INTRODUCTION TO STATISTICS

Session 10
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Introduction to hypothesis testing

General idea of hypothesis testing-

1) Make an initial assumption (state the null and alternative hypotheses).

2) Collect evidence (data).

3) *Make calculations- chose α level, critical value test statistic/p-value*

4) Based on the available evidence (data), <u>decide to reject or not reject the initial assumption.</u>

Step 1) Null (H_o) vs Alternative (H_1) Hypothesis

- **Null hypothesis**- No effect/change, relationship does not exist
- **Alternative hypothesis**- there will be an observed effect/change

  - typically the research hypothesis of interest and is the opposite of the null hypothesis

Example

Want to investigate if the mean body temperature is really the conventional 98.7 degrees – speculate that the mean body temperature is not 98.7

Null hypothesis: H_o : mean=98.7

The mean adult body temperature is 98.7 degrees Fahrenheit.

Alternative hypothesis: H_1 : mean ≠ 98.7

The mean adult body temperature is not 98.7 degrees Fahrenheit

If fail to reject the null hypothesis,

- working hypothesis remains that the average adult has temperature of 98.7 degrees

If the null hypothesis is rejected,

- conclude that the mean adult body temperature is not 98.7 degrees Fahrenheit

Two sided vs one sided hypothesis

- In the previous example

  -If the alternative hypothesis is true, the mean can either be above or below 98.7 degrees- thus is a two sided hypothesis

$$\boxed{\begin{array}{l} H_o: \text{mean}=98.7 \\ H_1: \text{mean} \neq 98.7 \end{array}}$$

- INSTEAD– speculate that the mean body temperature is less than 98.7

H_o: mean= 98.7

H_1: mean <98.7

-in this situation- we are only interested if the mean body temperature is less than 98.7 degrees thus is a one sided hypothesis

Two sided vs one sided hypothesis

- Need to decide whether to use the one-sided or two-sided probability/hypothesis before data collection
- A one-sided probability is more powerful- but should only use a one-sided probability when have a firm prediction about which direction of deviation is considered interesting

Errors in hypothesis testing

- Decisions in hypothesis testing are based on evidence
  - If we reject the null hypothesis, we do not prove that the alternative hypothesis is true.
  - If we do not reject the null hypothesis, we do not prove that the null hypothesis is true.

whatever the decision, there is always a chance of error

Types of Error

- **Type I error**- The null hypothesis is rejected when it is true.
  - The probability of committing a Type I error is called the **significance level**. This probability is also called **alpha** (α).
  - If $\alpha=0.05$, this is the same as saying there is a 5% chance of rejecting the null hypothesis, even if the null hypothesis is true
- **Type II error**- The null hypothesis is not rejected when it is false.
  - The probability of committing a Type II error is called **Beta** (β).
  - The probability of not committing a Type II error is called **Power** ($1-\beta$)
  - Since Power is a probability- can be a value between 0,1
  - power indicates the probability of a correct decision→ *the higher power, the better*

DECISION		TRUTH	
		Null Hypothesis is true	Alternative Hypothesis is true
	Do not reject null	Right decision	Type II error (β)
	Reject null	Type I error (α)	Right decision Power ($1-\beta$)

Which error is more serious?

- Depends on the situation
  - Medical screening for a disease
    - Type I- False positive- may give patient some anxiety, but this will lead to other testing procedures
    - Type II- False negative- give patient incorrect information that he does not have the disease when he in fact does

      In this case- Type I (false positive) is more desirable than a Type II (false negative)
  - Murder Trial
    - Type I error -found guilty of murder that you did not commit
    - Type II error -you are guilty but are found not guilty. This is a good outcome for you, but not for society

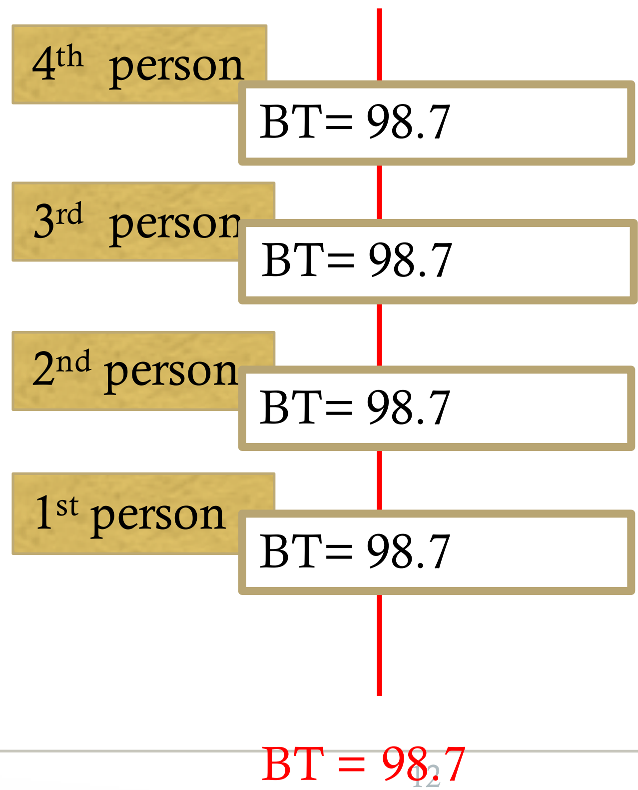
Step 3) Make calculations

- Test Statistic-standardized value calculated from sample data during a hypothesis test.
  - test statistic is used to determine whether to reject/not reject the null hypothesis.
  - The test statistic compares your data with what is expected under the **null hypothesis.**

Distribution under Null Hypothesis

Null Hypothesis

「The mean Body temperature is 98.7 degrees Fahrenheit 」



Distribution under Null Hypothesis

Null Hypothesis

「The mean Body temperature is 98.7 degrees Fahrenheit 」

Body temp= 98.7

Body temp <98.7 | *Body temp=* 98.7 | *Body temp >98.7*

↓extreme | *Body temp <<98.7* | *Body temp <98.7* | *Body temp=* 98.7 | *Body temp >98.7* | *Body temp >>98.7* | ↓extreme

Body temp <<<98.7 | *Body temp <<98.7* | *Body temp <98.7* | *Body temp=* 98.7 | *Body temp >98.7* | *Body temp>>98.7* | *Body temp >>>98.7*

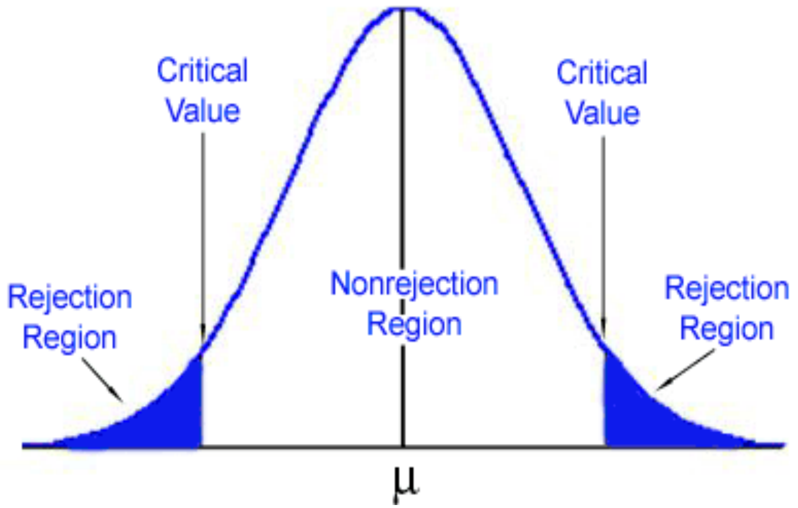
BT < 98.7

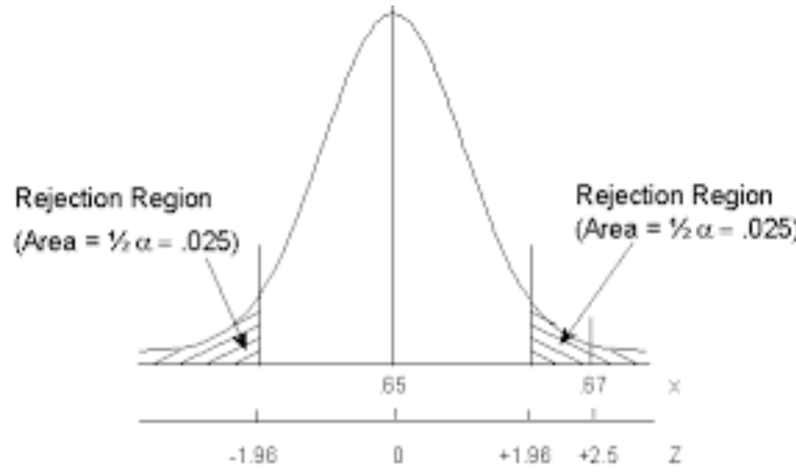
BT= 98.7

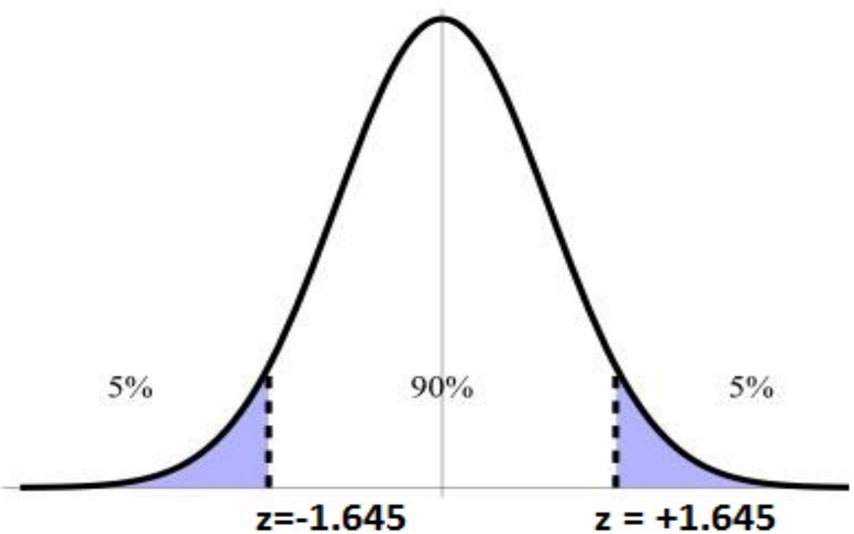
BT>98.7

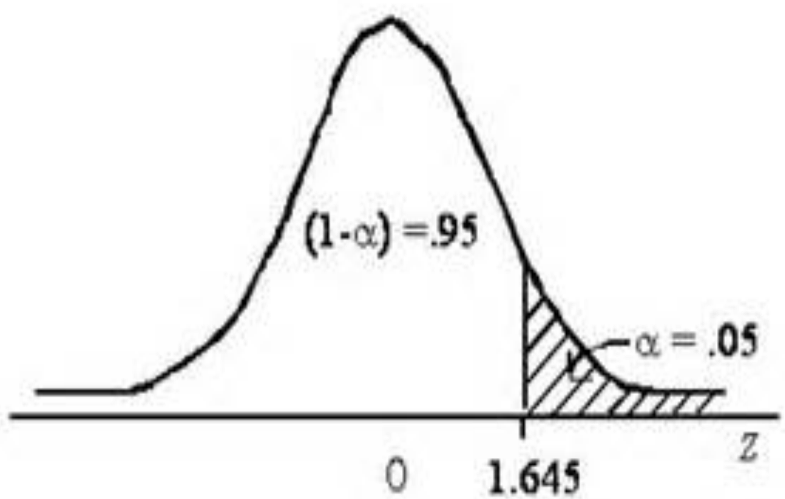
Rejection region and α level

- The rejection region is a part of the testing process. Specifically, the rejection region is an area of probability that tells you if your theory (your "hypothesis") is probably true, or probably not true.
- Rejection region depends on the α level
- Chose an acceptable α level
  - α level- probability of committing a Type I error – false positive, reject a null hypothesis when it is true
  - For example, if you wanted to be 95% confident that your results are significant, chose a 5% α level
- For a one tailed test, the rejection region would be 5% in one tail. For a two tailed test, the rejection region would be in two tails- 2.5% in each tail.


Critical
Value
Critical
Value
Rejection
Region
Nonrejection
Region
Rejection
Region
μ


Rejection Region
(Area = ½ α = .025)
Rejection Region
(Area = ½ α = .025)
65
67
x
-1.96
0
+1.96
+2.5
Z


5%
90%
5%
z=-1.645
z = +1.645


(1-α) =.95
α = .05
z
0
1.645

Critical value

- Determines whether or not the observed calculated test statistic is more extreme than would be expected if the null hypothesis were true
  - compare the observed test statistic to some cutoff value, called the "**critical value**."
- If the test statistic is more extreme than the critical value, then the null hypothesis is rejected in favor of the alternative hypothesis.
- If the test statistic is not as extreme as the critical value, then the null hypothesis is not rejected.

Critical Value

Alternative Hypothesis

「The mean Body temperature is not 98.7 degrees Fahrenheit」

likelihood of getting test statistic **more extreme =***p*-value

Body temp= 98.7

CRITICAL VALUE

CRITICAL VALUE

Body temp <98.7

Body temp= 98.7

Body temp >98.7

↓extreme

Body temp <<98.7

Body temp <98.7

Body temp= 98.7

Body temp >98.7

Body temp >>98.7

↓extreme

Body temp <<<98.7

Body temp <<98.7

Body temp <98.7

Body temp= 98.7

Body temp >98.7

Body temp>>98.7

Body temp <<<98.7

BT < 98.7

BT= 98.7

BT>98.7

Types of critical values

- Various types of critical values are used to calculate significance-
  - Z-test
  - Hypothesis test used when the data is normally distributed/ bell shaped
  - Use z-test when:
    - n>30
    - Data is randomly selected
    - Know the standard deviation of the population
    - One of the simplest tests to use

  - t-scores from Student`s t-test
  - chi-square

body temperature cont`d

- The researcher at SFC would like to test his hypothesis that the mean adult body temperature is not 98.7
- The researcher randomly measured the body temperature of 50 students and found the mean temperature to be 97.9
  - is there evidence to support the claim that the mean body temperature is not 98.7 degrees
    - the population SD is 0.2 degrees

Hypothesis testing- critical value approach

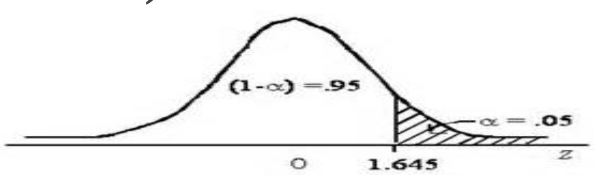
1. State the null and alternative hypothesis

H_0: mean=98.7 H_A: mean $\neq$98.7

two sided hypothesis

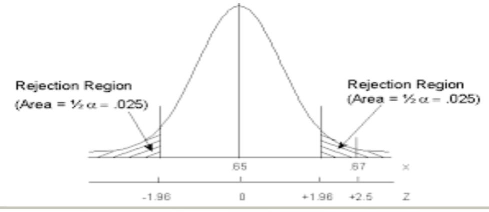
2. State the αlevel- if not given- use 5% (0.05)

3. Find the rejection region (given by the αlevel) from the z-table



one sided critical value= 1.645 or -1.645

two sided critical value= 1.96, -1.96



- Find the test statistic using the formula

x=*sample mean*

μ_o= *population mean*

σ = *standard deviation*

$$\boxed{Z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}}$$

- In the example

$$Z = \frac{97.9 - 98.6}{0.2 / \sqrt{50}} = \frac{-0.7}{0.03} = -23.33$$

- If the Z-score is less than -1.96 , reject the null hypothesis
- If one sided, reject the null hypothesis when it is greater than -1.96

since -23.33<-1.96→ we can reject the null hypothesis

p-value

- *p*-value- first formally introduced by Karl Pearson but popularized by Ronald Fischer
  - likelihood of getting test statistic or any test statistic **more extreme**, if the null hypothesis is true
    - "more extreme" is dependent on the way the hypothesis is tested
  - *p*-value helps determine the significance of your results
  - *p*-value refers only to the null hypothesis and does not make reference to or allow conclusions about the alternative hypothesis
- The *p*-value is ***not*** the probability that the null hypothesis is true or the probability that the alternative hypothesis is false

Step 4)- Draw conclusion/interpretation

- A small *p*-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis→ **reject the null hypothesis**

- A large *p*-value (> 0.05) indicates weak evidence against the null hypothesis→ **fail to reject the null hypothesis**

- *p*-values very close to the cutoff (0.05) are considered to be marginal (could go either way).

Always report the p-value so your readers can draw their own conclusions.

P-value approach

1. State the hypotheses
2. Calculate the test statistic
3. Look up the test statistic in the Z-score table and find the p-value
4. Make conclusion from the p-value

$$Z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$$

p-value approach: body temp exercise

In addition to the previous Body temperature exercise- The researcher randomly measured the body temperature of 100 students and found the mean temperature to be 98

- is there evidence to support the claim that the mean body temperature is not 98.7 degrees?
  - the population SD is 5 degrees

p-value approach: body temp exercise

1. State the null and alternative hypothesis

   H_0: mean=98.7 H_A: mean ≠98.7

   two sided hypothesis

2. The Z- test stastic= (98-98.7)/(5/10)

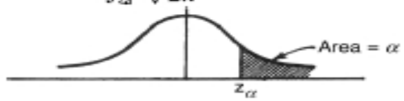
   =-.7/ .5

   =-1.4

3. Look up -1.4 in the z-table→ p=0.08

Z-table

$$\text{Value of } \alpha = \int_{z_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = \text{Prob}[z > z_\alpha]$$



z_α	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0539
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.09604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139