363 Winter 2017 Determination of an IIR Digital Notch Filter due March 3.

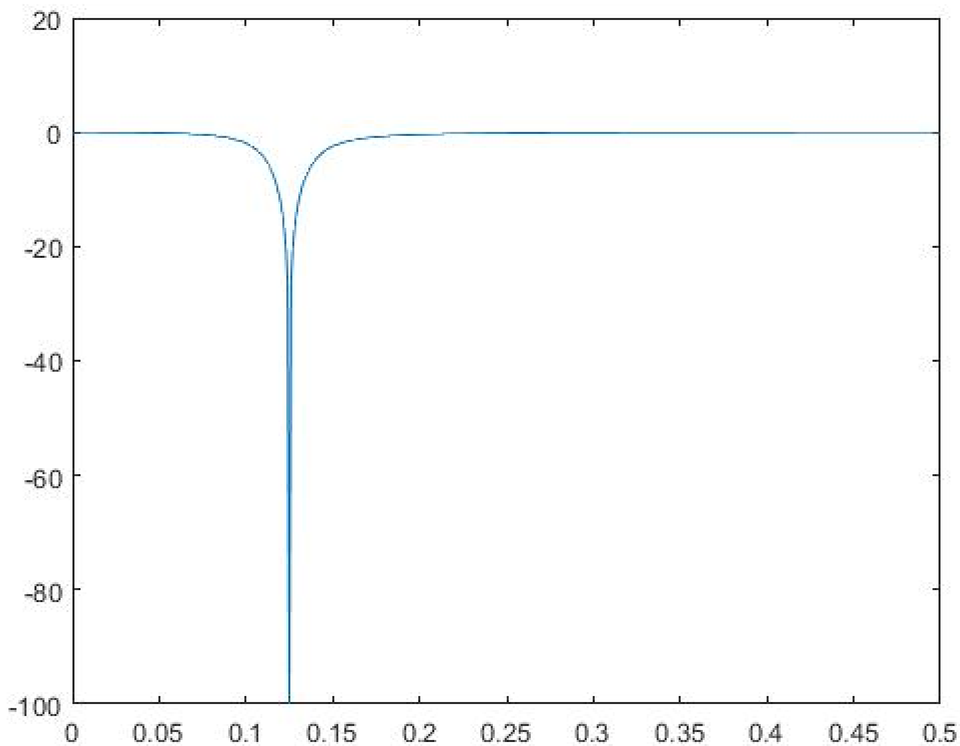
Matlab exercise in the determination of a notch IIR digital filter in the form of a single biquad section. You should say "`format long`" at the beginning of your calculations.

An analog biquad is a filter with transfer function

$$F(s) = \frac{c_0 s^2 + c_1 s + c_2}{s^2 + d_1 s + d_2}$$

We wish to first determine coefficients so that $F(0) = 1$ (therefore $c_2 = d_2$), $F(\infty) = 1$ (therefore $c_0 = 1$) , $F(j\Omega_0) = 0$, where $\Omega_0 = 12\pi$ (representing 6 kHz, so $c_1 = 0$, $c_2/c_0 = \Omega_0^2$), and $|F(j10\pi)| = 1/\sqrt{2}$ (representing 3 dB down at 5 kHz). This is an analog notch filter rejecting 6 kHz.

Determine all the coefficients of F. Then apply the bilinear z-transform to get a digital filter operating at sampling frequency 48 kHz and frequency of equivalence 6 kHz, to determine a digital biquad H constituting a notch filter rejecting the frequency 6 kHz if the sampling frequency is 48 kHz. For this purpose you can use the Matlab function `bilinear`; consult `doc bilinear` in Matlab. You need the version `[numd,dend] = bilinear(num,den,fs,fp)` (the "pre-warped" mode) to match the frequency response at 6 kHz (`fp`). Hand in your computations, both by hand and via Matlab. Also compute, plot and hand in the frequency response of the digital filter , for $0 \le f \le 0.5$, using the utility `freqzdB`. Here is the plot I got:



Yours should be the same. You should also do `zplane(numd,dend)` for a z-plane plot showing the pole near the zero that is exactly on the unit circle at the point corresponding to 6 kHz. Hand in your calculations, Matlab diary or m-files, frequency response graph, and z-plane plot.

Now take another approach to the notch filter problem which will not require the bilinear z-transform: direct determination of the notch filter via *z*-plane reasoning. The ideal we are aiming for would reject only the frequency 6 kHz and pass everything else without change. As an "ideal", of course, it is not attainable. Basically, we are going to place the pole closer to the zero than we did above.

The numerator of *H* should give you a zero at 1/8 the sampling frequency of 48 kHz:

$$H(z) = A\frac{1-\sqrt{2}z^{-1}+z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}$$

The denominator should give you a pole very close to that zero but slightly inside the unit circle; thus the negative coefficient a_1 is slightly less than √2 in magnitude and a_2 is slightly less than 1. How close? Aye, there's the rub. We are going to implement this version for a lab and the answer will depend mainly on the 24-bit limitation of the AIC3204 codec. As a practical matter, you should continue to think of this as a 16-bit situation, since the signals are still 16 bits. Place the pole so that you get a significantly better approximation to the ideal than the first approach gave you, but the notch still shows in the frequency response plot on 501 points. It should now be difficult to distinguish the locations of the pole and zero in the z-plane plot. Hand in your calculations, Matlab diary or m-files, a frequency response graph that shows a notch at 6 kHz, and the corresponding z-plane plot, in which it is difficult to distinguish the locations of the poles from those of the zeros.

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