which is the increment of stiffness per unit of depth, as indicated in Figure 2.3.

Together with the input of E_{inc} the input of y_{ref} becomes relevant. Above y_{ref} the stiffness is equal to E_{ref}. Below the stiffness is given by:

$$E(y) = E_{ref} + (y_{ref} - y) E_{inc} \qquad y < y_{ref} \tag{2.25}$$

The Linear Elastic model is usually inappropriate to model the highly non-linear behaviour of soil, but it is of interest to simulate structural behaviour, such as thick concrete walls or plates, for which strength properties are usually very high compared with those of soil. For these applications, the Linear Elastic model will often be selected together with *Non-porous* drainage type in order to exclude pore pressures from these structural elements.

2.4 UNDRAINED EFFECTIVE STRESS ANALYSIS (EFFECTIVE STIFFNESS PARAMETERS)

In PLAXIS it is possible to specify undrained behaviour in an effective stress analysis using effective model parameters. This is achieved by identifying the type of material behaviour (*Drainage type*) of a soil layer as *Undrained (A)* or *Undrained (B)* (Section 2.5 and Section 2.6). In this section, it is explained how PLAXIS deals with this special option.

The presence of pore pressures in a soil body, usually caused by water, contributes to the total stress level. According to Terzaghi's principle, total stresses $\underline{\sigma}$ can be divided into effective stresses $\underline{\sigma}'$, active pore pressure p_{active} and pore water pressures p_w (see also Eq. 2.3). However, water is supposed not to sustain any shear stress, and therefore the effective shear stresses are equal to the total shear stresses:

$$\underline{\sigma} = \underline{\sigma}' + \underline{m}\, p_{active} \tag{2.26a}$$

where,

$$\underline{m} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad p_{active} = \alpha_{Biot} S_{eff} p_w \tag{2.26b}$$

$$\sigma_{xx} = \sigma'_{xx} + \alpha_{Biot} S_{eff} p_w \tag{2.26c}$$

$$\sigma_{yy} = \sigma'_{yy} + \alpha_{Biot} S_{eff} p_w \tag{2.26d}$$

$$\sigma_{zz} = \sigma'_{zz} + \alpha_{Biot} S_{eff} p_w \tag{2.26e}$$

$$\sigma_{xy} = \sigma'_{xy} \tag{2.26f}$$

$$\sigma_{yz} = \sigma'_{yz} \tag{2.26g}$$

$$\sigma_{zx} = \sigma'_{zx} \tag{2.26h}$$

where α_{Biot} is Biot's pore pressure coefficient and S_{eff} is the effective degree of saturation. Considering incompressible grains, Biot's coefficient α_{Biot} is equal to unity (α_{Biot} = 1). The situation of compressible grains or compressible solid material (α_{Biot} < 1) is explained in more detail at the end of this section.

Note that, similar to the total and the effective stress components, p_w is considered negative for pressure.

The product $\alpha_{Biot} S_{eff} p_w$ is termed 'Active pore pressure', p_{active} in PLAXIS. A further distinction is made between steady state pore stress, p_{steady}, and excess pore stress, p_{excess}:

$$p_w = p_{steady} + p_{excess} \tag{2.27}$$

Steady state pore pressures are considered to be input data, i.e. generated on the basis of phreatic levels or by means of a groundwater flow calculation. Excess pore pressures are generated during plastic calculations for the case of undrained (A) or (B) material behaviour or during a consolidation analysis. Undrained material behaviour and the corresponding calculation of excess pore pressures are described below.

Since the time derivative of the steady state component equals zero, it follows:

$$\dot{p}_w = \dot{p}_{excess} \tag{2.28}$$

Hooke's law can be inverted to obtain:

$$\begin{bmatrix} \dot{\varepsilon}^e_{xx} \\ \dot{\varepsilon}^e_{yy} \\ \dot{\varepsilon}^e_{zz} \\ \dot{\gamma}^e_{xy} \\ \dot{\gamma}^e_{yz} \\ \dot{\gamma}^e_{zx} \end{bmatrix} = \frac{1}{E'} \begin{bmatrix} 1 & -\nu' & -\nu' & 0 & 0 & 0 \\ -\nu' & 1 & -\nu' & 0 & 0 & 0 \\ -\nu' & -\nu' & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu' & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu' & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu' \end{bmatrix} \begin{bmatrix} \dot{\sigma}'_{xx} \\ \dot{\sigma}'_{yy} \\ \dot{\sigma}'_{zz} \\ \dot{\sigma}'_{xy} \\ \dot{\sigma}'_{yz} \\ \dot{\sigma}'_{zx} \end{bmatrix} \tag{2.29}$$

Substituting Eq. (2.26) gives:

$$\begin{bmatrix} \dot{\varepsilon}^e_{xx} \\ \dot{\varepsilon}^e_{yy} \\ \dot{\varepsilon}^e_{zz} \\ \dot{\gamma}^e_{xy} \\ \dot{\gamma}^e_{yz} \\ \dot{\gamma}^e_{zx} \end{bmatrix} = \frac{1}{E'} \begin{bmatrix} 1 & -\nu' & -\nu' & 0 & 0 & 0 \\ -\nu' & 1 & -\nu' & 0 & 0 & 0 \\ -\nu' & -\nu' & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu' & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu' & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu' \end{bmatrix} \begin{bmatrix} \dot{\sigma}_{xx} - \alpha_{Biot}\dot{p}_w \\ \dot{\sigma}_{yy} - \alpha_{Biot}\dot{p}_w \\ \dot{\sigma}_{zz} - \alpha_{Biot}\dot{p}_w \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_{yz} \\ \dot{\sigma}_{zx} \end{bmatrix} \tag{2.30}$$

Considering slightly compressible water, the rate of excess pore pressure is written as:

$$\dot{p}_{excess} = \frac{\alpha_{Biot}\dot{\varepsilon}_v}{nC_w + (\alpha_{Biot} - n)C_s} \tag{2.31a}$$

$$C_w = \frac{1}{K_w} \tag{2.31b}$$

$$C_s = \frac{1}{K_s} \tag{2.31c}$$

in which K_w is the bulk modulus of the water, K_s is the bulk modulus of the solid material, C_w is the compressibility of the water, C_s is the compressibility of the solid material and n is the soil porosity.

$$n = \frac{e_0}{1 + e_0} \tag{2.32}$$

where e_0 is the initial void ratio as specified in the general soil properties.

The inverted form of Hooke's law may be written in terms of the total stress rates and the undrained parameters E_u and ν_u:

$$\begin{bmatrix} \dot{\varepsilon}^e_{xx} \\ \dot{\varepsilon}^e_{yy} \\ \dot{\varepsilon}^e_{zz} \\ \dot{\gamma}^e_{xy} \\ \dot{\gamma}^e_{yz} \\ \dot{\gamma}^e_{zx} \end{bmatrix} = \frac{1}{E_u} \begin{bmatrix} 1 & -\nu_u & -\nu_u & 0 & 0 & 0 \\ -\nu_u & 1 & -\nu_u & 0 & 0 & 0 \\ -\nu_u & -\nu_u & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu_u & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu_u & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu_u \end{bmatrix} \begin{bmatrix} \dot{\sigma}_{xx} \\ \dot{\sigma}_{yy} \\ \dot{\sigma}_{zz} \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_{yz} \\ \dot{\sigma}_{zx} \end{bmatrix} \tag{2.33}$$

where:

$$E_u = 2G(1 + \nu_u); \quad \nu_u = \frac{3\nu' + \alpha_{Biot}B(1 - 2\nu')}{3 - \alpha_{Biot}B(1 - 2\nu')}; \quad B = \frac{\alpha_{Biot}}{\alpha_{Biot} + n\left(\frac{K'}{K_w} + \alpha_{Biot} - 1\right)} \tag{2.34}$$

where B is Skempton's B-parameter.

Hence, the special option for undrained behaviour in PLAXIS (*Undrained (A)* or *Undrained (B)*) is such that the effective parameters G and ν' are transformed into undrained parameters E_u and ν_u according to Eq. (2.34). Note that the index u is used to indicate auxiliary parameters for undrained soil. Hence, E_u and ν_u should not be confused with E_{ur} and ν_{ur} as used to denote unloading / reloading.

Fully incompressible behaviour is obtained for $\nu_u = 0.5$. However, taking $\nu_u = 0.5$ leads to singularity of the stiffness matrix. In fact, water is not fully incompressible, but a realistic bulk modulus for water is very large. In order to avoid numerical problems caused by an extremely low compressibility, ν_u is, by default, taken as 0.495, which makes the undrained soil body slightly compressible. In order to ensure realistic computational results, the bulk modulus of the water must be high compared with the bulk modulus of the soil skeleton, i.e. $K_w >> nK'$. This condition is sufficiently ensured by requiring $\nu' \leq 0.35$.

Consequently, for material behaviour *Undrained (A)* or *Undrained (B)*, a bulk modulus for water is automatically added to the stiffness matrix. The bulk modulus of water is obtained in three ways: automatically from Eq. (2.35) (*Standard*), automatically but by specifying the Skempton's B-parameter (*Manual - Stiffness dependent K_w*) and manually by directly specifying K_w and α_{Biot} (*Manual - Constant K_w*).

$$\frac{K_w}{n} = \frac{3(\nu_u - \nu')}{(1 - 2\nu_u)(1 + \nu')} K' = \frac{0.495 - \nu'}{1 + \nu'} 300K' \geq 30K' \text{ (For } \alpha_{Biot} = 1) \tag{2.35}$$

Hint: In the UBC3D-PLM model, the implicit Poisson's ratio (ν') is defined based on the elastic bulk modulus K and the elastic shear modulus G at the current stress state, $\nu' = (3K - 2G)/(6K + 2G)$. For this reason, the value follows the evolution of the stiffnesses and it is not constant.

Hence, K_w/n is larger than $30K'$, at least for $\nu' \leq 0.35$ and $\alpha_{Biot} = 1$. The bulk stiffness of water K_w, calculated in this way, is a numerical value related to the soil stiffness. It is lower than or equal to the real bulk stiffness of pure water, K_w^0 ($2 \cdot 10^6$ kN/m^2). In retrospect it is worth mentioning here a review about the Skempton B-parameter.

Skempton B-parameter

When the *Drainage type* is set to *Undrained (A)* or *Undrained (B)*, PLAXIS automatically assumes an implicit undrained bulk modulus, K_u, for the soil as a whole (soil skeleton + water) and distinguishes between total stress rates, effective stress rates and rates of excess pore pressure:

Total stress: $\dot{p} = K_u \dot{\varepsilon}_\nu$

Excess pore pressure: $\dot{p}_{excess} = B\dot{p} = \dfrac{\alpha_{Biot}\dot{\varepsilon}_\nu}{nC_w + (\alpha_{Biot} - n)C_s}$

Effective stress: $\dot{p}' = (1 - \alpha_{Biot}B)\dot{p} = K'\dot{\varepsilon}_\nu$

Note that for *Undrained (A)* or *Undrained (B)* effective stiffness parameters should be entered in the material data set, i.e. E' and ν' and not E_u and ν_u, or the respective stiffness parameters in advanced models; the latter should be done for *Undrained (C)* behaviour in a total stress analysis (Section 2.7). The undrained bulk modulus is automatically calculated by PLAXIS using Hooke's law of elasticity:

$$K_u = \frac{2G(1+\nu_u)}{3(1-2\nu_u)} \qquad \text{where } G = \frac{E'}{2(1+\nu')}$$

When using the *Standard* option, $\nu_u = 0.495$ and $\alpha_{Biot} = 1$, whereas when using the *Manual* option with input of Skempton's B-parameter (*Manual - Stiffness dependent K_w*) or Biot's pore pressure coefficient (*Manual - Constant K_w*), ν_u is calculated as:

$$\nu_u = \frac{3\nu' + \alpha_{Biot} B(1-2\nu')}{3 - \alpha_{Biot} B(1-2\nu')}$$

If the soil is only partially saturated, the pores contain both air and water and the equivalent bulk modulus of the pore fluid K_w^{unsat} is expected to be much smaller compared to that of pure water, in the case of fully saturated soil K_w. PLAXIS assumes a constant gas pressure i.e. air is always drained, the drainage type of soil only relates to the drainage type of liquid water. Excess pore pressure change will therefore depend on the bulk modulus of liquid water and the surface tension (capillary effect). This assumption allows to establish the following rigorous relationship for the unsaturated bulk modulus of the pore fluid:

$$K_w^{unsat} = \frac{K_w}{1 + \dfrac{K_w}{S_w}\left(-\dfrac{\partial S_w}{\partial p_w}\right)} \tag{2.36}$$

where $\dfrac{\partial S_w}{\partial p_w}$ is the derivative of saturation w.r.t. pore water pressure (suction), i.e. the slope of water retention curve

The value of Skempton's B-parameter is calculated from the ratio of the bulk stiffnesses of the soil skeleton and the pore fluid, as already defined in Eq. (2.34):

$$B = \frac{\alpha_{Biot}}{\alpha_{Biot} + n\left(\dfrac{K'}{K_w} + \alpha_{Biot} - 1\right)}$$

The rate of excess pore pressure is calculated from the (small) volumetric strain rate, according to:

$$\dot{p}_{excess} = \frac{\alpha_{Biot}\dot{\varepsilon}_v}{nC_w + (\alpha_{Biot} - n)C_s} \tag{2.37}$$

The types of elements used in PLAXIS are sufficiently adequate to avoid mesh locking effects for nearly incompressible materials.

This special option to model undrained material behaviour on the basis of effective model parameters is available for most material models in PLAXIS. This enables undrained calculations to be executed with effective stiffness parameters, with explicit distinction between effective stresses and (excess) pore pressures. However, shear induced (excess) pore pressure may not be sufficiently included.

Such an analysis requires effective soil parameters and is therefore highly convenient when such parameters are available. For soft soil projects, accurate data on effective parameters may not always be available. Instead, in situ tests and laboratory tests may have been performed to obtain undrained soil parameters. In such situations measured undrained Young's moduli can be easily converted into effective Young's moduli based on Hooke's law:

$$E' = \frac{2(1 + \nu')}{3} E_u \tag{2.38}$$

For advanced models there is no such direct conversion possible. In that case it is recommended to estimate the required effective stiffness parameter from the measured undrained stiffness parameter, then perform a simple undrained test to check the resulting undrained stiffness and adapt the effective stiffness if needed. The *Soil test* facility (Reference Manual) may be used as a convenient tool to perform such test.

Biot pore pressure coefficient α_{Biot}

In general, for geotechnical applications, the compressibility of the soil skeleton is much higher than the compressibility of the individual grains, so deformations of the grains themselves can be ignored. However, in the case of very deep soil layers at very high pressures, the stiffness of the soil or rock matrix comes close to the stiffness of the solid material of which the soil grains or the rock is composed of, and, therefore, the compressibility of the solid material cannot be ignored. This has consequences for the division of total stress into effective stress and pore pressure. Considering compressible solid material, Terzaghi's effective stress definition changes into:

$$\underline{\sigma}' = \underline{\sigma} - \alpha_{Biot} S_{eff} \underline{m} p_w \tag{2.39}$$

Where α_{Biot} is Biot's pore pressure coefficient, S_{eff} is the effective degree of saturation, $\underline{m}$ is a vector with unity values (1) for the normal components and 0-values for the shear components, and p_w is the pore water pressure. The alpha coefficient is defined as:

$$\alpha_{Biot} = 1 - \frac{K'}{K_s} \tag{2.40}$$

Where K' is the effective bulk modulus of the soil matrix and K_s is the bulk modulus of the solid material. Indeed, for incompressible solid material ($K_s = \infty$) Terzaghi's original stress definition is retained. A lower value of α_{Biot} implies that for a given value of total stress and pore water pressure, the resulting effective stress is higher than when considering incompressible solid material ($\alpha_{Biot} = 1$).

In the case of undrained soil behaviour (*Undrained A* or *B* in PLAXIS), Biot's pore pressure coefficient also affects the undrained Poisson's ratio ν_u that is automatically calculated by PLAXIS based on a manual input of K_w parameter (see Eq. (2.34)).

The default value of Biot's pore pressure coefficient is 1.0 (*Standard* and *Manual - Stiffness dependent* K_w option), but users may change this value in the range of [0.001, 1.0] for the *Manual - Constant* K_w option.

2.5 UNDRAINED EFFECTIVE STRESS ANALYSIS WITH EFFECTIVE STRENGTH PARAMETERS (*UNDRAINED A*)

In principle, undrained effective stress analysis as described in Section 2.4 can be used in combination with effective strength parameters φ' and c' to model the material's undrained shear strength (*Undrained (A)*). In this case, the development of the pore pressure plays a crucial role in providing the right effective stress path that leads to failure at a realistic value of undrained shear strength (c_u or s_u). However, note that most soil models are not capable of providing the right effective stress path in undrained loading. As a result, they will produce the wrong undrained shear strength if the material strength has been specified on the basis of effective strength parameters. Another problem is that for undrained materials effective strength parameters are usually not available from soil investigation data. In order to overcome these problems, some models allow for a direct input of undrained shear strength. This approach is described in Section 2.6.

If the user wants to model the material strength of undrained materials using the effective strength parameters φ' and c', this can be done in PLAXIS in the same way as for drained materials. However, in this case the *Drainage type* must be set to *Undrained (A)*. As a result, PLAXIS will automatically add the stiffness of water to the stiffness matrix (see Section 2.4) in order to distinguish between effective stresses and (excess) pore pressures (= effective stress analysis). The advantage of using effective strength parameters in undrained loading conditions is that after consolidation a qualitatively increased shear strength is obtained, although this increased shear strength could also be quantitatively wrong, for the same reason as explained before.

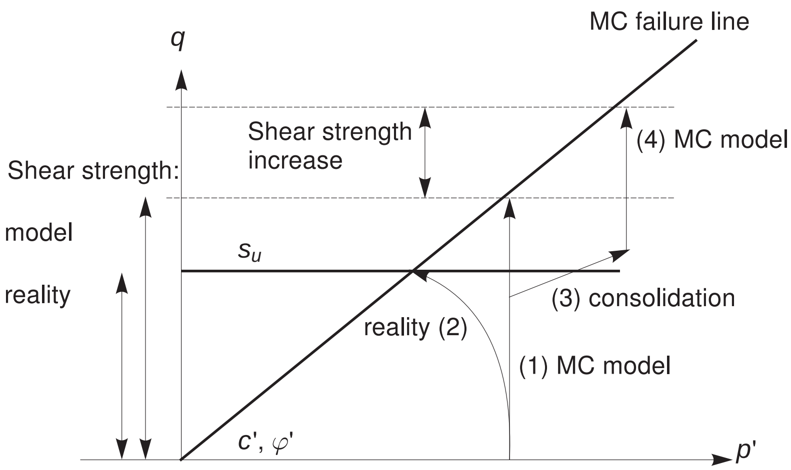

Figure 2.4 Illustration of stress paths; reality vs. Mohr-Coulomb model

Figure 2.4 illustrates an example using the Mohr-Coulomb model. When the *Drainage type* is set to *Undrained (A)*, the model will follow an effective stress path where the mean effective stress, p', remains constant all the way up to failure (1). It is known that especially soft soils, like normally consolidated clays and peat, will follow an effective stress path in undrained loading where p' reduces significantly as a result of shear induced pore pressure (2). As a result, the maximum deviatoric stress that can be reached in the model is over-estimated in the Mohr-Coulomb model. In other words, the mobilised shear strength in the model supersedes the available undrained shear strength.

If, at some stress state, the soil is consolidated, the mean effective stress will increase (3). Upon further undrained loading with the Mohr-Coulomb model, the observed shear strength will be increased (4) compared to the previous shear strength, but this increased shear strength may again be unrealistic, especially for soft soils.

On the other hand, advanced models do include, to some extent, the reduction of mean effective stress in undrained loading, but even when using advanced models it is generally advised to check the mobilised shear strength in the Output program against the available (undrained) shear strength when this approach is followed.

Note that whenever the *Drainage type* parameter is set to *Undrained (A)*, effective values must be entered for the stiffness parameters (Young's modulus E' and Poisson ratio ν' in case of the Mohr-Coulomb model or the respective stiffness parameters in the advanced models).

Care must be taken when using *Undrained (A)* together with a non-zero dilatancy angle ψ. The use of a positive dilatancy angle may lead to unrealistically large tensile pore stresses and, as a result, an unrealistically large shear strength. The use of a negative dilatancy angle may lead to unrealistically high pore pressure and unrealistic liquefication type of behaviour. Hence, for *Undrained (A)* it is recommended to use $\psi = 0$.

2.6 UNDRAINED EFFECTIVE STRESS ANALYSIS WITH UNDRAINED STRENGTH PARAMETERS (*UNDRAINED B*)

For undrained soil layers with a known undrained shear strength profile, PLAXIS offers for some models the possibility of an undrained effective stress analysis, as described in Section 2.4, with direct input of the undrained shear strength, i.e. setting the friction angle to zero and the cohesion equal to the undrained shear strength ($\varphi = \varphi_u = 0°$; $c = s_u$) (*Drainage type* = *Undrained (B)*). Also in this case, distinction is made between pore pressures and effective stresses. Although the pore pressures and effective stress path may not be fully correct, the resulting undrained shear strength is not affected, since it is directly specified as an input parameter.

The option to perform an undrained effective stress analysis with undrained strength properties is only available for the Mohr-Coulomb model, the Hardening Soil model, the HS small model and the NGI-ADP model. Since most soils show an increasing shear strength with depth, it is possible to specify the increase per unit of depth in PLAXIS in the *Advanced* subtree in the *Parameters* tabsheet of the *Soil* window.

Note that if the Hardening Soil model or the HS small model is used with $\varphi = 0°$, the stiffness moduli in the model are no longer stress-dependent and the model exhibits no compression hardening, although the model retains its separate unloading-reloading modulus and shear hardening. Also note that a direct input of undrained shear strength does not automatically give the increase of shear strength with consolidation.

Further note that whenever the *Drainage type* parameter is set to *Undrained (B)*, effective values must be entered for the stiffness parameters (Young's modulus E' and Poisson ratio ν' in case of the Mohr-Coulomb model or the respective stiffness parameters in the advanced models).

2.7 UNDRAINED TOTAL STRESS ANALYSIS WITH UNDRAINED PARAMETERS (*UNDRAINED C*)

If, for any reason, it is desired not to use the *Undrained (A)* or *Undrained (B)* options in PLAXIS to perform an undrained effective stress analysis, one may simulate undrained behaviour using a conventional total stress analysis with all parameters specified as undrained. In that case, stiffness is modelled using an undrained Young's modulus E_u and an undrained Poisson ratio ν_u, and strength is modelled using an undrained shear strength s_u and $\varphi = \varphi_u = 0°$. Typically, for the undrained Poisson ratio a value close to 0.5 is selected (between 0.495 and 0.499). A value of 0.5 exactly is not possible, since this would lead to singularity of the stiffness matrix.

In PLAXIS it is possible to perform a total stress analysis with undrained parameters if the Mohr-Coulomb model or the NGI-ADP model is used. In this case, one should select *Undrained (C)* as the *Drainage type*. The disadvantage of the undrained total stress analysis is that no distinction is made between effective stresses and pore pressures. Hence, all output referring to effective stresses should now be interpreted as total stresses and all pore pressures are equal to zero.

Note that a direct input of undrained shear strength does not automatically give the increase of shear strength with consolidation. In fact, it does not make sense to perform a consolidation analysis since there are no pore pressures to consolidate. Also note that the K_0-value to generate initial stresses refers to total stresses rather than effective stresses in this case. This type of approach is not possible for most advanced models.

Overview of models and allowable drainage types

Material model	Drainage type
Linear Elastic model	Drained Undrained (A) Undrained (C) Non-porous
Mohr-Coulomb model	Drained Undrained (A) Undrained (B) Undrained (C) Non-porous
Hardening Soil model	Drained Undrained (A) Undrained (B)

HS small model	Drained Undrained (A) Undrained (B)
UBC3D-PLM model	Drained Undrained (A)
Soft Soil model	Drained Undrained (A)
Soft Soil Creep model	Drained Undrained (A)
Jointed Rock model	Drained Non-porous
Modified Cam-Clay model	Drained Undrained (A)
NGI-ADP model	Drained Undrained (C)
UDCAM-S model	Undrained (C)
Hoek-Brown model	Drained Non-porous
Sekiguchi-Ohta model	Drained Undrained (A)
Concrete model	Drained Non-porous
User-defined soil models	Drained Undrained (A) Non-porous

2.8 THE INITIAL PRE-CONSOLIDATION STRESS IN ADVANCED MODELS

When using advanced models in PLAXIS an initial pre-consolidation stress has to be determined. In the engineering practice it is common to use a vertical pre-consolidation stress, σ_p, but PLAXIS needs an equivalent isotropic pre-consolidation stress, p_p^{eq} to determine the initial position of a cap-type yield surface. If a material is over-consolidated, information is required about the Over-Consolidation Ratio (OCR), i.e. the ratio of the greatest effective vertical stress previously reached, σ_p (see Figure 2.5), and the in-situ