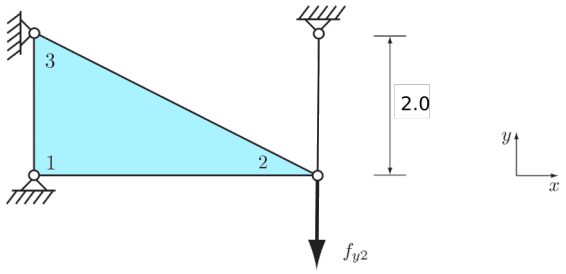
Assignment 4: FEM Structural analysis

1. Consider the following structural system consisting of a three-noded triangle and a cable element (i.e. two-noded one-dimensional element). The triangle element is fixed at the (local) nodes 1 and 3 and its stiffness matrix for the unconstrained degrees of freedom at node 2 is

$$\mathbf{K} = 10^9 \begin{bmatrix} 1.64 & 0.75 \\ 0.82 & 0.55 \end{bmatrix} \begin{matrix} u^e_{x2} \\ u^e_{y2} \end{matrix}$$

For the cable, the product of the Young's modulus and cross-sectional area is $EA = 1.0{\cdot}10^9$. Further, the system is loaded with a nodal force of $f_{y2} = -5000$.



- Determine the displacements u_{x2} and u_{y2} of node 2.
- Determine the force in the cable.

2. Using the Jupyter notebook for elastic-bar-linear.ipynb (available on Canvas) compute and plot the displacement profiles for the following loading conditions. Discretize the cantilever beam using linear elements.

- A uniformly distributed load of 1.
- A uniformly varying load with 1 at the fixed end, and 0 at the free end.
- A sinusoidal load described as $f(x) = \sin(x^2)$

(a) Calculate the error in the displacement, using the FE solution with 100 elements as the reference solution.

(b) Use at least 5 different discretizations and plot how the error changes with the number of elements.

(c) Based on the errors computed above, determine the number of linear elements required to perform a FE analysis for each of the three loading conditions shown above.