CE394M Advanced Analysis in Geotechnical Engineering: Introduction

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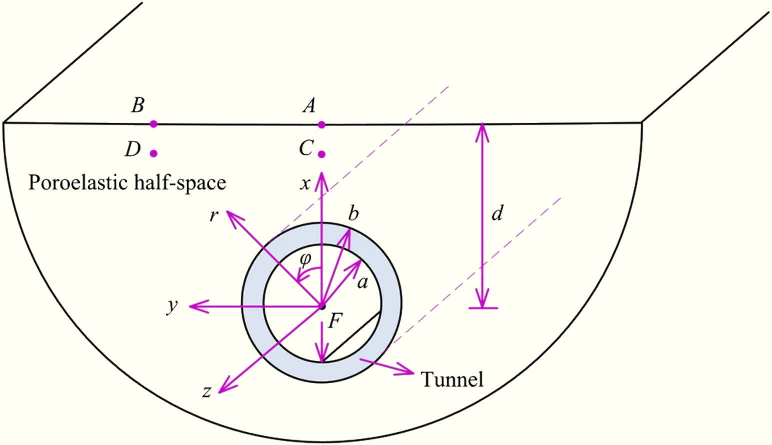
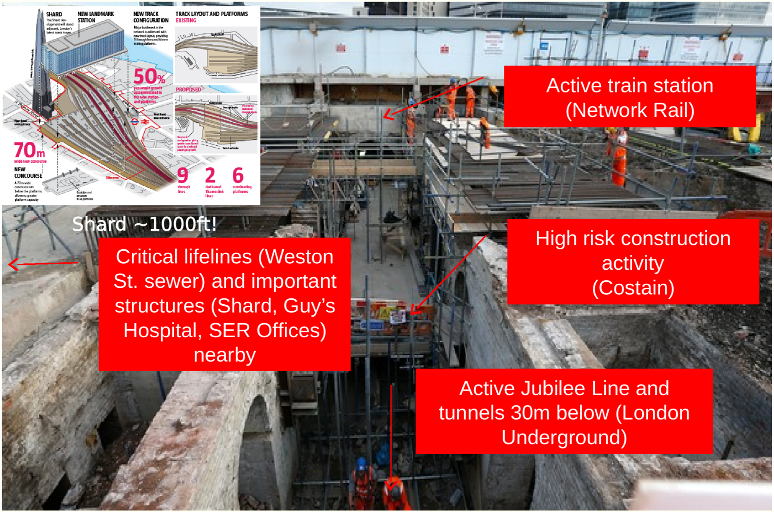
Overview

Is this model correct?



Geotechnical modeling of the complex world



London Bridge Station, London, UK

Geotechnical modeling of the complex world

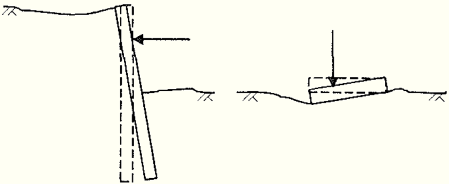
Fig. London Victoria station upgrade, London, UK

Geotechnical modeling of the complex world

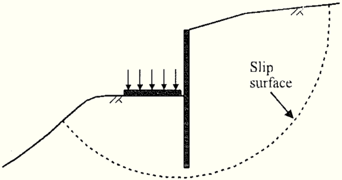
Fig. London Victoria station upgrade, London, UK

Movements must be estimated, both of the structure and of the ground. This is particularly important if there are adjacent buildings and for sensitive services. For example, if an excavation is to be made in an urban area close to existing services and buildings, one of the key design constraints is the effect that the excavation has on the adjacent structures and services. It may be necessary to predict any structural forces induced in these existing structures and/or services.

Local vs global stability

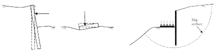


Local stability



Overall stability

Local vs global stability

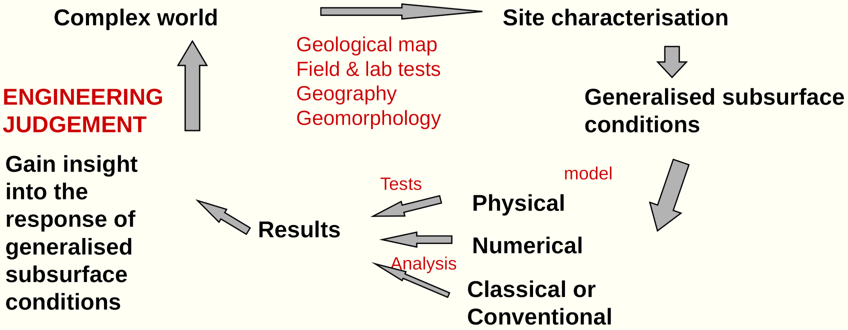


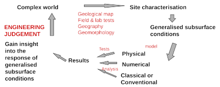
Local stability

Overall stability

When designing any geotechnical structure, the engineer must ensure that it is stable. Stability can take several forms. Firstly, the structure and support system must be stable as a whole. There must be no danger of rotational, vertical or translational failure (**local stability**). Secondly, **overall stability** must be established. For example, if a retaining structure supports sloping ground, the possibility of the construction promoting an overall slope failure should be investigated.

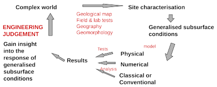
Geotechnical modeling





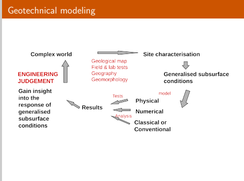
Design requirements: Before the design process can begin, a considerable amount of information must be assembled. The basic geometry and loading conditions must be established. These are usually defined by the nature of the engineering project.

A geotechnical site investigation is then required to establish the ground conditions. Both the soil stratigraphy and soil properties should be determined. In this respect it will be necessary to determine the strength of the soil and, if ground movements are important, to evaluate its stiffness too. The position of the ground water table and whether or not there is underdrainage or artesian conditions must also be established. The possibility of any changes to these water conditions should be investigated. For example, in many major cities around the world the ground water level is rising.



The site investigation should also establish the location of any services (gas, water, electricity, telecommunications, sewers andlor tunnels) that are in the vicinity of the proposed construction. The type (strip, raft andlor piled) and depth of the foundations of any adjacent buildings should also be determined. The allowable movements of these services and foundations should then be established.

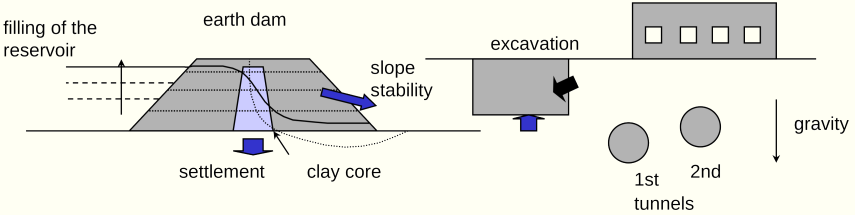
Any restrictions on the performance of the new geotechnical structure must be identified. Such restrictions can take many different forms. For example, due to the close proximity of adjacent services and structures there may be restrictions imposed on ground movements.



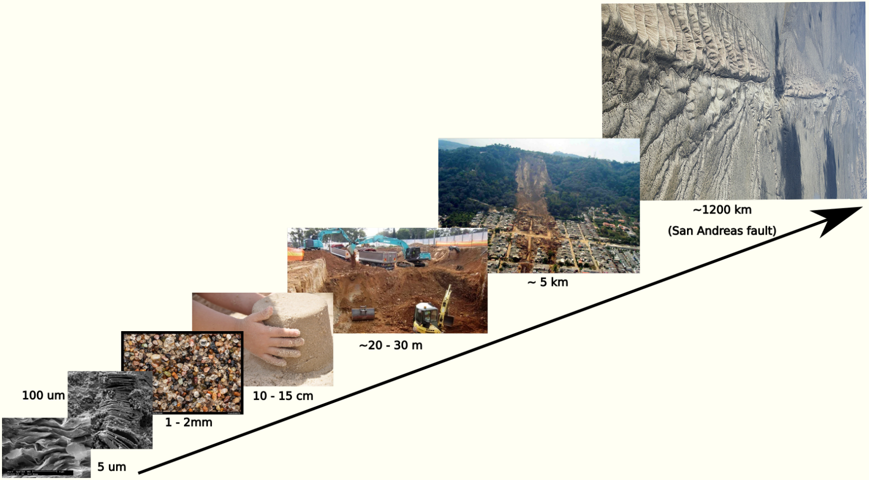
Once the above information has been collected, the design constraints on the geotechnical structure can be established. These should cover the construction period and the design life of the structure. This process also implicitly identifies which types of structure are and are not appropriate. For example, when designing an excavation, if there is a restriction on the movement of the retained ground, propped or anchored embedded retaining walls are likely to be more appropriate than gravity or reinforced earth walls. The design constraints also determine the type of design analysis that needs to be undertaken.

Geotechnical modeling: What should be modeled?

- Self weight effect of soils (This is why soil moves)
- Construction sequence (Complex geometry)
- Water movement (undrained, consolidation, drained)
- Insitu stresses (stiffness/strength depends on current stresses and stress history)
- Predict the ability of a design to withstand extreme loading conditions (you only have one chance)



Scales of modeling in geotechnical engineering

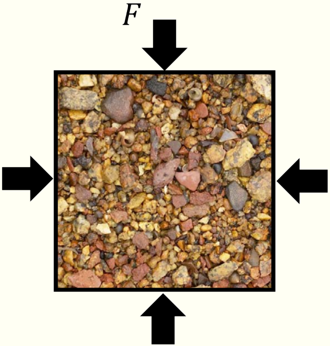


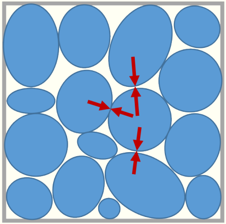
Soil behavior

- nonhomogeneous,
- anisotropic,
- non-linear,
- initial stress conditions,
- stress history
- Geometry - very complex

Soil Mechanics in practice - largely empirical

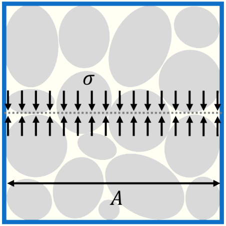
Numerical frameworks





Discrete mechanics

- Micro-scale
- Grain level description
- Intergranular forces



Continuum mechanics

- Macro-scale
- Stress definition

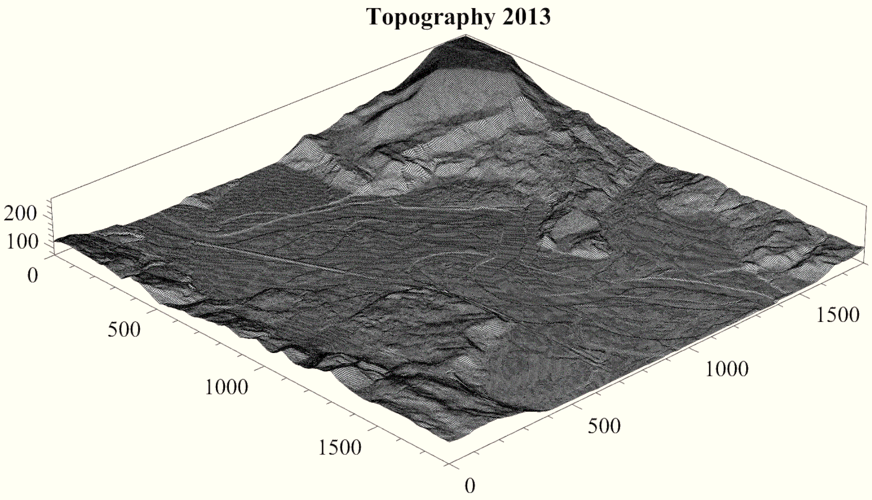
$$\sigma = \frac{F}{A}$$

Oso landslide: case study

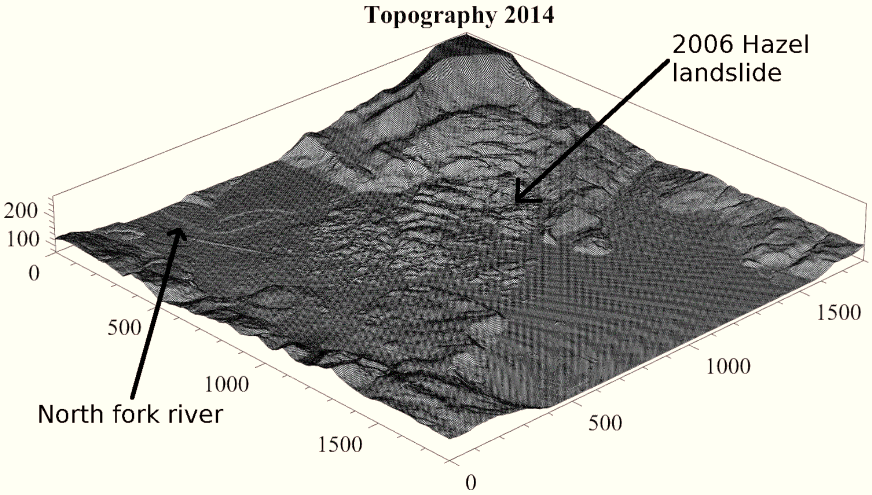
Oso landslide: case study

- 22nd March 2014 at 10:37 AM
- Volume approx 8 million cubic meters
- 43 casualties (deadliest landslide in the US)
- 1 neighborhood destroyed
- (unknown but) $>$ 150 million lost cost (DNR 2015), $+$ USD 65 million (lawsuit 2017) $+$ indirect costs
- Social tensions and Indian Snohomish tribe

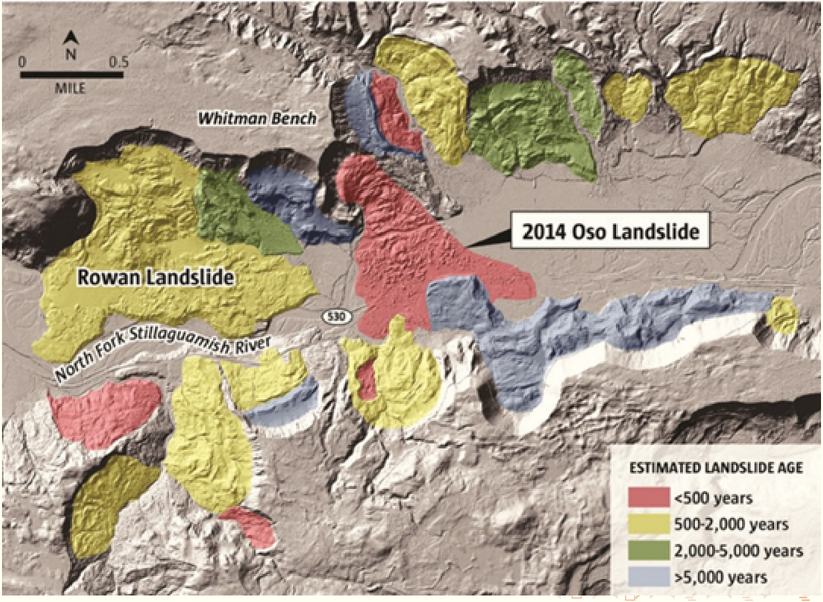
Oso landslide: topography



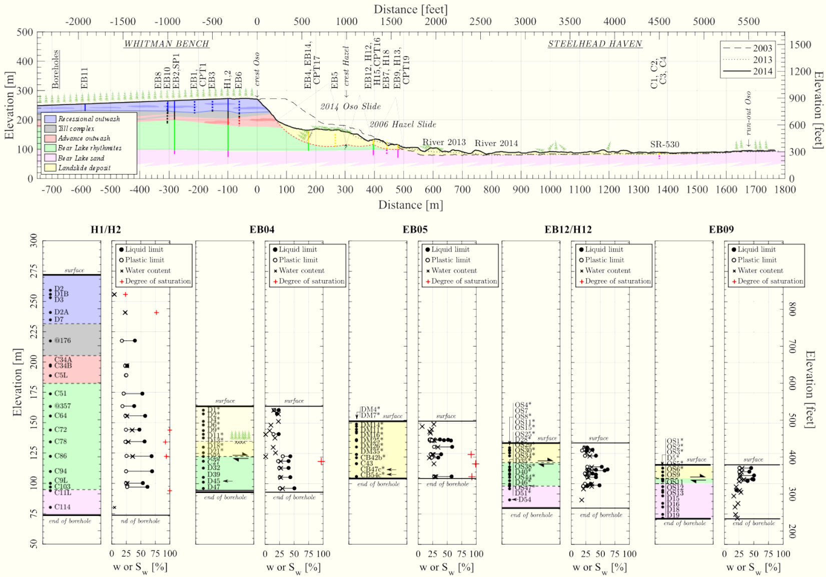
Oso landslide: topography



Oso landslide: historic slides



Oso landslide: Soil profile



Oso landslide: Geology. Identify the failure surface

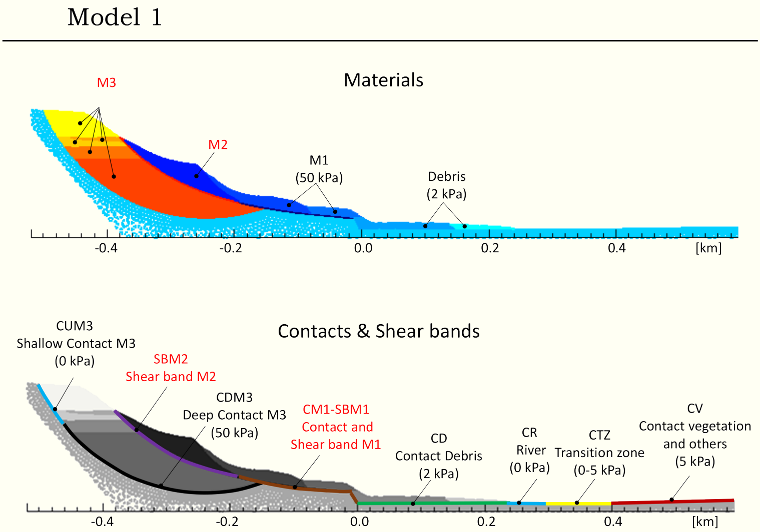
Lower portion of Bear Lake Rhythmites with failure surface. Courtesy of Dr Gunnar Schlieder

Oso landslide: Geology

Deformation till with flame structures in fine-grained glacio-lacustrine deposit EB7 (depth 65 ft). Courtesy of Dr Gunnar Schlieder

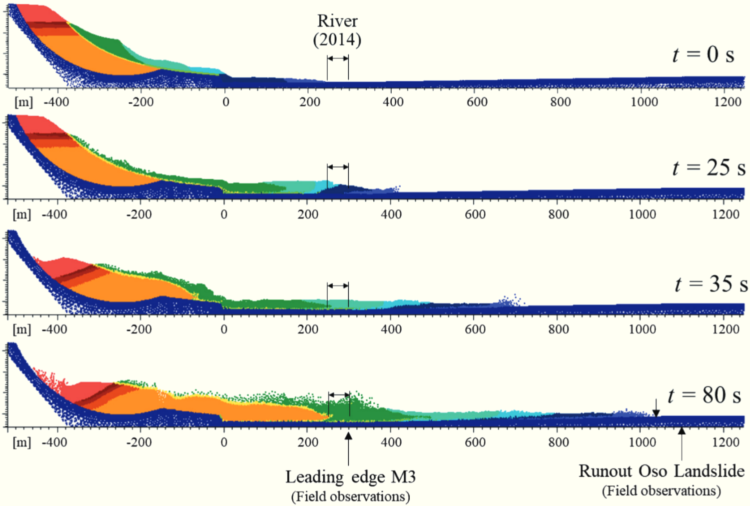
Oso landslide: Direct shear test (intact specimens)

- **Peak strength (effective stress)**
  - Friction angle $\phi'_{max} = 24^o (> \phi'_{cs} = 22^o)$
  - Cohesion $c'_{max} = 100 kPa (> c'_{cs} = 0)$
- **But some specimens**
  - Friction angle $\phi'_{max} = 12^o (< \phi'_{cs} = 22^o)$
  - Cohesion $c'_{max} = 0 kPa (= c'_{cs})$
- Natural soil with structure
  - Is it an intrinsic material property?
  - Or a soil structure property?



Alba et al., 2018

Oso landslide: Analysis



Alba et al., 2018

Advanced analysis in geotechnical engineering

Geotechnical design:

- Assess applied forces
- evaluate "performance" (stability & movements) under working and ultimate loads

Analysis:

- Mathematical framework to perform calculations for these quantities
- Requires idealization of: geometry, soil properties, and loading conditions
- Analysis is a tool in design, but design involves more: acceptable movements, constraints, site characterization, etc.

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- Analysis is a tool in design, but design involves more: acceptable movements, constraints, site characterization, etc.

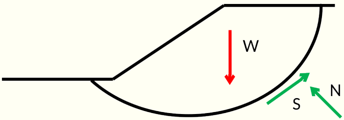
As part of the design process, it is necessary for an engineer to perform calculations to provide estimates of the above quantities. Analysis provides the mathematical framework for such calculations. A good analysis, which simulates real behaviour, allows the engineer to understand problems better. While an important part of the design process, analysis only provides the engineer with a tool to quantify effects once material properties and loading conditions have been set. The design process involves considerably more than analysis.

Classical geotechnical analysis: Slope stability

Assumptions

- *Mode:* Assume failure surface
- *Deformation:* Rigid soil block
- *Slip surface:* Define a failure criteria
- *Compute:* F_s Stability

This is the classical **limit equilibrium method**.



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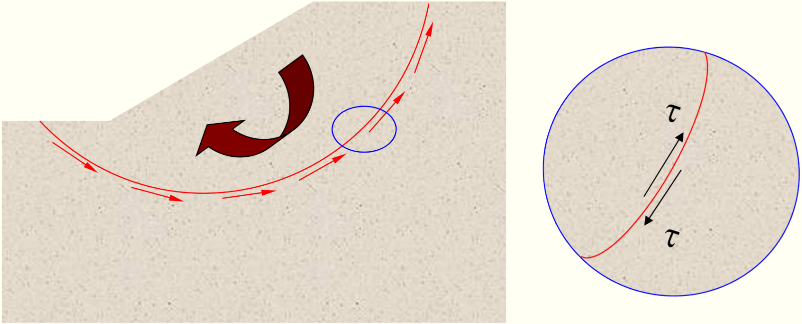
This is the classical **limit equilibrium method**.

Limit equilibrium In this method of analysis an 'arbitrary' failure surface is adopted (assumed) and equilibrium conditions are considered for the failing soil mass, assuming that the failure criterion holds everywhere along the failure surface. The failure surface may be planar, curved or some combination of these. Only the global equilibrium of the 'blocks' of soil between the failure surfaces and the boundaries of the problem are considered. The internal stress distribution within the blocks of soil is not considered. Coulomb's wedge analysis and the method of slices are examples of limit equilibrium calculations.

Only a global equilibrium, rather than the local equilibirium of every point in the soil, is satisified.

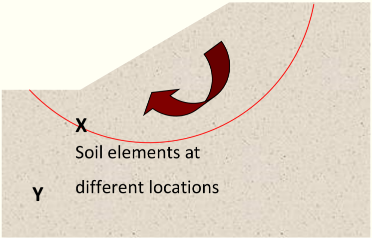
Shear failure plane

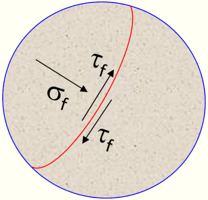


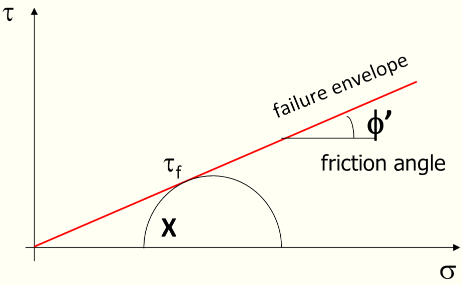
At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

Factor of Safety = Resistance (from soil shear strength)/Driving force (from total stress equilibrium (i.e. weight of the soil))

Dry slope (total stress = effective stress)





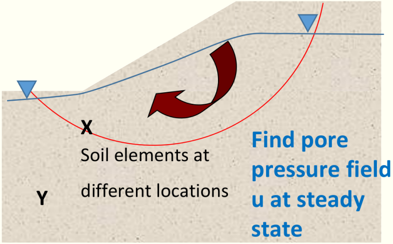


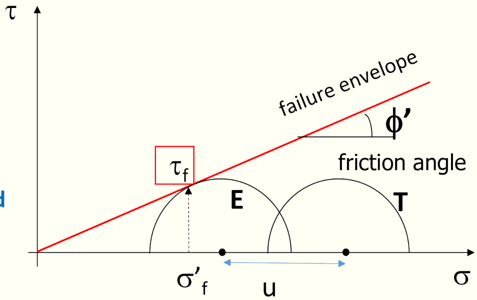
X ~ at failure

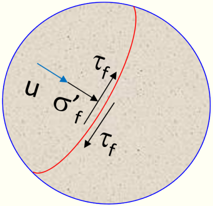
Y ~ stable

Saturated slope (total stress = effective stress + pwp)

Drained conditions - need to compute the steady state pore pressure field and then evaluate "effective stress-based" shear strength to find the overall stability (based on total stress equilibrium).



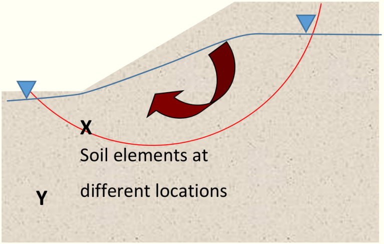


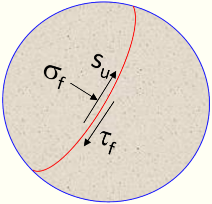


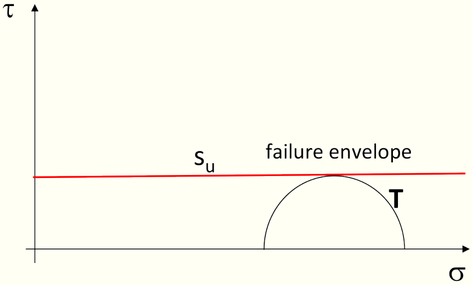
E Effective stress for soil shear resistance estimation

T Total stress for stress equilibrium calculation

Undrained conditions - (total stress approach) – Use "*total-stress based*" shear strength (s_u) to find the overall stability (based on total stress equilibrium).







T Total stress for stress equilibrium calculation

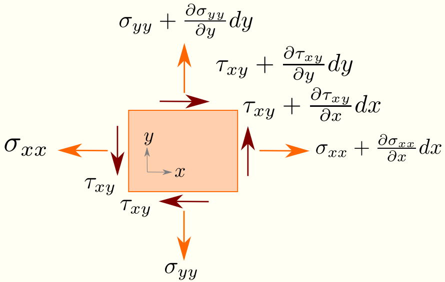
Governing equations in stress-deformation analysis

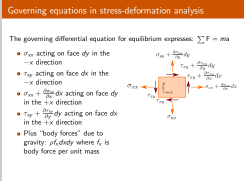
In stress-deformation analysis, we need to consider:

- **Equilibrium - static conditions**
  - forces and stress must agree across the region of interest. (geometric problem)
- **Compatibility-kinematic conditions**
  - geometry, displacement and strains must agree across the region of interest. (geometric problem)
- **Stress-strain relationship on physical conditions**
  - material dependent relationship between stress and strain must be specified. (element level)

The governing differential equation for equilibrium expresses: $\sum \mathsf{F} = ma$

- σ_{xx} acting on face dy in the $-x$ direction
- τ_{xy} acting on face dx in the $-x$ direction
- $\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx$ acting on face dy in the $+x$ direction
- $\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy$ acting on face dx in the $+x$ direction
- Plus "body forces" due to gravity: $\rho f_x dxdy$ where f_x is body force per unit mass





σ_{xy}, the term x denotes the direction of the stress component acts on a cut normal to the x-axis (denotes the plane). The y denotes the direction of the stress component.

Frequently in the literature the axes are labelled 1, 2 & 3 rather than x, y & z.

Equilibrium equations

Summing all this in the x-direction gives:

$$-\sigma_{xx}dy - \tau_{xy}dx + \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}dx\right)dy$$
$$+ \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y}dy\right)dx + \rho f_x dxdy = \rho dxdy a_x$$

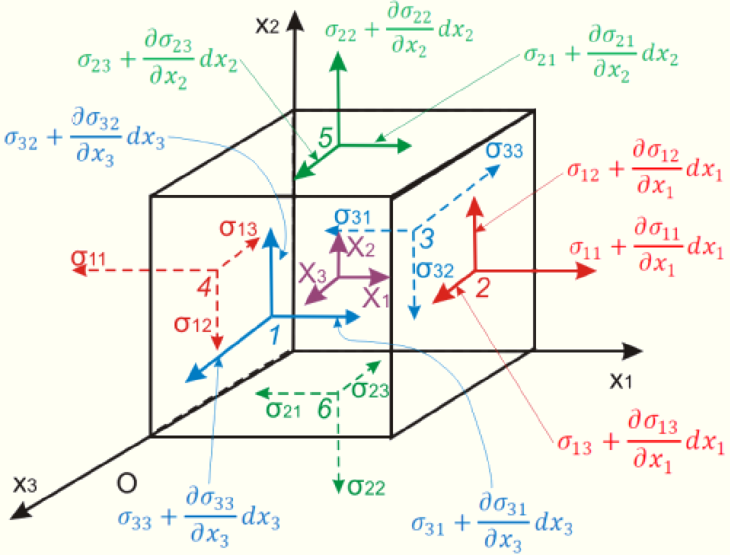
Cleaning up terms that cancel, and dividing through by $dxdy$ gives

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho f_x = \rho a_x$$

And summing forces in the y-direction leads to:

$$\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \rho f_y = \rho a_y$$

Equilibrium in 3D



Equilibrium in 3D

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \rho f_x = \rho a_x$$
$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + \rho f_y = \rho a_y$$
$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho f_z = \rho a_z$$

The governing differential equation for equilibrium expresses $\sum \mathrm{F} = m\mathrm{a}$ in terms of derivatives of the stress tensor as: $\nabla \cdot \boldsymbol{\sigma} + \rho \mathrm{f} = \rho \mathrm{a}$

$\boldsymbol{\sigma}$ is the stress tensor,

ρ is density,

f is the body force vector per unit mass and

a is the acceleration vector.

Stress equilibrium

If the object is in equilibrium, then $a = 0$ and $\sum F = 0$.

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0$$
$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = 0$$
$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0$$

Stresses: $[\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}]^T$.
Equilibrium equation: $\nabla^T \boldsymbol{\sigma} + b = 0$
Then:

$$\nabla^T = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}$$

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$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0$$
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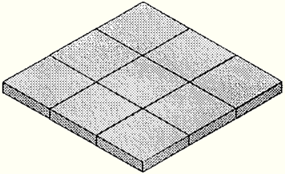
Divergence is a vector operator that produces a scalar field, giving the quantity of a vector field's source at each point. In physical terms, the extent to which there is more of some quantity exiting an infinitesimal region of space than entering it. If the divergence is nonzero at some point then there is compression or expansion at that point.

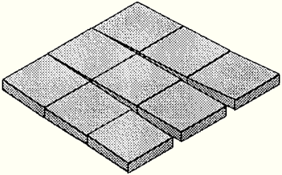
Compatibility

- compatibility relates the deformations of a structure so that its various parts (members, joints, and supports) fit together without any gaps or overlaps.
- ensure that the deformed shape of the structure is continuous (except at the locations of any internal hinges or rollers), and is consistent with the support conditions.

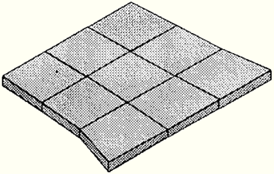
Governing equations: Compatibility



(a) original



(b) non-compatible



(c) compatible

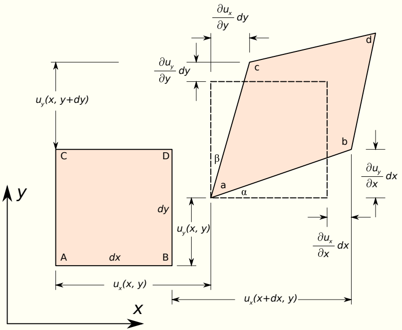
Governing equations: Displacement - strain relationship

Displacement - strain relationship: $\boldsymbol{\varepsilon} = \nabla \mathbf{u}$
Where,

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

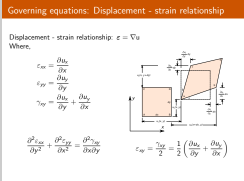
$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$

$$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

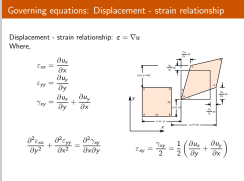


$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_{xy} = \frac{\gamma_{xy}}{2} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$



In the two-dimensional case, there are three strain-displacement relations but only two displacement components. This implies that the strains are not independent but are related in some way. The relations between the strains are called compatibility conditions.



Compatibility in 3D

$$
\begin{aligned}
\frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} &= 2\frac{\partial^2 \varepsilon_{yz}}{\partial y \partial z}, \quad & \frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} &= \frac{\partial}{\partial x}\left(-\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right) \\
\frac{\partial^2 \varepsilon_{zz}}{\partial x^2} + \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} &= 2\frac{\partial^2 \varepsilon_{zx}}{\partial z \partial x}, \quad & \frac{\partial^2 \varepsilon_{yy}}{\partial z \partial x} &= \frac{\partial}{\partial y}\left(+\frac{\partial \varepsilon_{yz}}{\partial x} - \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right) \\
\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} &= 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}, \quad & \frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y} &= \frac{\partial}{\partial z}\left(+\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} - \frac{\partial \varepsilon_{xy}}{\partial z}\right)
\end{aligned}
$$

Equilibrium and compatibility conditions

Combining the Equilibrium and Compatibility conditions gives:

- Unknowns: 6 stresses $+$ 6 strains $+$ 3 displacements $= 15$
- Equations: 3 equilibrium $+$ 6 compatibility $= 9$

To obtain a solution therefore requires 6 more equations. These come from the constitutive relationships

Governing equations: Stress-strain relationship

Stress - strain relationship: $\boldsymbol{\sigma} = \mathrm{D}\boldsymbol{\varepsilon}$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \mathrm{D(6x6)} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix}$$

Governing equations: Stress-strain relationship

Stress - strain relationship: $\boldsymbol{\sigma} = \mathrm{D}\varepsilon$

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{zx}\end{bmatrix} = \mathrm{D(6x6)} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{xy}\\ \varepsilon_{yz}\\ \varepsilon_{zx}\end{bmatrix}$$

Constitutive models describe the response behavior of materials under different mechanical and enviornmental conditions. The performance of materials is expressed in terms of stress, strain and internal state variables which describe the effect of the previous load histories on the current properties.

Governing equations in stress-deformation analysis

What are the variables used in the governing equations?

1. displacements u in the body
2. strains ε in the body or within the elements
3. stresses $\boldsymbol{\sigma}$ in the body or within the elements

Advanced analysis involves:

1. Equilibrium: External forces + internal stresses agree
2. Compatibility: Displacements fields agree (no gaps) + strains (derivatives)
3. Stress-strain relationship (constitutive behaviour)

Limit analysis: Lower and upper bound theorems

The exact determination of loads involved in the plastic deformation requires simultaneous solution of three sets of conditions:

1. equations of equilibrium
2. equations of compatibility
3. appropriate constitutive criteria (yield condition and flow rule)

Exact determination is often not easy, may be appropriate for simple shapes, but other cases we may have to use *numerical method*.

Limit analysis: Lower and upper bound theorems

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Calculations of slope stability are based on plasticity theory, which is held to be applicable to ductile materials such as soils. Strictly, plasticity theory applies only to perfectly plastic materials that adhere to the normality principle, which maximises the work done in plastic deformations.

Limit analysis: Lower bound theorem

"*If there is a set of external loads* **which are in equilibrium** *with a state of stress that nowhere exceeds the failure criterion for the material,* **collapse cannot occur** *and the external loads are a lower bound to the true collapse loads*"

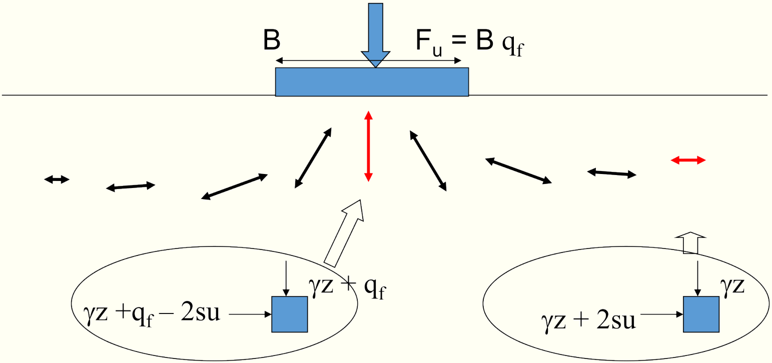
Any applied load is less than the actual limiting load, i.e., they will not cause collapse.

1. satisfies equilibrium everywhere
2. balances the externally applied loads, and
3. every point is within the yield locus.

Relax compatibility.

Lower bound solution

We need to determine a set of stresses in the ground that are in equilibrium with the external loads. Stress state everywhere should be: $\sigma_1 - \sigma_3 = 2s_u$



Limit analysis: Upper bound theorem

"*If there is a set of external loads and a mechanism of plastic collapse such that the increment of work done by the external loads in an* **increment of displacement** *equals the work done by the internal stresses,* **collapse must occur** *and the external loads are an upper bound to the true collapse loads*"

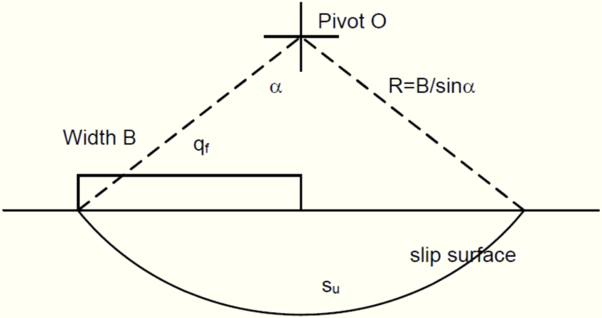
Apply enough load to achieve the desired change in component shape, e.g., process machinery.

1. satisfies the boundary displacement conditions, and
2. do not infringe incompressibility.

Relax equilibrium

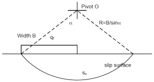
Upper bound solution

We need to determine a failure mechanism, which is kinematically feasible. The work done is equated by the plastic dissipation along the failure (or slip) surfaces.



Upper bound solution

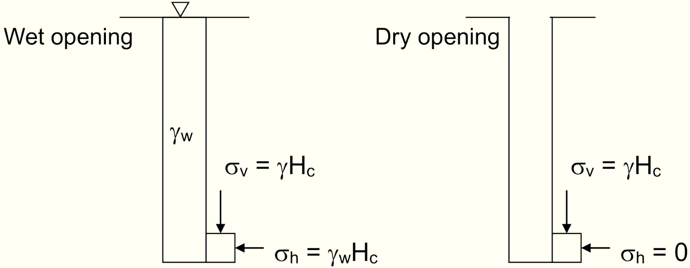
We need to determine a failure mechanism, which is kinematically feasible. The work done is equated by the plastic dissipation along the failure (or slip) surfaces.



Exact solution is between upper and lower bound.

Consider the equilibrium of an element of soil adjacent to the bottom of a cut, or crack. Assume that the major principal stress is vertical, and that the soil fails according to Tresca's criterion at a shear stress s_u (undrained strength).

Tresca's criterion: $\sigma_1 - \sigma_3 = 2s_u$

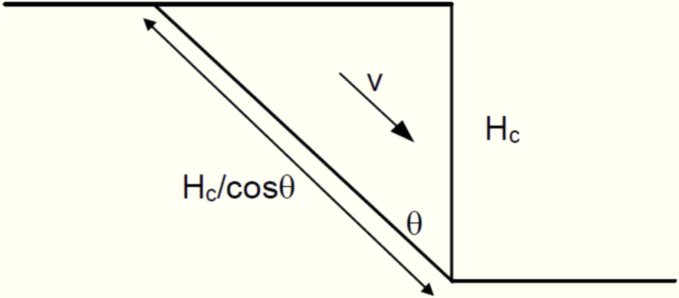


$\sigma_v - \sigma_h = 2s_u \Rightarrow H_c = 2s_u/(\gamma - \gamma_w)$

$\sigma_v - \sigma_h = 2s_u \Rightarrow H_c = 2s_u/\gamma$

The resulting maximum depth of crack, H_c, is different in wet and dry conditions. A deeper crack can be kept open if it fills with water.

Undrained cut: Upper bound solution - wedge mechanism

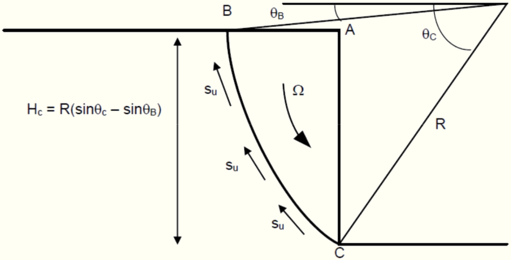


Potential energy loss = dissipation

$(1/2)\,\gamma\, H_c^2 \tan\theta \times v\cos\theta = s_u\,(H_c/\cos\theta) \times v$

Optimising, $\theta = \pi/4$, and hence $H_c = 4s_u/\gamma$

This simple upper bound solution involving a single sliding block gives a normalized cut depth, $N_s = \gamma H_c / s_u = 4$. A more refined upper bound solution involves a circular slip surface, passing through the base of the cut.

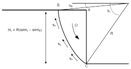


Change in potential energy of block ABC (Need to find the centroid and examine the vertical movement by $\Omega =$ Dissipation $= s_u R(\theta_c - \theta_b)R\Omega$

$$N_s = \frac{\gamma H_c}{s_u} = \frac{6(\theta_c - \theta_b)}{2\sin^2\theta_c - \sin\theta_c \sin\theta_b - \sin^2\theta_b}$$

Optimizing: $\theta_b = 27.4^o$ and $\theta_c = 57.5^o$. $H_c = 3.83 s_u/\gamma$.

Undrained cut: Upper bound solution - slip circle



Change in potential energy of block ABC (Need to find the centroid and examine the vertical movement by Ω = Dissipation = $s_u R(\theta_c - \theta_b) R\Omega$

$$N_s = \frac{\gamma H_c}{s_u} = \frac{6(\theta_c - \theta_b)}{2\sin^2\theta_c - \sin\theta_c \sin\theta_b - \sin^2\theta_b}$$

Optimizing: $\theta_b = 27.4^o$ and $\theta_c = 57.5^o$. $H_c = 3.83 s_u/\gamma$.

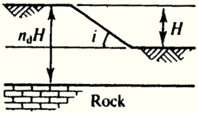
Based on this solution: how far back from the edge of an unstable vertical cut should you stand, to be safe if it fails? These upper bounds are far from the lower bound calculated. With numerical methods, the lower bound can be improved, and it is now accepted that the true solution is $\gamma H_c / s_u = 3.83$, to two significant figures, where the effective weight ($\gamma' = \gamma - \gamma_w$) should be used if the crack is water-filled.

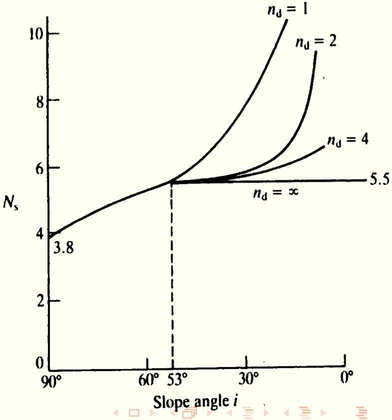
Taylor stability chart

For sloped cuts as shown on the right, the critical height at different slope angles (i), different excavation depths (H) and different rock depths ($n_d * H$) can be evaluated using the following generalised formula.

$$N_s = \frac{\gamma H_c}{s_u}$$

where N_s is the stability number ($N_s = 3.8$ for 90 degrees cut).





Methods of analysis

Method of analysis	Solution requirements				
	Equilibrium	**Compatibility**	**Constitutive law**	**Force**	**Displacement**
Closed form	✓	✓	Linear elastic	✓	✓
Limit equilibrium	✓	×	Rigid with a failure criterion	✓	×
Lower bound	✓	×	Plasticity + flow-rule	✓	×
Upper bound	×	✓	Plasticity + flow-rule	×	✓
Numerical analysis	✓	✓	Any	✓	✓