# CE394M Advanced Analysis in Geotechnical Engineering: Introduction

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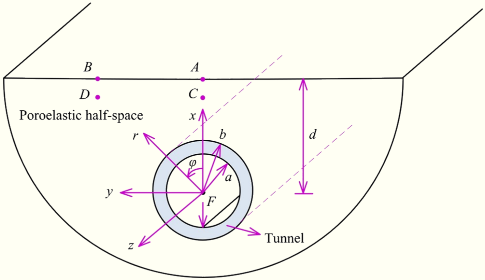
## Overview

# Is this model correct?






# Geotechnical modeling of the complex world

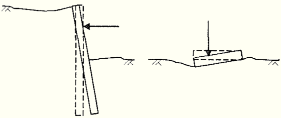
London Bridge Station, London, UK

## Geotechnical modeling of the complex world

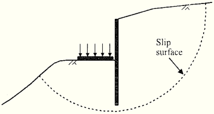

Fig. London Victoria station upgrade, London, UK



## Local vs global stability

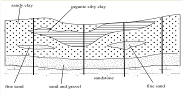


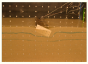
Local stability

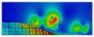



Overall stability

## Geotechnical modeling

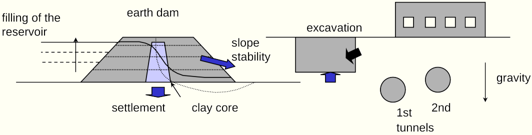


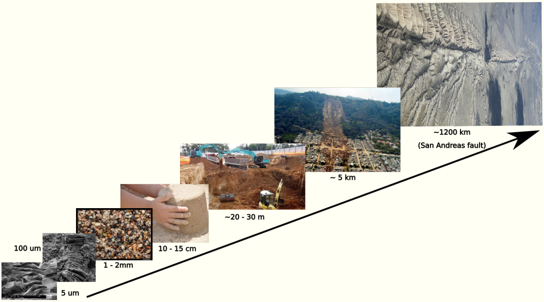




## Geotechnical modeling: What should be modeled?



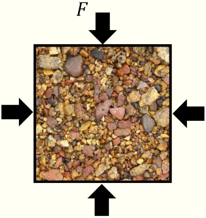
# Scales of modeling in geotechnical engineering

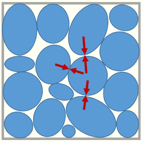





# Soil behavior

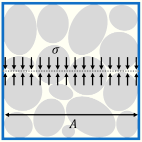
# Numerical frameworks



***Discrete mechanics***

- Micro-scale
- Grain level description
- Intergranular forces




***Continuum mechanics***

- Macro-scale
- Stress definition

$$\sigma = \frac{F}{A}$$

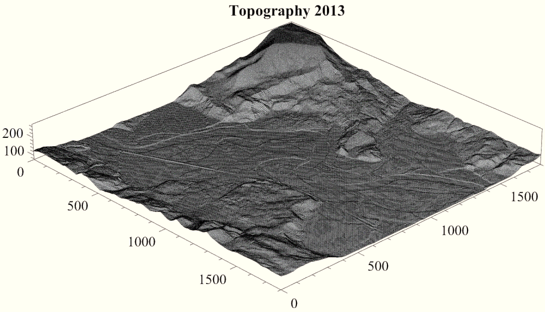


# Oso landslide: case study

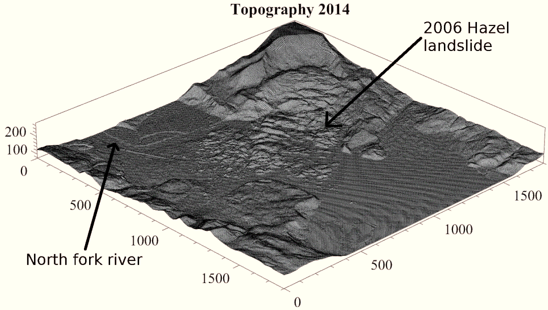
## Oso landslide: case study



## Oso landslide: topography

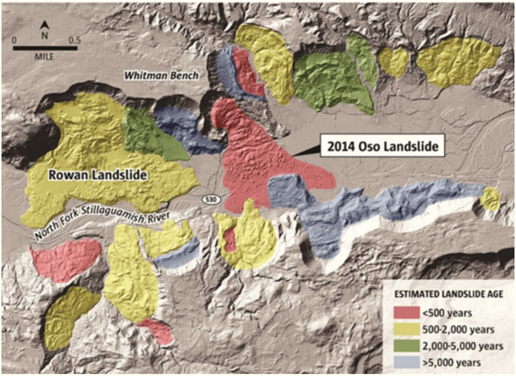


# Oso landslide: topography

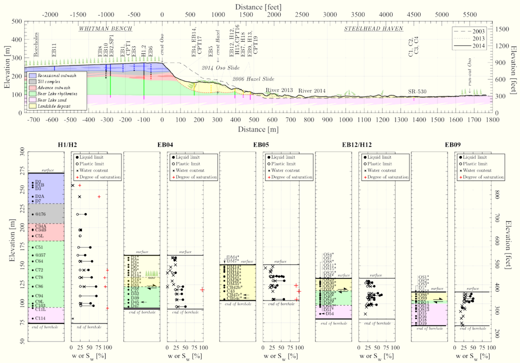





# Oso landslide: historic slides



# Oso landslide: Soil profile






# Oso landslide: Geology. Identify the failure surface

Lower portion of Bear Lake Rhythmites with failure surface. Courtesy of Dr Gunnar Schlieder

# Oso landslide: Geology

Deformation till with flame structures in fine-grained glacio-lacustrine deposit EB7 (depth 65 ft). Courtesy of Dr Gunnar Schlieder

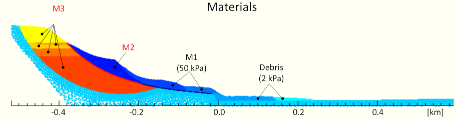


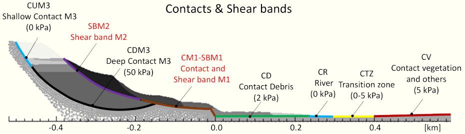
# Oso landslide: Direct shear test (intact specimens)

- **Peak strength (effective stress)**
  - Friction angle $\phi'_{max} = 24^o (> \phi'_{cs} = 22^o)$
  - Cohesion $c'_{max} = 100 kPa (> c'_{cs} = 0)$
- **But some specimens**
  - Friction angle $\phi'_{max} = 12^o (< \phi'_{cs} = 22^o)$
  - Cohesion $c'_{max} = 0 kPa (= c'_{cs})$
- Natural soil with structure
  - Is it an intrinsic material property?
  - Or a soil structure property?

# Oso landslide: Analysis

Model 1

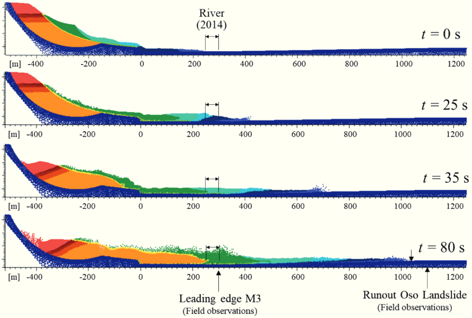



Alba et al., 2018



# Oso landslide: Analysis




Alba et al., 2018

## Advanced analysis in geotechnical engineering

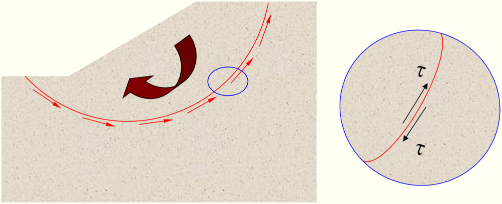
Geotechnical design:

Analysis:



## Classical geotechnical analysis: Slope stability

## Shear failure plane

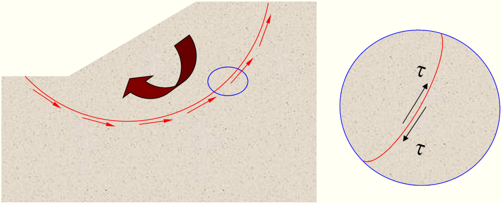



At failure, shear stress along the failure surface ($\tau$) reaches the shear strength ($\tau_f$).

Factor of Safety =

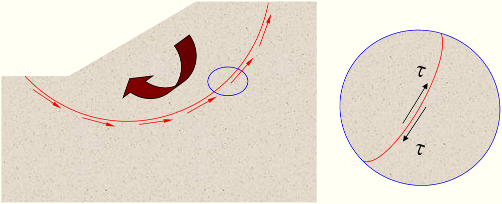


## Dry slope (total stress = effective stress)



## Saturated slope (total stress = effective stress + pwp)

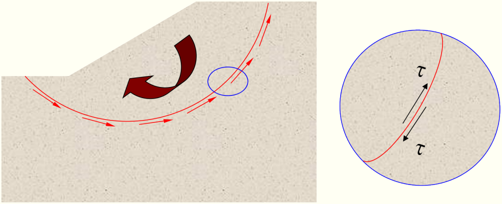
**Drained conditions** - need to compute the steady state pore pressure field and then evaluate "effective stress-based" shear strength to find the overall stability (based on total stress equilibrium).






## Saturated slope (total stress = effective stress + PWP)

**Undrained conditions** - (total stress approach) – Use "*total-stress based*" shear strength ($s_u$) to find the overall stability (based on total stress equilibrium).



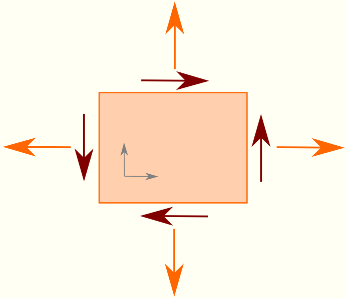
## Governing equations in stress-deformation analysis

In stress-deformation analysis, we need to consider:



## Governing equations in stress-deformation analysis

The governing differential equation for equilibrium expresses:



# Equilibrium equations

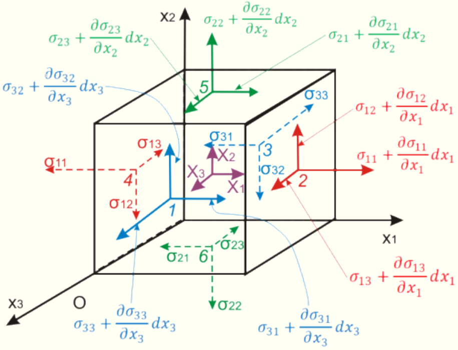
Summing all this in the x-direction gives:

Cleaning up terms that cancel, and dividing through by *dxdy* gives

And summing forces in the y-direction leads to:



# Equilibrium in 3D



## Equilibrium in 3D

The governing differential equation for equilibrium expresses $\sum \mathbf{F} = m\mathbf{a}$ in terms of derivatives of the stress tensor as:



## Stress equilibrium

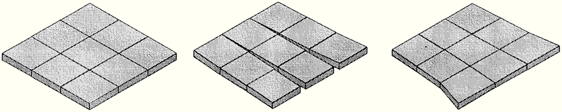
If the object is in equilibrium, then

Stresses:
Equilibrium equation:
Then:

# Compatibility



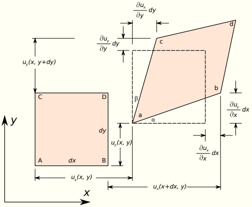
# Governing equations: Compatibility



(a) original (b) non-compatible (c) compatible

# Governing equations: Displacement - strain relationship

Displacement - strain relationship:






# Equilibrium and compatibility conditions

Combining the Equilibrium and Compatibility conditions gives:

- Unknowns:
- Equations:

# Governing equations: Stress-strain relationship

Stress - strain relationship:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \mathbf{D(6x6)} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{bmatrix}$$



# Governing equations in stress-deformation analysis

What are the variables used in the governing equations?

Advanced analysis involves:

## Limit analysis: Lower and upper bound theorems

The exact determination of loads involved in the plastic deformation requires simultaneous solution of three sets of conditions:

1. equations of equilibrium
2. equations of compatibility
3. appropriate constitutive criteria (yield condition and flow rule)

Exact determination is often not easy, may be appropriate for simple shapes, but other cases we may have to use *numerical method*.



## Limit analysis: Lower bound theorem

"*If there is a set of external loads* **which are in equilibrium** *with a state of stress that nowhere exceeds the failure criterion for the material,* **collapse cannot occur** *and the external loads are a lower bound to the true collapse loads*"

Any applied load is less than the actual limiting load, i.e., they will not cause collapse.

## Lower bound solution

We need to determine a set of stresses in the ground that are in equilibrium with the external loads. Stress state everywhere should be:



## Limit analysis: Upper bound theorem

"*If there is a set of external loads and a mechanism of plastic collapse such that the increment of work done by the external loads in an* **increment of displacement** *equals the work done by the internal stresses,* **collapse must occur** *and the external loads are an upper bound to the true collapse loads*"

Apply enough load to achieve the desired change in component shape, e.g., process machinery.

## Upper bound solution

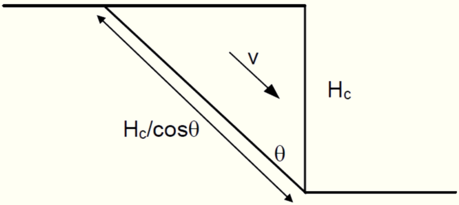
We need to determine a failure mechanism, which is kinematically feasible. The work done is equated by the plastic dissipation along the failure (or slip) surfaces.



## Undrained cut: Lower bound solution

Consider the equilibrium of an element of soil adjacent to the bottom of a cut, or crack. Assume that the major principal stress is vertical, and that the soil fails according to Tresca's criterion at a shear stress $s_u$ (undrained strength).

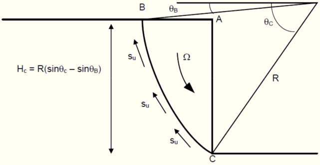
## Undrained cut: Upper bound solution - wedge mechanism




This simple upper bound solution involving a single sliding block gives a normalized cut depth, $N_s = \gamma H_c / s_u = 4$. A more refined upper bound solution involves a circular slip surface, passing through the base of the cut.



## Undrained cut: Upper bound solution - slip circle




Change in potential energy of block ABC (Need to find the centroid and examine the vertical movement by $\Omega$ = Dissipation = $s_u R(\theta_c - \theta_b) R\Omega$

$$N_s = \frac{\gamma H_c}{s_u} = \frac{6(\theta_c - \theta_b)}{2\sin^2\theta_c - \sin\theta_c \sin\theta_b - \sin^2\theta_b}$$

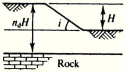
Optimizing: $\theta_b = 27.4^o$ and $\theta_c = 57.5^o$. $H_c = 3.83 s_u/\gamma$.

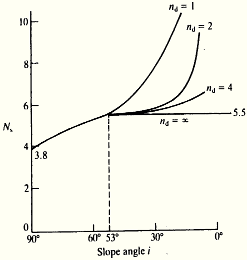
# Taylor stability chart

For sloped cuts as shown on the right, the critical height at different slope angles ($i$), different excavation depths ($H$) and different rock depths ($n_d * H$) can be evaluated using the following generalised formula.

$$N_s = \frac{\gamma H_c}{s_u}$$

where $N_s$ is the stability number ($N_s = 3.8$ for 90 degrees cut).






# Methods of analysis

| **Method of analysis** | **Solution requirements** | | | | |
|---|---|---|---|---|---|
| | **Equilibrium** | **Compatibility** | **Constitutive law** | **Force** | **Displacement** |
| **Closed form** | | | Linear elastic | | |
| **Limit equilibrium** | | | Rigid with a failure criterion | | |
| **Lower bound** | | | Plasticity + flow-rule | | |
| **Upper bound** | | | Plasticity + flow-rule | | |
| **Numerical analysis** | | | Any | | |