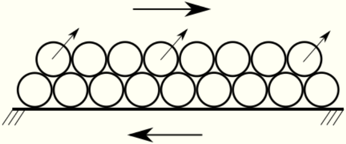
CE394M: Friction

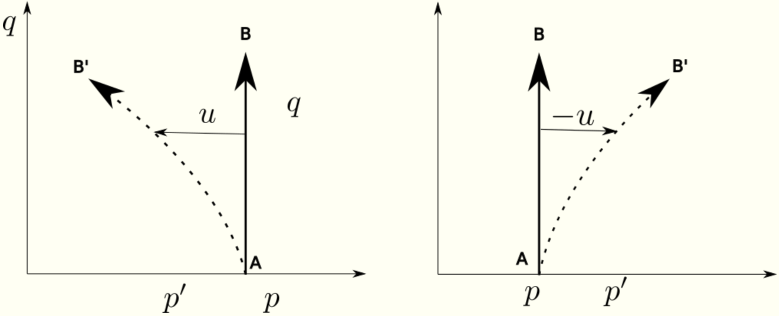
Krishna Kumar

University of Texas at Austin

krishnak@utexas.edu

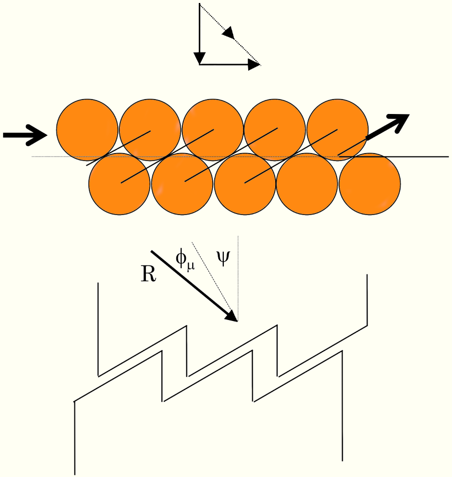
Triaxial compression undrained: loose v dense





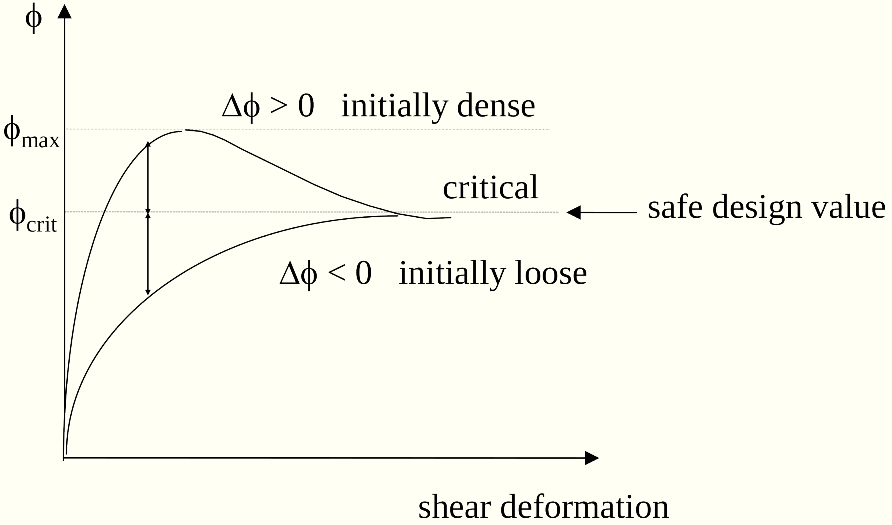
Total and effective stress paths for undrained triaxial test: (a) on soil that wishes to contract as it is sheared, and (b) on soil that wishes to expand as it is sheared.

Friction: Is this correct?



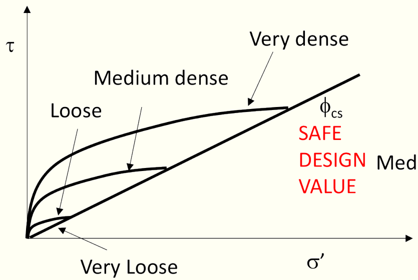
$$\phi_{ss} = \phi_\mu + \psi_{ss}$$

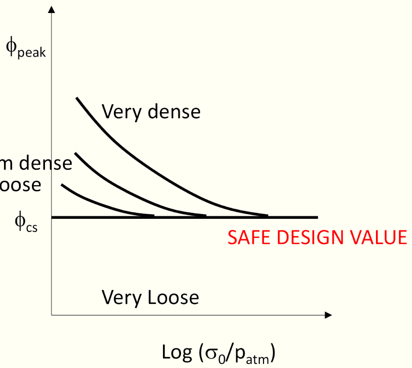
Macroscopically, as soil aggregates...



$$\phi = \phi_{cs} + \Delta\phi_{dilatancy}$$

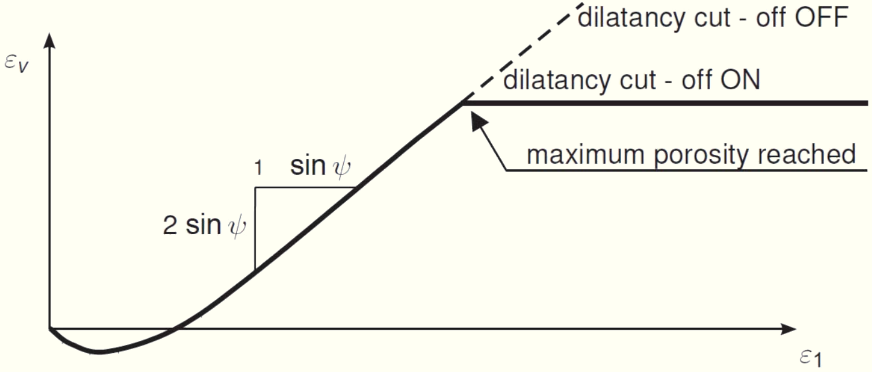
Critical state friction angle and density





$$\phi_{peak} = \phi_{cs} + \Delta\phi(\text{confining stress, initial void ratio})$$

Dilation angle ψ



1. Simulate CD or CU test and check the volume change behavior (for CD) and excess pore pressure behavior (for CU)!
2. For CU, if the computed undrained shear strength is greater than the actual value, use Undrained (B).

Friction angle (Bolton., 1986)

Friction angle

$$\begin{aligned}\phi_{peak} - \phi_{cs} &= 3I_R \quad \text{for triaxial compression} \\ &= 5I_R \quad \text{for plane strain}\end{aligned}$$

Relative density index

$$I_R = I_D \cdot I_c - 1$$

$$\text{Relative crushability } I_c = \ln\left(\frac{\sigma_c}{p'}\right)$$

$$\sigma_c \text{ for quartz} = 20\,GPa$$

$$\text{Relative density } I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

Dilation angle

$$\psi \approx 1.25(\phi_{peak} - \phi_{cs})$$

$$\psi = 0 \text{ when } \phi_{peak} = \phi_{cs}$$

Macroscopic friction angle

- ϕ_{crit} is the angle of friction measured at constant volume of a soil aggregate, and $\Delta\phi$ dilatancy is the extra dilatant contribution to friction angle ϕ. Typical values are:
- Critical state friction ϕ_{crit}:
  - clay: 22°
  - uniform rounded sand: 32°
  - well-graded angular sandy gravel: 38°
- peak strength of pre-compressed or uncrushable grains, densely compacted, and: shearing in plane strain $\Delta\phi$:
  - shearing in plane strain: $\Delta\phi_{max} = 20^\circ$
  - shearing in axial symmetry: $\Delta\phi_{max} = 12^\circ$

Discrete Element Method

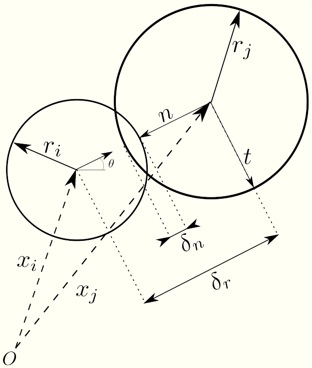
1. Particle level interaction based on Newton's equation of motion
2. The contact normal force is computed as:

$$F_n = \begin{cases} 0, & \delta_n > 0 \\ -k_n\delta_n - \gamma_n \frac{d\delta_n}{dt}, & \delta_n < 0 \end{cases}$$

3. The contact tangential force is computed in a similar way, but has a frictional limit.

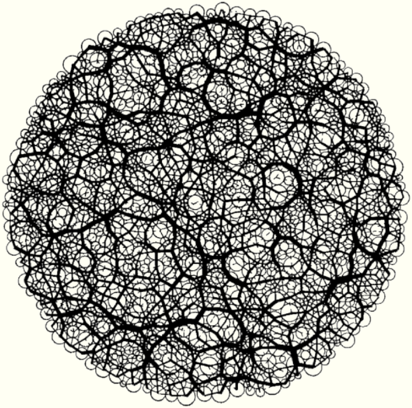
$$F_t \leq \mu F_n$$

4. Solve Newton's second law and the angular momentum equation (including rotational resistance).

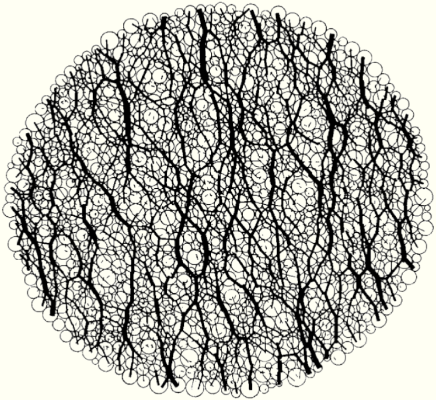


Strong force network vs weak clusters

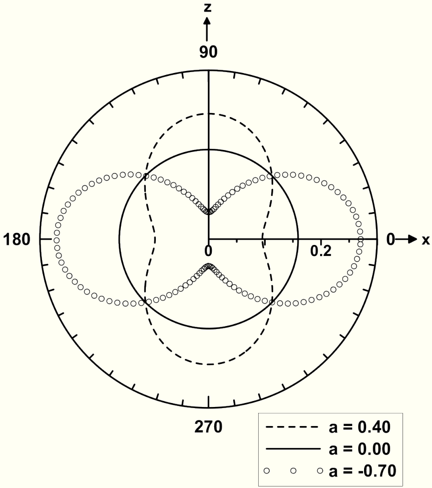
Isotropic Loading



Biaxial Loading



Fabric anisotropy

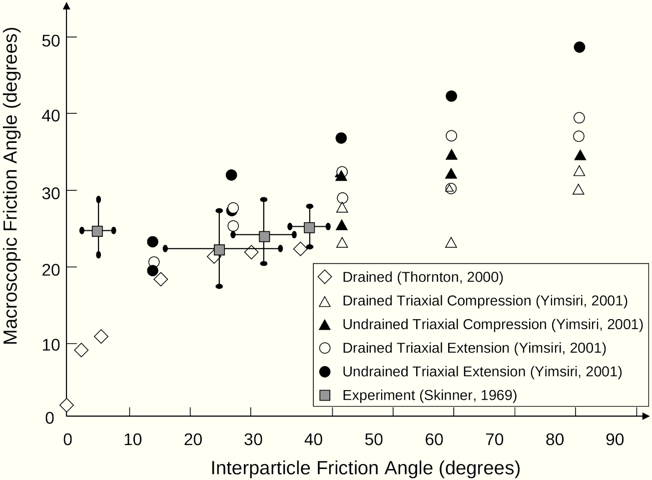


Fabric anisotropy under different stress conditions

Interparticle friction angles

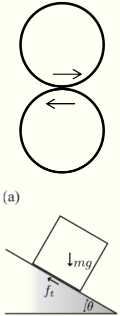
- For Quartz Sands: 26 degrees
- For Sheet Minerals (muscovite, phlogopite, biotite and chlorite): 7 - 13 degrees
  - Water acts as a lubricant
- Clay minerals: Probably 7 - 13 degrees
  - Similar to reported residual friction angles.
  - Sodium Montmorillonite: 4 degrees

Micro to Macroscopic friction angle

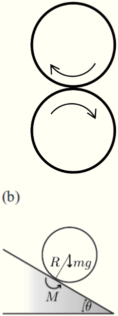


Relationship between macroscopic friction angle and interparticle friction angle (no rolling resistance) - Yimisir and Soga (2001)

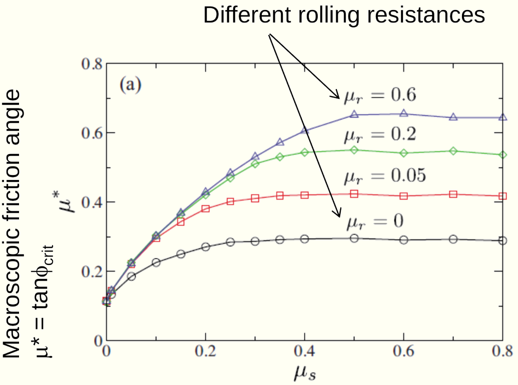
Micro to Macroscopic friction angle: Rolling resistance



Sliding friction μ_s



Rolling resistance μ_r

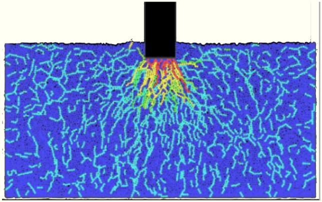


Microscopic sliding friction at particle contacts $\mu_s = \tan \phi_\mu$

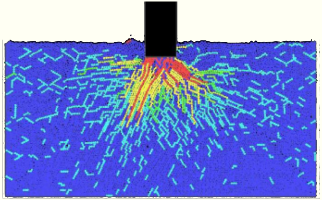
Estrada et al., (2001)

Interparticle friction angles

- The interparticle friction acts as a kinematic constraint of the strong force network and not as the direct source of macroscopic resistance to shear.
- Increased friction at the contacts increases the stability of the system (development of anisotropic fabric) and reduces the number of contacts required to achieve a stable condition.
- As long as the strong force network can be formed, the magnitude of the interparticle friction becomes of secondary importance.



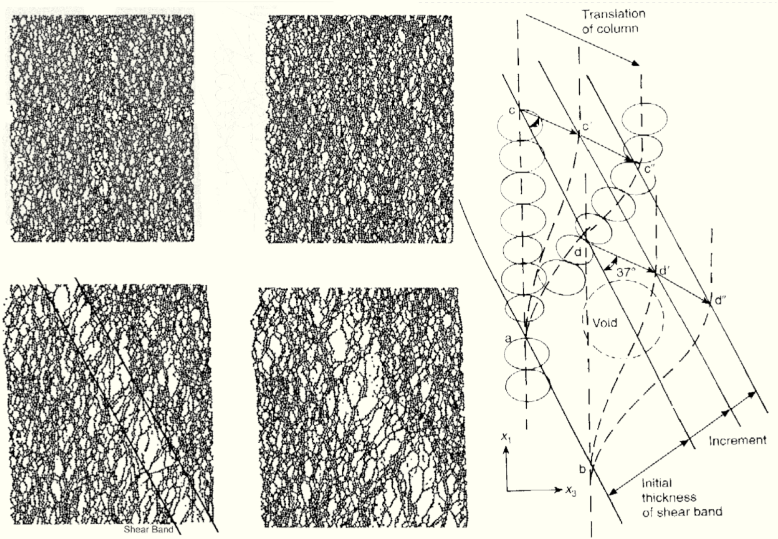
Loose



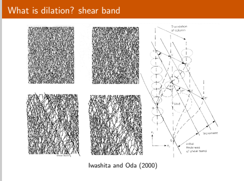
Dense

Muthuswamy and Tordesillas (2006)

What is dilation? shear band



Iwashita and Oda (2000)



Kuhn (1999) reports that their thicknesses are $1.5D_{50}$ to $2.5D_{50}$ in the early stages of shearing and increase to between $1.5D_{50}$ and $4D_{50}$ as deformation proceeds.

Fabric evolution at critical state

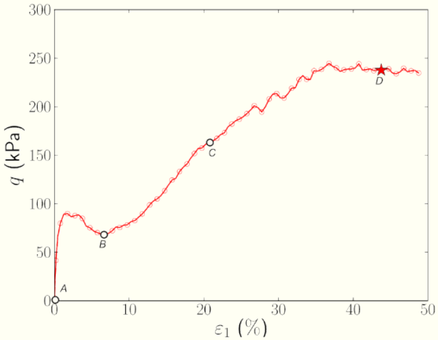


Fig.1 Undrained shear response of a medium dense sand and four stress states selected for examination of internal structure.

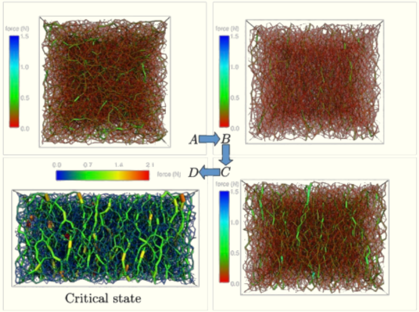
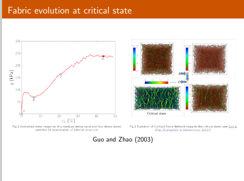


Fig.2 Evolution of Contact Force Network towards the critical state (see Guo & Zhao [Computers & Geotechnics, 2013]).

Guo and Zhao (2003)



As deformation progresses,

- The number of particles in the strong force network decreases.
- With fewer particles sharing the increased loads.
- Anisotropic fabric develops, showing the formation of strong force network. Fabric of particles associated with strong forces is different from that associated with weak clusters.
- At critical state, force chain forms and buckles continuously. Likely to buckle when a force chain has 8 particles