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Transient dynamics of granular piles: continuum v discrete modelling

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Abstract

Transient granular flows, such as rock falls, debris flows, and aerial and submarine avalanches, occur very often in nature. In the geotechnical context, transient movements of large granular slopes are a substantial factor of risk due to their destructive force and the transformations they may produce in the landscape. This paper investigates the ability of MPM, a continuum approach, to reproduce the evolution of a granular pile destabilised by an external energy source. In particular, a central issue is whether the power-law dependence of run-out distance and time observed with respect to the initial geometry or energy can be reproduced by a simple Mohr-Coulomb plastic behaviour for granular slopes subjected a horizontal excitation. The effects of different input parameters, such as the distribution of energy and base friction, on the run-out kinematics are studied by comparing the data obtained from DEM and MPM simulations. The mechanism of energy dissipation is primarily through friction and MPM is able to predict the run-out response in good agreement with DEM simulations. At very low energy, DEM simulations show longer run-out in the case of slope subjected to excitations due to local destabilization at the flow front. At low input energy, the gradient velocity distribution shows significantly longer run-out in comparison to uniform velocity distribution. At low input energies a larger fraction of the energy is consumed in the destabilisation process, hence the amount energy available for flow is less in uniform velocity distribution than the gradient velocity profile. However, at higher input energy, where most of the energy is dissipated during the spreading phase, the run-out distance has a weak dependence on the distribution of velocity in the granular mass.

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Keywords: MPM; Granuar dynamics; Tranient flows; Continuum; Discrete Element Modelling;

1. Introduction

Transient granular flows occur very often in nature. Well-known examples are rockfalls, debris flows, and aerial and submarine avalanches. They form a major element of lanscape reshape and their high destructive potential is a substantial factor of risk. Natural granular flows may be triggered as a result of different processes such as gradual degradation, induced by weathering or chemical reactions, liquefaction and external forces such as earthquakes.

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Granular flows have been studied in laboratory experiments in different geometries such as tilted piles leading to slope failure and surface avalanches [1?] or by considering vertical columns of grains collapsing and spreading under their own weight [2,3]. The dynamics observed in the experiments is often nontrivial in the sense that the final configurations after the dissipation of the whole kinetic energy can not be readily predicted by means of simple laws based on the Mohr-Coulomb nature of the material. For example, in collapsing columns, the runout distance is found to obey a power-law dependence upon the initial aspect ratio of the column.

The observed nontrivial transient dynamics is often correctly reproduced by the discrete element method (DEM), which provides a powerful tool for the grain-scale analysis of the trigger and its subsequent dynamics [4,5]. However, even in simplified geometries such asthose investigated in the experiments, the DEM suffers from a serious shortcoming in the number of particles that can be simulated in a reasonnable time. This is a critical issue when more complex geometries or long-time granular processes are considered, or when particle size distributions are broad. For this reason, most numerical studies are performed in 2D or simple particles shapes and size distributions are considered.

It is also obvious that classical modeling strategies based on the finite element method (FEM) cannot be used for the simulation of very large deformations. In various application of FEM, this problem is treated by means of technical tools such as remeshing. Such mthods are, however, not robust and lead to round-off errors and mesh-senstitivity. In contrast, the so-called Material Point Method (MPM) is an alternative approach for continuum problems that allows for indefinitely large deformations without remeshing Reference??. In this method, the material points carry the information on state variables and a background fixe grid is used to solve the governing equations. The information between the material points and the grid is exchanged via suitable shape functions. The MPM has been applied with success to a number of solid mechanics problems and its theoretical foundations have recently been invesitgated by several authors.

In this paper, we investigate the ability of the MPM, as a continuum approach, to reproduce the evolution of a granular pile under its own weight or when destabilized by energy input. In particular, a central issue is wheather power-law dependence of the runout distance and timewith respect to the initial geometry or energy can be reproduced by a simple prescription of the Mohr-Coulomb plastic behavior within a MPM code. We therefore perform extensive simulations by varying continuously different input parameters and compare the data with those obtained from DEM simulations of the same system. We compare in detail the evolution of the profile of the pile and its total kinetic energy between the two methods and for differerent initial states. As we shall see, the MPM can successfully simulate the transient evolution with a single input parameter, namely the internal angle of friction. This opens the way to the simulation of geological-scale flows on complex topographies.

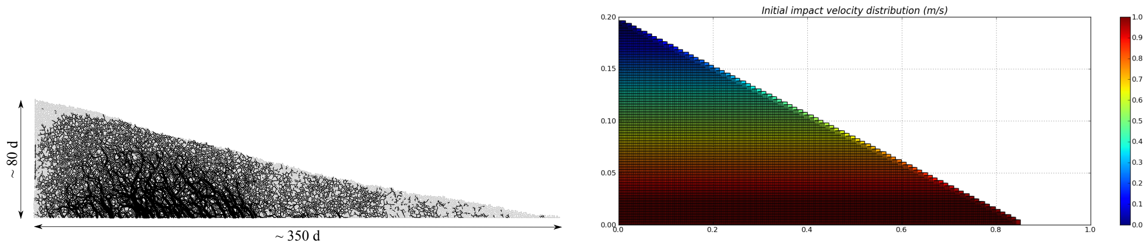
In the following, we first briefly introduce the MPM with a Mohr-Coulomb plastic behaviour. In section 3, we investigate the collapse dynamics and runout of granular columns. We then consider in section 4 the spreading of an initially static pile under the action of an impact energy. We conclude with a summary and discussion of the most salients results of this work.

1.1. Numerical set-up

The DEM sample was composed of $\sim$ 13000 disks with a uniform distribution of diameters by volume fractions ($d_{max} = 1.5 d_{min}$). The mean grain diameter and mass are $d \simeq 2.455$ mm and $m \simeq 0.0123$ kg, respectively. The grains are first poured uniformly into a rectangular box of given width and then the right-hand side wall is shifted far to the right to allow the grains to spread. A stable granular slope of 13.2 ° is obtained when all grains come to rest; see fig. 1. This procedure leads to a mean packing fraction $\simeq 0.82$. Soil grains with a mean density of 2600 kg/m^3 and internal friction coefficient of 0.4 between grains are considered. For grain-scale simulations, classical DEM approach was used.

The initial static pile was set into motion by applying a horizontal gradient velocity $v_{0x}(y) = k(y_{max} - y)$ with $k > 0$. The evolution of the pile geometry and the total kinetic energy as a function of the initial input energy E_0 is studied. The run-out distance L_f is the distance of the rightmost grain, which is still in contact with the main mass when the pile comes to rest. The run-out will be normalised by the initial length L_0 of the pile, as in the experiments of collapsing columns. The total run-out duration t_f is the time taken by the pile to reach its final run-out distance L_f.

In MPM simulations, the material point spacing is adopted to be the same as the mean grain diameter in DEM. A mesh size of 0.0125m is adopted with 25 material points per cell. The effect of mesh size and the number of



(a) DEM (b) MPM

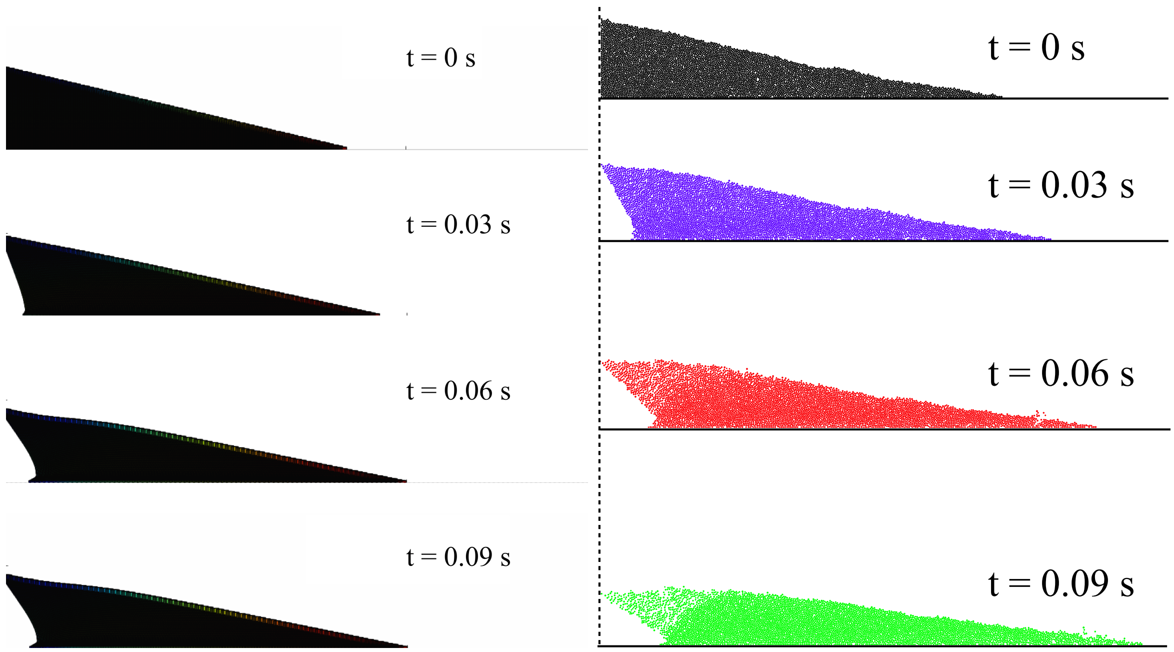
Fig. 1: Initial geometry and dimensions of the pile subjected to a horizontal velocity of 50J.

material points per cell was investigated by [6]. The initial configuration of the slope in MPM is shown in fig. 1b. Frictional boundary conditions are applied on the left and the bottom boundaries by applying constraints to the nodal acceleration. Mohr-Coulomb model with no dilation is used to simulate the continuum behaviour of the granular pile. Periodic shear tests in DEM reveals a macroscopic friction coefficient of 0.22.

The natural units of the system are the mean grain diameter d, the mean grain mass m and acceleration due to gravity g. For this reason, the length scales are normalised by d, time by $(d/g)^{1/2}$, velocities by $(gd)^{1/2}$ and energies by mgd.

1.2. Evolution of pile geometry and run-out

Figure 2a shows the initial evolution of the granular slope subjected to an initial horizontal energy $E_0 = 61$ (in dimensionless units) using MPM. As the granular slope is sheared along the bottom, the shear propagates to the top leaving a cavity in the vicinity of the left wall. This cavity gets partially filled as the granular mass at the top collapse behind the flowing mass due to inertia. Similar behaviour is observed during the initial stages of the flow evolution using CD technique (fig. 2b). Due to inertia, the grains at the top of the granular heap roll down to fill the cavity, while the pile continues to spread.

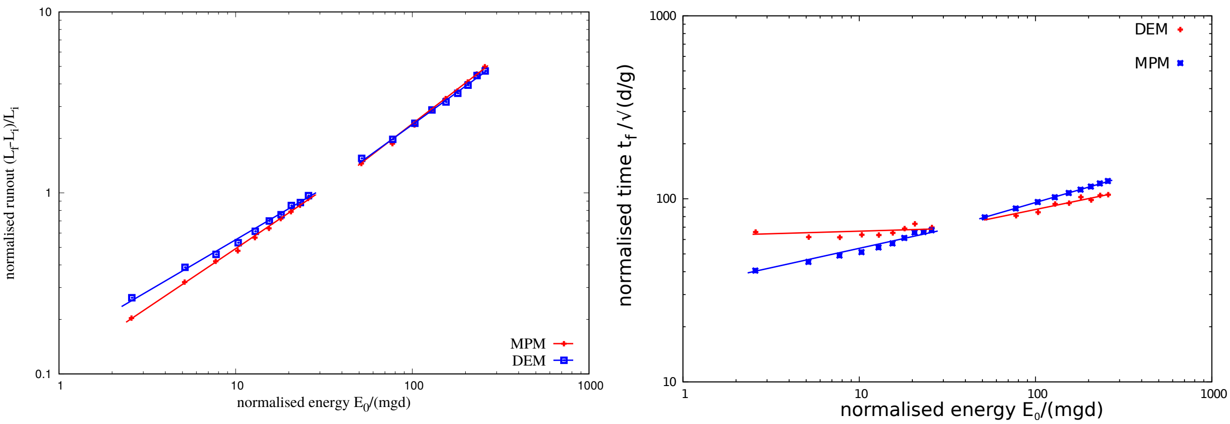


(a) MPM simulation of the initial stages of granular pile subjected to a gradient horizontal energy.

(b) CD simulation of the initial stages of granular pile subjected to a gradient horizontal energy. [7].

The flow involves a transient phase with a change in the geometry of the pile followed by continuous spreading. The gradient of input energy applied to the granular slope mimics a horizontal quake. Despite the creation of a cavity behind the flowing mass, the granular heap remains in contact with the left wall irrespective of the input energy. Figure 3a shows the normalised run-out distance $(L_f - L_0)/L_0$ and total run-out time t_f as a function of the input energy E_0. Two regimes characterised by a power-law relation between the run-out distance and time as a function of E_0 can be observed. In the first regime, corresponding to the range of low input energies $E_0 < 40\, mgd$, the run-out distance observed varies as $L_f \propto (E_0)^{\alpha}$ with $\alpha \simeq 0.206 \pm 0.012$ over nearly one decade. Overall, the run-out distance predicted by the continuum approach matches the DEM simulations. At very low energies, DEM simulations show longer run-out distance due to local fluidisation. The difference in the run-out between DEM and CD arise mainly from the scales of description and the inelastic nature of Contact Dynamics. Similar behaviour between DEM and CD approaches was observed by Radjai et al.[8].

While the run-out exhibits a power-law relation with the initial input energy, the DEM simulations show that the flow duration remains constant at a value $t_f \simeq 60\, (d/g)^{0.5}$ irrespective of the value of E_0. The constant run-out time, in grain-scale simulations, indicates the collapse of grain into the cavity left behind the pile. An average run-out speed can be defined as $v_s = (L_f - L_0)/t_f$. According to the data, $v_s \propto (E_0)^{0.52 \pm 0.012}$. The error on the exponent represents the error due to the linear fit on the logarithmic scale. Since the initial average velocity varies as $v_0 \propto (E_0)^{0.5}$, this difference between the values of the exponents suggests that the mobilised mass during run-out declines when the input energy is increased.



(a) Run-out distance as a function of normalised input kinetic energy.

(b) Duration of run-out as a function of normalised input kinetic energy.

Fig. 3: Evolution of run-out and time as a function of the normalised input energy for a pile subjected a gradient horizontal energy.

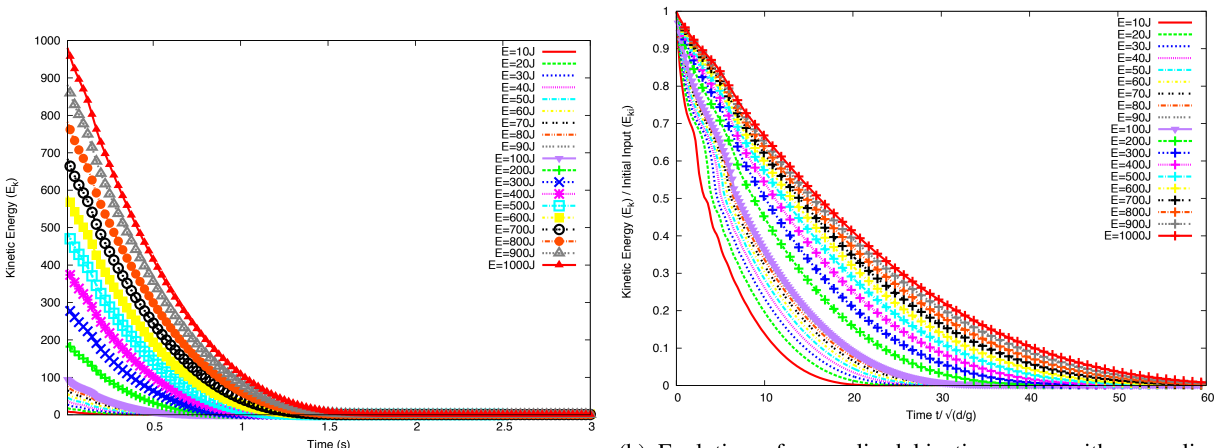
In the second regime, corresponding to the range of high input energies $E_0 > 40\, mgd$, the run-out distance varies as $L_f \propto (E_0)^{\alpha'}$ over one decade with $\alpha' \simeq 0.56 \pm 0.04$ while the duration increases as $t_f \propto (E_0)^{\beta'}$ with $\beta' \simeq 0.33 \pm 0.02$. Hence, in this regime the average run-out speed varies as $v_s \propto (E_0)^{0.498 \pm 0.01}$. This exponent is close to the value 0.5 in $v_0 \propto (E_0)^{0.5}$, and hence, within the confidence interval of the exponents. In the second regime, both DEM and MPM predict almost the same run-out behaviour. However, MPM predicts longer duration with increase in the input energy.

It is worth noting that a similar power-law dependence of the run-out distance and time are found in the case of granular column collapse with respect to the initial aspect ratio. In the column geometry, the grains spread away owing to the kinetic energy acquired during gravitational collapse of the column. Topin et al.[9] found that the run-out distance varies as a power-law of the available peak kinetic energy at the end of the free-fall stage with an exponent $\simeq 0.5$. This value of exponent is lower than the run-out evolution observed in the second regime. This is, however, physically plausible since the distribution of kinetic energies at the end of the collapse is more chaotic than in this case where the energy is supplied from the very beginning in a well-defined shear mode. As pointed out by Staron et al.[4], the distribution of kinetic energies is an essential factor for the run-out distance.

1.3. Decay of kinetic energy

The non-trivial evolution of the pile geometry in two regimes suggests that the energy supplied to the pile is not simply dissipated by shear and friction along the bottom plane. It is important to split the kinetic energy into vertical and horizontal components (K_{Ex} and K_{Ey}) of the velocity field. Although, the input energy is in the x component, a fraction of the energy is transferred to the vertical component of the velocity field and dissipated during the transient phase. The evolution of kinetic energy is studied to understand the behaviour of granular flow that is consistent with the evolution of the pile shape.

The evolution of total kinetic energies E_k with time for different values of the input energy E_{ki} based on MPM simulations are shown in fig. 4. The MPM simulation shows two distinct regimes in the normalised kinetic energy plot as a function of normalised time in fig. 4b. However, the DEM simulations (fig. 5) show that the energy evolution corresponding to the low energy regime nearly collapse on to a single time evolution. This is consistent with the observation of run-out time t_f being independent of the input energy. In contrast, MPM simulations predict a power law relation between the run-out duration and input energy. However, the plots corresponding to the high energy regime (fig. 4), collapse only at the beginning of the run-out i.e. for $t < t_1 \simeq 7.5\ (d/g)^{0.5}$. Although MPM simulations show a longer duration of run-out (fig. 4), the total kinetic energy is completely dissipated at $t = 60\sqrt{d/g}$. DEM simulations predict $t = 80\sqrt{d/g}$ for the kinetic energy to be completely dissipated. This is due to grain rearrangement at the free surface (fig. 6). The granular mass densifies as the flow progresses, after the initial dilation phase for $t = 20\sqrt{d/g}$.



(a) Evolution of total kinetic energy with time.

(b) Evolution of normalised kinetic energy with normalised time.

Fig. 4: Evolution of kinetic energy with time (MPM) for a pile subjected to gradient input velocities.

Figure 7 displays the evolution of kinetic energy in the translational (E_x and E_y) degrees of freedom. E_x decays similar to the total energy dissipation, but E_y increases and passes through a peak before decaying rapidly to a negligible level. The transient is best observed for E_y, which has significant values only for $t < t_1$. This energy represents the proportion of kinetic energy transferred to the y component of the velocity field due to the destabilisation of the pile and collapse of grains in the cavity behind the pile. Higher proportion of vertical acceleration E_{ky}/E_0 is observed for lower values of input energy E_0. This means that, at lower input energies a larger fraction of the energy is consumed in the destabilisation process. Whereas at a higher input energies, most of the energy is dissipated in the spreading phase. For this reason, the total duration t_1 of this destabilisation phase is nearly the same in both regimes and its value is controlled by gravity rather than the input energy. The height of the pile being of the order of 80 d, the total free-fall time for a particle located at this height is $\simeq 12\ (d/g)^{0.5}$, which is of the same order as t_1. DEM simulations show that the contribution of the rotational energy during the transient stage and the spreading stage is negligible.

To analyse the second phase for higher input energies, the kinetic energy E'_{kx0} at the end of the transient phase is considered. This energy is responsible for most of the run-out, hence it is expected to control the run-out distance and

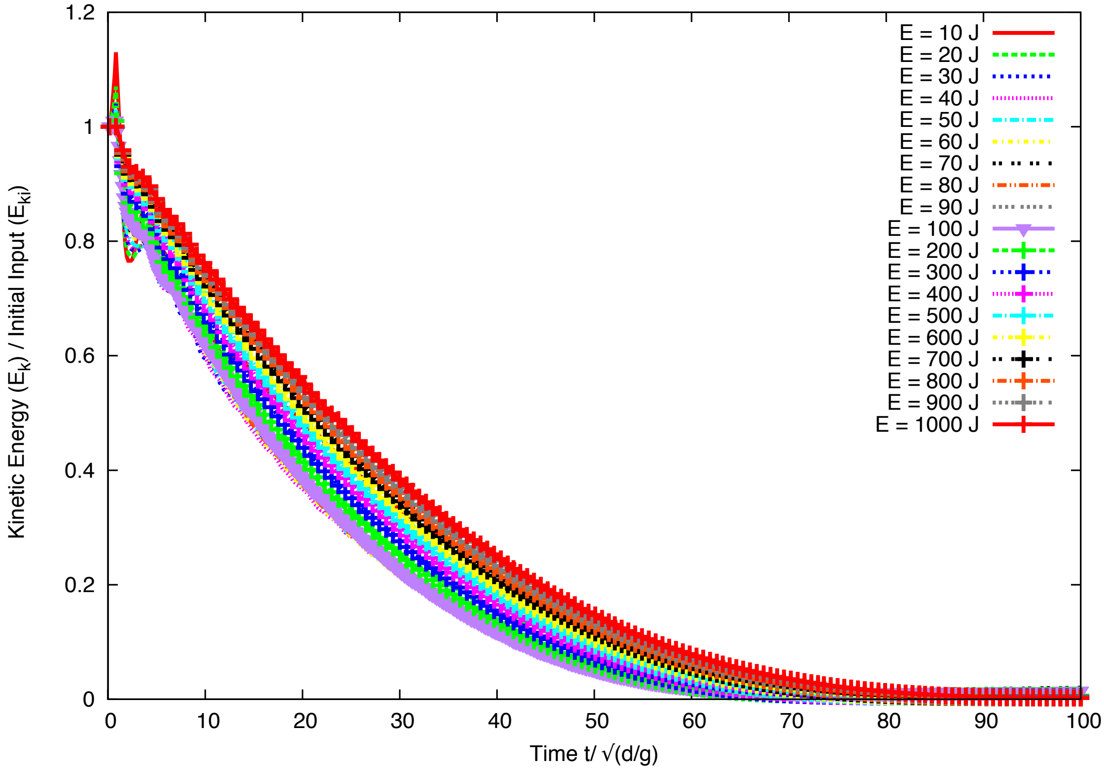


Fig. 5: Evolution of normalised kinetic energy with normalised time for a pile subjected to gradient input velocities.

time. Figure 8 shows the evolution of E_{kx} normalised by E'_{kx0} as a function of time. The plots have seemingly the same aspect but they show different decay times. A decay time τ can be defined as the time required for E_{kx} to decline by a factor $1/2$. Figure 9 shows the same data in which the time t' elapsed since t_1, normalised by τ. Interestingly, now all the data nicely collapse on to a single curve. However, this curve can not be fitted by simple functional forms such as variants of exponential decay. This means that the spreading of the pile is not a self-similar process in agreement with the fact that the energy fades away in a finite time t'_f.

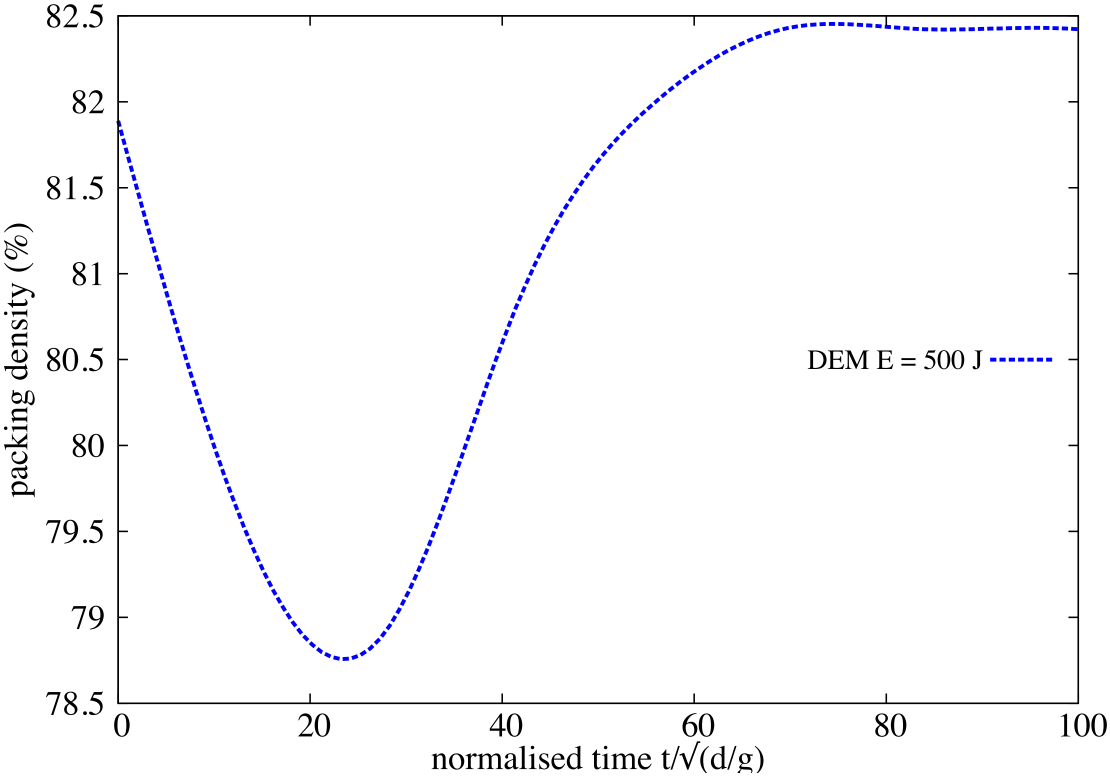
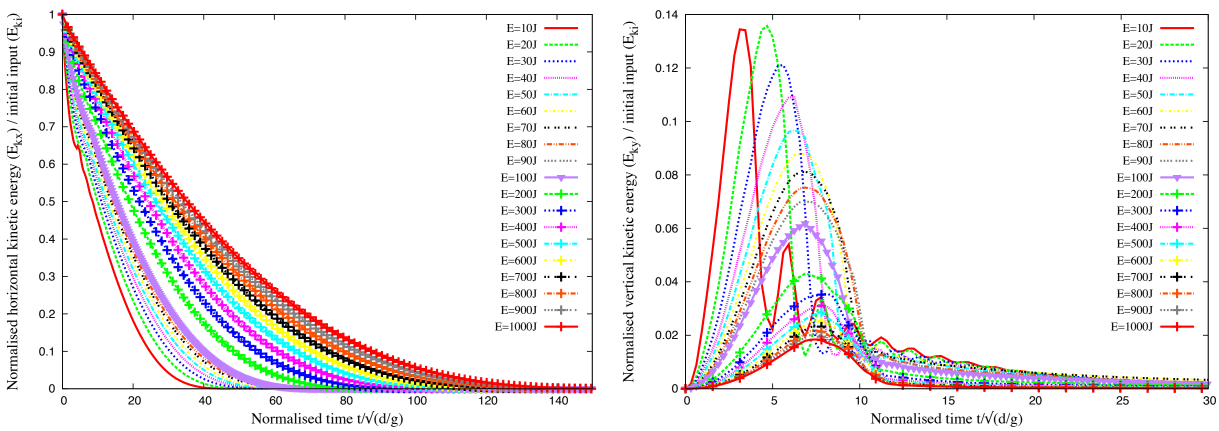


Fig. 6: Evolution of packing density with time $E_0 = 152mgd$ (DEM).



(a) Evolution of normalised horizontal kinetic energy with time. (b) Evolution of normalised vertical kinetic energy with time.

Fig. 7: Evolution of vertical and horizontal kinetic energy with time (MPM) for a pile subjected to gradient input velocities.

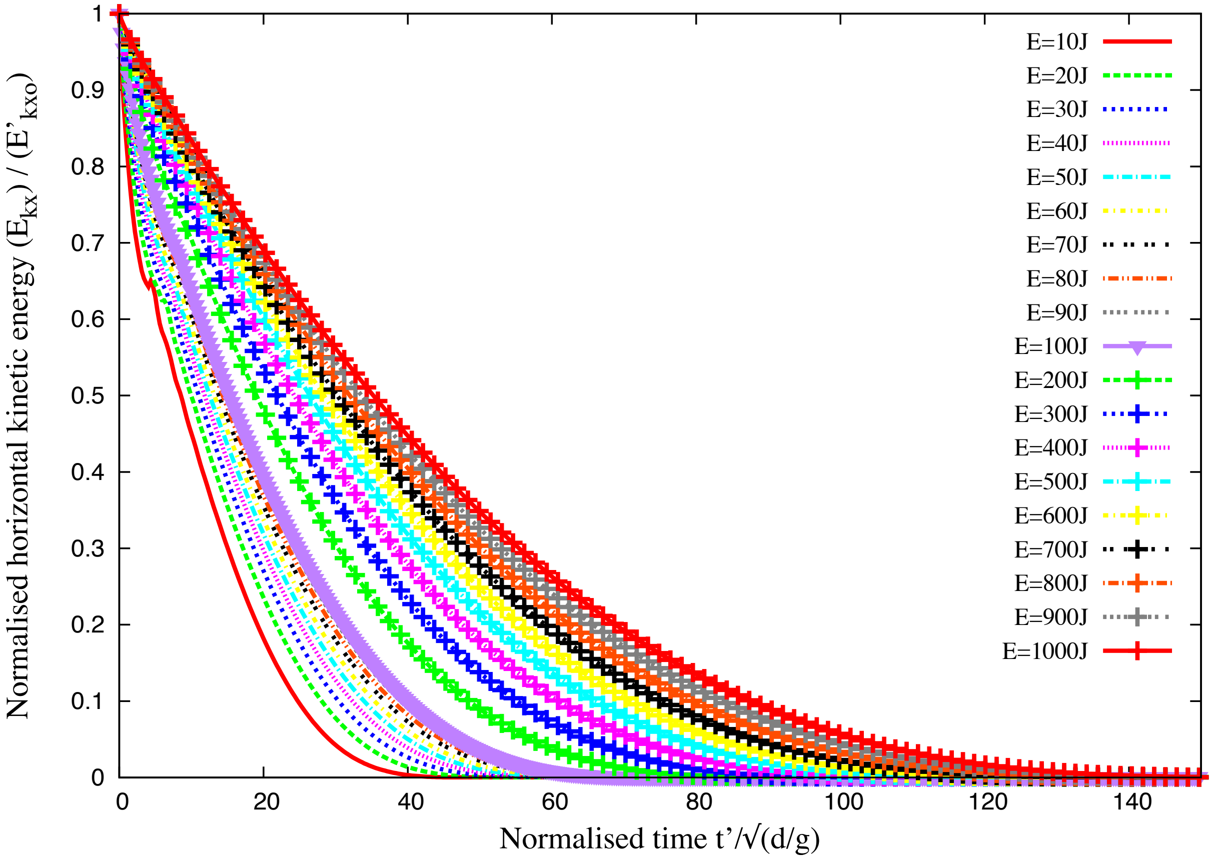


Fig. 8: Evolution of kinetic energy in the x component of the velocity field normalised by the available kinetic energy at the end of the transient phase as a function of time elapsed since the same instant (MPM).

The scaling of the data with the decay time τ suggests that the run-out time, since the beginning of the second phase, t'_f might be a simple function of τ. Figure 10a shows both t'_f and τ as a function of E'_{x0}, where a power-law relation can be observed for both time scales. The run-out time $t'_f \propto (E'_{x0})^{\beta'}$ has the same exponent $\beta' \simeq 0.33 \pm 0.02$ as t_f as a function of E_0. For the decay time we have $\tau \propto (E'_{x0})^{\beta''}$ with $\beta'' \simeq 0.38 \pm 0.03$. The relation between the two times can thus be expressed as (fig. 10b)

$$t'_f = k\,\tau\,(E'_{x0})^{\beta''-\beta'}\,, \tag{1}$$

where $k \simeq 5.0 \pm 0.4$ and $\beta'' - \beta' \simeq 0.05 \pm 0.05$. This value is small enough to be neglected. It is therefore plausible to assume that the run-out time is a multiple of the decay time and the spreading process is controlled by a single time. A weak dependence on the energy E'_{kx0} is consistent with the fact that the energy available at the beginning of the second phase is not dissipated in the spreading process (calculated from the position of the tip of the pile) since the pile keeps deforming by the movements of the grains at the free surface even when the tip comes to rest. This can explain the small difference between the two exponents as observed here.

1.4. Effect of friction

The run-out distance, duration of flow, and the dissipation of kinetic energy are controlled by the input energy and collective dynamics of the whole pile. However, the run-out behaviour is also expected to depend on the base friction. A series of simulations with different values of base friction was performed using MPM to analyse the influence of friction on the run-out behaviour. The influence of friction on the run-out behaviour for different input energies is shown in fig. 11a. The run-out distance decreases with increase in the basal friction. The exponent of the power-law

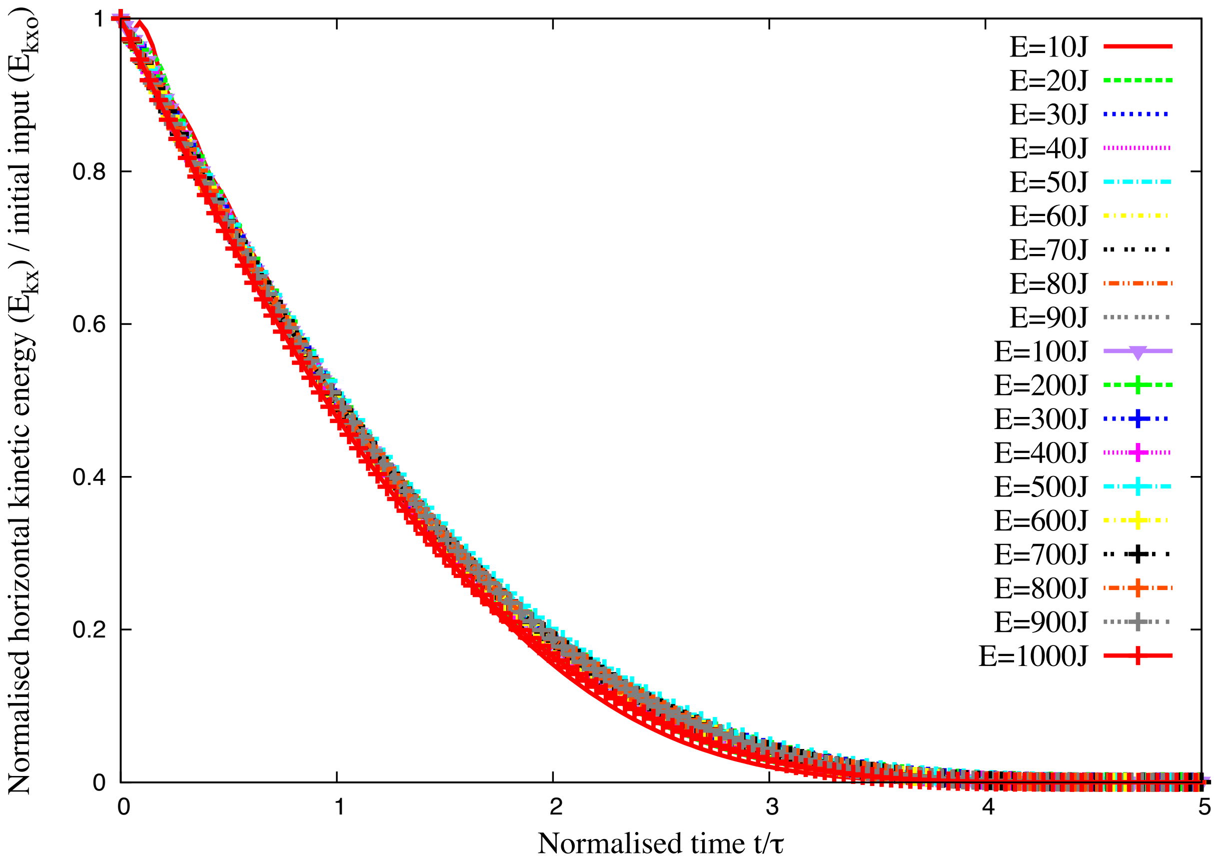
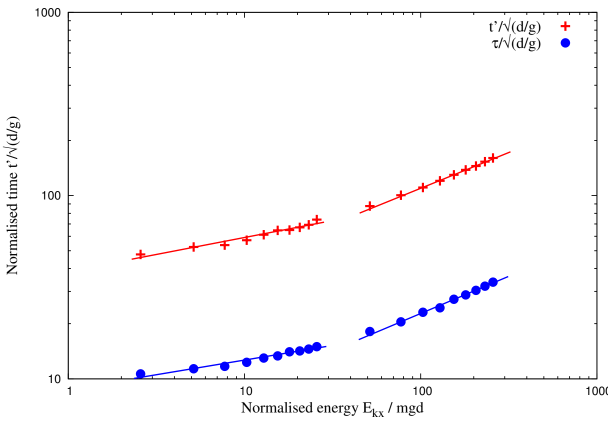
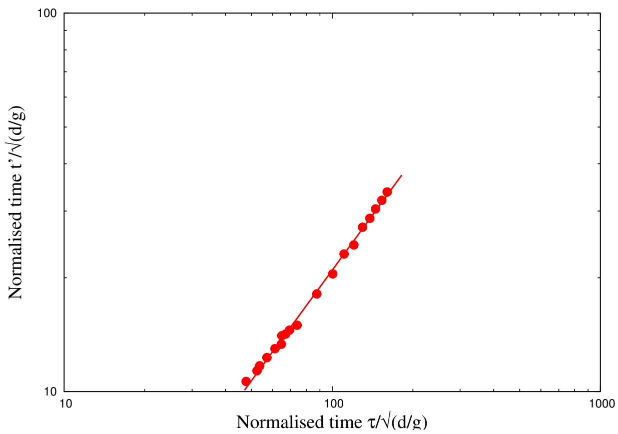


Fig. 9: Evolution of kinetic energy in the *x* component of the velocity field normalised by the available kinetic energy at the end of the transient as a function of normalised time (MPM).



(a) Power law evolution of t'_f and τ as a function of kinetic energy E'_{kx0}.



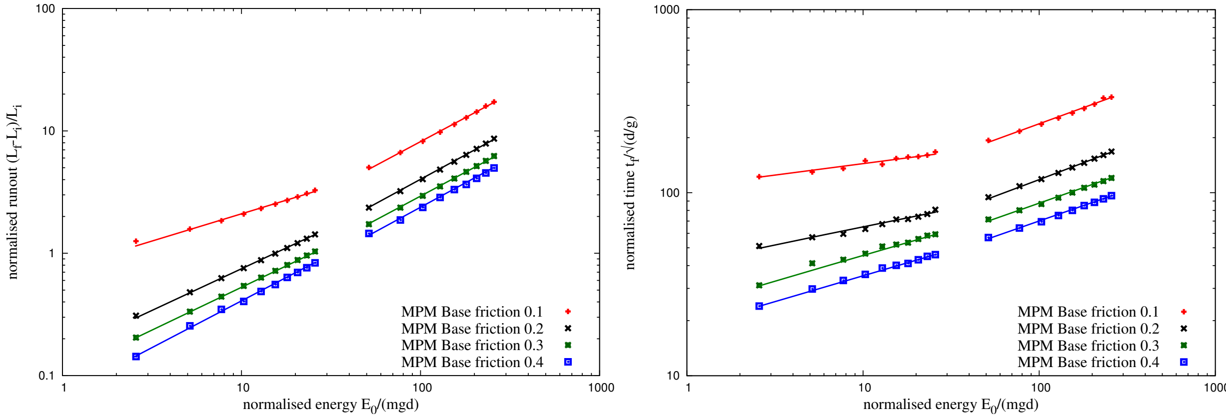
(b) Linear relationship between decay time and run-out time after the transient as a function of the normalised kinetic energy E_{kx0}.

Fig. 10: Decay time and run-out time as a function of the normalised kinetic energy E_{kx0}.

relation between the run-out and input energy has a weak dependence on the base friction, however, the proportionality constant is affected by the change in the base friction. This behaviour is similar to that observed in granular column collapse with varying initial properties [3,10].

CD simulations using different values of coefficient of restitution show no difference in the run-out behaviour. At large input energies, the pile remains in a dense state so that multiple collisions inside the pile occur at small time scales compared to the deformation time. When the restitution coefficients are increased, more collisions occur during a longer time interval but the overall energy dissipation rate by collisions remains the same. This effect is a seminal example of collective effects which erase the influence of local parameters at the macroscopic scale.

In contrast to the restitution coefficients, the effect of friction coefficient is quite important for the run-out. MPM simulations with varying friction coefficient show that, both the run-out distance and the decay time decrease as the friction coefficient is increased. This effect is much more pronounced at low values of the friction coefficient. The run-out time, for example, is reduced by a factor of approximately 4 as μ_s is increased from 0.1 to 0.2 while the change in the run-out and duration is less affected with increase in friction coefficient. This “saturation effect” can be observed in a systematic way in simple shear tests. The dissipation rate may reach a saturation point where the dilation of granular materials and rolling of the grains change in response to increase in friction coefficient [11].



(a) Effect of friction on the run-out distance

(b) Effect of friction on the duration of run-out.

Fig. 11: MPM simulations of effect of friction on the run-out behaviour of slopes subjected to horizontal excitation.

2. Conclusion

Natural granular flows are triggered by different mechanisms. The distribution of kinetic energy in the granular mass is found to have an effect on the flow kinematics. A multi-scale analyses of a granular slope subjected to horizontal velocities are performed and the following conclusions are derived:

- A power-law dependence of the run-out distance and time as a function of the input energy is observed. The power-law behaviour is found to be a generic feature of granular flow dynamics.
- The values of the power-law exponents are not simple functions of the geometry.
- Two regimes with different values of the exponents: a low-energy regime and a high-energy regime are observed.
- The low energy regime reflects mainly the destabilisation of the pile, with a run-out time independent of the input energy.
- The second regime is governed by the spreading dynamics induced by higher input energy. The evolution of granular slope in the high-energy regime can be described by a characteristic decay time, which is the time required for the input energy to decay by a factor of 0.5.

- The run-out distance and the decay time decrease as the friction increases. This effect is much more pronounced at low values of friction.
- At low input energy, the distribution of kinetic energy in the system is found have a significant effect on the run-out, as the energy is mostly consumed in the destabilisation process.
- At higher input energy, where most of the energy is dissipated during the spreading phase, the run-out distance has a weak dependence on the distribution of velocity in the granular mass.
- The duration of the flow shows similar behaviour to the run-out, however, a slope subjected to a gradient velocity flows quicker than a slope subjected to a uniform horizontal velocity.
- The material property and the distribution of kinetic energy in the system has a non-trivial influence on the flow kinematics and the internal flow structure.
- MPM is successfully able to simulate the transient evolution with a single input parameter, the macroscopic friction angle.

This study exemplifies the ability of MPM, a continuum approach, in modelling complex granular flow dynamics and opens the possibility of realistic simulation of geological-scale flows on complex topographies.

Acknowledgements

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