CE394M: Stress paths and invariants

Krishna Kumar

University of Texas at Austin

krishnak@utexas.edu

March 22, 2019

Overview

Stresses / strains

1D consolidation / simple shear

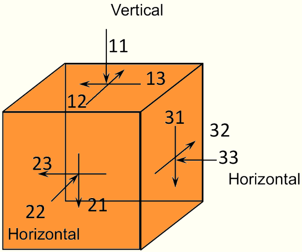
- Zero lateral strain ($\varepsilon_{22} = \varepsilon_{33} = 0$)
- Stresses: σ and τ
- Strains: $\varepsilon_{11} = \varepsilon_v$ and γ

2D plane strain

- Zero lateral strain ($\varepsilon_{22} = \gamma_{12} = \gamma_{23} = 0$)
- Stresses: s and t
- Strains: ε_v and ε_γ

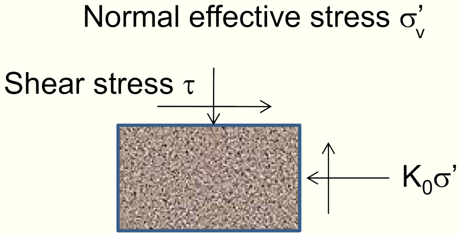
3D general (axi-symmetric as a special case)

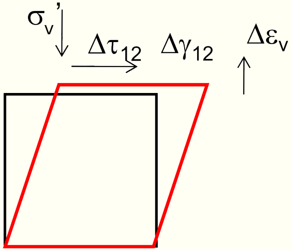
- Stresses: p and q
- Strains: ε_v and ε_s



1D simple shear

1. No lateral strain
2. Constant normal effective stress σ'_v
3. Increasing shear strain γ
4. Measure shear resistance τ
5. Measure volumetric strain ε_v or void ratio $e = e_0 - (1 + e_0)\varepsilon_v$
6. **No information for the lateral direction**



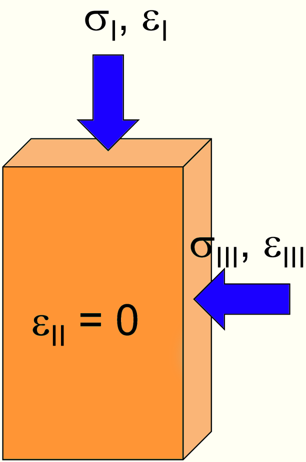


Stresses and strains: independent components

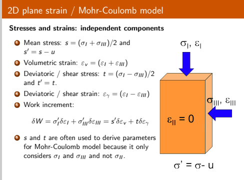
1. Mean stress: $s = (\sigma_I + \sigma_{III})/2$ and $s' = s - u$
2. Volumetric strain: $\varepsilon_v = (\varepsilon_I + \varepsilon_{III})$
3. Deviatoric / shear stress: $t = (\sigma_I - \sigma_{III})/2$ and $t' = t$.
4. Deviatoric / shear strain: $\varepsilon_\gamma = (\varepsilon_I - \varepsilon_{III})$
5. Work increment:

$$\delta W = \sigma'_I \delta\varepsilon_I + \sigma'_{III}\delta\varepsilon_{III} = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$$

6. s and t are often used to derive parameters for Mohr-Coulomb model because it only considers σ_I and σ_{III} and not σ_{II}.



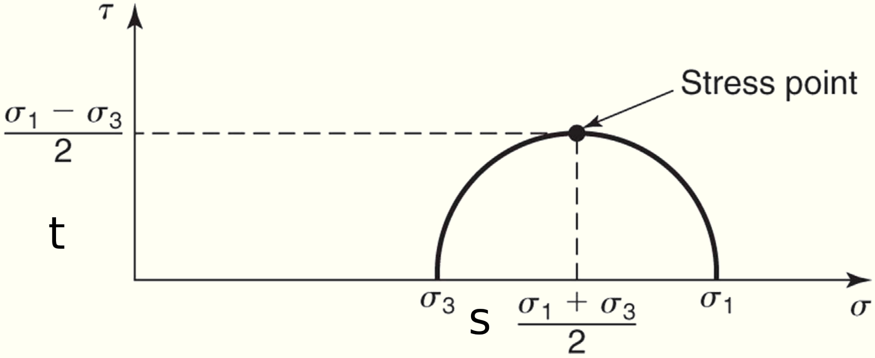
$\sigma' = \sigma - u$



Principal stresses $\sigma_I > \sigma_{II} > \sigma_{III}$. This equation holds good and is the definition of σ in principal stress notations, i.e., $I > II > III$.

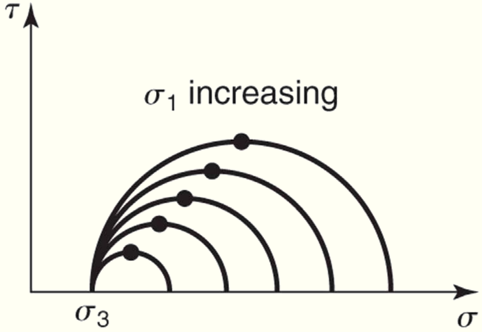
No shear stresses on these planes.

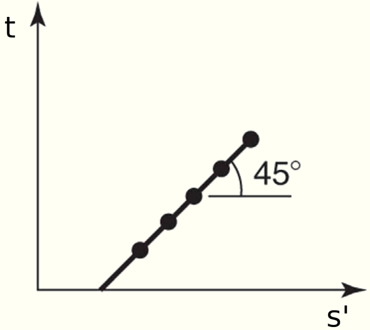
2D Mohr circle



Mohr circle to 2D stress components

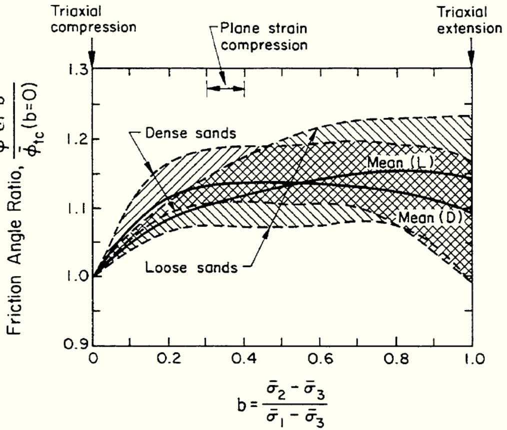
Successive Mohr circles and stress path for constant σ_3 and increasing σ_1:



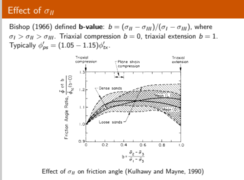


Effect of σ_{II}

Bishop (1966) defined **b-value**: $b = (\sigma_{II} - \sigma_{III})/(\sigma_I - \sigma_{III})$, where $\sigma_I > \sigma_{II} > \sigma_{III}$. Triaxial compression $b = 0$, triaxial extension $b = 1$. Typically $\phi'_{ps} = (1.05 - 1.15)\phi'_{tx}$.



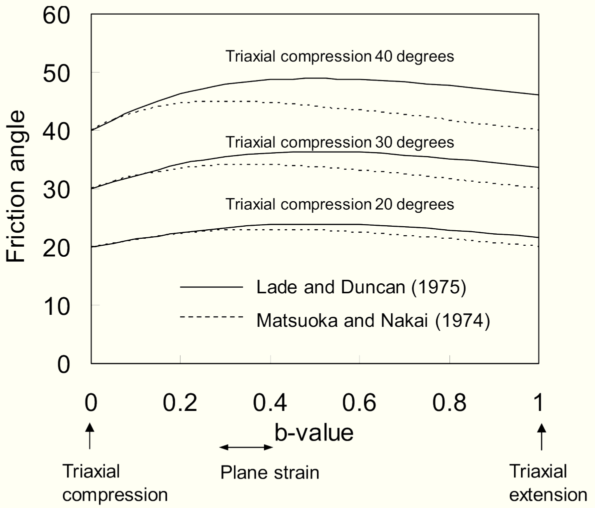
Effect of σ_{II} on friction angle (Kulhawy and Mayne, 1990)



σ_{II} do have an effect on soil behavior. For example, the friction angle depends on the loading condition: triaxial compression, plane-strain, triaxial extension and others ...., The effect of σ_{II} can be measured using true triaxial apparatus or hollow cylinder torsional shear apparatus. In general, the peak friction angle increases 10 to 15 percent from b = 0 (triaxial compression) to b = 0.3 to 0.4 (plane strain), and it stays constant or slightly decreases as b reaches 1 (tri-axial extension).

The variation of measured friction angle with changes in σ_{II} can be attributed to the effects of different mean stress and stress anisotropy on the dilatancy and particle rearrangement contributions to the total strength. For given maximum and minimum principal stresses, the TXE conditions have the largest mean effective stress, whereas the triaxial compression conditions have the smallest mean effective stress. The higher confinement for triaxial extension and plane strain conditions contributes to the increasing friction angle for these conditions. (Mitchell and Soga., 2005)

Effect of σ_{II} on friction



Chapter 11., Mitchell and Soga, 2005

Effect of σ_{II} on friction

Various models fit the experimental data showing the intermediate stress effect. Lade and Duncan (1975)

$$\frac{I_1^3}{I_3} = const$$

Matsuoka and Nakai (1974)

$$\frac{I_1 I_2}{I_3} = const$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$
$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$
$$I_3 = \sigma_1\sigma_2\sigma_3$$

Matsuoka and Nakai's model gives the same friction angle for compression and extension, whereas Lade and Duncan's model gives the ratio of the TXE friction angle (ϕ'_{TXE}) to the TXC friction angle (ϕ'_{TXC}) to be 1.08 at $\phi'_{TXC} = 20^o$ to 1.15 at $\phi'_{TXC} = 40^o$.

Effect of σ_{II} on friction

Various models fit the experimental data showing the intermediate stress effect. Lade and Duncan (1975)

$$\frac{I_1^3}{I_3} = const$$

Matsuoka and Nakai (1974)

$$\frac{I_1 I_2}{I_3} = const$$

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\ I_2 &= \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \\ I_3 &= \sigma_1\sigma_2\sigma_3 \end{aligned}$$

Matsuoka and Nakai's model gives the same friction angle for compression and extension, whereas Lade and Duncan's model gives the ratio of the TXE friction angle (ϕ'_{TXE}) to the TXC friction angle (ϕ'_{TXC}) to be 1.08 at $\phi'_{TXC} = 20^o$ to 1.15 at $\phi'_{TXC} = 40^o$.

Given the large scatter in the published experimental data, it is not possible to conclude that one model is better than the ohter.

Triaxial stresses and strains: independent components

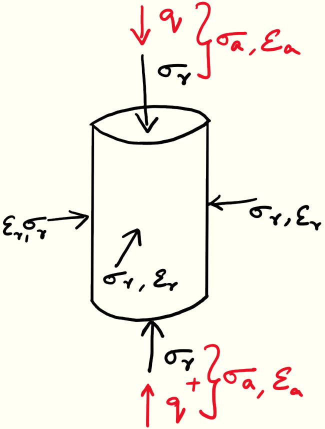
- We split the stress system into:
  - purely volumetric deformation 'p'
  - purely distortional deformation 'q'
- Mean stress:

$$p = (\sigma_a + 2\sigma_r)/3 \quad p' = p - u$$

- Volumetric strain: $\varepsilon_v = (\varepsilon_a + 2\varepsilon_r)$
- Deviatoric / shear stress (purely distortional deformation):

$$q = (\sigma_a - \sigma_r) \quad q' = q$$

- Deviatoric / shear strain: $\varepsilon_s = 2/3(\varepsilon_a - \varepsilon_r)$



Triaxial deviatoric strain

- Deviatoric / shear strain: $\varepsilon_s = 2/3(\varepsilon_a - \varepsilon_r)$
- Why 2/3?:
- Work equation: $\delta W = p'\delta\varepsilon_v + q'\delta\varepsilon_s$

$$
\begin{aligned}
\delta W &= \sigma'_1\delta\varepsilon_1 + \sigma'_2\Delta\varepsilon_2 + \sigma'_3\Delta\varepsilon_3 = \sigma'_a\delta\varepsilon_a + 2\sigma'_r\delta\varepsilon_r \\
\delta W &= (1/3\sigma'_a + 2/3\sigma'_r)(\varepsilon_a + 2\varepsilon_r) + (\sigma'_a - \sigma'_r)(2/3)(\varepsilon_a - \varepsilon_r) \\
&= \sigma'_a\delta\varepsilon_a + 2\sigma'_r\delta\varepsilon_r
\end{aligned}
$$

Triaxial stress - strain relationship

Relationship in terms of axial and radial direction:

$$\begin{bmatrix} \varepsilon_a \\ \varepsilon_r \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -2\nu \\ -\nu & (1-\nu) \end{bmatrix} \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \tag{1}$$

Relationship in terms of 'p' and 'q':

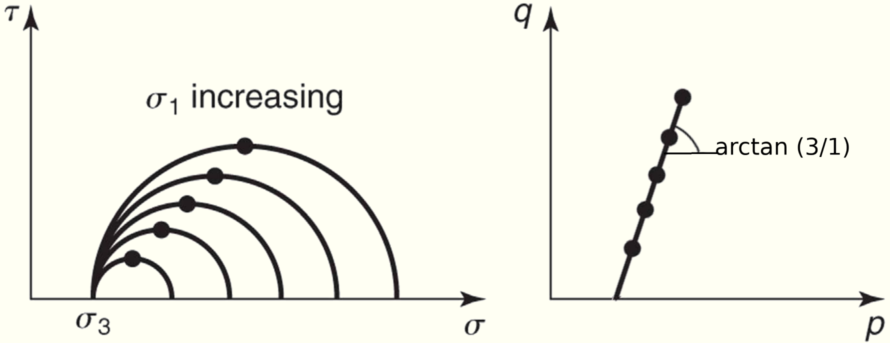
$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \quad \text{invert} \rightarrow \begin{bmatrix} \sigma_a \\ \sigma_r \end{bmatrix} \begin{bmatrix} 1 & 2/3 \\ 1 & -1/3 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \tag{2}$$

Strain:

$$\begin{bmatrix} \varepsilon_p \\ \varepsilon_q \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2/3 & -2/3 \end{bmatrix} \begin{bmatrix} \varepsilon_a \\ \varepsilon_r \end{bmatrix} \tag{3}$$

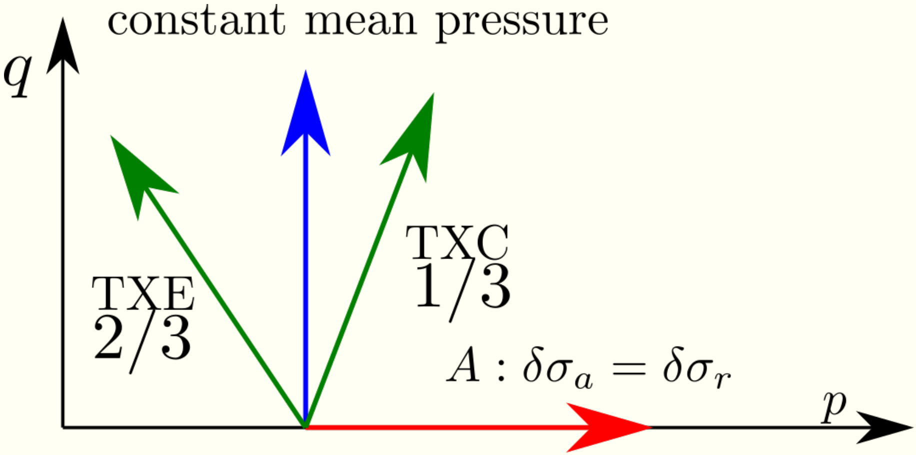
Mohr circle to 3D stress components

Successive Mohr circles and stress path for constant σ_3 and increasing σ_1:



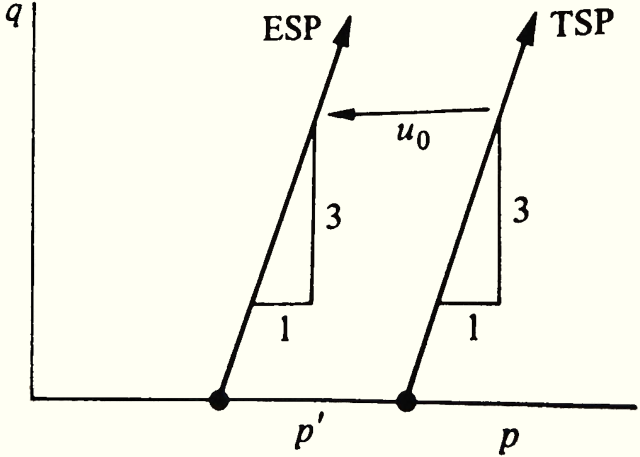
Stress paths p-q

Different stress paths for initially hydrostatic stress conditions:

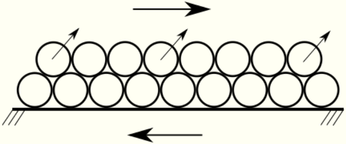


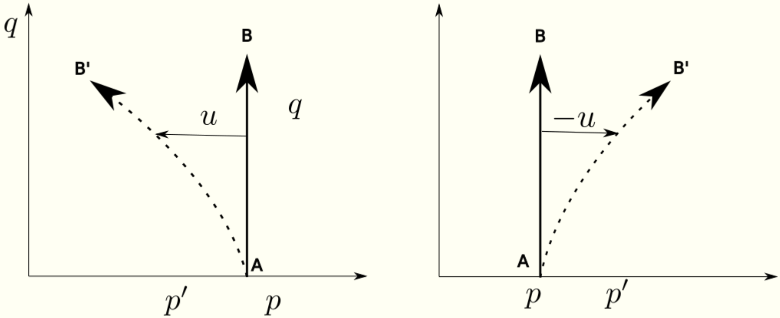
Triaxial compression

TXC with constant back pressure u_o (TSP: Total stress path; ESP: Effective stress path)



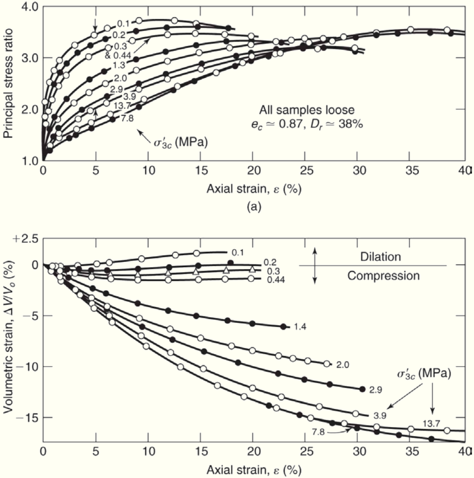
Triaxial compression undrained: loose v dense





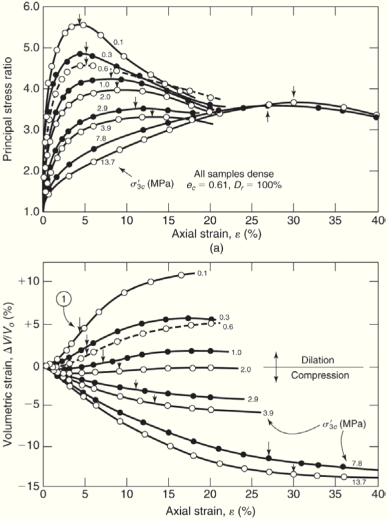
Total and effective stress paths for undrained triaxial test: (a) on soil that wishes to contract as it is sheared, and (b) on soil that wishes to expand as it is sheared.

Triaxial compression drained: loose



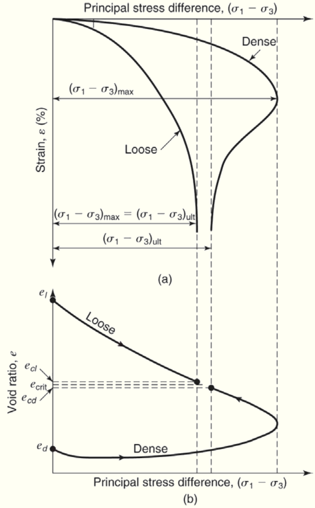
Loose Sacramento River sand: (a) principal stress ratio versus axial strain; (b) volumetric strain versus axial strain (Lee, 1965).

Triaxial compression drained: dense



Dense Sacramento River sand: (a) principal stress ratio versus axial strain; (b) volumetric strain versus axial strain (Lee, 1965).

Triaxial compression drained: loose v dense



Triaxial tests on "loose" and "dense" specimens of a typical sand: (a) stress-strain curves; (b) void ratio changes during shear (Hirschfeld, 1963).