

INTRODUCTION TO STATISTICS

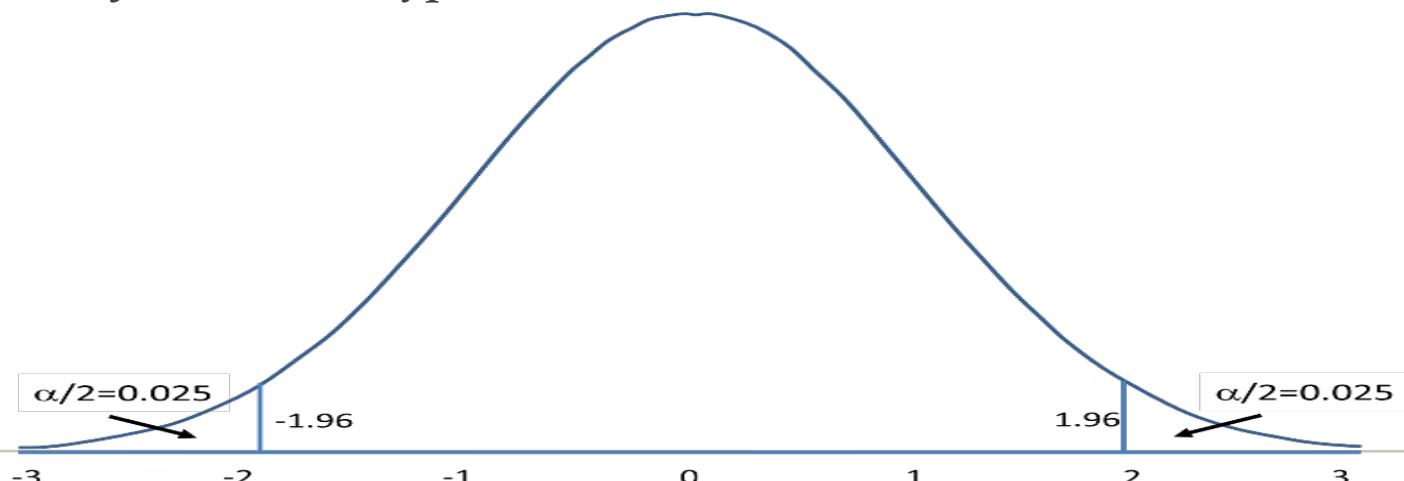
Session 11
June 26th 2017
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Review- Hypothesis testing

1. Write the null and alternative hypothesis
2. Identify if the test is one-sided or two sided
3. Write down all information from the problem
4. Compute the test statistic
5. Find the critical/ p-value from the table
6. Make a decision to reject or fail to reject the null hypothesis
7. Write the conclusion

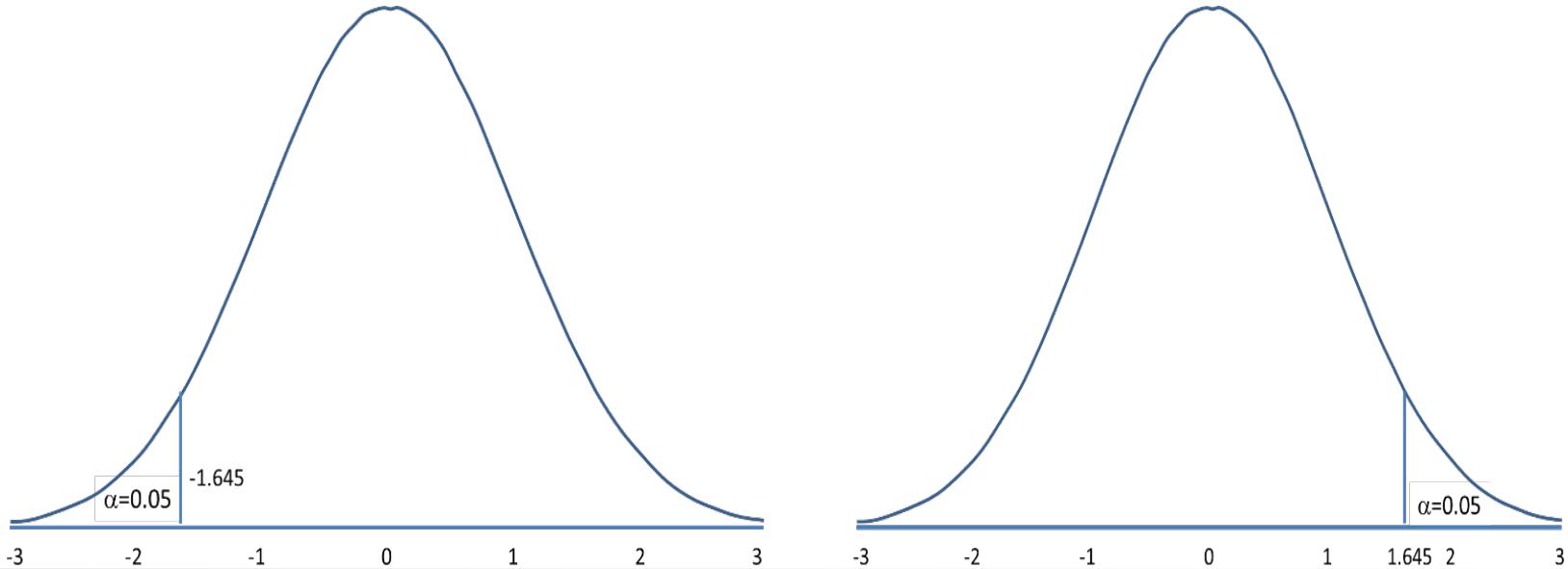
Two sided

- Two sided- if you are using a significance level (α) (*chance of rejecting the null hypothesis, even if the null hypothesis is true- type I error*) of 5%- then must allot half of the α to each tail.
- Our null hypothesis is that the parameter is equal to x . A two-tailed test will test both if the parameter is significantly greater than x and if the parameter significantly less than x .
- The parameter is considered significantly different from x if the test statistic is in the top 2.5% or bottom 2.5% of its probability distribution, resulting in a p-value less than 0.05.
- This is saying that if the calculated test statistic falls into the shaded region, one can reject the null hypothesis



One sided

- One sided- if one is using a significance level of 5%- allot all of the α to one tail.
 - In a one-tailed test, testing for the relation in only one direction and disregarding the possibility of a relation in the other direction
 - test if the parameter is significantly greater than x or if the parameter is significantly less than x , but not both



Example

A teacher gives a pop quiz for the 50 students in his class. The mean score is 60 out of 100 and the standard deviation 15.

John, a student in the class, scored 70 and wants to know how well he did compared to the other students- is his score significantly above the average?

- What is the null and alternative hypothesis??
- Is this a one sided or two sided test?
- What is the test statistic?

Given information

$$(X) = 70$$

$$\mu = 60$$

$$s = 15 \quad n = 50$$

Example-solution

- Test statistic

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

$$Z = (70 - 60) / (15 / \sqrt{50})$$

$$= 10 / (15 / 7.07)$$

$$= 4.72$$

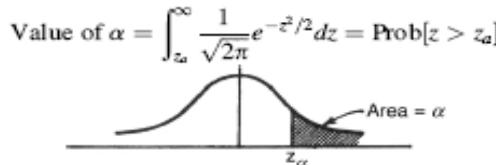
Reject H_0 if $Z \geq 1.645$ or if $Z \geq 1.96$?

What can we conclude?

P-value

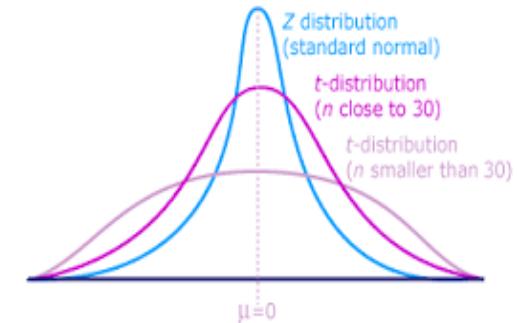
- How to find the p-value for a hypothesis test using the z-table?
 - For a one sided test, if the test statistic is 2- the p-value is 0.0228
 - For a two sided test, if the test statistic is 2, this implies that the test statistic is also -2, so must double the p-value from the one sided p-value → p-value is $0.0228 \times 2 = 0.0456$

Z-table



z_α	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0539
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139

Student's t-distribution



- When the population standard deviation is unknown, the parameter has a Student's t distribution.
- The Student's t distribution is similar to the normal distribution.
 - symmetric about its mean, and bell shaped
 - mean of zero
 - standard deviation and variance *greater* than 1.
- Unlike the normal distribution there are different t distributions, one for each degree of freedom (df)
- As the sample size increases, the t distribution approaches the normal distribution.

Degrees of Freedom

- Degrees of freedom (df) is principle to estimating statistics of populations from samples.
- The degrees of freedom (df) of an estimate is the number of independent pieces of information on which the estimate is based.
- Degrees of freedom = $n-1$

Why n-1?

A way to look at degrees of freedom is that they are the number of values that are free to vary in a data set.

What does “free to vary” mean? Here’s an example using the mean:

- Q. Pick a set of 3 numbers that have a mean of 5.
 - A. Some sets of numbers you might pick: 4, 5, 6 or 3, 5, 7
 - Once you have chosen the first two numbers in the set, the third is fixed. **you can't choose the third item in the set.**
 - The only numbers that are free to vary are the first two. Once you’ve made that decision you **must** choose a particular number that will give you the mean you are looking for.

So degrees of freedom for a set of three numbers is TWO. (n-1)

T-distribution test statistic

- The steps for hypothesis testing remain the same- test statistic is no longer the z-score, but the t-score

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{N}}}$$

- Z-score- σ =population standard deviation
- T-scores- s = sample standard deviation

T-distribution table

- The T-distribution table is slightly different from the z-distribution table
 - Must look up the df and the α to find the critical value, p-value

T-Distribution Table

df	$\alpha = 0.1$	0.05	0.025	0.01	0.005
∞	$t_{\alpha} = 1.282$	1.645	1.960	2.306	2.576
1	3.078	6.314	12.706	31.821	63.656
2	1.886	2.920	4.201	6.965	9.925
3	1.638	2.351	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.996	3.499
8	1.397	1.860	2.306	2.896	3.255
9	1.383	1.833	2.262	2.821	3.290
10	1.372	1.812	2.228	2.764	3.169

Values of the *t* Distribution

	<i>Level of Significance for One-Tailed Test</i>					
	.10	.05	.025	.01	.005	.0005
<i>df</i>	<i>Level of Significance for Two-Tailed Test</i>					
	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
10	1.372	1.812	2.228	2.764	3.169	4.587
15	1.341	1.753	2.131	2.602	2.947	4.073
20	1.325	1.725	2.086	2.528	2.845	3.850
25	1.316	1.708	2.060	2.485	2.787	3.725
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
80	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

Example

- The CEO of light bulbs manufacturing company claims that an average light bulb lasts 300 days. A researcher does not believe the claim of the CEO and decides to conduct a quick study. The researcher randomly selects 15 bulbs for testing. The sampled bulbs last an average of 290 days, with a standard deviation of 50 days. What can the researcher conclude?

solution

1. What information is given?
 - Population mean (300), sample mean(290), sample standard deviation(50), sample size(15)
2. What are the hypotheses?
3. What is the test statistic? $t = \frac{290-300}{(50/\sqrt{15})} = 0.7746$
4. What are the degrees of freedom? (n-1) $15-1=14$
5. What is the critical value for an α of 0.05 with df 14 2.228
6. How does the calculated test statistic compare to the critical value?
7. Can we reject the null hypothesis? What is our decision?

Comparison of Two Means

- We are often interested in gathering information about two populations in order to compare them.
- There are three major types of comparison of means tests:
 - One sample test
 - Two Independent sample test
 - Paired/repeated measure test

Comparison of two means

- **One sample test:** Make an inference to a population in comparison to some set value.
 - For example, might be interest in knowing whether the mean bacteria levels in a lake meet the national standards.
- **Two independent sample test:** collect two independent samples to test whether there is a difference in the means between two populations (or if one population mean is greater or less than the other).
 - For example compare GRE scores between men and women.
- **Paired or Repeated measure test:** This test compares paired data, such as data collected before and after a treatment. (dependent data)
 - Example: a comparison of gas emissions from automobiles before and after an engine is fixed.

Are the Data Independent Samples or Dependent Samples?

- Independent data- the samples selected from one of the populations has no relationship with the samples selected from the other population.
- Dependent data- (also called paired data) one sample is matched or paired with a particular measurement in the other sample.
 - Values in one sample affect the values in the other sample

- Another way to consider this is how many measurements are taken off each subject.
 - If only one measurement, then independent; if two measurements, then paired.
 - Exceptions are in familial situations such as in a study of spouses or twins. In such cases the data is almost always treated as paired data.
- i.e- Want to compare the effectiveness of treatment A and treatment B- have 12 subjects
 - independent data- assign 6 subjects treatment A and 6 subjects treatment B
 - dependent data- assign all of the subjects treatment A first and then treatment B

Comparison of two means- basics

- The methods of comparisons of two means is the extension on confidence intervals and hypothesis testing for one-parameter
- Calculate the specific test statistic and compare this to a critical value (rejection region approach) or find the probability of observing this test statistic or one more extreme (p-value approach).
- The decision process will be the same
 - if the test statistic falls in the rejection region we will reject the null hypothesis;
 - if the p -value is less than the preset level of significance we will reject the null hypothesis.

Comparison of two means- basics Cont'd

- The difference in the comparison of two parameters vs one parameter is how the null hypothesis is treated
- When testing of one parameter, the null hypothesis could vary- could be one sided or two sided.
- When comparing two parameters, null hypothesis is the value of 0 meaning "no difference".
- When comparing two means- **use the t-test**

Comparing 2 means- Independent samples

- Suppose that two samples are independently drawn- meaning that there is no connection between the 2 samples

Sample 1= $x_1, x_2, x_3, x_4, \dots, x_n$

Sample 2= $y_1, y_2, y_3, y_4, \dots, y_n$

- We want to compare the means of the two samples

(H_0): The mean of sample 1 = mean of sample 2

(H_A): The mean of sample 1 does not equal sample 2

*The Null Hypothesis can be re-written as: mean sample 1- mean sample 2=0

Comparing 2 means- independent sample

- Since we are using 2 samples, there will be two variances
- If the sample sizes are small, the estimates may not be that accurate and one may get a better estimate for the common standard deviation by pooling the data from both samples
 - This is assuming that the the standard deviations for the two samples are not that different.

Pooled vs non-pooled variances

- There are two options for estimating the variances for the 2-sample t -test with independent samples:
 1. 2-sample t -test using pooled/combined variances
 2. 2-sample t -test using separate variances

Q. When do we use which variance method?

A. When we are reasonably sure that the two samples have similar variances, then use the pooled variances. Otherwise, use the separate variances

Pooled Variances

- If there is evidence or reason to believe that the variances of the two samples are similar, estimate and use the common variance by pooling information from the two samples .
- There is a formal test for equality of variance, but as a *rule of thumb* if the sample sizes are *similar* and the ratio of the SDs are greater than 0.5 or less than 2, then can pool the samples to find a common SD of the sample

Pooled Variance and test statistic

- Let n_1 be the sample size from population 1, s_1 be the sample standard deviation of population 1.
- Let n_2 be the sample size from population 2, s_2 be the sample standard deviation of population 2
- Then the common standard deviation can be estimated by the pooled standard deviation:

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

- The Test Statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- The df= $n_1 + n_2 - 2$

Example

- In a factory a new machine is being developed to shorten production time. To test whether the new machine performs better, 50 old machines and 50 new machines were time tested.
 - The average production time of the new machines was 14 seconds with a $sd= 1.1$ seconds
 - The average production time of the old machines was 15 seconds with a $sd=1.2$ seconds
- Do the data provide sufficient evidence to conclude that, on the average, the new machine was faster? Perform the required hypothesis test at 5 % significance level

Checking to see if variances can be pooled

Q. Are the samples independent?

A. YES- samples from the two types of machines are not related

Q. Are the sample sizes similar

A. YES- both 50

Q. Do the populations have equal variance?

A. Yes- 1.2 and 1.1 are similar but test the rule of thumb- $s_1/s_2 \rightarrow 1.2/1.1 = 1.09$ which is very close to 1

Since the samples are independent, the same size and have equal variance can use the pooled variance approach

solution

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

1. null and alternative hypothesis

- Null: $\mu_1 - \mu_2 = 0$

- Alternative: $\mu_1 - \mu_2 < 0$

μ_1 is the old machine μ_2 is the new machine

2. Significance level= 5% (0.05)

3. Compute t statistic and pooled SD

The average time for the old machines was 15 seconds with the sd= 1.2 n=50

The average time for the new machines was 14 seconds with the sd= 1.1 n=50

$$\begin{aligned} S_p &= \sqrt{(49 * (1.2)^2 + 49 * (1.1)^2) / (50+50-2)} \\ &= \sqrt{(70.56 + 59.29) / 98} \\ &= \sqrt{1.325} \\ &= 1.15 \end{aligned}$$

$$\begin{aligned} t \text{ stat} &= 15 - 14 / s_p \sqrt{1/n_1 + 1/n_2} \\ &= 1 / 1.15 \sqrt{1/50 + 1/50} \\ &= 1 / 1.15 * 0.2 \\ &= 1 / .23 \\ &= 4.35 \end{aligned}$$

solution

4. Critical value $t_{0.05}$ $df = 50+50-2 \rightarrow 98$
 - look in table to find value of $df=98$ and $\alpha=0.05 \rightarrow 1.66$
 5. Check to see if the value of the test statistic falls in the rejection region and decide whether to reject H_0 .

$4.35 > 1.66$ thus we can reject the null hypothesis at $\alpha=0.05$
- At 5% level of significance, the data provide sufficient evidence that the new machine packs faster than the old machine on average.

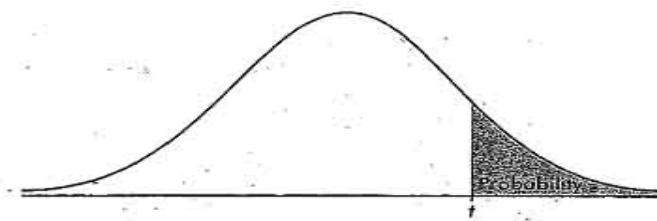


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291

Non-pooled variances

- If the variances are not equal then the variances can not be combined/pooled so must use different variances.
 - From the previous example-
 - The average time for the old machines was 15 seconds with the $sd = \underline{5}$
 - The average time for the new machines was 14 seconds with the $sd = 1.1$
- The rule of thumb- $s_1/s_2 \rightarrow 5/1.1 = 5.5$ which is very far from 1
- The test statistic for non pooled variances is
- The df is

$$df^* = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{\left(s_1^2 \right)^2}{n_1 - 1} + \frac{\left(s_2^2 \right)^2}{n_2 - 1}}$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

In class exercise

- To research the hours spent studying per week between gender, a survey was taken.
 - 99 female students took the survey and it was found that the average time spent studying was 16.4 hours with a standard deviation of 10.85 hours.
 - 95 male students took the survey and the average time spent studying was 11.9 hours with a standard deviation of 7.41 hours.
- Is the mean time spent studying different between females and males?
- Do we use the pooled variance or separate variance approach?
 - Are the samples independent? Is the sample size large enough? Are the SD's similar?

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

solution

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

1. null and alternative hypothesis
 - Null: $\mu_1 - \mu_2 = 0$
 - Alternative: $\mu_1 - \mu_2 < 0$ - μ_1 is females μ_2 is male
2. Significance level= 5% (0.05)
3. Compute t statistic and pooled SD

$$\begin{aligned} s_p &= \sqrt{(99-1) * (10.85)^2 + (95-1) * (7.41)^2 / (99+94-2)} \\ &= \sqrt{115367 + 5161.36} / 191 \\ &= \sqrt{631} \\ &= 25.12 \end{aligned}$$

$$\begin{aligned} t \text{ stat} &= 16.4 - 11.9 / s_p \sqrt{1/n_1 + 1/n_2} \\ &= 4.51 / 25.12 \sqrt{1/99 + 1/94} \\ &= 4.51 / 25.12 * .14 \\ &= 4.51 / 3.5168 \\ &= 1.28 \end{aligned}$$

Steps to solution

4. Critical value $t_{0.05}$ df= $99+95-2 \rightarrow 191$ look in table to find value $\rightarrow 1.66$
5. Check to see if the value of the test statistic falls in the rejection region and decide whether to reject H_o .

$1.28 < 1.66$ thus we fail to reject the null hypothesis at $\alpha=0.05$

- At 5% level of significance, the data does not provide sufficient evidence that the amount spent studying is different between the genders