CE394M: Finite Element Analysis in Geotechnical Engineering

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Overview

1. Geotechnical FEA
   - Element types
   - Discretization
   - Boundary conditions
   - Errors in FEA

FE Disclaimer

IMPORTANT WARNING AND DISCLAIMER

PLAXIS is a finite element program for geotechnical applications in which soil models are used to simulate the soil behaviour. The PLAXIS code and its soil models have been developed with great care. Although a lot of testing and validation have been performed, it cannot be guaranteed that the PLAXIS code is free of errors. Moreover, the simulation of geotechnical problems by means of the finite element method implicitly involves some inevitable numerical and modelling errors. The accuracy at which reality is approximated depends highly on the expertise of the user regarding the modelling of the problem, the understanding of the soil models and their limitations, the selection of model parameters, and the ability to judge the reliability of the computational results. Hence, PLAXIS may only be used by professionals that possess the aforementioned expertise. The user must be aware of his/her responsibility when he/she uses the computational results for geotechnical design purposes. The PLAXIS organization cannot be held responsible or liable for design errors that are based on the output of PLAXIS calculations.

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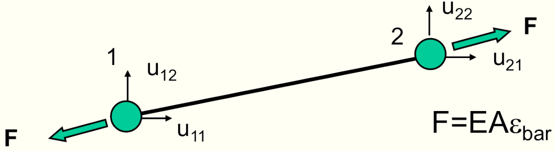
Consistent system of units

	SI			
Length	m	m	m	cm
Density	kg/m^3	10^3 kg/m^3	10^6 kg/m^3	10^6 g/cm^3
Force	N	kN	MN	Mdynes
Stress	Pa	kPa	MPa	bar
Gravity	m/sec^2	m/sec^2	m/sec^2	cm/s^2
Stiffness*	Pa / m	kPa / m	MPa / m	bar / cm

Problem definition

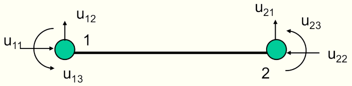
1D Finite Elements: Bar element

Two node element with axial stiffness only (no flexural or shear resistance).



1D Finite Elements: Beam element

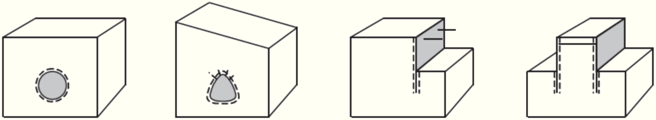
two node structure element with axial and bending stiffness (no transverse shear deformation). Three degrees of freedom for 2D beam element (1, 2 displacements and a moment).



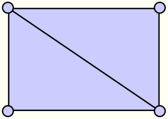
$F_a = EA\,\varepsilon_a$

$V = -EI(u_{12}-u_{21})/L^3 - 6EI(u_{13}+u_{23})/L^2$

$M = EI(u_{13}-u_{23})/L$



2D plane-strain / axisymmetric elements

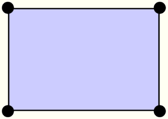


3 nodes element

linear variation of displacement within the element = constant strain in the element

$$d_1 = \alpha_1 + \alpha_2 x + \alpha_3 y$$

$$d_2 = \beta_1 + \beta_2 x + \beta_3 y$$

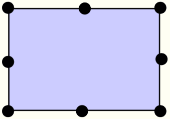


4 nodes element

linear variation of displacement in both x and y directions

$$d_1 = \alpha_1 + \alpha_2\xi + \alpha_3\eta + \alpha_4\xi\eta$$

$$d_2 = \beta_1 + \beta_2 x + \beta_3 y + \beta_4\xi\eta$$

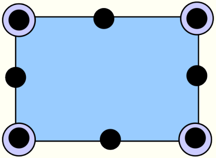


8 nodes element

quadratic variation of displacement in both x and y directions.

$$d_1 = \alpha_1 + \alpha_2\xi + \alpha_3\eta + \alpha_4\xi^2 + \alpha_5\xi\eta + \alpha_6\eta^2 + \alpha_7\xi^2\eta + \alpha_8\xi\eta^2$$
$$d_2 = \beta_1 + \beta_2\xi + \beta_3\eta + \beta_4\xi^2 + \beta_5\xi\eta + \beta_6\eta^2 + \beta_7\xi^2\eta + \beta_8\xi\eta^2$$

2D Consolidation element

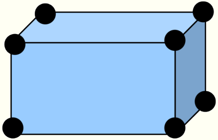


Pore pressure and displacements

Displacements

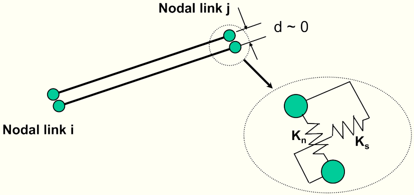
Linear variation of pore pressures and quadratic variation of displacements in x and y directions

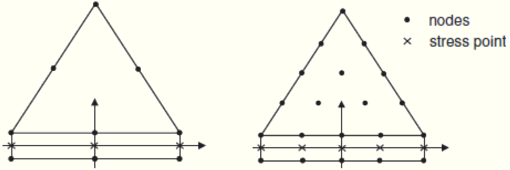
8 node 3D brick element



Linear variation of displacements in x, y and z directions

Interface element





a. 6-node soil element

b. 15-node soil element

13 Distribution of nodes and stress points in interface elements and their connection to soil elements

13

Use a reduced strength at the interface

Interface elements for Soil Structure Interactions

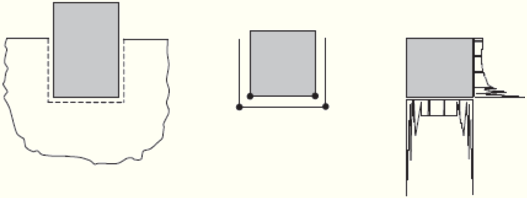


Figure 3.14 Inflexible corner point, causing poor quality stress results

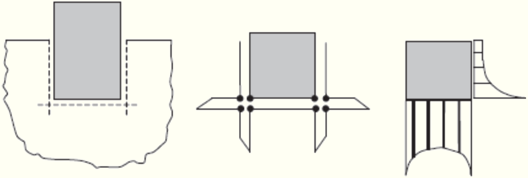
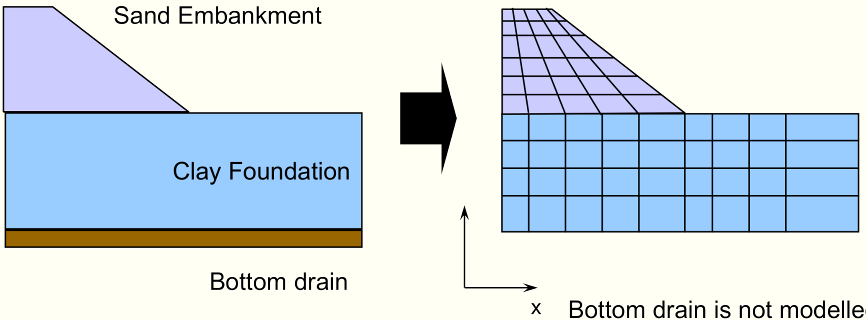


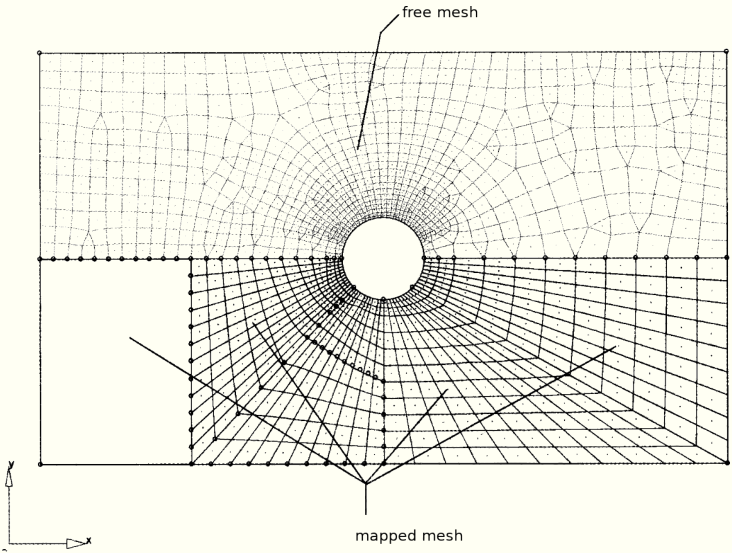
Figure 3.15 Flexible corner point with improved stress results

FE discretization

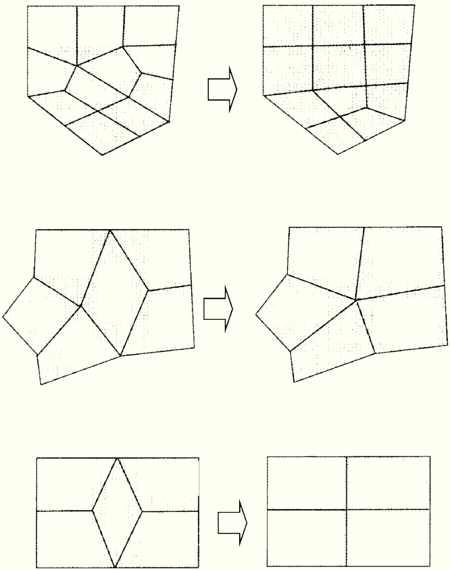


Bottom drain is not modelled because it is assumed to behave as a rigid material.

FE discretization

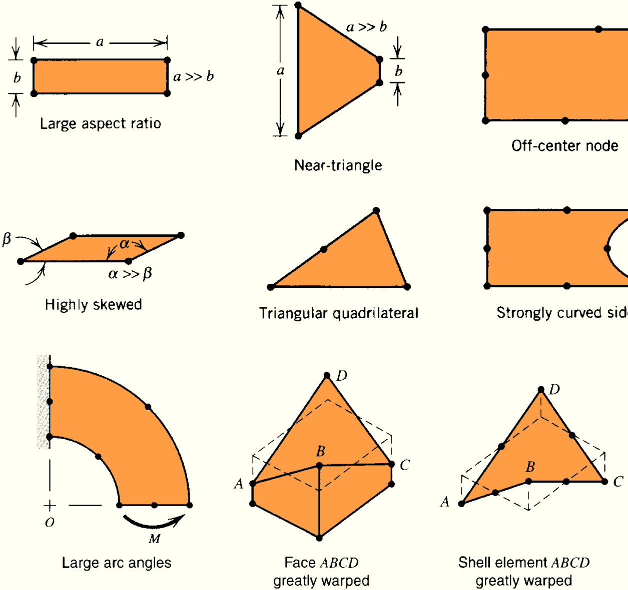


FE discretization

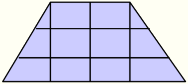


1. Node valence less or equal to two and greater than or equal to six should be eliminated.
2. The number of nodes with valence of three or five should be minimized.
3. Angles greater than 160 degrees should be eliminated.
4. The aspect ratio should be less than 3 for stress analysis and 10 for displacement analysis.

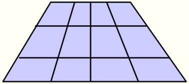
FE discretization



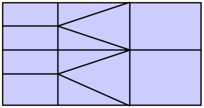
FE discretization



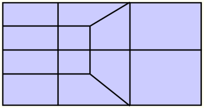
Not recommended



Better

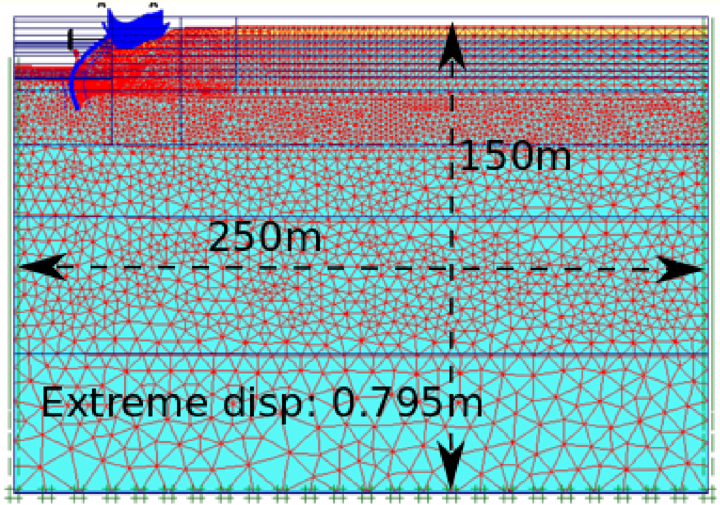


Not recommended



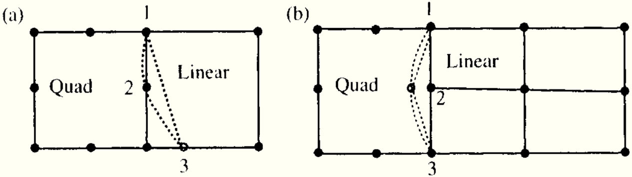
Better

FE discretization: Refining



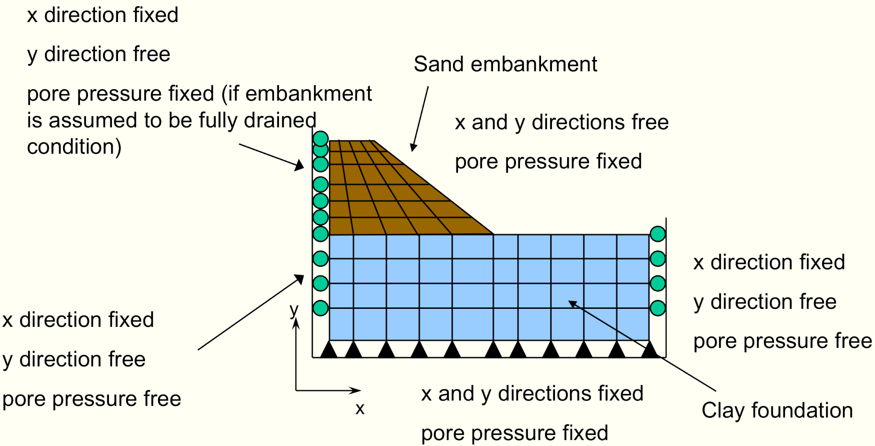
Avoid large jumps in element size: size jump should be < 3

FE mesh compatibility



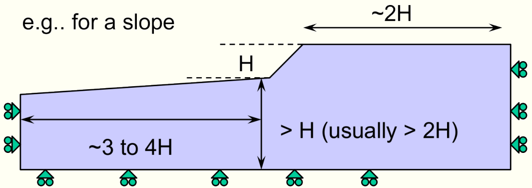
Incompatible mesh

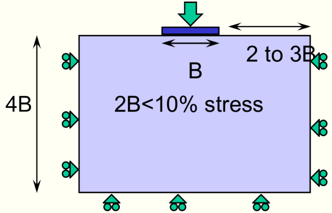
FE boundary conditions



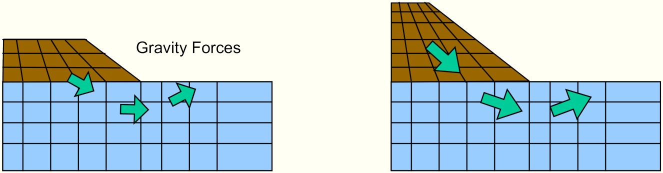
Pore pressure free = no water flow perpendicular to the boundary

FE boundary conditions





FE boundary conditions

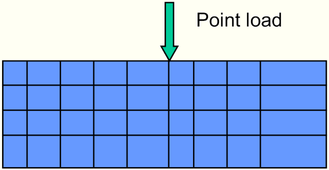


Construction sequence

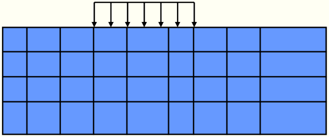
FE loading conditions

External forces

Point load

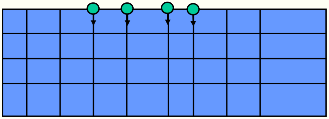


Distributed load

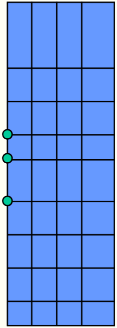


Apply displacements

Rigid foundation

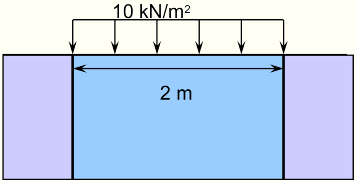


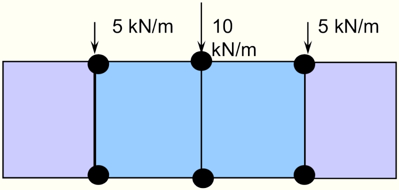
Excess pore pressure



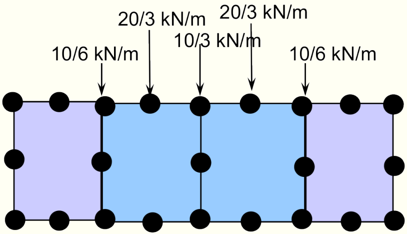
Water pumping

FE loading conditions



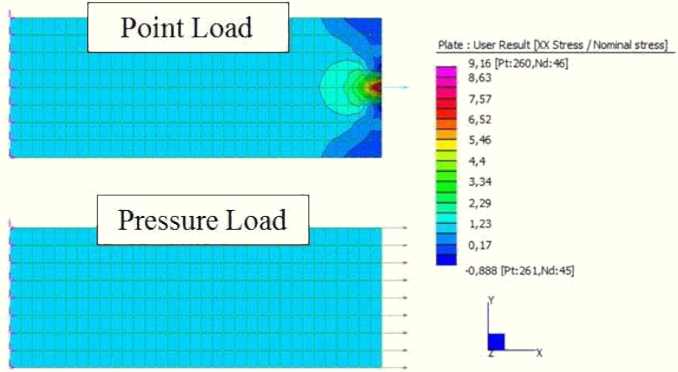


(1) 4 nodes elements



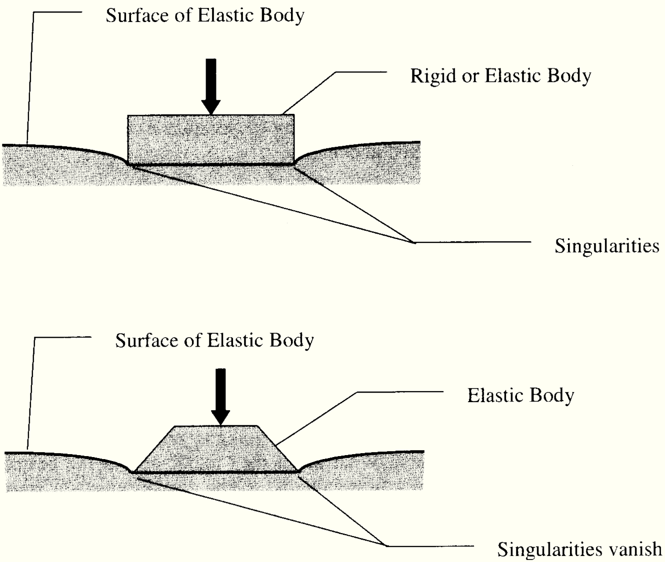
(2) 8 nodes element

FE stress singularity



2D uniaxial bar model. A single point load leads to stress singularities whereas an edge pressure results in a uniform stress field. (Gonzalez., 2015)

FE stress singularity

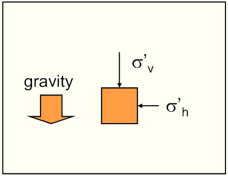


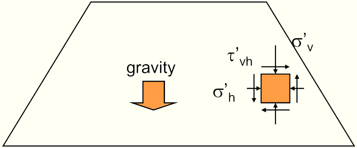
Geostatic stresses

Before conducting your analysis, you need to make sure that the stresses in the ground are the correct values, as the soil behavior depends on the current in-situ stresses.

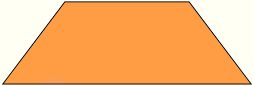
1. **list your estimated stresses in the input file**: hopefully the system is in equilibrium – difficult to find the in-situ stresses in the sloping ground.

Horizontally levelled ground

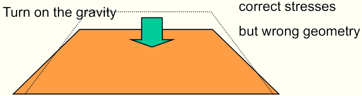




2. **Zero displacement approach** - ask the program to compute the in-situ stresses from the equilibrium condition (very few programs allow you to do this)



(a) define model geometry and assign a soil model



(b) displacement by self weight

and obtain the equilibrium condition

(c) Zero the displacement

But keep the computed stresses

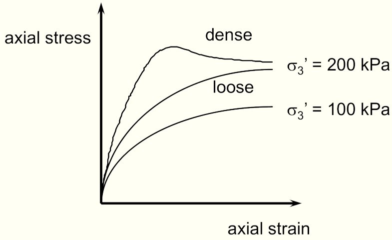
3. **Intermediate approach** - Guess the insitu stress distribution, apply gravity and perform the equilibrium check (hopefully the displacements are zero) - ABAQUS GEOSTATIC approach

The coefficient of earth pressure at rest K_0

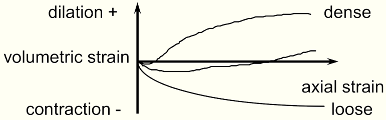
- **Normally consolidated soils**

- **Over consolidated soils**
  - **Cam-clay predictions** $K_{\mathrm{oc}} = (1 - \sin\phi') \times (\mathrm{OCR})$ (Mayne et al., 1982) up to $K_p = (1 + \sin\phi')/(1 - \sin\phi')$
  - **Wroth's method (1975)** $K_{\mathrm{oc}} = (\mathrm{OCR})K_{\mathrm{nc}} - \left(\frac{\nu'}{1-\nu'}\right)(\mathrm{OCR} - 1)$ for OCR < 5 and ν' is the poisson's ratio $=$ 0.254 - 0.371.

Soil models



axial stress
dense
σ_3' = 200 kPa
loose
σ_3' = 100 kPa
axial strain



dilation +
dense
volumetric strain
axial strain
contraction -
loose

Verification of Design Parameters (Atkinson., 1995)

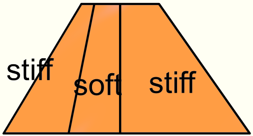
1. Are the soil models and analyses being used appropriate for the soils and for the structure?
2. Are the design parameters being measured appropriate for the soil models being used?
3. Are the tests being carried out the appropriate ones to determine the required design parameters?
4. Is the laboratory where the tests are being done capable of doing the required tests? Ideally, the engineer should inspect the laboratory and equipment, observe tests and check the procedures being used to analyse and interpret the results.
5. Are the correct samples being used? Are the appropriate methods of sample preparation being used? Do you need high quality samples or reconstituted samples?

Verification of Design Parameters (Atkinson., 1995)

1. Have any tests been done which investigate whether the soils reasonably follow the critical state theories or the elasto-plastic theories being used in the analyses?
2. Have tests being carried out at stress levels corresponding to the range of stresses in the ground? Is it necessary to follow special stress paths representing the previous stress history and the current loading?
3. Have linear parameters been fitted to non-linear test results over appropriate ranges?
4. Are the design parameters internally self consistent? Do parameters determined from triaxial or other loading tests correspond to parameters estimated form the grading and nature of the grains? How the values compare to the values obtained from empirical equations?

FE errors

Check! Check! Check!

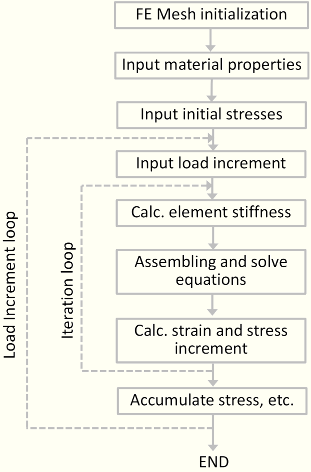


Beware : σ_v here lower than γh due to stress transfer to stiff material - hanging up

Steps to perform Finite Element Analysis

1. **Define the problem**
   - Total stress analysis, effective stress analysis or consolidation analysis?
   - What are the unknowns?
   - Spatial dimension (plane strain, axisymmetric, 3D)
   - Element types
2. **Create finite element mesh**
3. Proper discretization and avoid bad elements
4. **Define analysis (or construction sequence) steps**
   - apply loading and boundary conditions
5. **Define materials**
   - determine material properties
6. **Assign materials and element types to elements**
7. **Define in-situ stress conditions**
8. **Run the analysis**
9. **Check! Check! Check!**

Summary of FE analysis



$\boldsymbol{\sigma}$

$\Delta \boldsymbol{f}$

$$K_{ij} \approx \sum_{p}^{n_p} B_i^p D^p B_j^p W_p$$

$\Delta \boldsymbol{u} = \boldsymbol{K}^{-1} \cdot \Delta \boldsymbol{f}$ **Equilibrium eq.**

$\Delta \boldsymbol{\varepsilon} = \boldsymbol{B} \cdot \Delta \boldsymbol{u}$ **Compatibility eq.**

$\Delta \boldsymbol{\sigma} = \boldsymbol{D} \cdot \Delta \boldsymbol{\varepsilon}$ **Constitutive eq.**

$$\boldsymbol{u} = \boldsymbol{u} + \Delta \boldsymbol{u}$$
$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon} + \Delta \boldsymbol{\varepsilon}$$
$$\boldsymbol{\sigma} = \boldsymbol{\sigma} + \Delta \boldsymbol{\sigma}$$