CE394M Advanced Analysis in Geotechnical Engineering: Introduction

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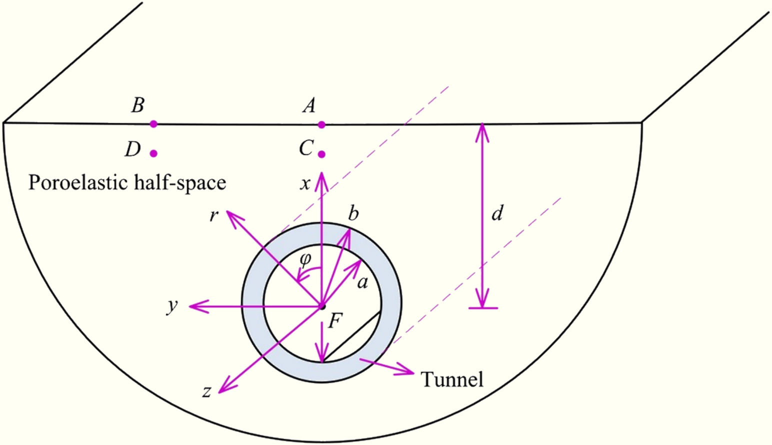
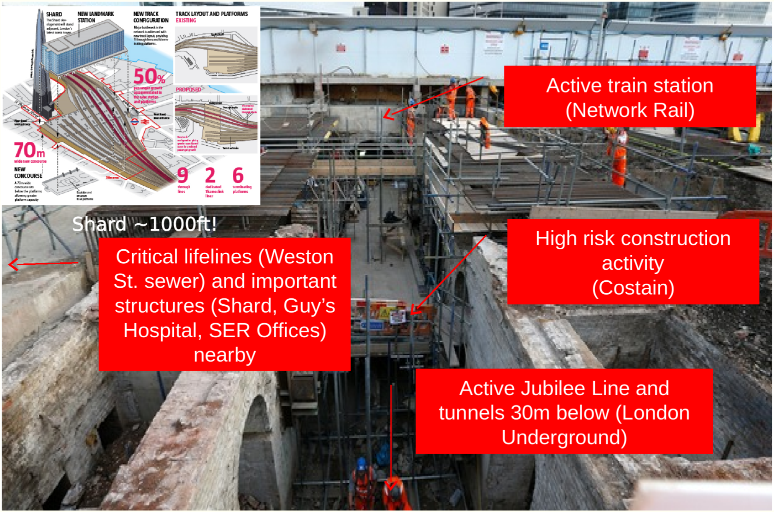
Overview

1. Geotechnical modeling
   - Complexity in Geotechnical modeling
   - Oso landslide
2. Geotechnical analysis
3. Governing equations in stress-deformation analysis
   - Stress equilibrium
   - Compatibility condition
   - Stress-strain relationship

Is this model correct?



Geotechnical modeling of the complex world



London Bridge Station, London, UK

Geotechnical modeling of the complex world

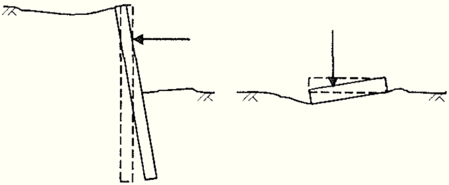
Fig. London Victoria station upgrade, London, UK

Geotechnical modeling of the complex world

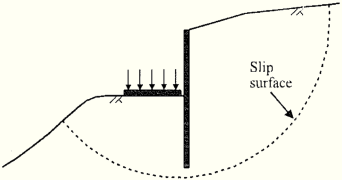
Fig. London Victoria station upgrade, London, UK

Movements must be estimated, both of the structure and of the ground. This is particularly important if there are adjacent buildings and for sensitive services. For example, if an excavation is to be made in an urban area close to existing services and buildings, one of the key design constraints is the effect that the excavation has on the adjacent structures and services. It may be necessary to predict any structural forces induced in these existing structures and/or services.

Local vs global stability

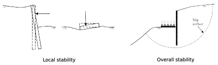


Local stability



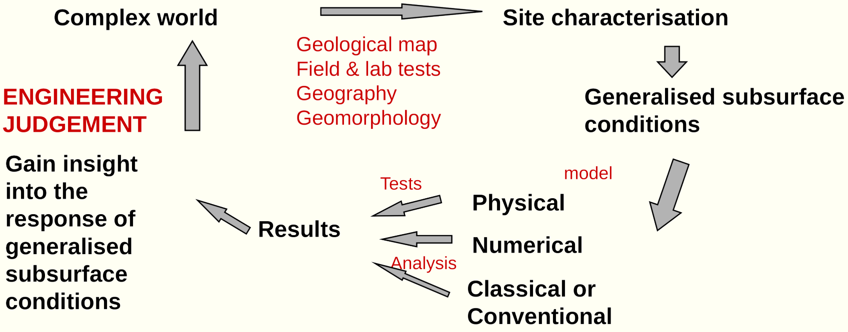
Overall stability

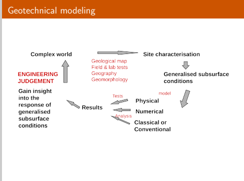
Local vs global stability



When designing any geotechnical structure, the engineer must ensure that it is stable. Stability can take several forms. Firstly, the structure and support system must be stable as a whole. There must be no danger of rotational, vertical or translational failure (**local stability**). Secondly, **overall stability** must be established. For example, if a retaining structure supports sloping ground, the possibility of the construction promoting an overall slope failure should be investigated.

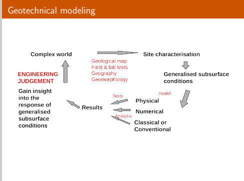
Geotechnical modeling





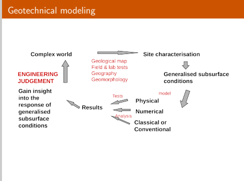
Design requirements: Before the design process can begin, a considerable amount of information must be assembled. The basic geometry and loading conditions must be established. These are usually defined by the nature of the engineering project.

A geotechnical site investigation is then required to establish the ground conditions. Both the soil stratigraphy and soil properties should be determined. In this respect it will be necessary to determine the strength of the soil and, if ground movements are important, to evaluate its stiffness too. The position of the ground water table and whether or not there is underdrainage or artesian conditions must also be established. The possibility of any changes to these water conditions should be investigated. For example, in many major cities around the world the ground water level is rising.



The site investigation should also establish the location of any services (gas, water, electricity, telecommunications, sewers andlor tunnels) that are in the vicinity of the proposed construction. The type (strip, raft andlor piled) and depth of the foundations of any adjacent buildings should also be determined. The allowable movements of these services and foundations should then be established.

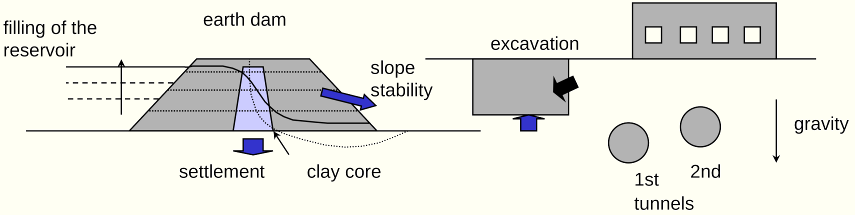
Any restrictions on the performance of the new geotechnical structure must be identified. Such restrictions can take many different forms. For example, due to the close proximity of adjacent services and structures there may be restrictions imposed on ground movements.



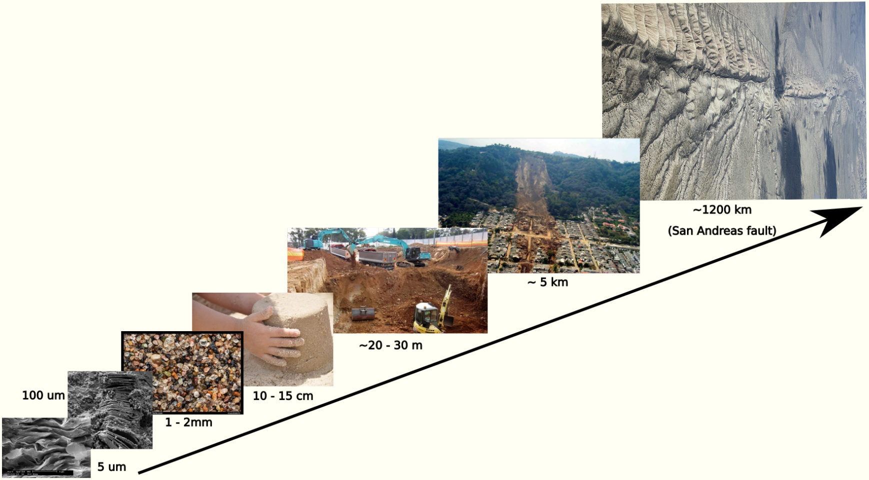
Once the above information has been collected, the design constraints on the geotechnical structure can be established. These should cover the construction period and the design life of the structure. This process also implicitly identifies which types of structure are and are not appropriate. For example, when designing an excavation, if there is a restriction on the movement of the retained ground, propped or anchored embedded retaining walls are likely to be more appropriate than gravity or reinforced earth walls. The design constraints also determine the type of design analysis that needs to be undertaken.

Geotechnical modeling: What should be modeled?

- Self weight effect of soils (This is why soil moves)
- Construction sequence (Complex geometry)
- Water movement (undrained, consolidation, drained)
- Insitu stresses (stiffness/strength depends on current stresses and stress history)
- Predict the ability of a design to withstand extreme loading conditions (you only have one chance)



Scales of modeling in geotechnical engineering



Soil behavior

- nonhomogeneous,
- anisotropic,
- non-linear,
- initial stress conditions,
- stress history
- Geometry - very complex

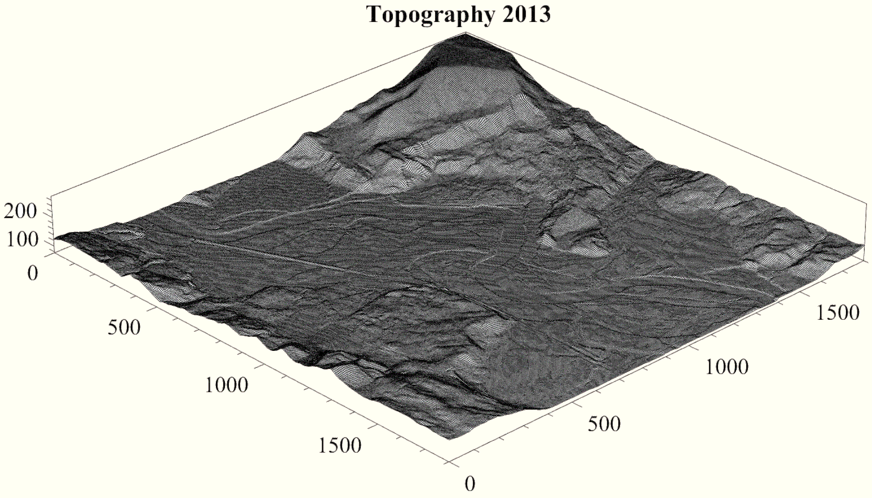
Soil Mechanics in practice - largely empirical

Oso landslide: case study

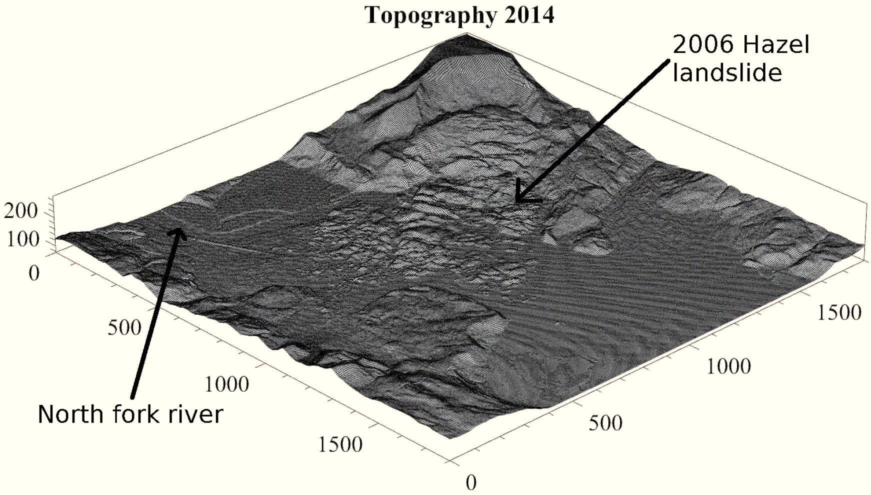
Oso landslide: case study

- 22nd March 2014 at 10:37 AM
- Volume approx 8 million cubic meters
- 43 casualties (deadliest landslide in the US)
- 1 neighborhood destroyed
- (unknown but) $>$ 150 million lost cost (DNR 2015), $+$ USD 65 million (lawsuit 2017) $+$ indirect costs
- Social tensions and Indian Snohomish tribe

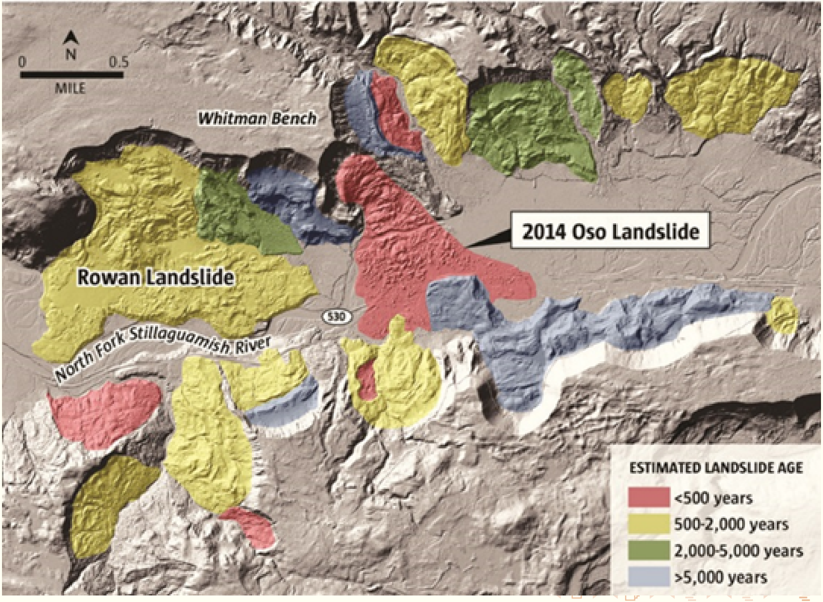
Oso landslide: topography



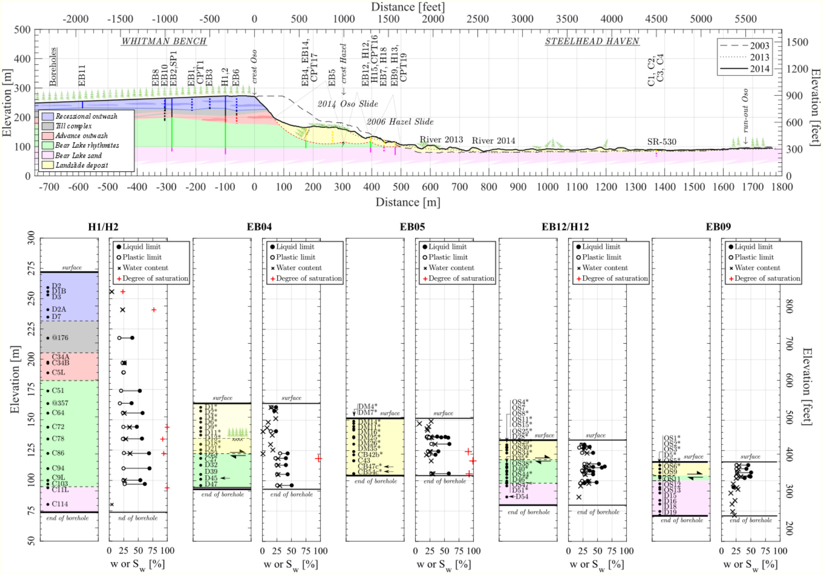
Oso landslide: topography



Oso landslide: historic slides



Oso landslide: Soil profile



Oso landslide: Geology. Identify the failure surface

Lower portion of Bear Lake Rhythmites with failure surface. Courtesy of Dr Gunnar Schlieder

Oso landslide: Geology

Deformation till with flame structures in fine-grained glacio-lacustrine deposit EB7 (depth 65 ft). Courtesy of Dr Gunnar Schlieder

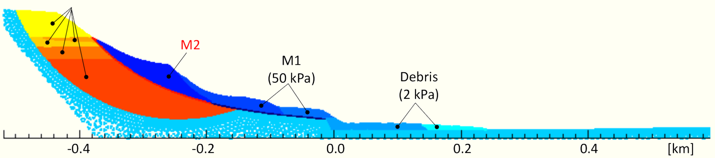
Oso landslide: Direct shear test (intact specimens)

- **Peak strength (effective stress)**
  - Friction angle $\phi'_{max} = 24^o (> \phi'_{cs} = 22^o)$
  - Cohesion $c'_{max} = 100 kPa (> c'_{cs} = 0)$
- **But some specimens**
  - Friction angle $\phi'_{max} = 12^o (< \phi'_{cs} = 22^o)$
  - Cohesion $c'_{max} = 0 kPa (= c'_{cs})$
- Natural soil with structure
  - Is it an intrinsic material property?
  - Or a soil structure property?

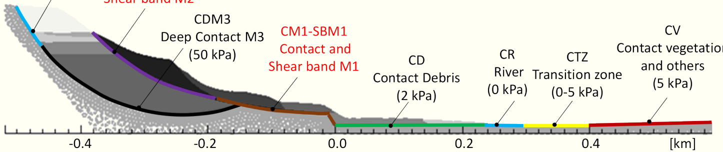
Oso landslide: Analysis

Model 1

Materials

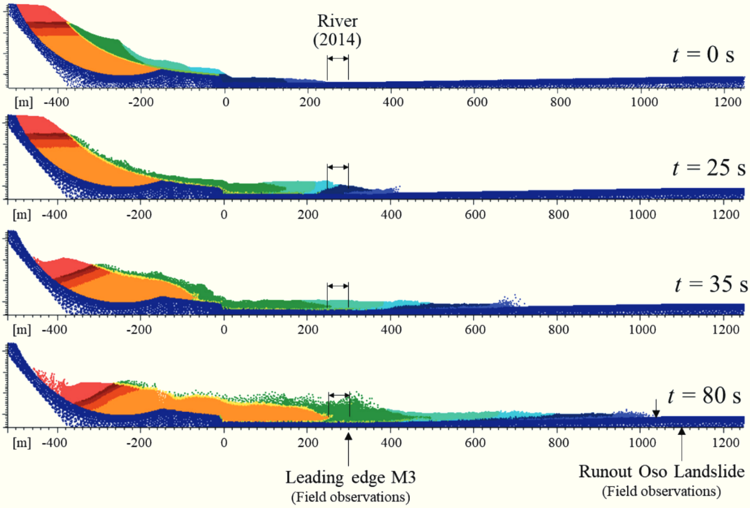


Contacts & Shear bands



Alba et al., 2018

Oso landslide: Analysis



Alba et al., 2018

Advanced analysis in geotechnical engineering

Geotechnical design:

- Assess applied forces
- evaluate “performance” (stability & movements) under working and ultimate loads

Analysis:

- Mathematical framework to perform calculations for these quantities
- Requires idealization of: geometry, soil properties, and loading conditions
- Analysis is a tool in design, but design involves more: acceptable movements, constraints, site characterization, etc.

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Analysis:

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As part of the design process, it is necessary for an engineer to perform calculations to provide estimates of the above quantities. Analysis provides the mathematical framework for such calculations. A good analysis, which simulates real behaviour, allows the engineer to understand problems better. While an important part of the design process, analysis only provides the engineer with a tool to quantify effects once material properties and loading conditions have been set. The design process involves considerably more than analysis.

Governing equations in stress-deformation analysis

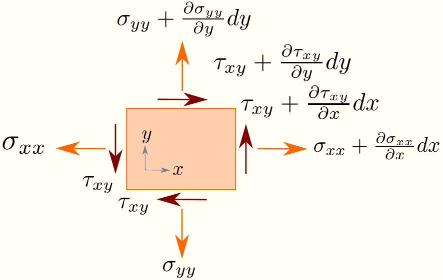
In stress-deformation analysis, we need to consider:

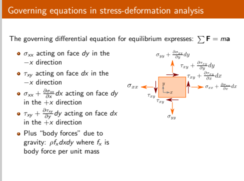
- **Equilibrium - static conditions**
  - forces and stress must agree across the region of interest. (geometric problem)
- **Compatibility-kinematic conditions**
  - geometry, displacement and strains must agree across the region of interest. (geometric problem)
- **Stress-strain relationship on physical conditions**
  - material dependent relationship between stress and strain must be specified. (element level)

Governing equations in stress-deformation analysis

The governing differential equation for equilibrium expresses: $\sum \mathbf{F} = m\mathbf{a}$

- σ_{xx} acting on face dy in the $-x$ direction
- τ_{xy} acting on face dx in the $-x$ direction
- $\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx$ acting on face dy in the $+x$ direction
- $\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy$ acting on face dx in the $+x$ direction
- Plus "body forces" due to gravity: $\rho f_x dx dy$ where f_x is body force per unit mass





σ_{xy}, the term x denotes the direction of the stress component acts on a cut normal to the x-axis (denotes the plane). The y denotes the direction of the stress component.

Frequently in the literature the axes are labelled 1, 2 & 3 rather than x, y & z.

Equilibrium equations

Summing all this in the x-direction gives:

$$-\sigma_{xx}dy - \tau_{xy}dx + \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}dx\right)dy + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y}dy\right)dx + \rho f_x dxdy = \rho dxdy a_x$$

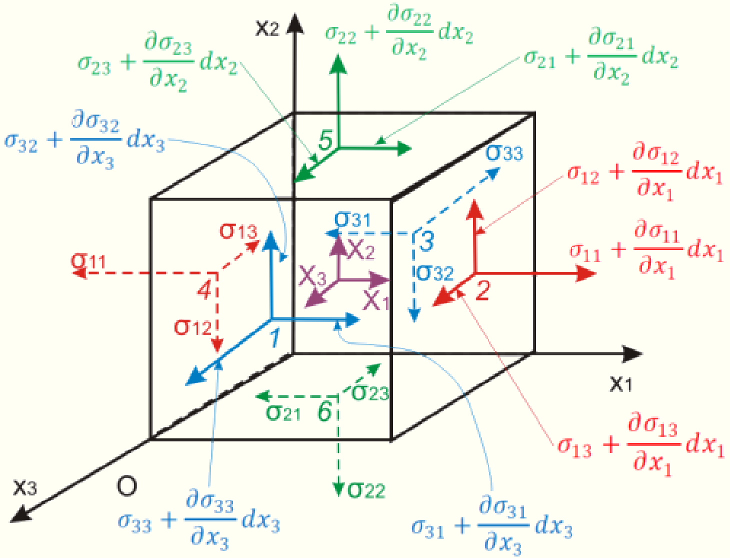
Cleaning up terms that cancel, and dividing through by *dxdy* gives

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho f_x = \rho a_x$$

And summing forces in the y-direction leads to:

$$\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \rho f_y = \rho a_y$$

Equilibrium in 3D



Equilibrium in 3D

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \rho f_x = \rho a_x$$
$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + \rho f_y = \rho a_y$$
$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho f_z = \rho a_z$$

The governing differential equation for equilibrium expresses $\sum \mathbf{F} = m\mathbf{a}$ in terms of derivatives of the stress tensor as: $\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{f} = \rho \mathbf{a}$

$\boldsymbol{\sigma}$ is the stress tensor,

ρ is density,

$\mathbf{f}$ is the body force vector per unit mass and

$\mathbf{a}$ is the acceleration vector.

Stress equilibrium

If the object is in equilibrium, then $\mathbf{a} = 0$ and $\sum \mathbf{F} = 0$.

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0$$
$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = 0$$
$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0$$

Stresses: $[\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}]^T$.
Equilibrium equation: $\nabla^T \boldsymbol{\sigma} + \mathbf{b} = 0$
Then:

$$\nabla^T = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}$$

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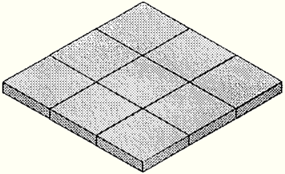
Divergence is a vector operator that produces a scalar field, giving the quantity of a vector field's source at each point. In physical terms, the extent to which there is more of some quantity exiting an infinitesimal region of space than entering it. If the divergence is nonzero at some point then there is compression or expansion at that point.

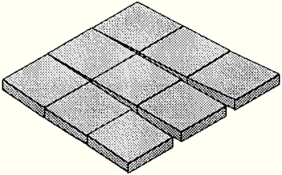
Matrix analysis of structures: Compatibility

- compatibility relates the deformations of a structure so that its various parts (members, joints, and supports) fit together without any gaps or overlaps.
- ensure that the deformed shape of the structure is continuous (except at the locations of any internal hinges or rollers), and is consistent with the support conditions.

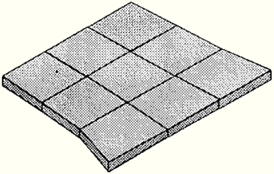
Governing equations: Compatibility



(a) original



(b) non-compatible



(c) compatible

Governing equations: Displacement - strain relationship

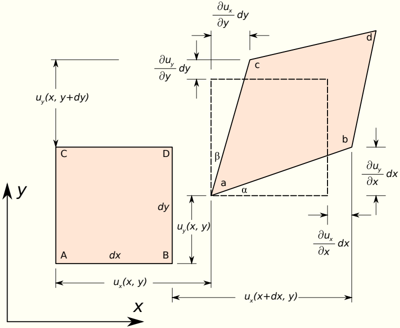
Displacement - strain relationship: $\boldsymbol{\varepsilon} = \nabla \mathbf{u}$

Where,

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

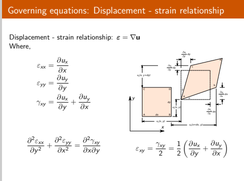
$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$

$$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$



$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_{xy} = \frac{\gamma_{xy}}{2} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$



Compatibility in 3D

$$\frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} = 2\frac{\partial^2 \varepsilon_{yz}}{\partial y \partial z}, \quad \frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial x^2} + \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} = 2\frac{\partial^2 \varepsilon_{zx}}{\partial z \partial x}, \quad \frac{\partial^2 \varepsilon_{yy}}{\partial z \partial x} = \frac{\partial}{\partial y}\left(+\frac{\partial \varepsilon_{yz}}{\partial x} - \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}, \quad \frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y} = \frac{\partial}{\partial z}\left(+\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} - \frac{\partial \varepsilon_{xy}}{\partial z}\right)$$

Equilibrium and compatibility conditions

Combining the Equilibrium and Compatibility conditions gives:

- Unknowns: 6 stresses + 6 strains + 3 displacements = 15
- Equations: 3 equilibrium + 6 compatibility = 9

To obtain a solution therefore requires 6 more equations. These come from the constitutive relationships

Governing equations: Stress-strain relationship

Stress - strain relationship: $\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \begin{bmatrix} D_{xxxx} & D_{xxyy} & D_{xxzz} & D_{xxxy} & D_{xxyz} & D_{xxzx} \\ D_{yyxx} & D_{yyyy} & D_{yyzz} & D_{yyxy} & D_{yyyz} & D_{yyzx} \\ D_{zzxx} & D_{zzyy} & D_{zzzz} & D_{zzxy} & D_{zzyz} & D_{zzzx} \\ D_{xyxx} & D_{xyyy} & D_{xyzz} & D_{xyxy} & D_{xyyz} & D_{xyzx} \\ D_{yzxx} & D_{yzyy} & D_{yzzz} & D_{yzxy} & D_{yzyz} & D_{yzzx} \\ D_{zxxx} & D_{zxyy} & D_{zxzz} & D_{zxxy} & D_{zxyz} & D_{zxzx} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix}$$

Governing equations in stress-deformation analysis

What are the variables used in the governing equations?

1. displacements $\mathbf{u}$ in the body
2. strains ε in the body or within the elements
3. stresses $\boldsymbol{\sigma}$ in the body or within the elements

Advanced analysis involves:

1. Equilibrium: External forces + internal stresses agree
2. Compatibility: Displacements fields agree (no gaps) + strains (derivatives)
3. Stress-strain relationship (constitutive behaviour)

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Lower and upper bound theorems: The exact determination of loads involved in the plastic deformation requires simultaneous solution of three sets of conditions:

1. equations of equilibrium
2. equations of compatibility
3. appropriate consitutive criteria (yield condition and flow rule)

Exact dertermination is often not easy, may be appropriate for simple shapes, but other cases we may have to use *numerical method*.

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Lower bound theorem: Any applied load is less than the actual limiting load, i.e., they will not cause collapse.
In rigid-plastic continua ther can be no plastic deformation under loads for which a stress distribution can be found that:

1. satisfies equilibrium everywhere
2. balances the externally applied loads, and
3. is everywhere within the yield locus.

Relax compatibility.

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Upper bound theorem: Apply enough load to acheive the desired change in component shape, e.g., process machinery
In rigid-plastic continua, plastic deformation must occur for any system of load calculated by equating the external work done by loads to the internal plastic work calcuated from a distribution of strain increments that:

1. satisfies the boundary displacement conditions, and
2. do not infringe incompressibility.

Relax equilibrium

Governing equations in stress-deformation analysis

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Limit equilibrium In this method of analysis an 'arbitrary' failure surface is adopted (assumed) and equilibrium conditions are considered for the failing soil mass, assuming that the failure criterion holds everywhere along the failure surface. The failure surface may be planar, curved or some combination of these. Only the global equilibrium of the 'blocks' of soil between the failure surfaces and the boundaries of the problem are considered. The internal stress distribution within the blocks of soil is not considered. Coulomb's wedge analysis and the method of slices are examples of limit equilibrium calculations.

Only a global equilibrium, rather than the local equilibirium of every point in the soil, is satisified.

What are the variables used in the governing equations?

- displacements $\mathbf{u}$ in the body
- strains ε in the body or within the elements
- stresses $\boldsymbol{\sigma}$ in the body or within the elements

Advanced analysis involves:

- Equilibrium: External forces + internal stresses agree
- Compatibility: Displacements fields agree (no gaps) + strains (derivatives)
- Stress-strain relationship (constitutive behaviour)

Method of analysis	**Solution requirements**				
	Equilibrium	**Compatibility**	**Constitutive law**	**Force**	**Disp**
Closed form	✓	✓	Linear elastic	✓	✓
Limit equilibrium	✓	×	Rigid with a failure criterion	✓	×
Lower bound	✓	×	Plasticity + flow-rule	✓	×
Upper bound	×	✓	Platicity + flow-rule	×	✓
Numerical analysis	✓	✓	Any	✓	✓