CE394M Advanced Analysis in Geotechnical Engineering: Introduction

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Overview

1. Geotechnical modeling
   - Complexity in Geotechnical modeling
   - Classical vs advanced analysis
2. Numerical methods for differential equations
   - Direct method: Matrix analysis of structures
   - Numerical analysis of engineering problems
3. Governing equations in stress-deformation analysis
   - Stress equilibrium
   - Stress-strain relationship

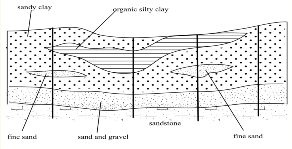
Geotechnical modeling of the complex world

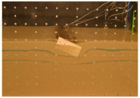
Fig. London Bridge Station, London, UK

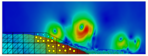
Geotechnical modeling of the complex world

Fig. London Victoria station upgrade, London, UK

Geotechnical modeling





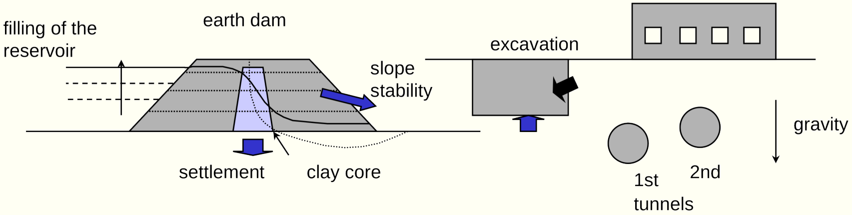


Soil behavior

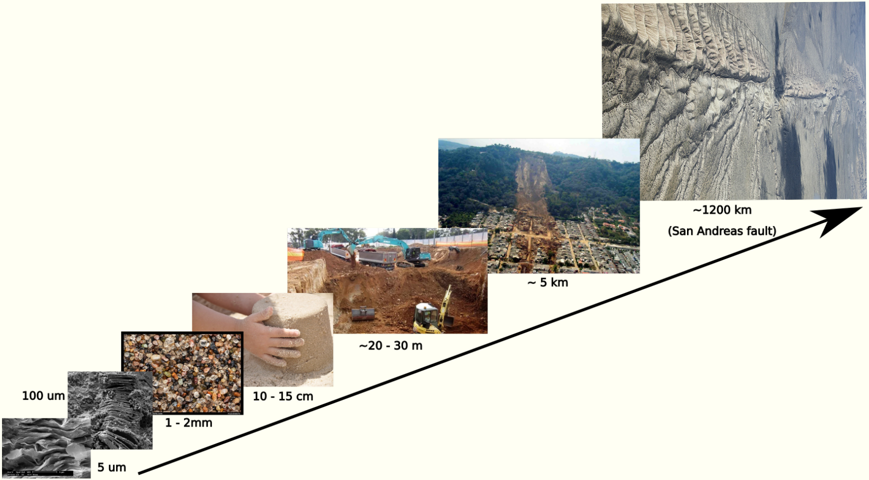
- nonhomogeneous,
- anisotropic,
- non-linear,
- initial stress conditions,
- stress history
- Geometry - very complex

Soil Mechanics in practice - largely empirical

Geotechnical modeling: What should be modeled?



Scales of modeling in geotechnical engineering



Advanced analysis in geotechnical engineering

Geotechnical design:

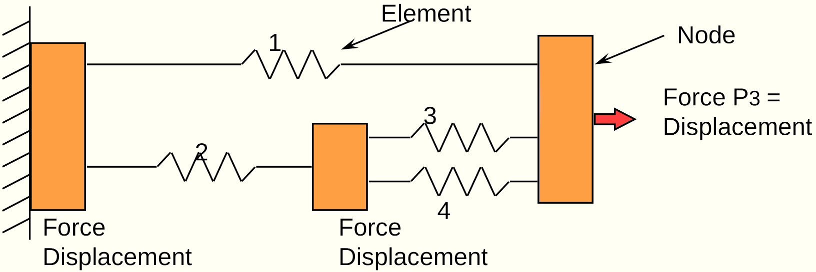
Analysis:

Classical vs advanced analysis

Classical approach:

Advanced analysis:

Matrix analysis of structures



- What are the known variables?
- What are the unknowns?
- What do we know?

Matrix analysis of structures: Equilibrium

- $P_1 =$
- What are the unknowns?
- What do we know?

Matrix analysis of structures: Compatibility

Matrix analysis of structures: Compatibility

v = internal spring distortion δ = nodal displacement

- $v_1 =$
- $v_2 =$
- $v_3 =$
- $v_4 =$

Matrix analysis of structures: Physical condition

Force-distance relationship: spring constant

spring #	1	2	3	4
stiffness $(F.L^{-1})$	3	2	1	2

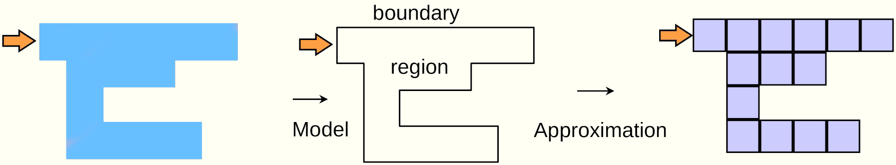
Matrix analysis of structures: Direct Method

Combine all the equations: $\mathbf{P} =$
where $\mathbf{K} =$

Matrix analysis of structures

Numerical analysis of engineering problems

Physical phenomenon $\xrightarrow[\text{Model}]{}$ Differential equation $\xrightarrow[\text{Approximation}]{}$ Equations of the numerical technique



Boundary value problems

Differential equations coupled with boundary conditions

- Steady state (time-independent)

- Transient (time-dependent)

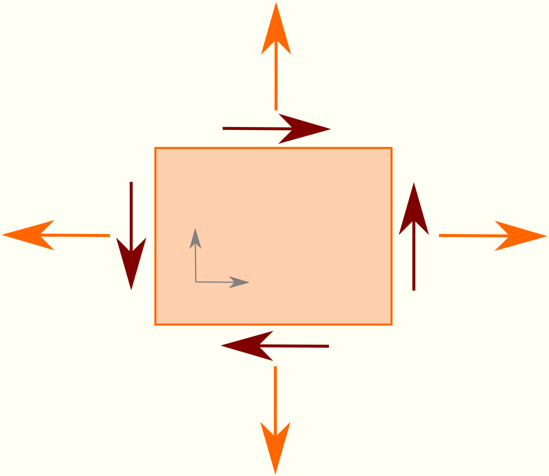
Numerical solutions to differential equations

Governing equations in stress-deformation analysis

In stress-deformation analysis, we need to consider:

Governing equations in stress-deformation analysis

The governing differential equation for equilibrium expresses:



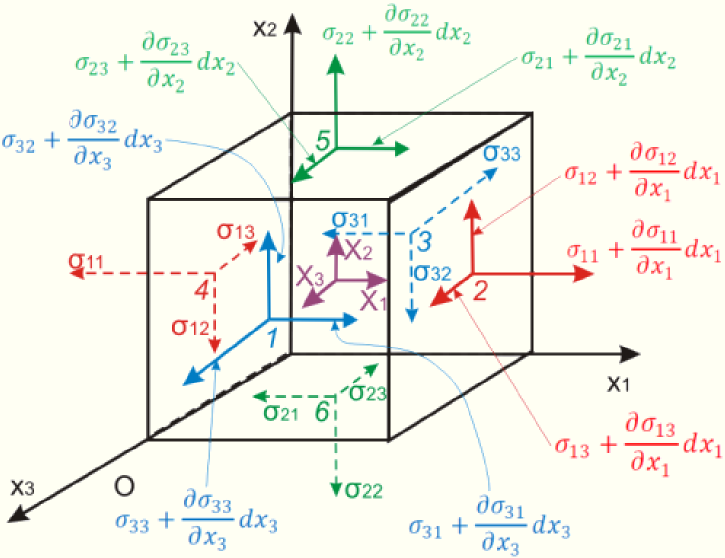
Equilibrium equations

Summing all this in the x-direction gives:

Cleaning up terms that cancel, and dividing through by *dxdy* gives

And summing forces in the y-direction leads to:

Equilibrium in 3D



Equilibrium in 3D

The governing differential equation for equilibrium expresses $\sum \mathbf{F} = m\mathbf{a}$ in terms of derivatives of the stress tensor as:

Stress equilibrium

If the object is in equilibrium, then

Stresses in Voigt notation:
Equilibrium equation:
Then:

Governing equations: Stress-strain relationship

Stress - strain relationship:

$$
\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \begin{bmatrix} D_{xxxx} & D_{xxyy} & D_{xxzz} & D_{xxxy} & D_{xxyz} & D_{xxzx} \\ D_{yyxx} & D_{yyyy} & D_{yyzz} & D_{yyxy} & D_{yyyz} & D_{yyzx} \\ D_{zzxx} & D_{zzyy} & D_{zzzz} & D_{zzxy} & D_{zzyz} & D_{zzzx} \\ D_{xyxx} & D_{xyyy} & D_{xyzz} & D_{xyxy} & D_{xyyz} & D_{xyzx} \\ D_{yzxx} & D_{yzyy} & D_{yzzz} & D_{yzxy} & D_{yzyz} & D_{yzzx} \\ D_{zxxx} & D_{zxyy} & D_{zxzz} & D_{zxxy} & D_{zxyz} & D_{zxzx} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix}
$$

Governing equations in stress-deformation analysis

What are the variables used in the governing equations?

Advanced analysis involves: