CE394M: Finite Element Analysis in Geotechnical Engineering

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Overview

1. Geotechnical FEA
   - Element types
   - Discretization
   - Boundary conditions
   - Errors in FEA
2. Slope stability

FE Disclaimer

IMPORTANT WARNING AND DISCLAIMER

PLAXIS is a finite element program for geotechnical applications in which soil models are used to simulate the soil behaviour. The PLAXIS code and its soil models have been developed with great care. Although a lot of testing and validation have been performed, it cannot be guaranteed that the PLAXIS code is free of errors. Moreover, the simulation of geotechnical problems by means of the finite element method implicitly involves some inevitable numerical and modelling errors. The accuracy at which reality is approximated depends highly on the expertise of the user regarding the modelling of the problem, the understanding of the soil models and their limitations, the selection of model parameters, and the ability to judge the reliability of the computational results. Hence, PLAXIS may only be used by professionals that possess the aforementioned expertise. The user must be aware of his/her responsibility when he/she uses the computational results for geotechnical design purposes. The PLAXIS organization cannot be held responsible or liable for design errors that are based on the output of PLAXIS calculations.

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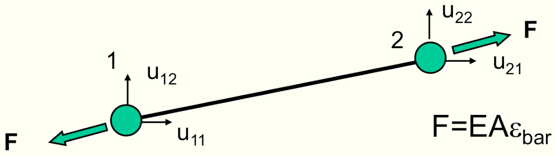
Consistent system of units

	SI			
Length	m	m	m	cm
Density	kg / m^3	10^3 kg / m^3	10^6 kg / m^3	10^6 g / cm^3
Force	N	kN	MN	Mdynes
Stress	Pa	kPa	MPa	bar
Gravity	m / sec^2	m / sec^2	m / sec^2	cm / s^2
Stiffness*	Pa / m	kPa / m	MPa / m	bar / cm

Problem definition

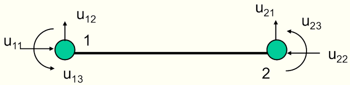
1D Finite Elements: Bar element

Two node element with axial stiffness only (no flexural or shear resistance).



1D Finite Elements: Beam element

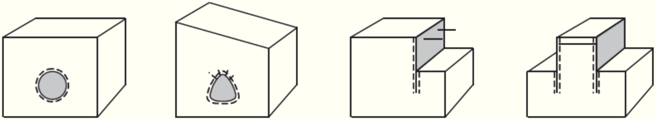
two node structure element with axial and bending stiffness (no transverse shear deformation). Three degrees of freedom for 2D beam element (1, 2 displacements and a moment).



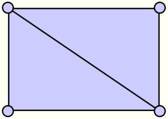
$F_a = EA\,\varepsilon_a$

$V = -EI(u_{12}-u_{21})/L^3 - 6EI(u_{13}+u_{23})/L^2$

$M = EI(u_{13}-u_{23})/L$



2D plane-strain / axisymmetric elements

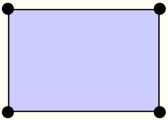


3 nodes element

linear variation of displacement within the element = constant strain in the element

$$d_1 = \alpha_1 + \alpha_2 x + \alpha_3 y$$

$$d_2 = \beta_1 + \beta_2 x + \beta_3 y$$

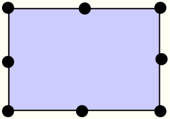


4 nodes element

linear variation of displacement in both x and y directions

$$d_1 = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi\eta$$

$$d_2 = \beta_1 + \beta_2 x + \beta_3 y + \beta_4 \xi\eta$$



8 nodes element

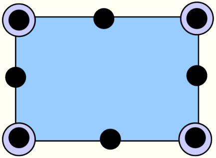
quadratic variation of displacement in both x and y directions.

$$d_1 = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi^2 + \alpha_5 \xi\eta + \alpha_6 \eta^2 + \alpha_7 \xi^2\eta + \alpha_8 \xi\eta^2$$

$$d_2 = \beta_1 + \beta_2 \xi + \beta_3 \eta + \beta_4 \xi^2 + \beta_5 \xi\eta + \beta_6 \eta^2 + \beta_7 \xi^2\eta + \beta_8 \xi\eta^2$$

2D/3D Finite elements

2D Consolidation element

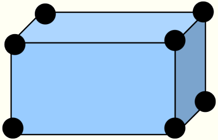


Pore pressure and displacements

Displacements

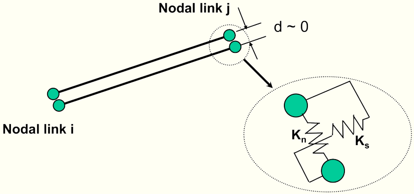
Linear variation of pore pressures and quadratic variation of displacements in x and y directions

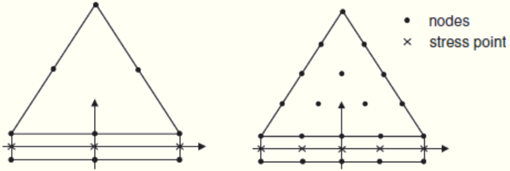
8 node 3D brick element



Linear variation of displacements in x, y and z directions

Interface element





a. 6-node soil element   b. 15-node soil element

13 Distribution of nodes and stress points in interface elements and their connection to soil elements

13

Use a reduced strength at the interface

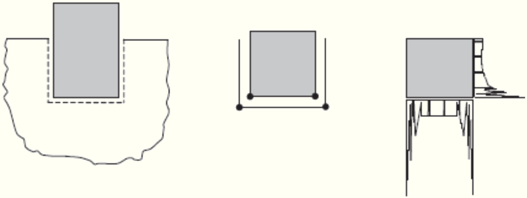


Figure 3.14 Inflexible corner point, causing poor quality stress results

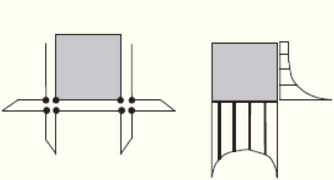
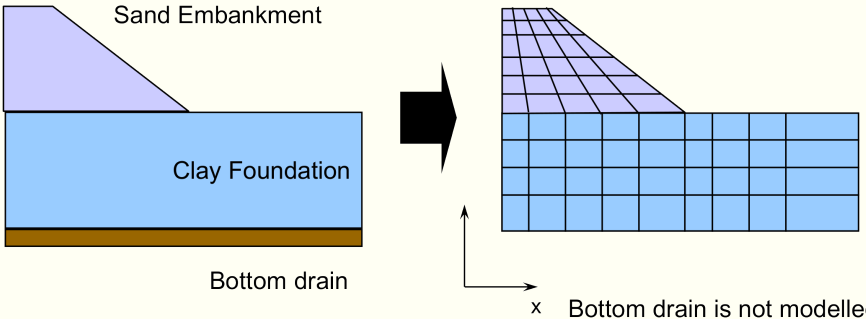


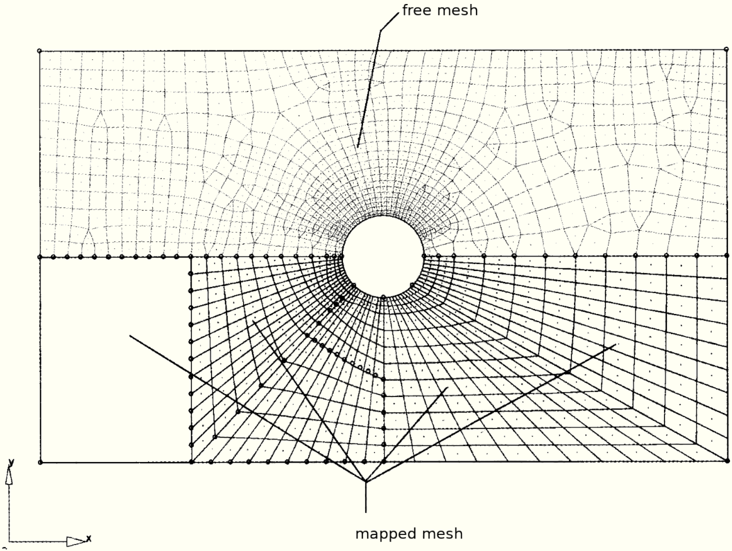
Figure 3.15 Flexible corner point with improved stress results

FE discretization

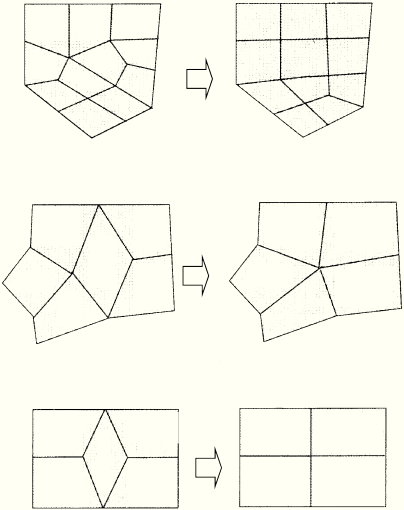


Bottom drain is not modelled because it is assumed to behave as a rigid material.

FE discretization

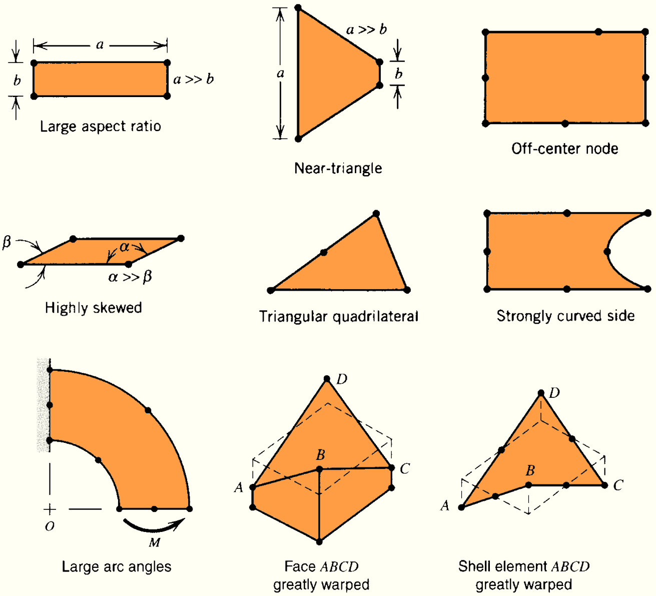


FE discretization

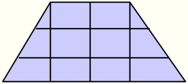


1. Node valence less or equal to two and greater than or equal to six should be eliminated.
2. The number of nodes with valence of three or five should be minimized.
3. Angles greater than 160 degrees should be eliminated.
4. The aspect ratio should be less than 3 for stress analysis and 10 for displacement analysis.

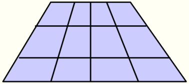
FE discretization



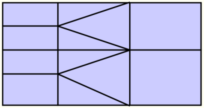
FE discretization



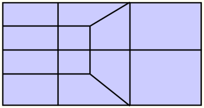
Not recommended



Better

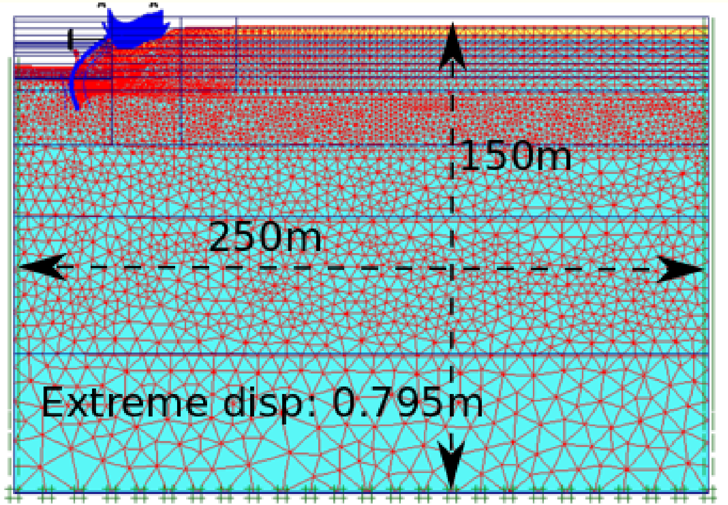


Not recommended



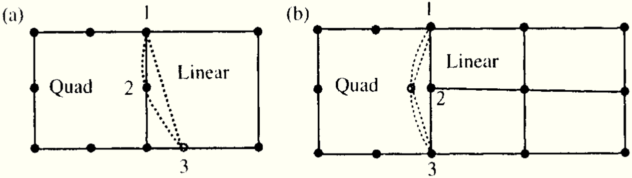
Better

FE discretization: Refining



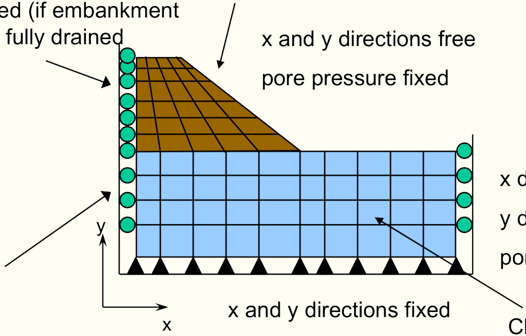
Avoid large jumps in element size: size jump should be < 3

FE mesh compatibility



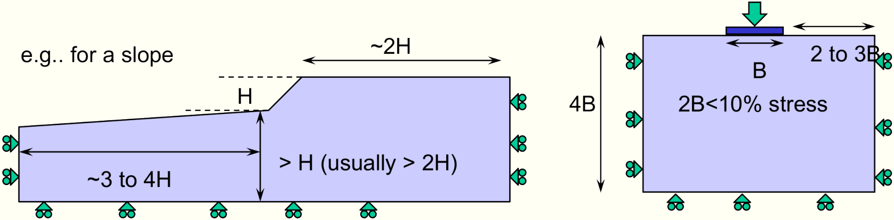
Incompatible mesh

FE boundary conditions

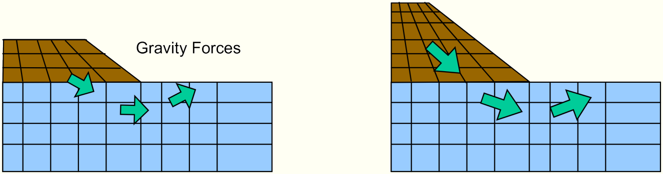


Pore pressure free = no water flow perpendicular to the boundary

FE boundary conditions

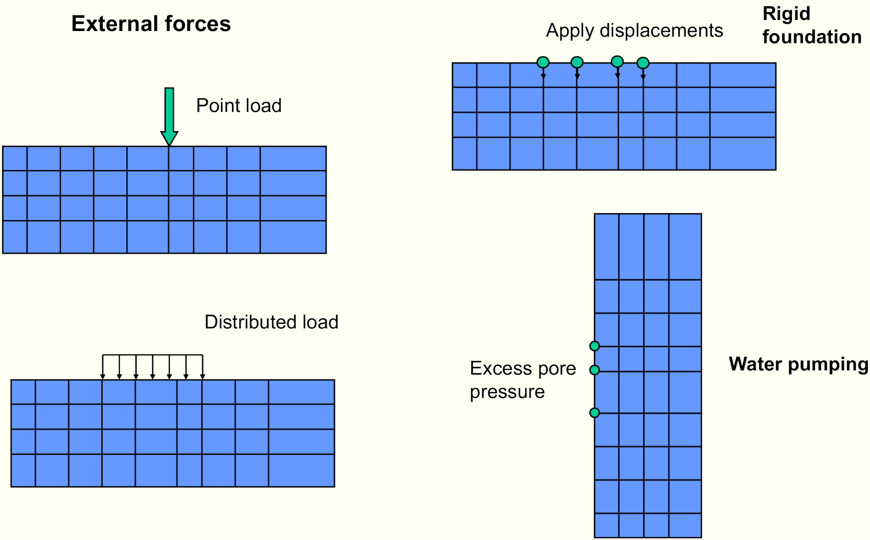


FE boundary conditions

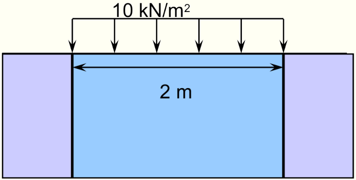


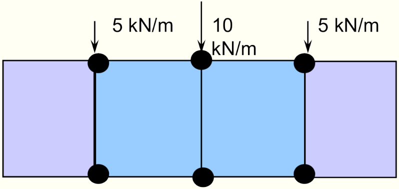
Construction sequence

FE loading conditions

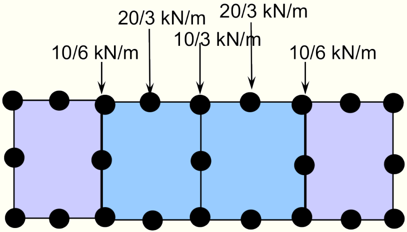


FE loading conditions



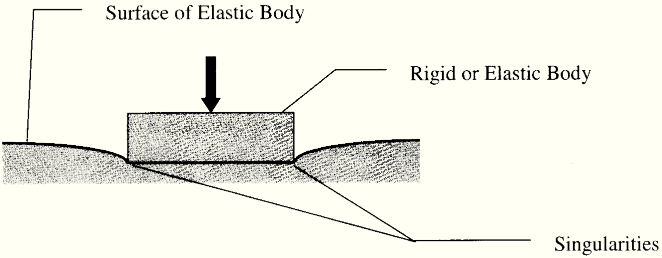


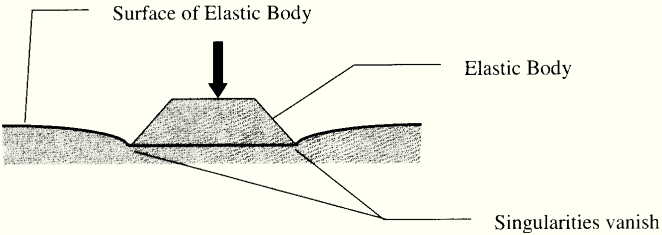
(1) 4 nodes elements



(2) 8 nodes element

FE stress singularity

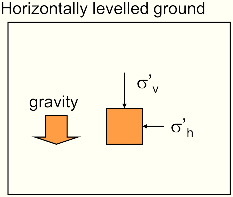


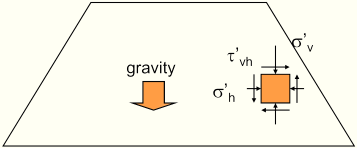


Geostatic stresses

Before conducting your analysis, you need to make sure that the stresses in the ground are the correct values, as the soil behavior depends on the current in-situ stresses.

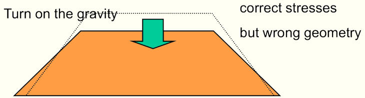
1. **list your estimated stresses in the input file**: hopefully the system is in equilibrium – difficult to find the in-situ stresses in the sloping ground.





2. **Zero displacement approach** - ask the program to compute the in-situ stresses from the equilibrium condition (very few programs allow you to do this)

(a) define model geometry and assign a soil model



(b) displacement by self weight and obtain the equilibrium condition

(c) Zero the displacement But keep the computed stresses

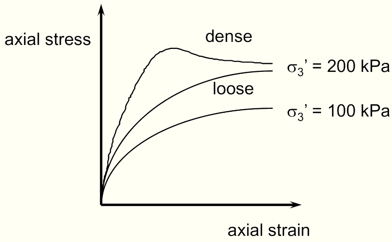
3. **Intermediate approach** - Guess the insitu stress distribution, apply gravity and perform the equilibrium check (hopefully the displacements are zero) - ABAQUS GEOSTATIC approach

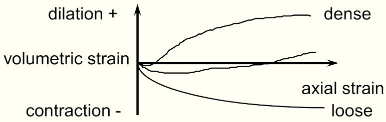
The coefficient of earth pressure at rest K_0

- **Normally consolidated soils**

- **Over consolidated soils**
  - **Cam-clay predictions** $K_{\mathrm{oc}} = (1 - \sin\phi') \times (\mathrm{OCR})$ (Mayne et al., 1982) up to $K_p = (1 + \sin\phi')/(1 - \sin\phi')$
  - **Wroth's method (1975)** $K_{\mathrm{oc}} = (\mathrm{OCR})K_{\mathrm{nc}} - \left(\frac{\nu'}{1-\nu'}\right)(\mathrm{OCR} - 1)$ for OCR < 5 and ν' is the poisson's ratio $=$ 0.254 - 0.371.

Soil models


axial stress
dense
σ_3' = 200 kPa
loose
σ_3' = 100 kPa
axial strain

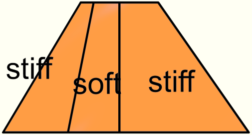

dilation +
dense
volumetric strain
axial strain
contraction -
loose

Verification of Design Parameters (Atkinson., 1995)

Verification of Design Parameters (Atkinson., 1995)

FE errors

Check! Check! Check!



Beware : σ_v here lower than γh due to stress transfer to stiff material - hanging up

Pore-pressure analysis in geotechnical engineering

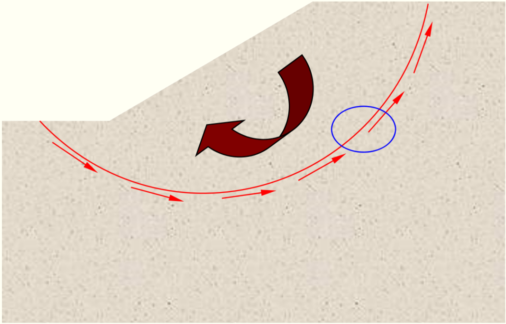
Pore-pressure analysis in geotechnical engineering

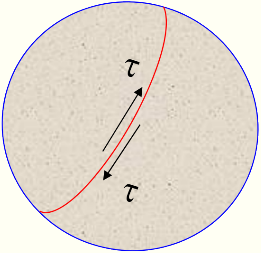
Undrained effective stress analysis

Steps to perform Finite Element Analysis

2014 Oso landslide

Shear failure plane

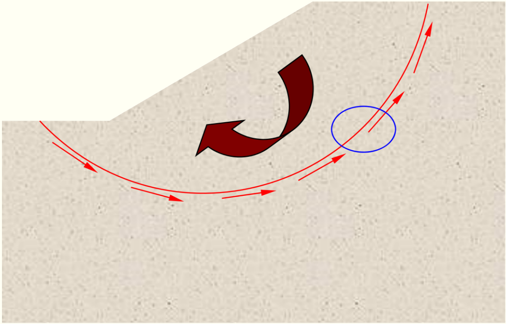


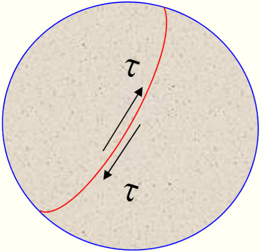


At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

Factor of Safety $=$

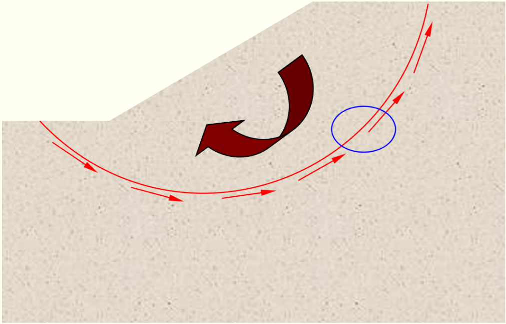
Dry slope (total stress = effective stress)

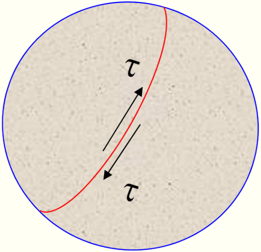




Saturated slope (total stress = effective stress + pwp)

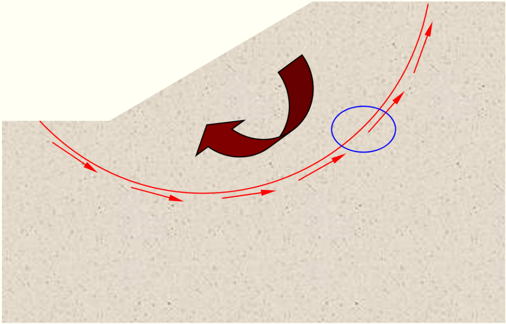
Drained conditions - need to compute the steady state pore pressure field and then evaluate "effective stress-based" shear strength to find the overall stability (based on total stress equilibrium).

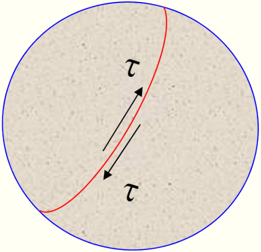




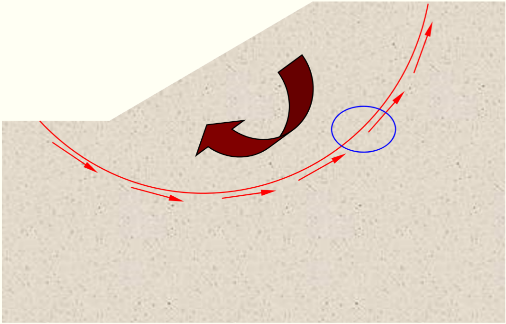
Saturated slope (total stress = effective stress + PWP)

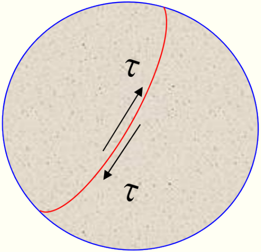
Undrained conditions - (total stress approach) – Use "*total-stress based*" shear strength (s_u) to find the overall stability (based on total stress equilibrium).



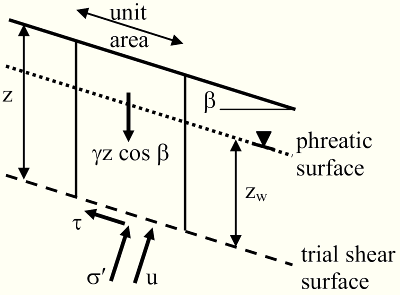


Undrained conditions - (Effective stress approach-not common) - need to compute the pore pressure (including excess pore pressure) field and then evaluate "*effective stress -based*" shear strength to find the overall stability (based on total stress equilibrium).





Infinite slopes

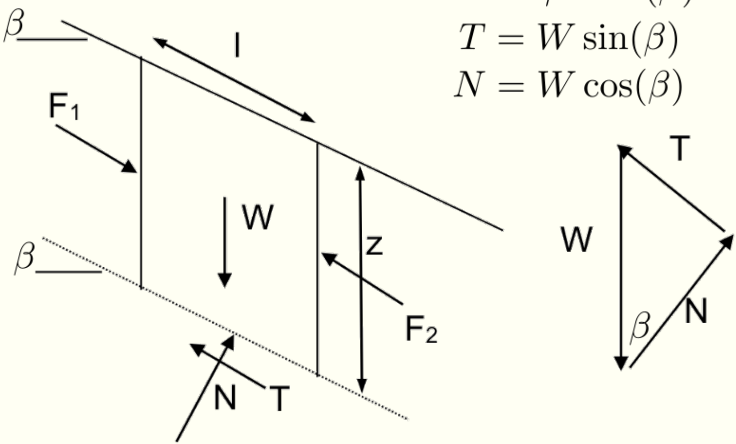


$$u = \gamma_w z_w \cos^2\beta$$
$$\sigma = \gamma z \cos^2\beta$$
$$\sigma' = (\gamma z - \gamma_w z_w) \cos^2\beta$$
$$\tau = \gamma z \cos\beta \sin\beta$$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

Soil fails when (dry):
Soil fails when (submerged):

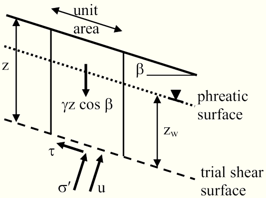
Undrained infinite slope (total stress approach)



$$W = \gamma z l \cos(\beta)$$
$$T = W \sin(\beta)$$
$$N = W \cos(\beta)$$

But also the shear stress:

Infinite slope: Summary



$$u = \gamma_w z_w \cos^2\beta$$
$$\sigma = \gamma z \cos^2\beta$$
$$\sigma' = (\gamma z - \gamma_w z_w)\cos^2\beta$$
$$\tau = \gamma z \cos\beta \sin\beta$$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

1. Factor of Safety = resistance / driving
2. Dry FoS $= \tan(\phi_{\mathrm{mob}})/\tan(\beta)$
3. Submerged FoS $= \tan(\phi_{\mathrm{mob}})/\tan(\beta)$
4. Undrained FoS $= 2s_u/\gamma z \sin(2\beta)$
5. Steady state seepage FoS $= (1 - \gamma_w/\gamma)\tan(\phi_{\mathrm{mob}})/\tan(\beta)$ where the water table is located at the slope surface