

Assignment 2: Stress equilibrium, compatibility and stiffness matrix

1. Prove that the following stress-field could be a valid lower-bound solution.

$$\begin{aligned}\sigma_{xx} &= c_1 x^3 y - 2c_2 xy + c_3 y \\ \sigma_{yy} &= c_1 xy^3 - 2c_1 x^3 y \\ \sigma_{xy} &= -\frac{3}{2}c_1 x^2 y^2 + c_2 y^2 + \frac{1}{2}c_1 x^4 + c_4\end{aligned}$$

where c_1 , c_2 , c_3 and c_4 are constants.

2. Determine and describe the stress-state given by the following Airy stress functions:

(a) $\phi = Ay^2$

(b) $\phi = Bxy$

Are these stresses valid for an isotropic elastic element? Note: please refer https://en.wikiversity.org/wiki/Airy_stress_function on how to determine cauchy stress components from an Airy function.

3. Determine whether the following strain fields are possible in a two-dimensional continuous body:

(a) $\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} c_1(x^2 + y^2) & c_1 xy \\ c_1 xy & c_2 y^2 \end{bmatrix}$

(b) $\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} c_1(x^2 + y^3) & 3c_1 xy^2/2 \\ 3c_1 xy^2/2 & c_2 x^3 \end{bmatrix}$

4. Using Optum G2 perform limit analyses of a vertical cut in clay with the following properties. Excavate to a vertical depth of H_c , where H_c is determined using the stability charts. A preliminary stage involving development of initial stresses of a rectangular domain before excavation is required. Set this phase as the starting phase for both the lower and upper bound analyses. Compare the factor of safety from lower and upper bound solutions. Plot the displacement profiles.

Properties	Values
cohesion (c)	30 kPa
friction (ϕ)	0°
unit weight (γ)	20 kN/m ³
K0	0.6

5. Repeat the previous problem using a slope angle of 30°, instead of a vertical cut, and the depth to bed rock $n_d * H_c$ as H_c and $2 * H_c$.