CE394M: Stress paths and invariants

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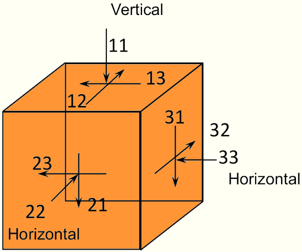
Overview

1. Stresses / strains in typical geotechnical lab tests
2. Friction
3. Stress invariants

Stresses / strains

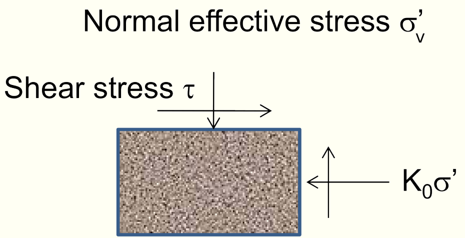
1D consolidation / simple shear

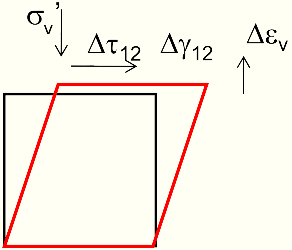


2D plane strain

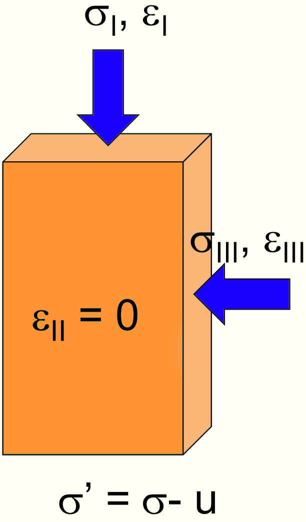
3D general (axi-symmetric as a special case)

1D simple shear





Stresses and strains: independent components



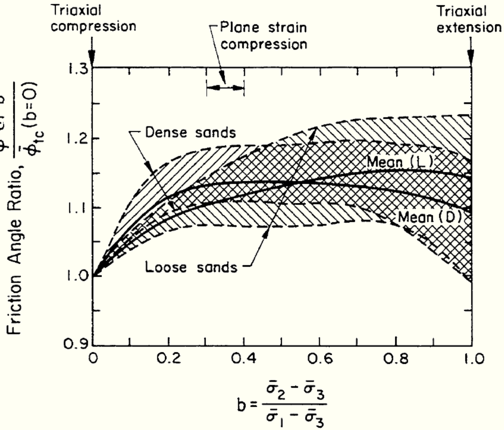
2D Mohr circle

Mohr circle to 2D stress components

Successive Mohr circles and stress path for constant σ_3 and increasing σ_1:

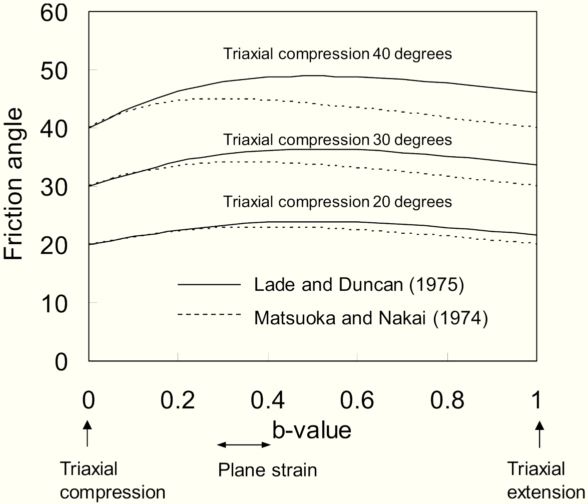
Effect of σ_{II}

Bishop (1966) defined **b-value**:



Effect of σ_{II} on friction angle (Kulhawy and Mayne, 1990)

Effect of σ_{II} on friction



Chapter 11., Mitchell and Soga, 2005

Effect of σ_{II} on friction

Various models fit the experimental data showing the intermediate stress effect. Lade and Duncan (1975)

$$\frac{I_1^3}{I_3} = const$$

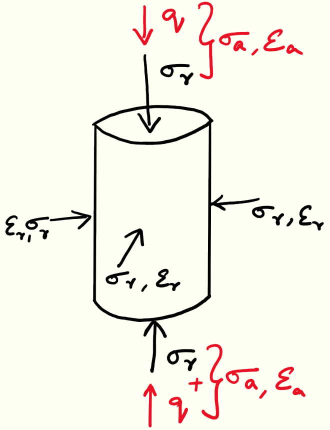
Matsuoka and Nakai (1974)

$$\frac{I_1 I_2}{I_3} = const$$

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\ I_2 &= \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \\ I_3 &= \sigma_1\sigma_2\sigma_3 \end{aligned}$$

Matsuoka and Nakai's model gives the same friction angle for compression and extension, whereas Lade and Duncan's model gives the ratio of the TXE friction angle (ϕ'_{TXE}) to the TXC friction angle (ϕ'_{TXC}) to be 1.08 at $\phi'_{TXC} = 20^o$ to 1.15 at $\phi'_{TXC} = 40^o$.

Triaxial stresses and strains: independent components



Triaxial deviatoric strain

- Deviatoric / shear strain: $\varepsilon_s = 2/3(\varepsilon_a - \varepsilon_r)$
- Why 2/3?:

Triaxial stress - strain relationship

Relationship in terms of axial and radial direction:

$$\begin{bmatrix}\varepsilon_a \\ \varepsilon_r\end{bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -2\nu \\ -\nu & (1-\nu)\end{bmatrix}\begin{bmatrix}\sigma_a \\ \sigma_r\end{bmatrix} \tag{1}$$

Relationship in terms of 'p' and 'q':

$$\begin{bmatrix}p \\ q\end{bmatrix} = \begin{bmatrix}1/3 & 2/3 \\ 1 & -1\end{bmatrix}\begin{bmatrix}\sigma_a \\ \sigma_r\end{bmatrix} \quad \text{invert} \ \rightarrow \begin{bmatrix}\sigma_a \\ \sigma_r\end{bmatrix}\begin{bmatrix}1 & 2/3 \\ 1 & -1/3\end{bmatrix}\begin{bmatrix}p \\ q\end{bmatrix} \tag{2}$$

Strain:

$$\begin{bmatrix}\varepsilon_p \\ \varepsilon_q\end{bmatrix} = \begin{bmatrix}1 & 2 \\ 2/3 & -2/3\end{bmatrix}\begin{bmatrix}\varepsilon_a \\ \varepsilon_r\end{bmatrix} \tag{3}$$

Volume - shear coupling

Equation (1) and eq. (2) in eq. (3) to relate $\varepsilon_p, \varepsilon_q$ to p, q:

$$\begin{bmatrix} \varepsilon_p \\ \varepsilon_q \end{bmatrix} = \begin{bmatrix} 3(1-2\nu)/E & 0 \\ 0 & 2(1+\nu)/3E \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}$$

Volume shear coupling

Usually volume change and shear deformation are decoupled (i.e. bulk modulus and shear modulus), but in soils they are coupled:

Mohr circle to 3D stress components

Successive Mohr circles and stress path for constant σ_3 and increasing σ_1:

Stress paths p-q

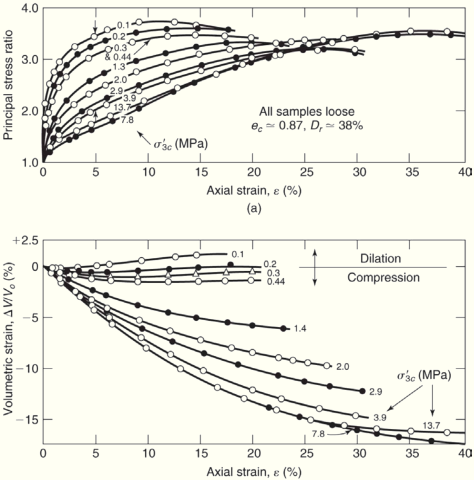
Different stress paths for initially hydrostatic stress conditions:

Triaxial compression

TXC with constant back pressure u_o (TSP: Total stress path; ESP: Effective stress path)

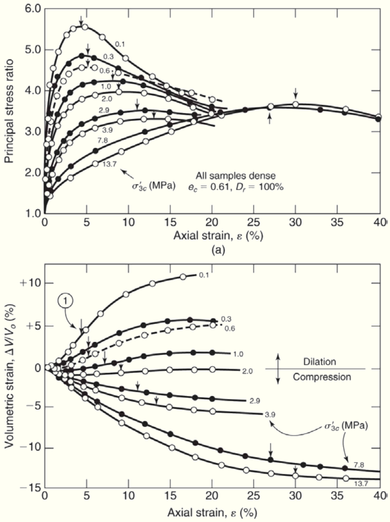
Triaxial compression undrained: loose v dense

Triaxial compression drained: loose



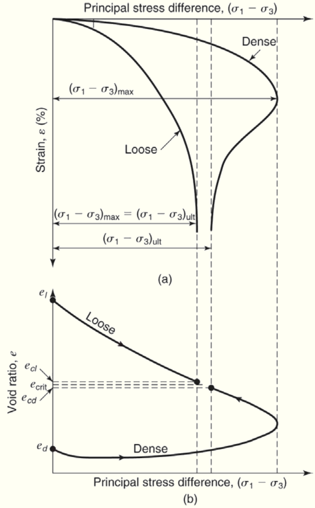
Loose Sacramento River sand: (a) principal stress ratio versus axial strain; (b) volumetric strain versus axial strain (Lee, 1965).

Triaxial compression drained: dense



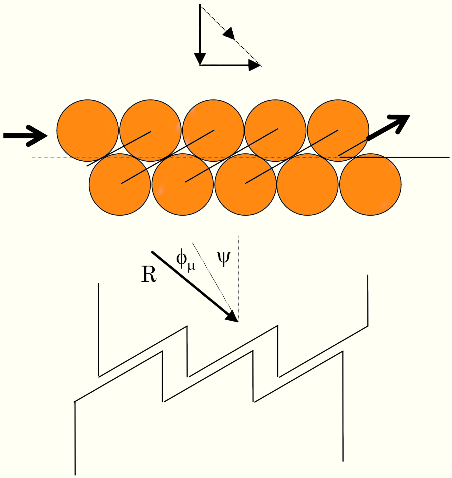
Dense Sacramento River sand: (a) principal stress ratio versus axial strain; (b) volumetric strain versus axial strain (Lee, 1965).

Triaxial compression drained: loose v dense



Triaxial tests on "loose" and "dense" specimens of a typical sand: (a) stress-strain curves; (b) void ratio changes during shear (Hirschfeld, 1963).

Friction: Is this correct?



$$\phi_{ss} = \phi_\mu + \psi_{ss}$$

Discrete Element Method

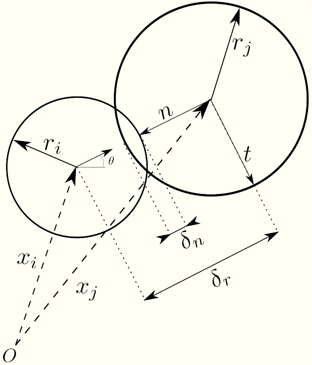
1. Particle level interaction based on Newton's equation of motion
2. The contact normal force is computed as:

$$F_n = \begin{cases} 0, & \delta_n > 0 \\ -k_n\delta_n - \gamma_n \frac{d\delta_n}{dt}, & \delta_n < 0 \end{cases}$$

3. The contact tangential force is computed in a similar way, but has a frictional limit.

$$F_t \leq \mu F_n$$

4. Solve Newton's second law and the angular momentum equation (including rotational resistance).

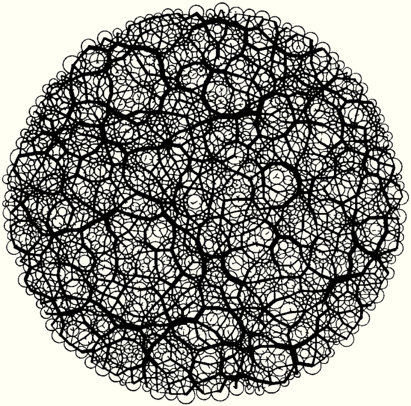


Interparticle friction angles

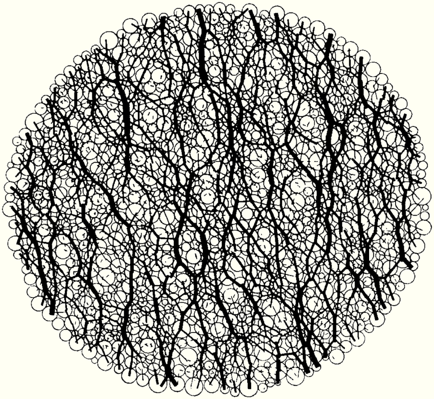
- For Quartz Sands: 26 degrees
- For Sheet Minerals (muscovite, phlogopite, biotite and chlorite): 7 - 13 degrees
    - Water acts as a lubricant
- Clay minerals: Probably 7 - 13 degrees
    - Similar to reported residual friction angles.
    - Sodium Montmorillonite: 4 degrees

Strong force network vs weak clusters

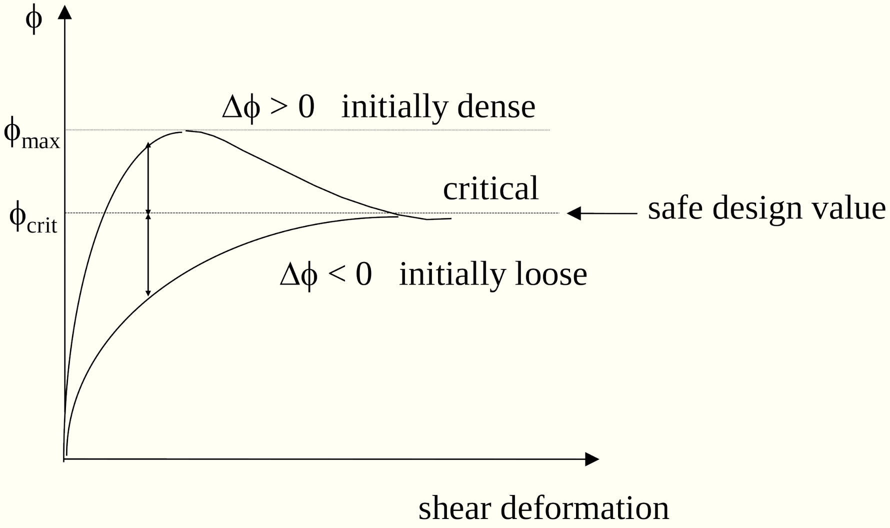
Isotropic Loading



Biaxial Loading



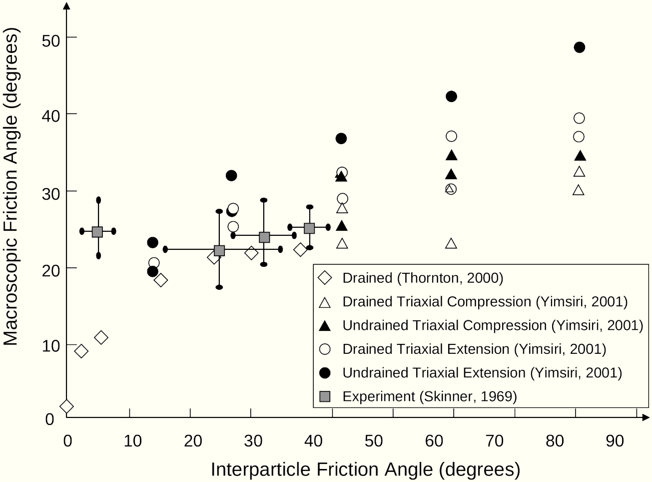
Macroscopically, as soil aggregates...



Macroscopic friction angle

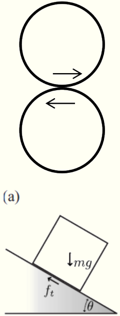
- ϕ_{crit} is the angle of friction measured at constant volume of a soil aggregate, and $\Delta\phi$ dilatancy is the extra dilatant contribution to friction angle ϕ. Typical values are:
- Critical state friction ϕ_{crit}:
  - clay: 22°
  - uniform rounded sand: 32°
  - well-graded angular sandy gravel: 38°
- peak strength of pre-compressed or uncrushable grains, densely compacted, and: shearing in plane strain $\Delta\phi$:
  - shearing in plane strain: $\Delta\phi_{max} = 20^\circ$
  - shearing in axial symmetry: $\Delta\phi_{max} = 12^\circ$

Micro to Macroscopic friction angle

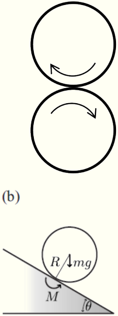


Relationship between macroscopic friction angle and interparticle friction angle (no rolling resistance) - Yimisir and Soga (2001)

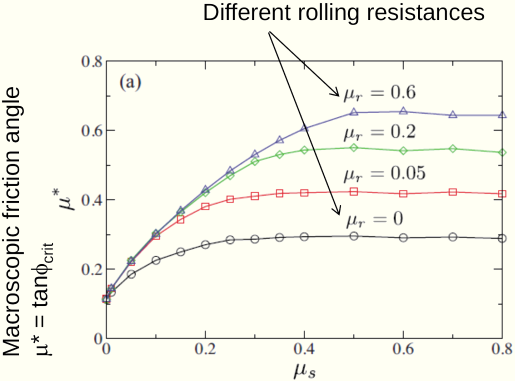
Micro to Macroscopic friction angle: Rolling resistance



Sliding friction μ_s



Rolling resistance μ_r

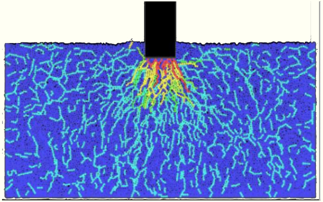


Microscopic sliding friction at particle contacts $\mu_s = \tan \phi_\mu$

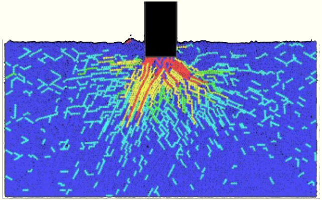
Estrada et al., (2001)

Interparticle friction angles

- The interparticle friction acts as a kinematic constraint of the strong force network and not as the direct source of macroscopic resistance to shear.
- Increased friction at the contacts increases the stability of the system (development of anisotropic fabric) and reduces the number of contacts required to achieve a stable condition.
- As long as the strong force network can be formed, the magnitude of the interparticle friction becomes of secondary importance.



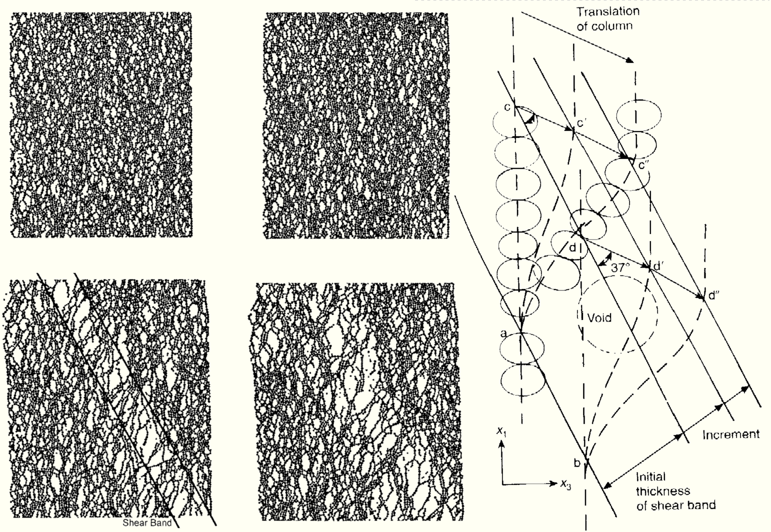
Loose



Dense

Muthuswamy and Tordesillas (2006)

What is dilation? shear band



Iwashita and Oda (2000)

Fabric evolution at critical state

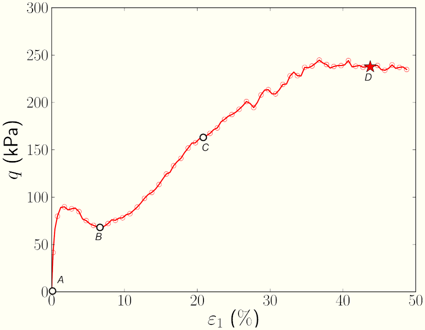


Fig.1 Undrained shear response of a medium dense sand and four stress states selected for examination of internal structure.

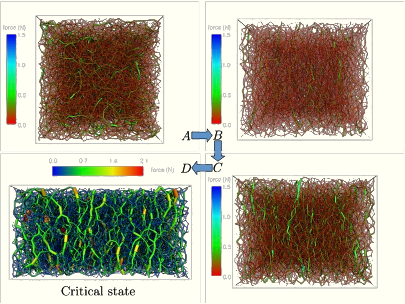


Fig.2 Evolution of Contact Force Network towards the critical state (see Guo & Zhao [Computers & Geotechnics, 2013]).

Guo and Zhao (2003)

6 stresses and strains

- Mean pressure:
- Deviator stress: $q = \sqrt{3/2}\sqrt{s_{11}^2 + s_{22}^2 + s_{33}^2 + 2\tau_{12}^2 + 2\tau_{23}^2 + 2\tau_{31}^2}$
- where $s_{11} = \sigma'_{11} - p', s_{22} = \sigma'_{22} - p', s_{33} = \sigma'_{33} - p'$
- Volumetric strain:
- Deviatoric strain:
  $\varepsilon_s = \sqrt{2/3}\sqrt{e_{11}^2 + e_{22}^2 + e_{33}^2 + \gamma_{12}^2/2 + \gamma_{23}^2/2 + \gamma_{31}^2/2}$
- where $e_{11} = \varepsilon_{11} - \varepsilon_v/3, e_{22} = \varepsilon_{22} - \varepsilon_v/3, e_{33} = \varepsilon_{33} - \varepsilon_v/3$

Stress and strain invariants in 3D

3 Principal stresses and strains

- Mean pressure:
- Deviator stress: $q = \sqrt{1/2}\sqrt{(\sigma'_I - \sigma'_{II})^2 + (\sigma'_{II} - \sigma'_{III})^2 + (\sigma'_{III} - \sigma'_I)^2}$
- Volumetric strain:
- Deviatoric strain: $\varepsilon_s = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon'_I - \varepsilon'_{II})^2 + (\varepsilon'_{II} - \varepsilon'_{III})^2 + (\varepsilon'_{III} - \varepsilon'_I)^2}$

In triaxial condition (principal stresses/strains)

The missing stress invariant

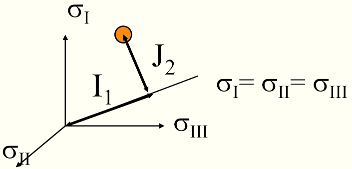
- **General stresses:**

- **Principal stresses:**

Stress invariant

Principal stresses (3 components) $\Leftrightarrow$ Invariants (3 components)

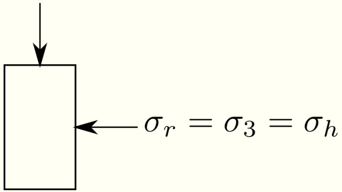
- $I_1 =$
- $s_{ij} =$
- $J_2 =$
- $J_2 =$
- $q =$

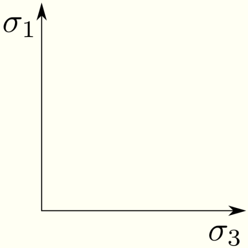


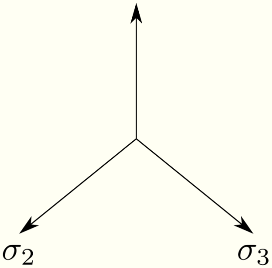
Lode angle:

- $\sin 3\theta = \frac{-3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}}$.
- $\theta = \tan^{-1}\left[\frac{1}{\sqrt{3}}\left(2\frac{(\sigma_2-\sigma_3)}{(\sigma_1-\sigma_3)} - 1\right)\right]$
-
-

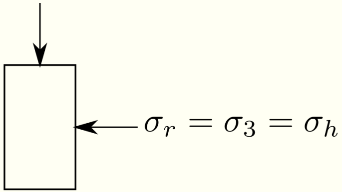
π plane: Triaxial compression

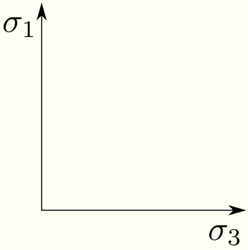


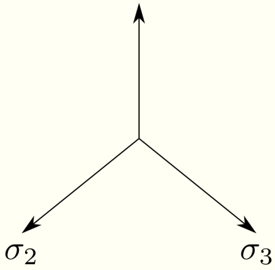




π plane: Triaxial extension







π plane: Random stress paths