluid is emerging as gas (air), then part of the voume change of the sample will not be observed as a change in the level of liquid in a burette. It is often desirable to ensure that the pore fluid is indeed saturated by subjecting the whole pore fluid system to a pressure, called a *back pressure*, which is held constant at a value of u_0 during the test in order to ensure that the gaseous phase remains in solution. Drainage can occur freely, but against this back pressure. In such a test, the total and effective stress paths will be separated by a constant distance u_0.

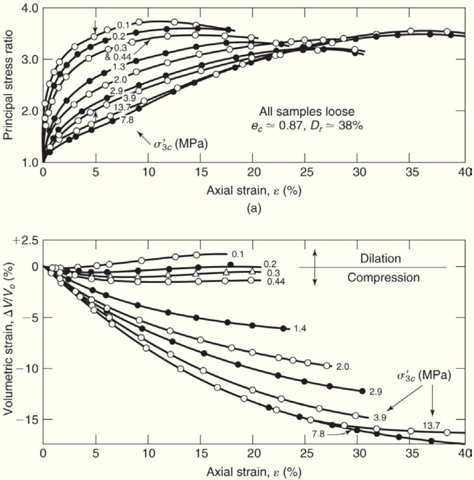
Triaxial compression undrained: loose v dense





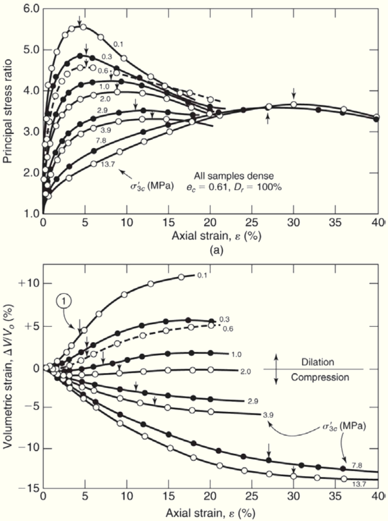
Total and effective stress paths for undrained triaxial test: (a) on soil that wishes to contract as it is sheared, and (b) on soil that wishes to expand as it is sheared.

Triaxial compression drained: loose



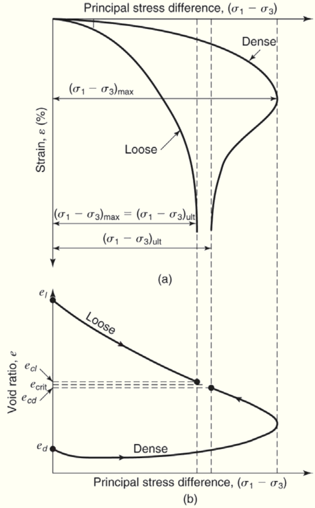
Loose Sacramento River sand: (a) principal stress ratio versus axial strain; (b) volumetric strain versus axial strain (Lee, 1965).

Triaxial compression drained: dense



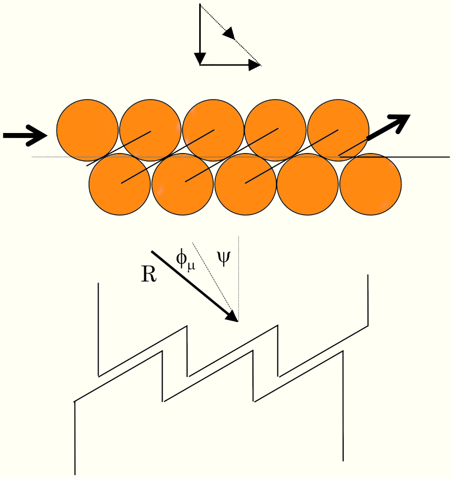
Dense Sacramento River sand: (a) principal stress ratio versus axial strain; (b) volumetric strain versus axial strain (Lee, 1965).

Triaxial compression drained: loose v dense



Triaxial tests on "loose" and "dense" specimens of a typical sand: (a) stress-strain curves; (b) void ratio changes during shear (Hirschfeld, 1963).

Friction: Is this correct?



$$\phi_{ss} = \phi_\mu + \psi_{ss}$$

Discrete Element Method

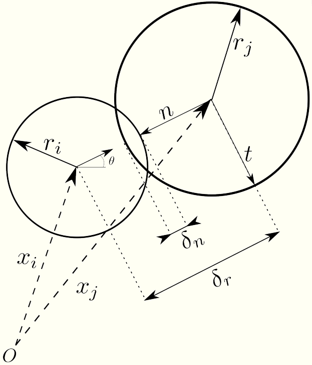
1. Particle level interaction based on Newton's equation of motion
2. The contact normal force is computed as:

$$F_n = \begin{cases} 0, & \delta_n > 0 \\ -k_n \delta_n - \gamma_n \frac{d\delta_n}{dt}, & \delta_n < 0 \end{cases}$$

3. The contact tangential force is computed in a similar way, but has a frictional limit.

$$F_t \leq \mu F_n$$

4. Solve Newton's second law and the angular momentum equation (including rotational resistance).

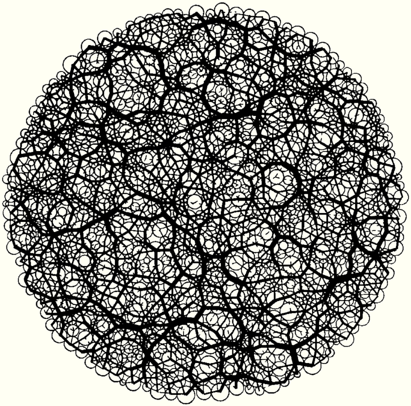


Interparticle friction angles

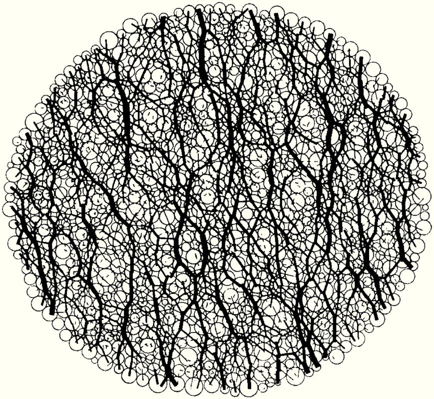
- For Quartz Sands: 26 degrees
- For Sheet Minerals (muscovite, phlogopite, biotite and chlorite): 7 - 13 degrees
  - Water acts as a lubricant
- Clay minerals: Probably 7 - 13 degrees
  - Similar to reported residual friction angles.
  - Sodium Montmorillonite: 4 degrees

Strong force network vs weak clusters

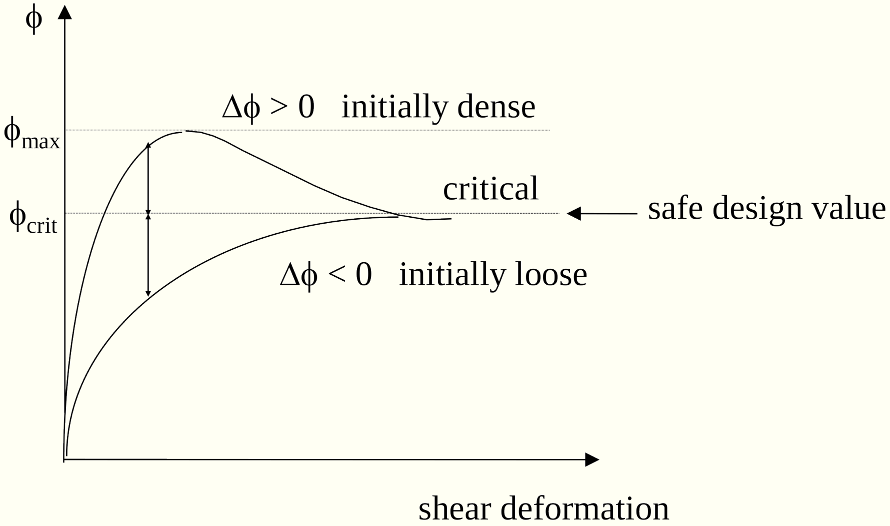
Isotropic Loading



Biaxial Loading



Macroscopically, as soil aggregates...

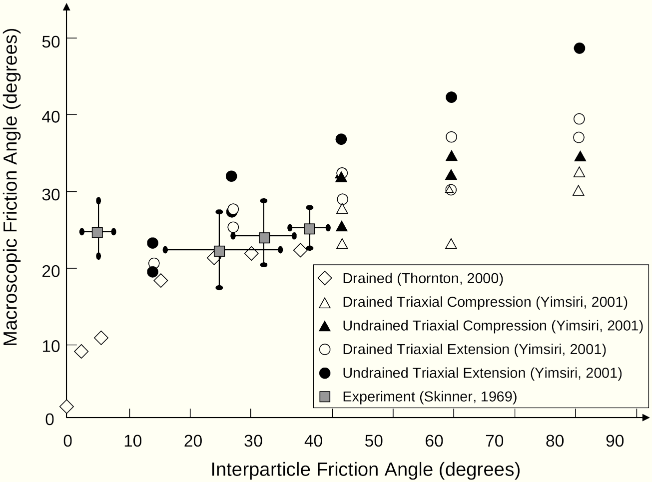


$$\phi = \phi_{crit} + \Delta\phi_{dilatancy}$$

Macroscopic friction angle

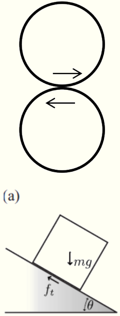
- ϕ_{crit} is the angle of friction measured at constant volume of a soil aggregate, and $\Delta\phi$ dilatancy is the extra dilatant contribution to friction angle ϕ. Typical values are:
- Critical state friction ϕ_{crit}:
  - clay: 22°
  - uniform rounded sand: 32°
  - well-graded angular sandy gravel: 38°
- peak strength of pre-compressed or uncrushable grains, densely compacted, and: shearing in plane strain $\Delta\phi$:
  - shearing in plane strain: $\Delta\phi_{max} = 20^\circ$
  - shearing in axial symmetry: $\Delta\phi_{max} = 12^\circ$

Micro to Macroscopic friction angle

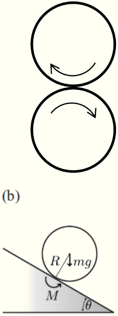


Relationship between macroscopic friction angle and interparticle friction angle (no rolling resistance) - Yimisir and Soga (2001)

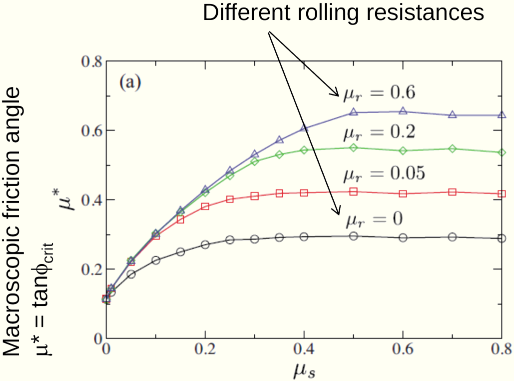
Micro to Macroscopic friction angle: Rolling resistance



Sliding friction μ_s



Rolling resistance μ_r

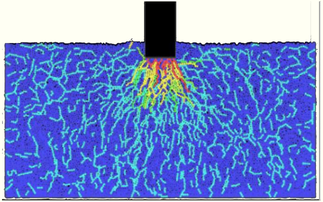


Microscopic sliding friction at particle contacts $\mu_s = \tan\phi_\mu$

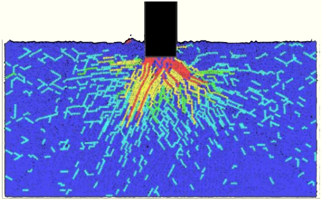
Estrada et al., (2001)

Interparticle friction angles

- The interparticle friction acts as a kinematic constraint of the strong force network and not as the direct source of macroscopic resistance to shear.
- Increased friction at the contacts increases the stability of the system (development of anisotropic fabric) and reduces the number of contacts required to achieve a stable condition.
- As long as the strong force network can be formed, the magnitude of the interparticle friction becomes of secondary importance.



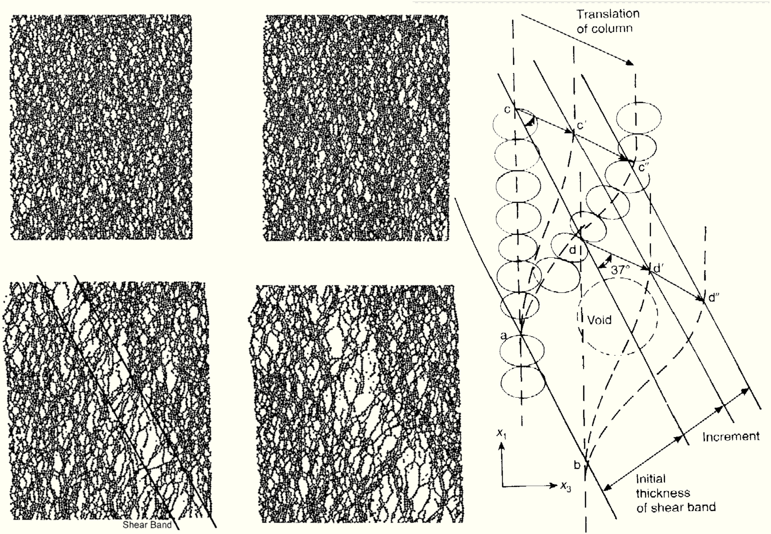
Loose



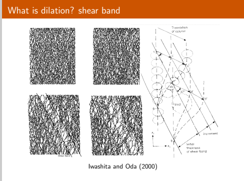
Dense

Muthuswamy and Tordesillas (2006)

What is dilation? shear band



Iwashita and Oda (2000)



Kuhn (1999) reports that their thicknesses are $1.5D_{50}$ to $2.5D_{50}$ in the early stages of shearing and increase to between $1.5D_{50}$ and $4D_{50}$ as deformation proceeds.

Fabric evolution at critical state

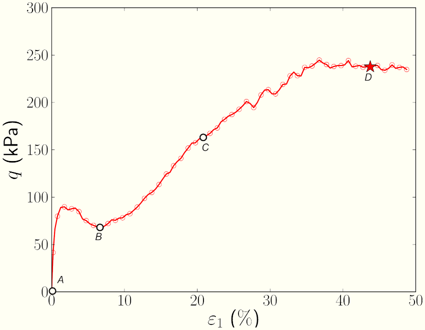


Fig.1 Undrained shear response of a medium dense sand and four stress states selected for examination of internal structure.

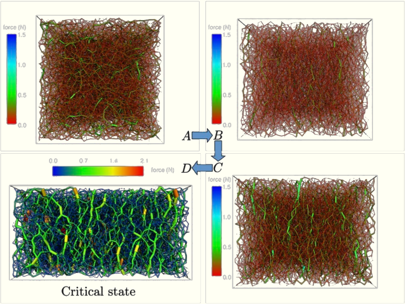
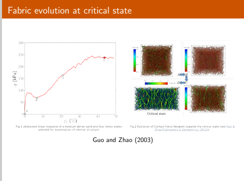


Fig.2 Evolution of Contact Force Network towards the critical state (see Guo & Zhao [Computers & Geotechnics, 2013]).

Guo and Zhao (2003)



As deformation progresses,

- The number of particles in the strong force network decreases.
- With fewer particles sharing the increased loads.
- Anisotropic fabric develops, showing the formation of strong force network. Fabric of particles associated with strong forces is different from that associated with weak clusters.
- At critical state, force chain forms and buckles continuously. Likely to buckle when a force chain has 8 particles

Stress and strain invariants in 3D

6 stresses and strains

- Mean pressure: $p' = (\sigma'_{11} + \sigma'_{22} + \sigma'_{33})/3$
- Deviator stress: $q = \sqrt{3/2}\sqrt{s_{11}^2 + s_{22}^2 + s_{33}^2 + 2\tau_{12}^2 + 2\tau_{23}^2 + 2\tau_{31}^2}$
- where $s_{11} = \sigma'_{11} - p', s_{22} = \sigma'_{22} - p', s_{33} = \sigma'_{33} - p'$
- Volumetric strain: $\varepsilon_v = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$
- Deviatoric strain: $\varepsilon_s = \sqrt{2/3}\sqrt{e_{11}^2 + e_{22}^2 + e_{33}^2 + \gamma_{12}^2/2 + \gamma_{23}^2/2 + \gamma_{31}^2/2}$
- where $e_{11} = \varepsilon_{11} - \varepsilon_v/3, e_{22} = \varepsilon_{22} - \varepsilon_v/3, e_{33} = \varepsilon_{33} - \varepsilon_v/3$

Stress and strain invariants in 3D

3 Principal stresses and strains

- Mean pressure: $p' = (\sigma'_I + \sigma'_{II} + \sigma'_{III})/3$

- Deviator stress: $q = \sqrt{1/2}\sqrt{(\sigma'_I - \sigma'_{II})^2 + (\sigma'_{II} - \sigma'_{III})^2 + (\sigma'_{III} - \sigma'_I)^2}$

- Volumetric strain: $\varepsilon_v = \varepsilon_I + \varepsilon_{II} + \varepsilon_{III}$

- Deviatoric strain: $\varepsilon_s = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon'_I - \varepsilon'_{II})^2 + (\varepsilon'_{II} - \varepsilon'_{III})^2 + (\varepsilon'_{III} - \varepsilon'_I)^2}$

In triaxial condition (principal stresses/strains)

$$p' = (\sigma_I + 2\sigma_{III})/3 \quad q = \sigma_I - \sigma_{III} \quad \varepsilon_v = \varepsilon_I + 2\varepsilon_{III} \quad \varepsilon_s = 2(\varepsilon_I - \varepsilon_{III})/3$$

Stress and strain invariants in 3D

3 Principal stresses and strains

- Mean pressure: $p' = (\sigma'_I + \sigma'_{II} + \sigma'_{III})/3$
- Deviator stress: $q = \sqrt{1/2}\sqrt{(\sigma'_I - \sigma'_{II})^2 + (\sigma'_{II} - \sigma'_{III})^2 + (\sigma'_{III} - \sigma'_I)^2}$
- Volumetric strain: $\varepsilon_v = \varepsilon_I + \varepsilon_{II} + \varepsilon_{III}$
- Deviatoric strain: $\varepsilon_s = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon'_I - \varepsilon'_{II})^2 + (\varepsilon'_{II} - \varepsilon'_{III})^2 + (\varepsilon'_{III} - \varepsilon'_I)^2}$

In triaxial condition (principal stresses/strains)

$p' = (\sigma_I + 2\sigma_{III})/3 \quad q = \sigma_I - \sigma_{III} \quad \varepsilon_v = \varepsilon_I + 2\varepsilon_{III} \quad \varepsilon_s = 2(\varepsilon_I - \varepsilon_{III})/3$

The magnitudes of the components of the stress vector (i.e. $\sigma_{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{xz}, \tau_{yz}}$) depend on the chosen direction of the coordinate axes. The principal stresses $(\sigma_I, \sigma_{II}, and \sigma_{III})$ however, always act on the same planes and have the same magnitude, no matter which direction is chosen for the coordinate axes. They are therefore invariant to the choice of axes. Consequently, the state of stress can be fully defined by either specifying the six component values for a fixed direction of the coordinate axis, or by specifying the magnitude of the principal stresses and the directions of the three planes on which these principal stresses act. In either case six independent pieces of information are required.

The missing stress invariant

- **General stresses:** $\sigma_{ij}(i,j=1,2,3) \quad \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}$

$$\Updownarrow$$

- **Principal stresses:** $\sigma_I, \sigma_{II}, \sigma_{III} + 3 angles$

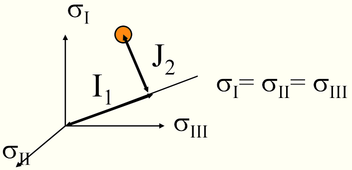
$$\Downarrow \quad \not\Uparrow$$

Two stress invariant (mean pressure p or s, deviator stress q or t). One more stress invariant is needed to go back to 3 principal stresses: **Lode angle!**

Stress invariant

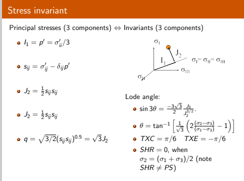
Principal stresses (3 components) $\Leftrightarrow$ Invariants (3 components)

- $I_1 = p' = \sigma'_{ii}/3$
- $s_{ij} = \sigma'_{ij} - \delta_{ij} p'$
- $J_2 = \frac{1}{2} s_{ij} s_{ij}$
- $J_2 = \frac{1}{2} s_{ij} s_{ij}$
- $q = \sqrt{3/2}(s_{ij} s_{ij})^{0.5} = \sqrt{3} J_2$



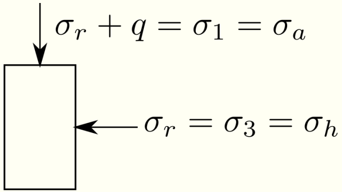
Lode angle:

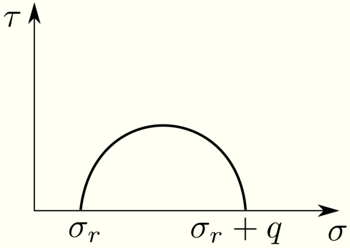
- $\sin 3\theta = \frac{-3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}}$.
- $\theta = \tan^{-1}\left[\frac{1}{\sqrt{3}}\left(2\frac{(\sigma_2 - \sigma_3)}{(\sigma_1 - \sigma_3)} - 1\right)\right]$
- $TXC = \pi/6 \quad TXE = -\pi/6$
- $SHR = 0$, when $\sigma_2 = (\sigma_1 + \sigma_3)/2$ (note $SHR \neq PS$)

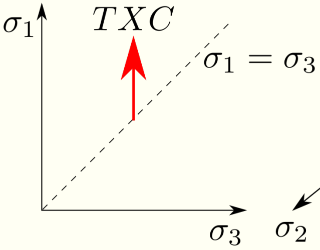


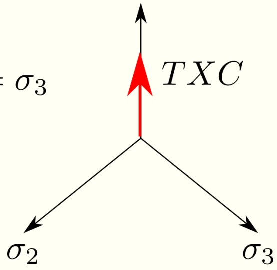
If one is interested in the overall magnitude of the stress, all three principal stresses would be needed, but not the directions of the planes on which they act. In geotechnical engineering it is often convenient to work with alternative invariant quantities which are combinations of the principal effective stresses.

π plane: Triaxial compression

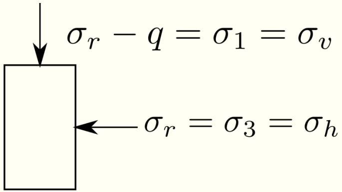


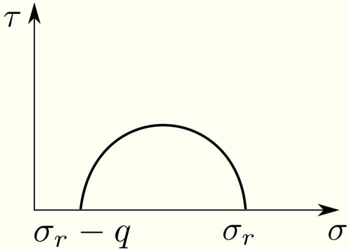


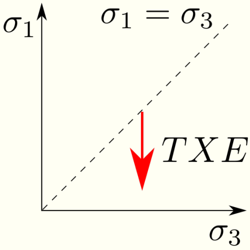


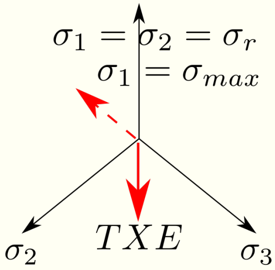


π plane: Triaxial extension



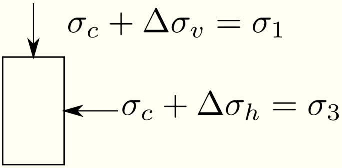


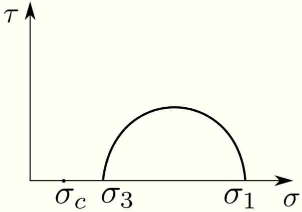


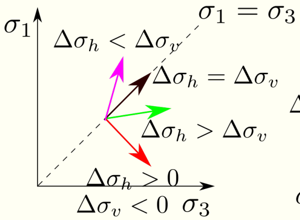


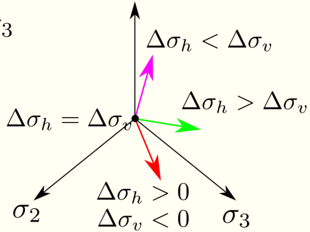
w/o forcing $\sigma_1 = \sigma_{max}$

π plane: Random stress paths

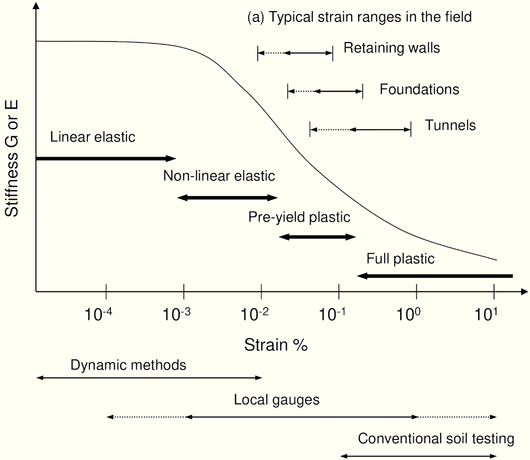


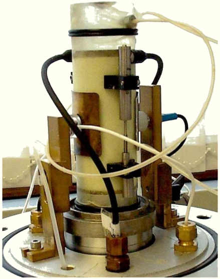




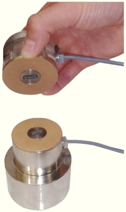


Stiffness: small to large strains



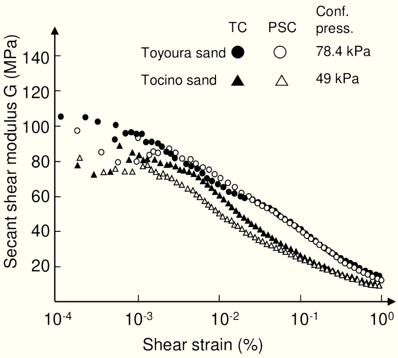


Local gauges

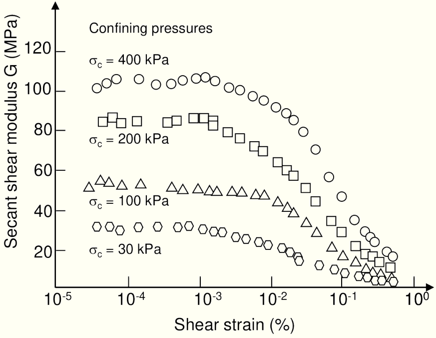


Bender element (GDS)

Stiffness: small to large strains



(a) Toyoura and Tocino sands

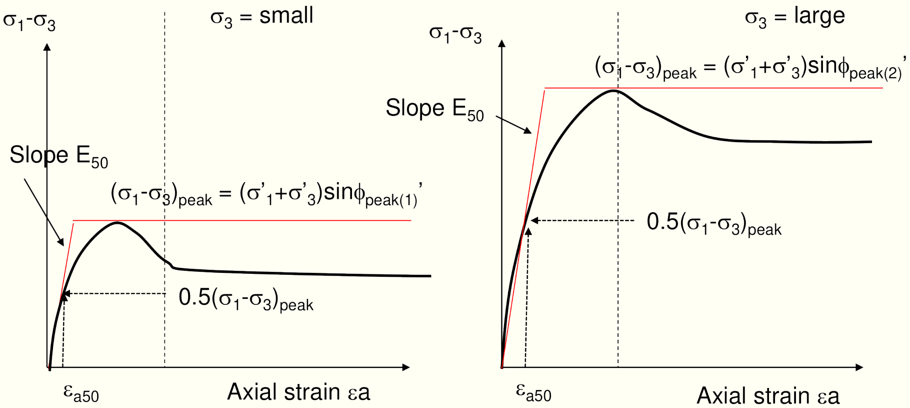


(b) Kaolin clay

E or G is a function of:

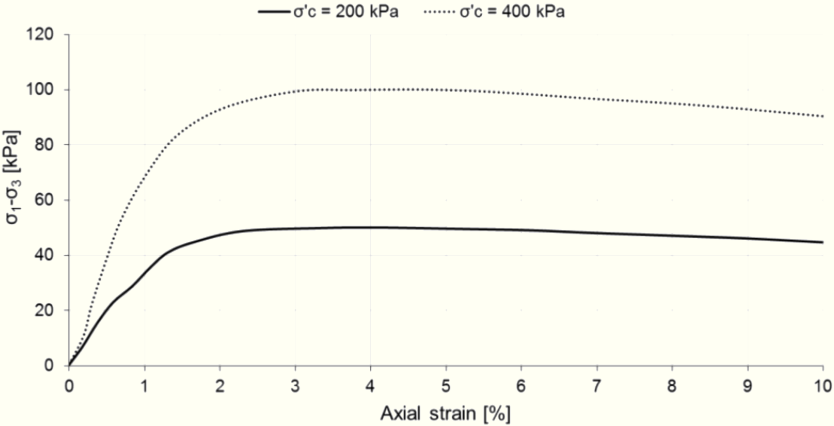
1. confining stress and void ratio (relative density)
2. strain level

Stiffness at intermediate strains



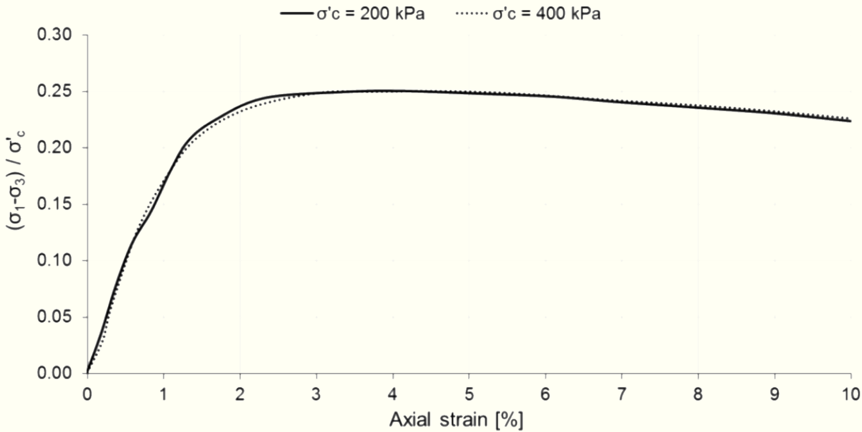
$E_{50} = E$ at 50% of $()\sigma_1 - \sigma_3)_{peak}$ Note: $\phi'_{peak1} < \phi'_{peak2}$

Undrained strength



Triaxial compression test data of homogeneous clay (Ladd & Foott, 1974)

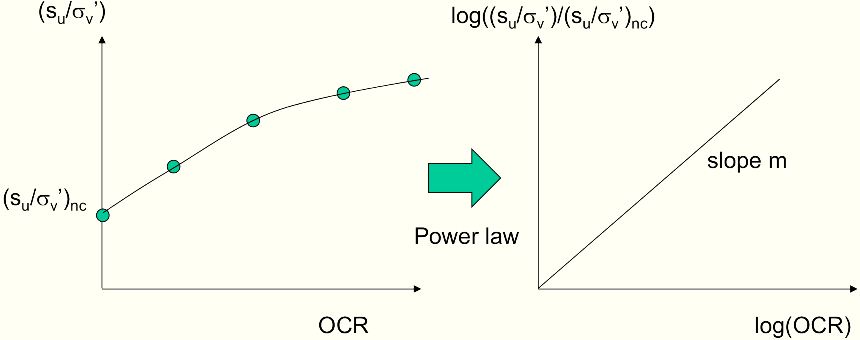
Normalized undrained strength

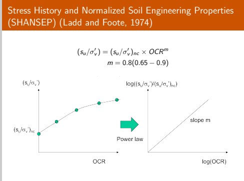


Normalised triaxial compression test data of homogeneous clay (Ladd & Foott, 1974)

Stress History and Normalized Soil Engineering Properties (SHANSEP) (Ladd and Foote, 1974)

$$(s_u/\sigma'_v) = (s_u/\sigma'_v)_{nc} \times OCR^m$$
$$m = 0.8(0.65 - 0.9)$$

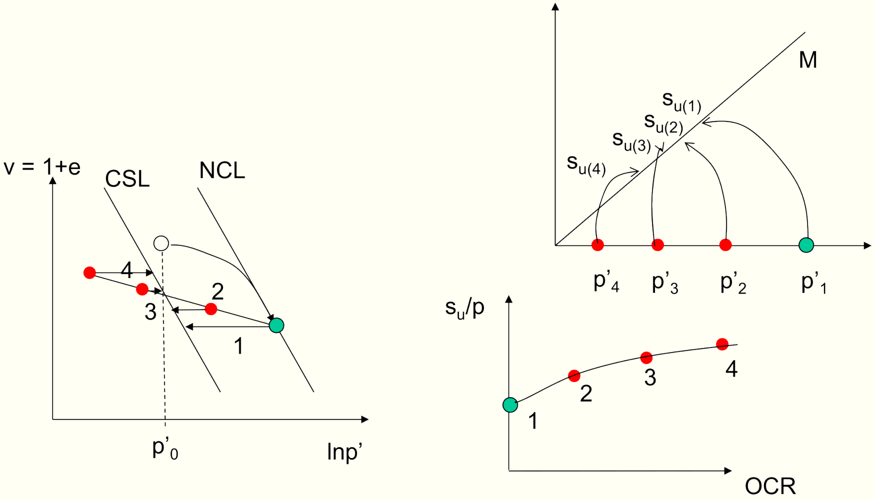


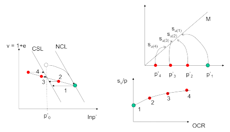


Laboratory tests conducted at the Imperial College using remolded clays (Henkel (1960) and Parry (1960)) and at the Massachusetts Institute of Technology on a wide range of clays, give evidence that clay samples with the same over-consolidation ratio (OCR), but different consolidation stress σ'_c and therefore different pre-consolidation stress σ'_p, exhibit very similar strength and stress-strain characteristics when the results are normalized over the consolidation stress σ'_c.

In practice, normalized behaviour is not as perfect as shown, there is discrepancy in the normalized plots caused by different consolidation stresses, soil deposit heterogeneity or even the fact that the conditions from one soil test to another are not identical. However, this discrepancy is reported to be quite small.

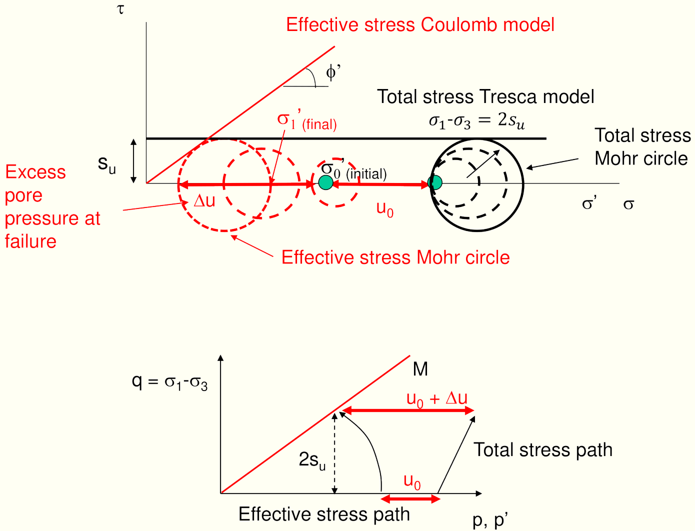
SHANSEP procedure





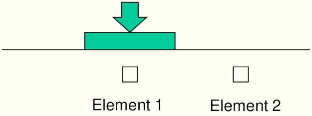
1. obtain a sample of “clayey” soil
2. Reconsolidate to 1.5 to 2.0 of insitu effective mean pressure p'_0 , or reconsolidate back to the virgin normal compression line
3. Rebound to desired OCR
4. Perform undrained shearing and find s_u .
5. Perform multiple tests at different OCRs.
6. Evaluate $(s_u/\sigma'_v) = (s_u/\sigma'_v)_{nc} \times OCR^m$

TSP v ESP footing: Tresca failure

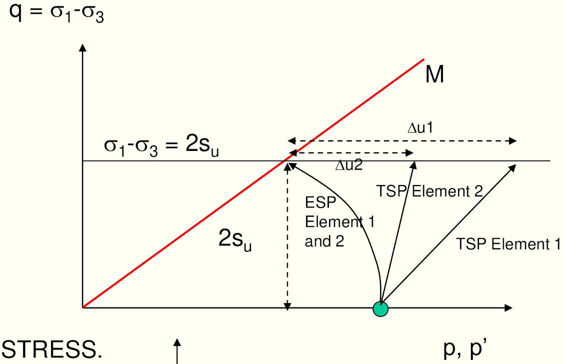


Tresca criteria: $\sigma_1 - \sigma_3 = 2s_u$

TSP v ESP footing: Tresca failure



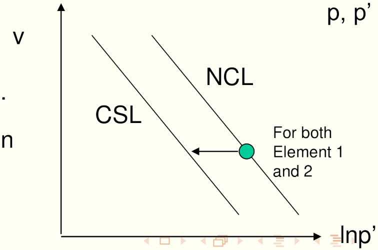
Let's assume K_0 =1
But the same argument can be made for $K_0 \neq 1$

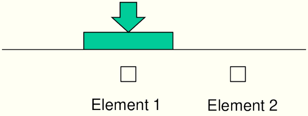


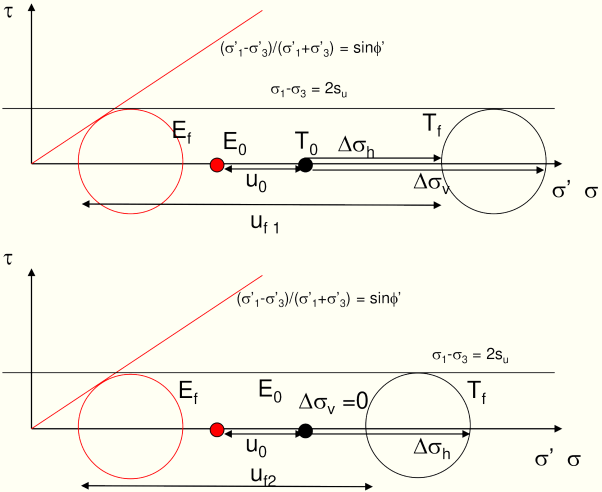
Stress equilibrium is in TOTAL STRESS.

1. Total stress – Both elements fail when σ_1-σ_3 = $2s_u$. Use this for stress equilibrium.

2. Effective stress – Both element fail when q=Mp' but excess pore pressures need to be assigned separately for each element for stress equilibrium.







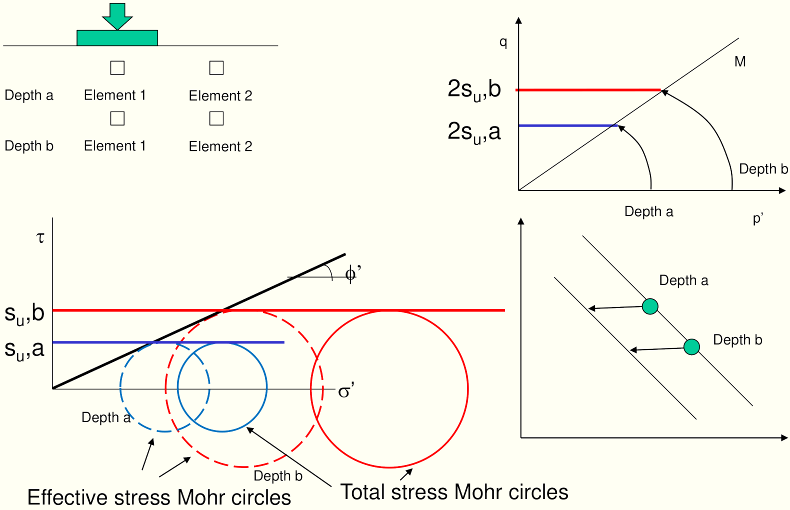
Both elements use the same total stress or effective stress failure criterion.

But for effective stress analysis, excess pore pressure ($u_{f1} \neq u_{f2}$) need to be computed separately.

No need to do this for the total stress analysis.

16

TSP v ESP footing: Tresca failure



However, s_u needs to be assigned at different depths even for the same soil.

TSP v ESP footing: Tresca failure

- Better! Conduct Total Stress Analysis: $s_u = \sigma_1 - \sigma_3$
  - But s_u needs to be defined at different locations even if it is the same soil.
  - We can do quick undrained (UU) tests to obtain spatial variation of s_u. Hence, practical.
  - No information on pore pressure, so cannot assess the long-term consolidation deformation behavior.
- Effective analysis $= (\sigma_1' - \sigma_3')/(\sigma_1' + \sigma_3') = \sin \phi'$:
  - If it is the same soil, can use the same soil properties (such as ϕ') and need to conduct CU tests.
  - But need to compute the excess pore pressure at different locations.
  - This is difficult: only a good effective stress constitutive model that can predict the excess pore pressure correctly can do this.
  - If pore pressure profile can be computed, then it can be used to evaluate the subsequent consolidation process.