CE394M Advanced Analysis in Geotechnical Engineering: Introduction

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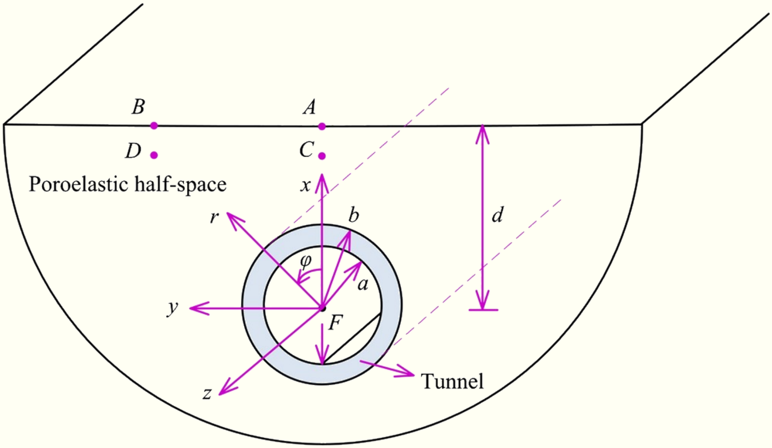
Overview

1. Geotechnical modeling
   - Complexity in Geotechnical modeling
   - Oso landslide
2. Geotechnical analysis
3. Governing equations in stress-deformation analysis
   - Stress equilibrium
   - Compatibility condition
   - Stress-strain relationship
4. Limit analysis

Is this model correct?



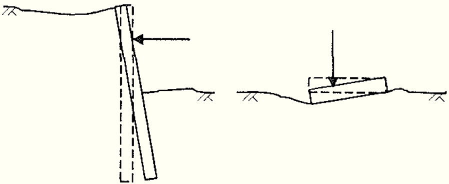
Geotechnical modeling of the complex world

London Bridge Station, London, UK

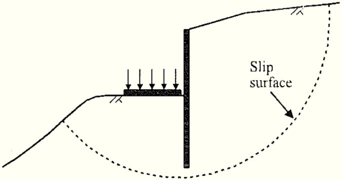
Geotechnical modeling of the complex world

Fig. London Victoria station upgrade, London, UK

Local vs global stability

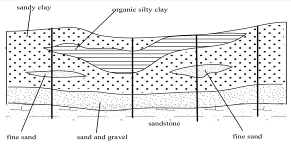


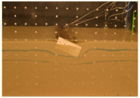
Local stability

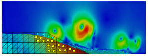


Overall stability

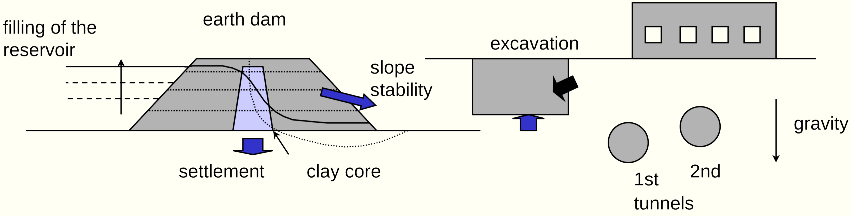
Geotechnical modeling



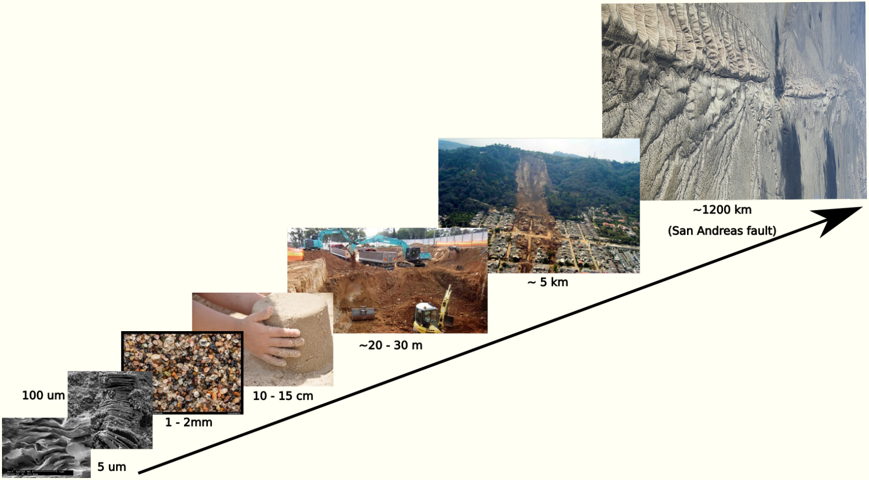




Geotechnical modeling: What should be modeled?



Scales of modeling in geotechnical engineering

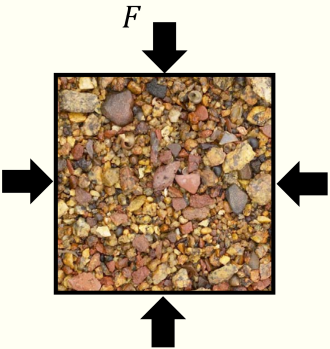


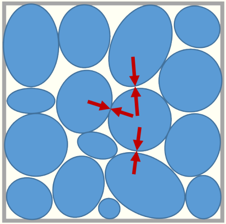
Soil behavior

- nonhomogeneous,
- anisotropic,
- non-linear,
- initial stress conditions,
- stress history
- Geometry - very complex

Soil Mechanics in practice - largely empirical

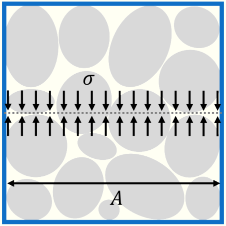
Numerical frameworks





Discrete mechanics

- Micro-scale
- Grain level description
- Intergranular forces



Continuum mechanics

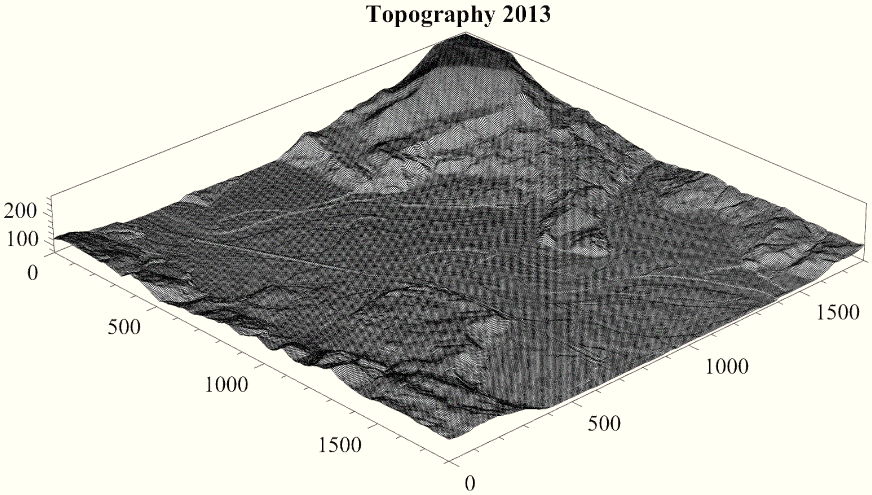
- Macro-scale
- Stress definition

$$\sigma = \frac{F}{A}$$

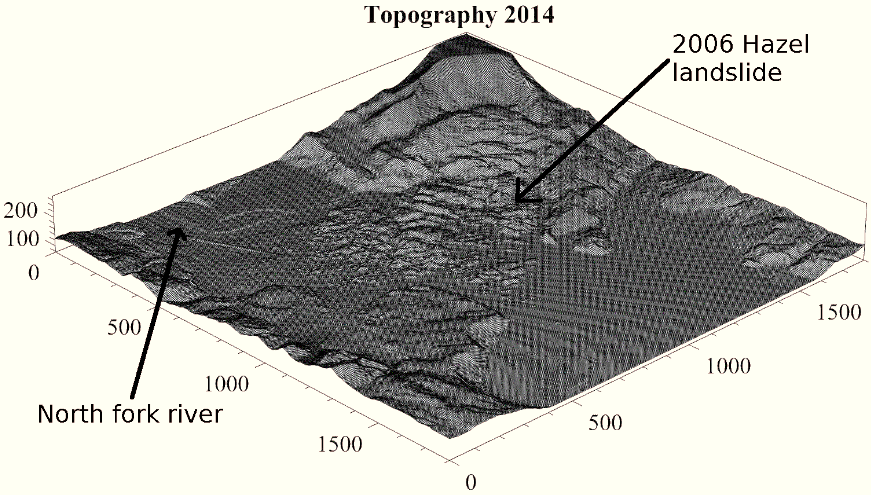
Oso landslide: case study

Oso landslide: case study

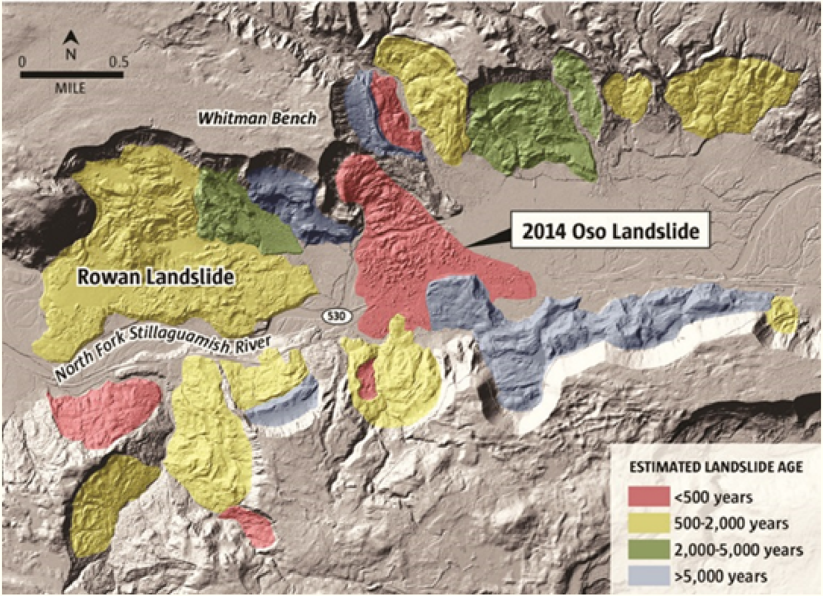
Oso landslide: topography



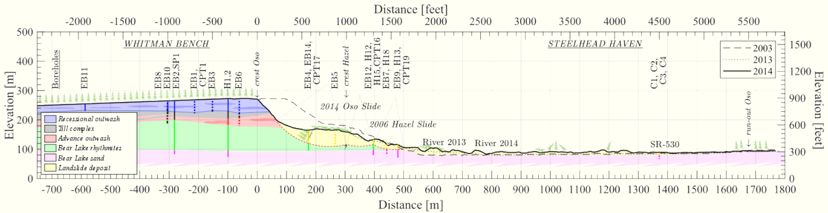
Oso landslide: topography

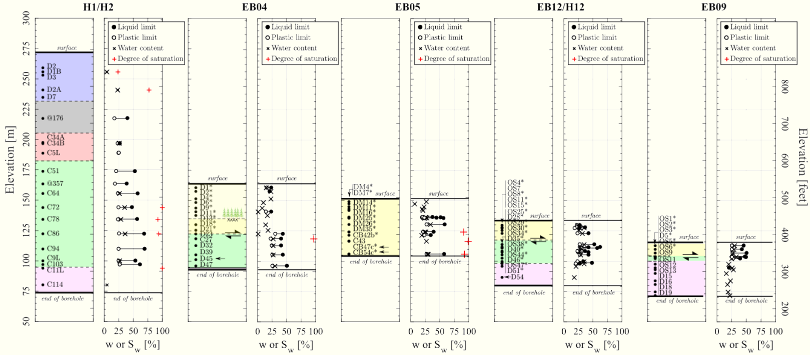


Oso landslide: historic slides



Oso landslide: Soil profile





Oso landslide: Geology. Identify the failure surface

Lower portion of Bear Lake Rhythmites with failure surface. Courtesy of Dr Gunnar Schlieder

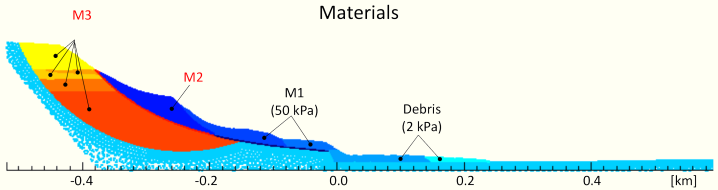
Oso landslide: Geology

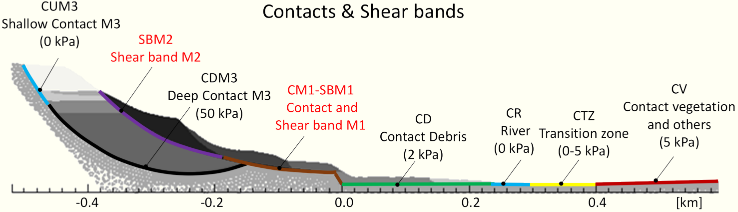
Deformation till with flame structures in fine-grained glacio-lacustrine deposit EB7 (depth 65 ft). Courtesy of Dr Gunnar Schlieder

Oso landslide: Direct shear test (intact specimens)

- **Peak strength (effective stress)**
  - Friction angle $\phi'_{max} = 24^o (> \phi'_{cs} = 22^o)$
  - Cohesion $c'_{max} = 100 kPa (> c'_{cs} = 0)$
- **But some specimens**
  - Friction angle $\phi'_{max} = 12^o (< \phi'_{cs} = 22^o)$
  - Cohesion $c'_{max} = 0 kPa (= c'_{cs})$
- Natural soil with structure
  - Is it an intrinsic material property?
  - Or a soil structure property?

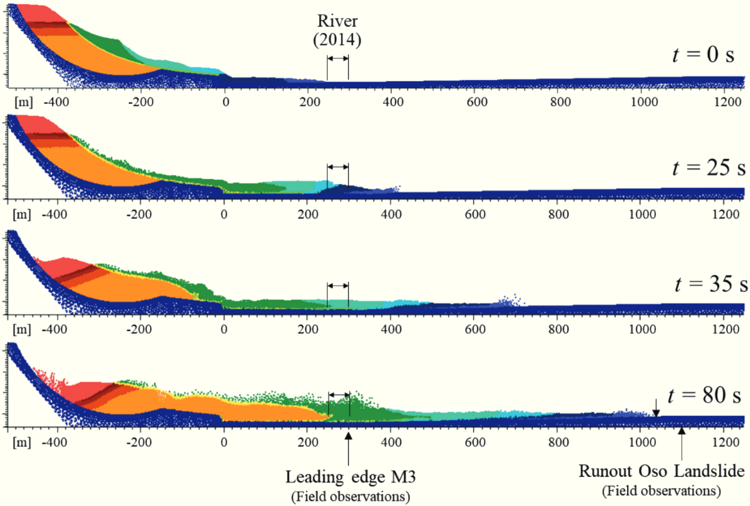
Model 1





Alba et al., 2018

Oso landslide: Analysis



Alba et al., 2018

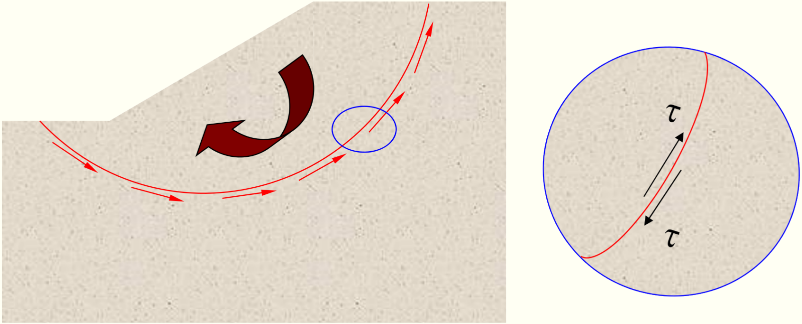
Advanced analysis in geotechnical engineering

Geotechnical design:

Analysis:

Classical geotechnical analysis: Slope stability

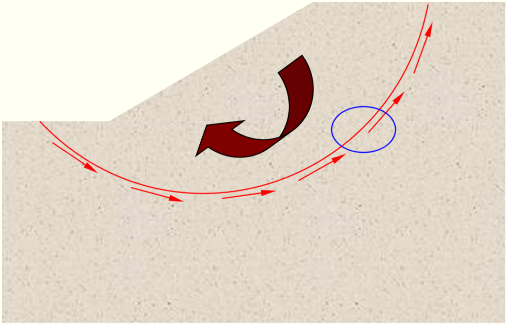
Shear failure plane

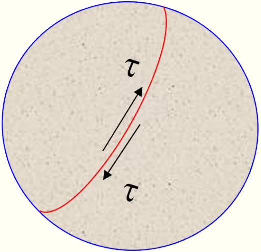


At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

Factor of Safety $=$

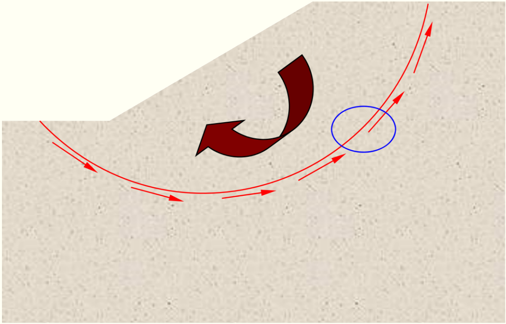
Dry slope (total stress = effective stress)

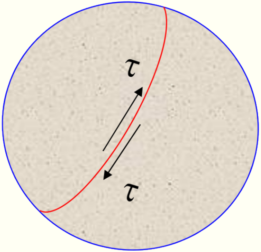




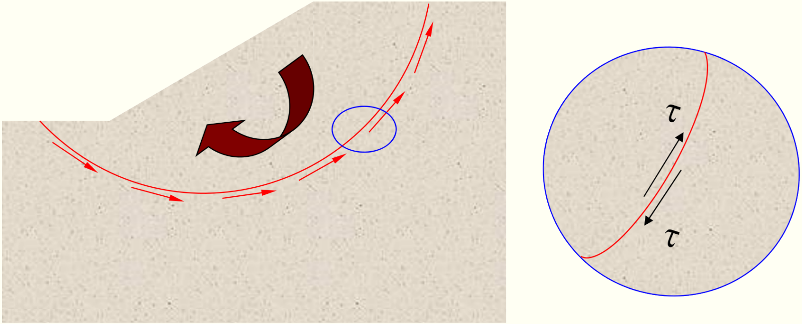
Saturated slope (total stress = effective stress + pwp)

Drained conditions - need to compute the steady state pore pressure field and then evaluate "effective stress-based" shear strength to find the overall stability (based on total stress equilibrium).





Undrained conditions - (total stress approach) – Use "*total-stress based*" shear strength (s_u) to find the overall stability (based on total stress equilibrium).

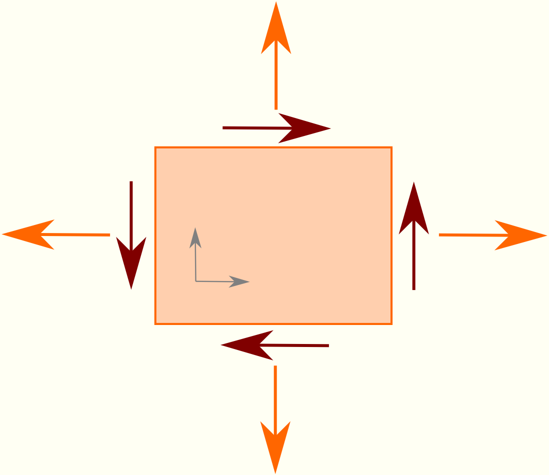


Governing equations in stress-deformation analysis

In stress-deformation analysis, we need to consider:

Governing equations in stress-deformation analysis

The governing differential equation for equilibrium expresses:



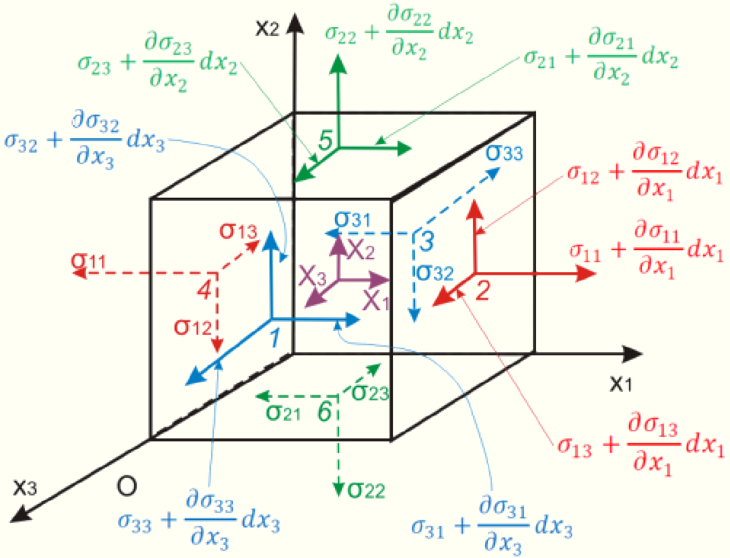
Equilibrium equations

Summing all this in the x-direction gives:

Cleaning up terms that cancel, and dividing through by *dxdy* gives

And summing forces in the y-direction leads to:

Equilibrium in 3D



Equilibrium in 3D

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \rho f_x = \rho a_x$$
$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + \rho f_y = \rho a_y$$
$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho f_z = \rho a_z$$

The governing differential equation for equilibrium expresses $\sum \mathrm{F} = m\mathrm{a}$ in terms of derivatives of the stress tensor as:

Stress equilibrium

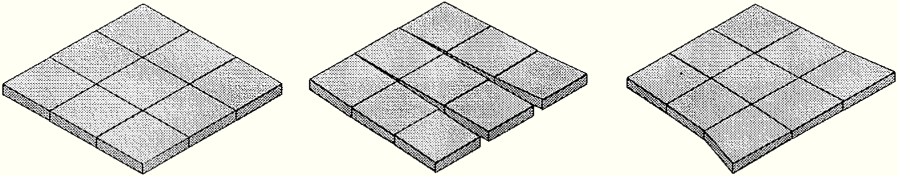
If the object is in equilibrium, then

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0$$
$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = 0$$
$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0$$

Stresses:
Equilibrium equation:
Then:

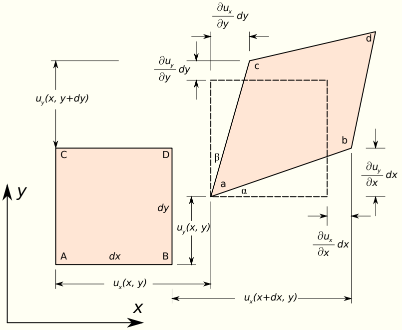
Compatibility

Governing equations: Compatibility



(a) original (b) non-compatible (c) compatible

Displacement - strain relationship:



Equilibrium and compatibility conditions

Combining the Equilibrium and Compatibility conditions gives:

- Unknowns:
- Equations:

Governing equations: Stress-strain relationship

Stress - strain relationship:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \text{D(6x6)} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix}$$

Governing equations in stress-deformation analysis

What are the variables used in the governing equations?

Advanced analysis involves:

Limit analysis: Lower and upper bound theorems

The exact determination of loads involved in the plastic deformation requires simultaneous solution of three sets of conditions:

1. equations of equilibrium
2. equations of compatibility
3. appropriate constitutive criteria (yield condition and flow rule)

Exact determination is often not easy, may be appropriate for simple shapes, but other cases we may have to use *numerical method*.

Limit analysis: Lower bound theorem

"*If there is a set of external loads* which are in equilibrium *with a state of stress that nowhere exceeds the failure criterion for the material,* collapse cannot occur *and the external loads are a lower bound to the true collapse loads*"

Any applied load is less than the actual limiting load, i.e., they will not cause collapse.

Lower bound solution

We need to determine a set of stresses in the ground that are in equilibrium with the external loads. Stress state everywhere should be:

Limit analysis: Upper bound theorem

"*If there is a set of external loads and a mechanism of plastic collapse such that the increment of work done by the external loads in an* increment of displacement *equals the work done by the internal stresses,* collapse must occur *and the external loads are an upper bound to the true collapse loads*"

Apply enough load to achieve the desired change in component shape, e.g., process machinery.

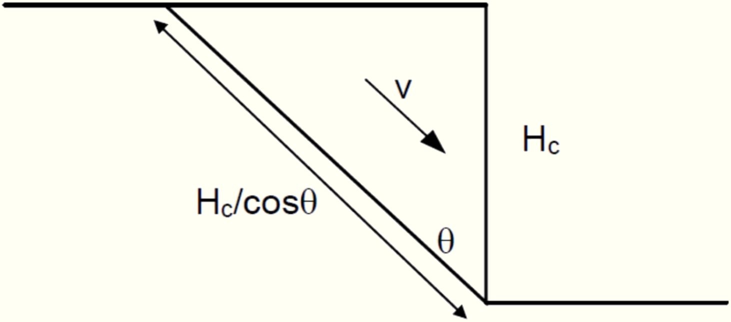
Upper bound solution

We need to determine a failure mechanism, which is kinematically feasible. The work done is equated by the plastic dissipation along the failure (or slip) surfaces.

Undrained cut: Lower bound solution

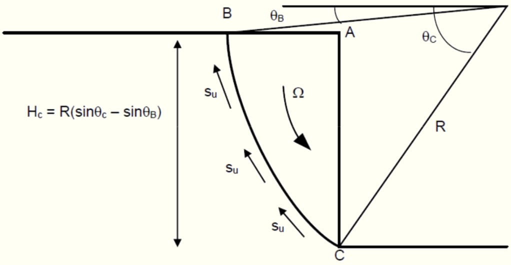
Consider the equilibrium of an element of soil adjacent to the bottom of a cut, or crack. Assume that the major principal stress is vertical, and that the soil fails according to Tresca's criterion at a shear stress s_u (undrained strength).

Undrained cut: Upper bound solution - wedge mechanism



This simple upper bound solution involving a single sliding block gives a normalized cut depth, $N_s = \gamma H_c / s_u = 4$. A more refined upper bound solution involves a circular slip surface, passing through the base of the cut.

Undrained cut: Upper bound solution - slip circle



Change in potential energy of block ABC (Need to find the centroid and examine the vertical movement by Ω = Dissipation $= s_u R(\theta_c - \theta_b) R\Omega$

$$N_s = \frac{\gamma H_c}{s_u} = \frac{6(\theta_c - \theta_b)}{2\sin^2\theta_c - \sin\theta_c \sin\theta_b - \sin^2\theta_b}$$

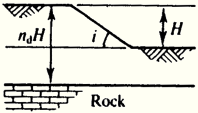
Optimizing: $\theta_b = 27.4^o$ and $\theta_c = 57.5^o$. $H_c = 3.83 s_u/\gamma$.

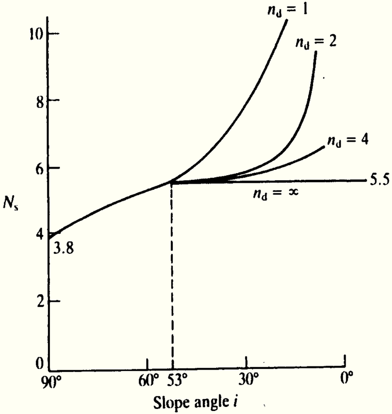
Taylor stability chart

For sloped cuts as shown on the right, the critical height at different slope angles (i), different excavation depths (H) and different rock depths ($n_d * H$) can be evaluated using the following generalised formula.

$$N_s = \frac{\gamma H_c}{s_u}$$

where N_s is the stability number ($N_s = 3.8$ for 90 degrees cut).





Methods of analysis

Method of analysis	Solution requirements				
	Equilibrium	Compatibility	Constitutive law	Force	Displacement
Closed form			Linear elastic		
Limit equilibrium			Rigid with a failure criterion		
Lower bound			Plasticity + flow-rule		
Upper bound			Plasticity + flow-rule		
Numerical analysis			Any		