

LeetCode Practice

Outline

Divide and Conquer

Dynamic Programming

Median of Two Sorted Array (#4)

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You can do the following:

- ▶ The middle element of the first array is the $\frac{m}{2}$ -th element. Then determine its position in the 2nd array with binary search, say k . Now, it's $\frac{m}{2} + k$ -th element in the merged array. If this is smaller than $\frac{m+n}{2}$, we should proceed with the right half of the first array, otherwise, left half.

Median of Two Sorted Array (#4) Solution

```
class Solution {
public:
    int findKth(vector<int>& nums1, vector<int>& nums2, int s1, int e1, int k) {
        if (s1 == e1)
            return nums2[k - e1 - 1];
        int mid = (s1 + e1) / 2;
        auto it = lower_bound(nums2.begin(), nums2.end(), nums1[mid]);
        int rank = mid + distance(nums2.begin(), it) + 1;
        if (rank == k)
            return nums1[mid];
        if (rank > k)
            return findKth(nums1, nums2, s1, mid, k);
        return findKth(nums1, nums2, mid + 1, e1, k);
    }

    double findMedianSortedArrays(vector<int>& nums1, vector<int>& nums2) {
        int sz = nums1.size() + nums2.size();
        if (sz % 2 == 0) {
            return (findKth(nums1, nums2, 0, nums1.size(), sz / 2) +
                    findKth(nums1, nums2, 0, nums1.size(), sz / 2 + 1)) / 2.0;
        }
        return findKth(nums1, nums2, 0, nums1.size(), sz / 2 + 1);
    }
};
```

Edit Distance (#72)

Edit distance refers to a group of dynamic programming problems that mostly contains alignment of two or more sequences. The edit distance between two string can be described with the following formula:

$$dist(i, j) = \begin{cases} dist(i-1, j-1) & \text{if } A_i = B_j \\ \min(dist(i-1, j), dist(i, j-1), dist(i-1, j-1)) + 1 & \end{cases}$$

(1)

Edit Distance (#72) Solution

```
class Solution {
    public int minDistance(String word1, String word2) {
        int[][] dist = new int[word1.length() + 1][word2.length() + 1];
        for (int i = 0; i < word1.length(); i++) {
            for (int j = 0; j < word2.length(); j++) {
                dist[i + 1][j + 1] = Integer.MAX_VALUE;
            }
        }
        // Usually, using additional [0][0] can simplify subscript initialization.
        dist[0][0] = 0;
        for (int i = 0; i < word1.length(); i++)
            dist[i + 1][0] = i + 1;
        for (int i = 0; i < word2.length(); i++)
            dist[0][i + 1] = i + 1;
        for (int i = 0; i < word1.length(); i++) {
            for (int j = 0; j < word2.length(); j++) {
                if (word1.charAt(i) == word2.charAt(j)) {
                    dist[i + 1][j + 1] = dist[i][j];
                } else {
                    dist[i + 1][j + 1] = Math.min(Math.min(dist[i][j + 1],
                                                                dist[i + 1][j]), dist[i][j]) + 1;
                }
            }
        }
        return dist[word1.length()][word2.length()];
    }
}
```

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 - ▶ "ab?" and "abc"

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 - ▶ "ab?" and "abc"
- ▶ **Hint 3:** You can simplify cases like "***" to "*"
- ▶ **Hint 4:** The formula is:

$$match(i, j) = \begin{cases} \text{false if } A_i \neq B_j \wedge A_i \neq * \wedge A_i \neq ? \\ match(i-1, j-1) \text{ if } A_i = B_j \vee A_i = ? \\ match(i, j-1) \vee match(i-1, j) \text{ if } A_i = * \end{cases} \quad (2)$$

Wildcard Matching (#44) Solution

```
public boolean isMatch(String s, String p) {  
    boolean[][] dp = new boolean[s.length() + 1][p.length() + 1];  
    dp[0][0] = true;  
    for (int i = 0; i < p.length(); i++) {  
        dp[0][i + 1] = p.charAt(i) == '*' && dp[0][i];  
    }  
    for (int i = 0; i < s.length(); i++) {  
        for (int j = 0; j < p.length(); j++) {  
            dp[i + 1][j + 1] =  
                (dp[i][j] && (s.charAt(i) == p.charAt(j) || p.charAt(j) == '?'))  
                || (p.charAt(j) == '*' && (dp[i][j + 1] || dp[i + 1][j]));  
        }  
    }  
    return dp[s.length()][p.length()];  
}
```

Interleaving String (#97)

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- ▶ **Hint 2:** Let i, j be the length of the prefix $S1', S2'$. The last character of $S3'$ at this point is $S3_{(i+j-1)}$. What is the formula?

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- ▶ **Hint 2:** Let i, j be the length of the prefix $S1', S2'$. The last character of $S3'$ at this point is $S3_{(i+j-1)}$. What is the formula?
- ▶ **Hint 3:** The formula is:

```
interleave(i, j) =  
  true  (i = 0, j = 0)  
  interleave(0, j - 1) && S2[j-1] == S3[j - 1] (i = 0)  
  interleave(i - 1, 0) && S1[i-1] == S3[i - 1] (j = 0)  
  interleave(i - 1, j) || interleave(i, j - 1) (S3[i+j-1] == S1[i-1] == S2[j-1])  
  interleave(i - 1, j) (S3[i+j-1] == S1[i-1])  
  interleave(i, j - 1) (S3[i+j-1] == S2[j-1])
```

Interleaving String (#97) Solution

```
public boolean isInterleave(String s1, String s2, String s3) {
    if (s1.length() + s2.length() != s3.length()) {
        return false;
    }
    boolean[][] dp = new boolean[s1.length() + 1][s2.length() + 1];
    dp[0][0] = true;
    for (int i = 0; i < s1.length(); i++) {
        dp[i + 1][0] = dp[i][0] && s1.charAt(i) == s3.charAt(i);
    }
    for (int j = 0; j < s2.length(); j++) {
        dp[0][j + 1] = dp[0][j] && s2.charAt(j) == s3.charAt(j);
    }
    for (int i = 0; i < s1.length(); i++) {
        for (int j = 0; j < s2.length(); j++) {
            if (s3.charAt(i + j + 1) == s1.charAt(i) && s3.charAt(i + j + 1) == s2.charAt(j)) {
                dp[i + 1][j + 1] = dp[i][j + 1] || dp[i + 1][j];
                continue;
            }
            if (s3.charAt(i + j + 1) == s1.charAt(i)) {
                dp[i + 1][j + 1] = dp[i][j + 1];
                continue;
            }
            if (s3.charAt(i + j + 1) == s2.charAt(j)) {
                dp[i + 1][j + 1] = dp[i + 1][j];
                continue;
            }
            dp[i + 1][j + 1] = false;
        }
    }
    return dp[s1.length()][s2.length()];
}
```

Minimum ASCII Delete Sum for Two Strings (#712)

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Minimum ASCII Delete Sum for Two Strings (#712)

- ▶ **Hint 1:** Similar to edit distance. What will happen if $S1_i == S2_j$ or $S1_i \neq S2_j$?
- ▶ **Hint 2:** The formula is:

```
minimum(i, j) =  
    minimum(i - 1, j - 1) (S1[i] == S2[j])  
    min(minimum(i - 1, j) + S1[i], minimum(i, j - 1) + S2[j]) (otherwise)
```

Minimum ASCII Delete Sum for Two Strings (#712)

Solution

```
class Solution {
    public int minimumDeleteSum(String s1, String s2) {
        int[] [] dp = new int[s1.length() + 1][s2.length() + 1];
        for (int i = 0; i < s1.length(); i++) {
            dp[i + 1][0] = dp[i][0] + s1.charAt(i);
        }
        for (int i = 0; i < s2.length(); i++) {
            dp[0][i + 1] = dp[0][i] + s2.charAt(i);
        }
        for (int i = 0; i < s1.length(); i++) {
            for (int j = 0; j < s2.length(); j++) {
                if (s1.charAt(i) == s2.charAt(j)) {
                    dp[i + 1][j + 1] = dp[i][j];
                    continue;
                }
                dp[i + 1][j + 1] = Math.min(dp[i][j + 1] + s1.charAt(i),
                    dp[i + 1][j] + s2.charAt(j));
            }
        }
        return dp[s1.length()][s2.length()];
    }
}
```


Regular Expression Matching (#10)

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► **Hint 1:** Todo

Regular Expression Matching (#10)

```
// TODO
```

Single Sequence Styled DP

There is only one sequence. Current state is often determined by 1 or more previous states.

Best Time to Buy and Sell Stock I (#121)

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- ▶ **Hint 2:** We can keep a minimum value seen so far and check if $P_i - \text{min}$ is greater than current maximum.

Best Time to Buy and Sell Stock I (#121) Solution

```
public int maxProfit(int[] prices) {  
    int minPrice = Integer.MAX_VALUE;  
    int maxProfit = 0;  
    for (int p : prices) {  
        if (p - minPrice > maxProfit)  
            maxProfit = p - minPrice;  
        if (minPrice > p)  
            minPrice = p;  
    }  
    return maxProfit;  
}
```


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- ▶ **Hint 2:** You can use two states: bought and sold. How should they transfer upon seeing a new price? e.g. What will happen if bought \rightarrow sold at p ? Or sold \rightarrow bought at p ?

Best Time to Buy and Sell Stock II (#122)

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- ▶ **Hint 2:** You can use two states: bought and sold. How should they transfer upon seeing a new price? e.g. What will happen if bought \rightarrow sold at p ? Or sold \rightarrow bought at p ?
- ▶ **Hint 3:** At price p , we could have:

```
sold = max(bought + p, sold)
bought = max(bought, sold - p)
```

Best Time to Buy and Sell Stock II (Solution)

```
public int maxProfit(int[] prices) {  
    int maxBought = Integer.MIN_VALUE;  
    int maxSold = 0;  
  
    for (int p : prices) {  
        if (maxBought + p > maxSold)  
            maxSold = maxBought + p;  
        if (maxSold - p > maxBought)  
            maxBought = maxSold - p;  
    }  
    return maxSold;  
}
```

Best Time to Buy and Sell Stock III (#123)

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- ▶ **Hint 1:** What status can you have? Is it still two Bought / Sold or more?
- ▶ **Hint 2:** We can use the following states:
 - ▶ Bought1, in 1st transaction, holding 1 stock.
 - ▶ Sold1, 1 transaction completed and not holding anything.
 - ▶ Bought2, in 2nd transaction, holding 1 stock.
 - ▶ Sold2, 2 transaction completed and not holding anything.

Best Time to Buy and Sell Stock III (#123)

- ▶ **Hint 1:** What status can you have? Is it still two Bought / Sold or more?
- ▶ **Hint 2:** We can use the following states:
 - ▶ Bought1, in 1st transaction, holding 1 stock.
 - ▶ Sold1, 1 transaction completed and not holding anything.
 - ▶ Bought2, in 2nd transaction, holding 1 stock.
 - ▶ Sold2, 2 transaction completed and not holding anything.
- ▶ **Hint 3:** The state transfer would be: 0 - (buy) -> Bought1 - (sell) -> Sold1 - (buy) -> Bought2 -> (sell) -> Sold2
At each price P , the above sequence could happen and we'll take the max of each.

Best Time to Buy and Sell Stock III (#123) Solution

```
public int maxProfit(int[] prices) {  
    int maxBought_1 = Integer.MIN_VALUE;  
    int maxSold_1 = 0;  
    int maxBought_2 = Integer.MIN_VALUE;  
    int maxSold_2 = 0;  
  
    for (int p : prices) {  
        maxBought_1 = Math.max(maxBought_1, -p);  
        if (maxBought_1 + p > maxSold_1)  
            maxSold_1 = maxBought_1 + p;  
        if (maxSold_1 - p > maxBought_2)  
            maxBought_2 = maxSold_1 - p;  
        if (maxBought_2 + p > maxSold_2)  
            maxSold_2 = maxBought_2 + p;  
    }  
    return maxSold_2;  
}
```

Best Time to Buy and Sell Stock IV (#188)

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Best Time to Buy and Sell Stock IV (#188)

- ▶ **Hint 1:** Similar to III, what are the states?
- ▶ **Hint 2:** Now we have k states instead of 2. How do you represent them?
- ▶ **Hint 3:** Still the states could be represented as:

```
maxBought[0] = max(maxBought[0], -p)
maxBought[i] = max(maxSold[i - 1] - p, maxBought[i])
maxSold[i] = max(maxBought[i] + p, maxSold[i])
```

Best Time to Buy and Sell Stock IV (#188) Solution

```
public int maxProfit(int k, int[] prices) {  
    if (k == 0 || prices.length == 0) {  
        return 0;  
    }  
    // When k > prices.length / 2, this problem is simplified to  
    // Best Time to Buy and Sell Stock II as you can complete as  
    // many transactions as you like. This is here only to handle  
    // LeetCode's corner cases.  
    if (k > prices.length / 2) {  
        int maxBought = Integer.MIN_VALUE;  
        int maxSold = 0;  
  
        for (int p : prices) {  
            if (maxBought + p > maxSold)  
                maxSold = maxBought + p;  
            if (maxSold - p > maxBought)  
                maxBought = maxSold - p;  
        }  
        return maxSold;  
    }  
  
    int[] maxBought = new int[k];  
    int[] maxSold = new int[k];  
    Arrays.fill(maxBought, Integer.MIN_VALUE);  
    for (int p : prices) {  
        maxBought[0] = Math.max(maxBought[0], -p);  
        for (int i = 0; i < k - 1; i++) {  
            maxSold[i] = Math.max(maxBought[i] + p, maxSold[i]);  
            maxBought[i + 1] = Math.max(maxSold[i] - p, maxBought[i + 1]);  
        }  
        maxSold[k - 1] = Math.max(maxSold[k - 1], maxBought[k - 1] + p);  
    }  
    return maxSold[k - 1];  
}
```

Paint House II

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Paint House II

- ▶ **Hint 1:** What are the states? Do you need to examine all colors in each step?
- ▶ **Hint 2:** You don't have to examine all colors in each step – using the colors with lowest 2 values would be sufficient, since the next set of lowest values would exactly come from these 2 values + current price.
- ▶ **Hint 3:** The formula is:

```
let i = 0..k such that prices(n - 1, i) is smallest
    j = 0..k such that prices(n - 1, j) is second smallest
prices(n, k) =
    prices(n - 1, i) + cost[n][k], if i != k
    prices(n - 1, j) + cost[n][k], if i == k
```

Do you need $O(nk)$ storage space?

Paint House II (Solution)

```
public int minCostII(int[][] costs) {
    if (costs.length == 0)
        return 0;

    int [] cost = new int[costs[0].length];

    for (int i = 0; i < costs[0].length; i++)
        cost[i] = costs[0][i];

    for (int i = 1; i < costs.length; i++) {
        int[] prices = costs[i];

        // Find the lowest 2 cost.
        int minCost1 = Integer.MAX_VALUE, minColor1 = -1;
        int minCost2 = Integer.MAX_VALUE;
        for (int j = 0; j < cost.length; j++) {
            if (cost[j] < minCost1) {
                minCost2 = minCost1;
                minCost1 = cost[j];
                minColor1 = j;
                continue;
            }
            if (cost[j] < minCost2) {
                minCost2 = cost[j];
            }
        }

        for (int j = 0; j < prices.length; j++) {
            if (j == minColor1) {
                cost[j] = minCost2 + prices[j];
            } else {
                cost[j] = minCost1 + prices[j];
            }
        }
    }

    return Arrays.stream(cost).min().orElse(-1);
}
```

Max Consecutive Ones (#485)

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- ▶ **Hint 2:** You can keep two numbers: current consecutive ones and a max.

Max Consecutive Ones (#485) Solution

```
public int findMaxConsecutiveOnes(int[] nums) {  
    int max = 0;  
    int current = 0;  
    for (int x : nums) {  
        if (x == 0) current = 0;  
        else current += 1;  
        max = Math.max(max, current);  
    }  
    return max;  
}
```


Climbing Stairs (#70)

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- ▶ **Hint 2:** It only depends on 2 previous stairs: $i - 1$ and $i - 2$.
- ▶ **Hint 3:** The formula is:

`step(i) = step(i - 1) + step(i - 2)`

So it only depends on 2 variables. And yes, it's same as getting n-th element from Fibonacci sequence.

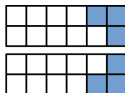
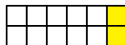
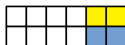
Climbing Stairs (#70) Solution

```
public int climbStairs(int n) {  
    int a0 = 0;  
    int a1 = 1;  
    for (int i = 0; i < n; i++) {  
        int a2 = a0 + a1;  
        a0 = a1;  
        a1 = a2;  
    }  
    return a1;  
}
```

Domino and Tromino Tiling (#790)

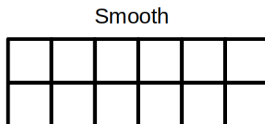
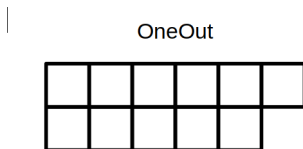
Domino and Tromino Tiling (#790)

- ▶ **Hint 1:** Consider the following graph. How many types of states are there?



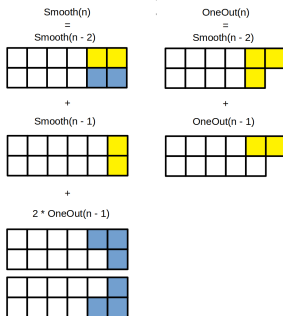
Domino and Tromino Tiling (#790)

- **Hint 2:** We can classify them into two categories: OneOut and Smooth. How can you construct the subproblem for both types?



Domino and Tromino Tiling (#790)

► **Hint 3:** You can have the following formula:



Domino and Tromino Tiling (#790) Solution

```
public int numTilings(int N) {  
    final int MOD = 1000000007;  
  
    if (N == 0) return 0;  
    if (N == 1) return 1;  
    if (N == 2) return 2;  
  
    // Use long to avoid overflow during addition.  
    long[] smooth = new long[N];  
    long[] oneOut = new long[N];  
  
    smooth[0] = 1;  
    smooth[1] = 2;  
  
    oneOut[1] = 1;  
  
    for (int i = 2; i < N; i++) {  
        smooth[i] = (smooth[i - 1] + smooth[i - 2] + 2 * oneOut[i - 1]) % MOD;  
        oneOut[i] = (smooth[i - 2] + oneOut[i - 1]) % MOD;  
    }  
  
    return (int) smooth[N - 1];  
}
```

Bonus Point: Can you solve it with $O(1)$ space?

Domino and Tromino (#790) Solution

Notice that $\text{smooth}_i / \text{oneOut}_i$ only depends on smooth_{i-1} , smooth_{i-2} and oneOut_{i-1} .

```
public int numTilings(int N) {  
    if (N == 0) return 0;  
    if (N == 1) return 1;  
  
    final int MOD = 1000000007;  
  
    long smooth0 = 1;  
    long smooth1 = 2;  
    long oneOut0 = 1;  
  
    for (int i = 2; i < N; i++) {  
        long smooth2 = (smooth1 + smooth0 + 2 * oneOut0) % MOD;  
        oneOut0 = (smooth0 + oneOut0) % MOD;  
        smooth0 = smooth1;  
        smooth1 = smooth2;  
    }  
  
    return (int) smooth1;  
}
```

Frog Jump (#403)

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Frog Jump (#403)

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- ▶ **Hint 2:** If the frog jumps from stone_j to stone_i, it must have jumped from another stone_k to stone_j, where $|i - j - (j - k)| \leq 1$.
- ▶ **Hint 3:** One way to express this is:

```
jump(i) = {j | j = 0..i - 1 and (i - j - 1 in jump[j] or  
                                i - j in jump[j] or i - j + 1 in jump[j])}  
jump(0) = {0}  
The frog can reach the end iff jump(last) is not empty.
```

Frog Jump (#403) Solution

```
public boolean canCross(int[] stones) {  
    ArrayList<HashSet<Integer>> dp = new ArrayList<>();  
    for (int ignored : stones) dp.add(new HashSet<>());  
    dp.get(0).add(0);  
    for (int i = 1; i < stones.length; i++) {  
        for (int j = 0; j < i; j++) {  
            int step = stones[i] - stones[j];  
            if (dp.get(j).contains(step - 1) ||  
                dp.get(j).contains(step) ||  
                dp.get(j).contains(step + 1)) {  
                dp.get(i).add(step);  
            }  
        }  
    }  
    return !dp.get(stones.length - 1).isEmpty();  
}
```


Knapsack Styled DP

Knapsack problems are pseudo-polynomial time. They require DP over the value domain of some of the parameters. The characteristic of the problems of this kind is they are often quite small on value range. For example, in subset sum, the largest number is usually in terms of 100s.

Coin Change II (#518)

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- ▶ **Hint 2:** For a specific coin, I can use it or not use it. What is the difference?
- ▶ **Hint 3:** The formula is:

```
// # of ways to make value k from coins 0..n:  
coin(n, k) =  
    // We don't use coin[n] or use it  
    coin(n - 1, k) + coin(n, k - value[n])
```

Coin Change II (#518) Solution

```
public int change(int amount, int[] coins) {  
    int[][] dp = new int[coins.length + 1][amount + 1];  
    for (int i = 0; i <= coins.length; i++) {  
        dp[i][0] = 1;  
    }  
  
    for (int i = 0; i < coins.length; i++) {  
        for (int j = 0; j <= amount; j++) {  
            int useCoin = (j >= coins[i]) ? dp[i + 1][j - coins[i]] : 0;  
            dp[i + 1][j] = useCoin + dp[i][j];  
        }  
    }  
    return dp[coins.length][amount];  
}
```

Coin Change I (#322)

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- ▶ **Hint 2:** We can use change as state. What is the formula?
- ▶ **Hint 3:** The formula is:

`changes[i] = min(changes[i - coins[j]] + 1) for j = 0 to coins.length.`

You'll need to work out the corner cases.

Coin Change I (#322) Solution

```
public int coinChange(int[] coins, int change) {  
    int[] changes = new int[change + 1];  
    Arrays.fill(changes, Integer.MAX_VALUE);  
  
    changes[0] = 0;  
    for (int i = 0; i < coins.length; i++) {  
        if (coins[i] <= change) {  
            changes[coins[i]] = 1;  
        }  
    }  
  
    for (int i = 1; i <= change; i++) {  
        for (int coin : coins) {  
            if (i >= coin && changes[i - coin] != Integer.MAX_VALUE) {  
                changes[i] = Math.min(changes[i], changes[i - coin] + 1);  
            }  
        }  
    }  
  
    return changes[change] == Integer.MAX_VALUE ? -1 : changes[change];  
}
```

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- ▶ **Hint 2:** The target change is $sum/2$. The constraint is each coin can only be used once. How should you encode such info in the formula?
- ▶ **Hint 3:** The formula is:

```
// canSum(i, target) represents whether we can select nums[0..i] to  
// get the sum target.  
canSum(i, target) = canSum(i - 1, target) || canSum(i - 1, target - nums[i])
```

Again, please work out the edge cases.

Partition Equal Subset Sum (#416) Solution

```
public boolean canPartition(int[] nums) {  
    int sum = Arrays.stream(nums).sum();  
    if (sum % 2 != 0) {  
        return false;  
    }  
    int target = sum / 2;  
  
    boolean[][] canSum = new boolean[nums.length + 1][target + 1];  
  
    for (int i = 0; i <= nums.length; i++) {  
        canSum[i][0] = true;  
    }  
  
    for (int i = 1; i <= nums.length; i++) {  
        for (int j = 1; j <= target; j++) {  
            canSum[i][j] = (j >= nums[i - 1] && canSum[i - 1][j - nums[i - 1]])  
                || canSum[i - 1][j];  
        }  
    }  
  
    return canSum[nums.length][target];  
}
```


Tree Style DP

This should not be very common. Each tree node represents one optimal value when we apply the operation within that subtree.

House Robber III (#337)

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- ▶ **Hint 1:** This is a little bit tricky. For each root node, you have 2 options rob it or no. If root is robbed, you should not rob its left child and right child. Otherwise, you can choose to rob either child, both children or none. How do you represent the state?

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- ▶ **Hint 2:** You can use two hashmap: *hasRoot*<TreeNode, Int>, *noRoot*<TreeNode, Int>. Then you can establish a connection between its a node and its children and get the formula.

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- ▶ **Hint 2:** You can use two hashmap: *hasRoot*<TreeNode, Int>, *noRoot*<TreeNode, Int>. Then you can establish a connection between its a node and its children and get the formula.
- ▶ **Hint 3:** The formula is:

```
hasRoot(root) = root.val + noRoot(root.left) + noRoot(root.right);  
noRoot(root) = max(noRoot(root.left), hasRoot(root.left)) +  
                max(noRoot(root.right), hasRoot(root.right))
```

House Robber III (#337) Solution

// Do a level order traversal so that we could manipulate nodes bottom-up.

```
private ArrayList<TreeNode> addNodes(TreeNode root) {
    ArrayList<TreeNode> nodes = new ArrayList<>();
    int index = 0;
    nodes.add(root);
    while (index < nodes.size()) {
        TreeNode cur = nodes.get(index);
        if (cur.left != null) nodes.add(cur.left);
        if (cur.right != null) nodes.add(cur.right);
        index++;
    }
    return nodes;
}

private int getOrZero(TreeNode node, HashMap<TreeNode, Integer> map) {
    if (node != null && map.containsKey(node)) return map.get(node);
    return 0;
}

public int rob(TreeNode root) {
    if (root == null) return 0;

    HashMap<TreeNode, Integer> hasRoot = new HashMap<>();
    HashMap<TreeNode, Integer> noRoot = new HashMap<>();

    ArrayList<TreeNode> nodes = addNodes(root);

    for (int i = nodes.size() - 1; i >= 0; i--) {
        TreeNode node = nodes.get(i);
        int noRootLeft = getOrZero(node.left, noRoot);
        int noRootRight = getOrZero(node.right, noRoot);
        hasRoot.put(node, noRootLeft + noRootRight + node.val);
        int hasRootLeft = getOrZero(node.left, hasRoot);
        int hasRootRight = getOrZero(node.right, hasRoot);
        noRoot.put(node, Math.max(hasRootLeft, noRootLeft)
            + Math.max(hasRootRight, noRootRight));
    }
    return Math.max(getOrZero(root, hasRoot), getOrZero(root, noRoot));
}
```

Coordinate Style DP

This normally consists of a grid-like structure, with coordinates representing the states.

Unique Paths (#62)

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- ▶ **Hint 2:** The robot can come from $(x - 1, y)$ or $(x, y - 1)$.
What is the formula?
- ▶ **Hint 3:** The formula is:

$$\text{pos}(x, y) = \text{pos}(x - 1, y) + \text{pos}(x, y - 1)$$

Unique Paths (#62) Solution

```
public int uniquePaths(int m, int n) {  
    int[][] dp = new int[m][n];  
    for (int i = 0; i < m; i++) {  
        dp[i][0] = 1;  
    }  
    for (int i = 0; i < n; i++) {  
        dp[0][i] = 1;  
    }  
    for (int i = 1; i < m; i++) {  
        for (int j = 1; j < n; j++) {  
            dp[i][j] = dp[i - 1][j] + dp[i][j - 1];  
        }  
    }  
    return dp[m - 1][n - 1];  
}
```

Unique Paths II (#63)

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- ▶ **Hint 2:** If $\text{grid}(x, y) == 1$ then $\text{pos}(x, y) = 0$. The rest are the same.
- ▶ **Hint 3:** The formula is:

```
pos(x, y) = 0 if grid(x, y) == 1  
pos(x, y) = pos(x - 1, y) + pos(x, y - 1) otherwise
```


Unique Paths II (#63) Solution

```
public int uniquePathsWithObstacles(int[][] obstacleGrid) {  
    int m = obstacleGrid.length;  
    int n = obstacleGrid[0].length;  
    int[][] dp = new int[m][n];  
    dp[0][0] = obstacleGrid[0][0] == 0 ? 1 : 0;  
    for (int i = 1; i < m; i++) {  
        dp[i][0] = obstacleGrid[i][0] == 0 ? dp[i - 1][0] : 0;  
    }  
  
    for (int i = 1; i < n; i++) {  
        dp[0][i] = obstacleGrid[0][i] == 0 ? dp[0][i - 1] : 0;  
    }  
  
    for (int i = 1; i < m; i++) {  
        for (int j = 1; j < n; j++) {  
            dp[i][j] = (obstacleGrid[i][j] == 0) ? dp[i - 1][j] + dp[i][j - 1] : 0;  
        }  
    }  
    return dp[m - 1][n - 1];  
}
```

Triangle (#120)

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- ▶ **Hint 2:** The formula is:

`a(i, j) = min(a(i - 1, j - 1), a(i - 1, j)) + triangle(i, j)`

Triangle (#120)

- ▶ **Hint 1:** Similar to unique paths, you can still use coordinates as state.

- ▶ **Hint 2:** The formula is:

`a(i, j) = min(a(i - 1, j - 1), a(i - 1, j)) + triangle(i, j)`

- ▶ **Hint 3:** For layer N, you may only care about layer N - 1, which saves you from using $O(n^2)$ space

Triangle (#120) Solution

```
public int minimumTotal(List<List<Integer>> triangle) {  
    int[] dp = new int[triangle.size()];  
    for (int i = 0; i < triangle.size(); i++) {  
        int[] tmp = new int[triangle.size()];  
        List<Integer> row = triangle.get(i);  
        for (int j = 0; j < row.size(); j++) {  
            if (i == 0 || j == 0) tmp[j] = dp[j] + row.get(j);  
            else if (j == row.size() - 1) tmp[j] = dp[j - 1] + row.get(j);  
            else tmp[j] = Math.min(dp[j], dp[j - 1]) + row.get(j);  
        }  
        dp = tmp;  
    }  
    int min = Integer.MAX_VALUE;  
    for (int a : dp) min = Math.min(min, a);  
    return min;  
}
```

Dungeon Game (#174)

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- ▶ **Hint 1:** Traditionally if we walk forward, we don't know what the start HP is. It's not easy to go right / down. It's probably better to go backwards, since we know at the point we reach the princes, we should have at least 1 HP to spare.

Dungeon Game (#174)

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- ▶ **Hint 2:** The formula is:

```
// minHp represents the minimum Hp we need *before* we step on cell[i][j].  
minHp(i, j) = min(minHp(i + 1, j) ? dungeon(i, j), minHp(i, j + 1) ? dungeon(i, j))
```

Can you guess what ? should represent?

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// minHp represents the minimum Hp we need *before* we step on cell[i][j].  
minHp(i, j) = min(minHp(i + 1, j) ? dungeon(i, j), minHp(i, j + 1) ? dungeon(i, j))
```

Can you guess what ? should represent?

- ▶ **Hint 3:** ? could be represented as the following function:

```
// lastCell is either [i+1][j] or [i][j+1]. currentCell is dungeon[i][j].  
// This says that if we want to move from currentCell to lastCell, the Hp we need  
// before we step onto [i][j] so that we can finally reach the bottom / right cell.  
private int getValue(int lastCell, int currentCell) {  
    // If [i][j] is negative, we'll need to add that to our budget.  
    if (currentCell < 0) return lastCell - currentCell;  
    // If lastCell's required amount is less than the amount we can gain from  
    // current cell, we only need to be alive before we step on it.  
    if (lastCell <= currentCell) return 1;  
    // Otherwise, we can charge up at current cell to the point that lastCell  
    // requires.  
    return lastCell - currentCell;  
}
```

Dungeon Game (#174) Solution

```
private int getValue(int lastCell, int currentCell) {
    if (currentCell < 0) return lastCell - currentCell;
    if (lastCell <= currentCell) return 1;
    return lastCell - currentCell;
}

public int calculateMinimumHP(int[][] dungeon) {
    int[][] minHp = new int[dungeon.length][dungeon[0].length];
    int h = dungeon.length - 1;
    int w = dungeon[0].length - 1;
    for (int i = h; i >= 0; i--) {
        for (int j = w; j >= 0; j--) {
            if (i == h && j == w) minHp[i][j] = getValue(1, dungeon[i][j]);
            else if (i == h) minHp[i][j] = getValue(minHp[i][j + 1], dungeon[i][j]);
            else if (j == w) minHp[i][j] = getValue(minHp[i + 1][j], dungeon[i][j]);
            else minHp[i][j] = Math.min(getValue(minHp[i + 1][j], dungeon[i][j]),
                                         getValue(minHp[i][j + 1], dungeon[i][j]));
        }
    }
    return minHp[0][0];
}
```

Higher Ordered DP

I'm calling this type "higher ordered DP" because it generally requires $O(n^3)$ to solve. One example of it would be matrix multiplication:

Given matrix chain ABCDEFG, calculate the product of it with least number of multiplications. Different number of multiplications could be achieved by adding parentheses. For example, $(A(B(CD)E)(FG))$ and $((ABCD)(EFG))$ may require different number of multiplications.

Read

<https://home.cse.ust.hk/~dekai/271/notes/L12/L12.pdf>
for more information on matrix multiplication.

Burst Balloons (#312)

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- ▶ **Hint 2:** The formula:

$\text{max}(i, j) = \text{max}\{\text{max}(i, k) + \text{max}(k, j) + \text{balloons}[i] * \text{balloons}[k] * \text{balloons}[j]\}.$

- ▶ **Hint 3:** It helps to prepend 1 and append 1 to both sides of the array.

Burst Balloons (#312) Solution

```
public int maxCoins(int[] nums) {  
    int[] balloons = new int[nums.length + 2];  
    balloons[0] = balloons[balloons.length - 1] = 1;  
    System.arraycopy(nums, 0, balloons, 1, nums.length);  
    int[][] dp = new int[balloons.length][balloons.length];  
    // As indicated in Page 19 of https://home.cse.ust.hk/~dekai/271/notes/L12/L12.pdf,  
    // we'll need to calculate sequences ordered by length. So k represents the length  
    // of the sequence.  
    for (int k = 2; k <= balloons.length - 1; k++) {  
        for (int i = 0; i < balloons.length - k; i++) {  
            int end = i + k;  
            for (int j = i + 1; j <= end - 1; j++) {  
                int score = dp[i][j] + dp[j][end] + balloons[i] * balloons[j] * balloons[end];  
                if (score > dp[i][end]) dp[i][end] = score;  
            }  
        }  
    }  
    return dp[0][balloons.length - 1];  
}
```