ECE 513 Fall 2016 Project # 1

READ CAREFULLY ALL THE INSTRUCTIONS

Instructions:

1. **Work independently.**
2. **The project part is 10% of the grade.**
2. **There are 14 mandatory questions.**
3. **If in doubt about any question, make suitable assumptions and proceed.**
4. **No points will be awarded unless results are supported by the corresponding derivation.**

Due Date: Wednesday September 26th by the end of the class.

Submission guideline: IMPORTANT

1- **ALL your work (including codes and handworks) should be submitted in ONLY ONE pdf file.**
2- **Use black ink**
3- **Your answers need to be well organized and neat. Will be a 10% penalty if not sequentially and neatly (readable and clean) written.**
4- **Sign this page and place at the beginning of your answer sheet.**

I certified that only I have done work in this exam

Signature: Karthikeyan

Printed Name: Karthikeyan Kumar

Date: 09/25/2016

ECE - 513

Section 1

Q.1

a) $E_1 = \{2, 2, 4\}$

b) $E_2 = \{2, 2, 3, 3, 4\}$

c) $E_3 = \{1, 1, 2, 2\}$

d) $E_4 = E_1 \cup E_3 = \{1, 1, 2, 2, 4\}$

e) $E_5 = E_1 \cup (E_2 \cap E_3) \Rightarrow \cancel{E_1 \cup (} \{2, 2, 4\}$.

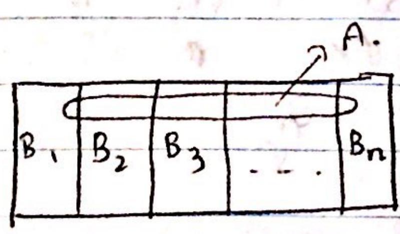
Q.2

Sample Space $= \{1, 1, 2, 2, 3, 3, 4\}$

$P(E_1) = 3/7$; $P(E_2) = 5/7$; $P(E_3) = 4/7$; $P(E_4) = 5/7$

$P(E_5) = 3/7$.

Q.3



Now, we know that all the events of B from B_1 to B_n are disjoint in nature according to set theory.

And also B_1 to B_n union gives us the sample space 'S':

$$\therefore S = B_1 \cup B_2 \cup \dots \cup B_N$$

Assume that the event 'A' occurs in the entire sample space 'S'.

Hence the event of 'A' in the sample space is given by the intersection

(i.e.) $A \cap B$. But $B = B_1 \cup B_2 \cup \dots \cup B_n$

$\therefore A = (A \cap B_1) \cup (A \cap B_2) \cup \cdots (A \cap B_n)$.

b) Let us assume Event 'A' conditioned upon event 'B'.

So $P(A) = P(A \cap \Omega)$. where $\Omega \to$ Sample space

But from previous result.

$$= P(A \cap (B_1 \cup B_2 \cup \dots B_n))$$

$$= P(A \cap B_1) \cup P(A \cap B_2) \cup \dots P(A \cap B_n)$$

(By disjoint property)

$$= P(A \cap B_1) + P(A \cap B_2) + \dots + P(A \cap B_n) \to ①$$

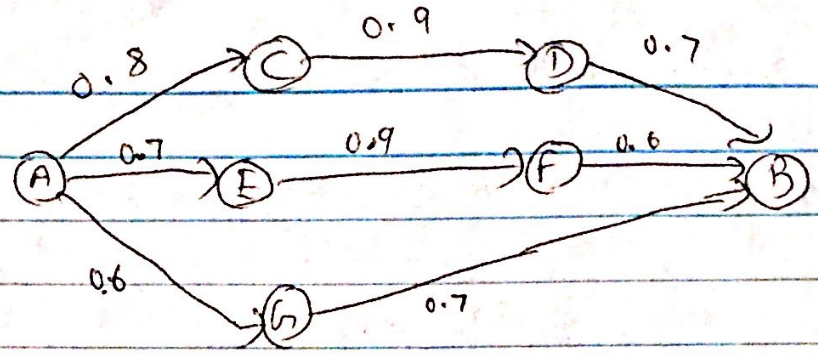
By rule of Conditional property, where,

$$P(A \cap B) = P(B)\, P(A/B) \to ②$$

Applying ② in ①.

$$\Rightarrow P(B_1)\, P(A/B_1) + P(B_2)\, P(A/B_2) + \dots + P(B_n)\, P(A/B_n).$$

Q4



Say Path 1 is 'A' to 'B' via 'G'
Path 2 is 'A' to 'B' via 'E' & 'F'
Path 3 via 'C' & 'D'.

Now the failure of Path 1 = $0.6 \times 0.3 + 0.4 \times 0.7 + .3 \times .4$
$\Rightarrow 0.58$

Path 2 = $1 - (0.7 \times 0.9 \times 0.6)$
$= 0.622$

Path 3 = $1 - (0.8 \times 0.9 \times 0.7)$
$= 0.496$.

$\therefore$ Total failure probability $= 0.58 \times 0.622 \times 0.496$
$= 0.1789$.

b) Probability of complete failure $= 0.1789$

a) Probability of success $= 0.8211$.

Q5. $f_X(x) = \frac{x}{\sigma^2} e^{-\left[\frac{x^2}{2\sigma^2}\right]}$ for $x \geq 0$ & $\sigma > 0$.

a) $E[x] = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$

$$\int_0^\infty \frac{x^2}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right) dx$$

Multiply & divide by $\sqrt{2\pi}$

$$= \sqrt{2\pi} \int_0^\infty \frac{x^2}{\sigma^2\sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma^2}\right) dx \rightarrow ①$$

$\int_0^\infty \frac{x^2}{\sqrt{2\pi}\cdot\sigma} \exp\left(\frac{-x^2}{2\sigma^2}\right) dx$ is the variance of the gaussian pdf. within 0 to ∞. & the value $= \frac{\sigma^2}{2}$

$\therefore$

$$\int_0^\infty \frac{x^2}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-x^2}{2\sigma^2}\right) dx = \frac{\sigma^2}{2}. \rightarrow ②$$

Sub ② in ①.

$$= \frac{\sqrt{2\pi}}{\sigma} \times \frac{\sigma^2}{2} = \sigma\sqrt{\pi/2}.$$

b) $Var[x] = E[x^2] - (E[x])^2$

$$(E[x])^2 = \frac{\sigma^2\pi}{2}.$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx.$$

$$= \int_0^\infty \frac{x^2}{\sigma^2} x \exp\left(\frac{-x^2}{2\sigma^2}\right) dx.$$

$u = x^2$ ~~$u = \frac{x^2}{\sigma^2}$~~ $dv = \frac{x}{\sigma^2} \exp\left(-x^2/2\sigma^2\right) dx.$

$uv - \int v\,du$;

③

$$= x^2 \int \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} dx - \int 2x \int \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} dx\, dx$$

$$= \left[x^2 \frac{1}{2\sigma^2} (-2\sigma^2) e^{-x^2/2\sigma^2} \right]_0^\infty - \int_0^\infty 2x \frac{1}{\sigma^2} \frac{1}{2} (-2\sigma^2) e^{-x^2/2\sigma^2} dx$$

$$= 0 + 2 \int_0^\infty x e^{-x^2/2\sigma^2} dx.$$

$$= \left[2(-2\sigma^2) \frac{1}{2} e^{-x^2/2\sigma^2} \right]_0^\infty = 2\sigma^2.$$

$$Var[x] = 2\sigma^2 - \frac{\sigma^2 \pi}{2} \Rightarrow \sigma^2 (2 - \pi/2).$$

c) $F_X(x) = \int f_X(x)\, dx.$

$$= \int_0^\infty \frac{x}{\sigma^2} e^{-x^2/2\sigma^2} dx$$

$$t = \frac{x^2}{2\sigma^2} \qquad dt = \frac{2x}{2\sigma^2} dx$$

$$dt = \frac{x}{\sigma^2} dx.$$

$$= \int_0^\infty e^{-t} dt$$

$$= \left[\frac{e^{-t}}{-1} \right]_0^\infty = 0 + 1.$$

$\therefore$ CDF $= 1 - e^{-t}$

$= 1 - e^{-x^2/2\sigma^2}$

Q.6 $f_x(x) = \begin{cases} \frac{1}{2}\lambda \exp(-\lambda x) & x \geq 0 \\ \frac{1}{2}\lambda \exp(\lambda x) & x < 0. \end{cases}$ for $\lambda > 0$.

Find the mean & Variance of x.

Mean $E(x) = \int_{-\infty}^{\infty} f_x(x)\, x\, dx$

$$= \int_{-\infty}^{0} x \frac{1}{2}\lambda e^{+\lambda x} dx + \int_{0}^{\infty} \frac{x}{2} \lambda e^{-\lambda x} dx.$$

$u = x \quad dv = e^{\lambda x} dx$

$v = \frac{e^{\lambda x}}{\lambda}$

$$\frac{x e^{\lambda x}}{\lambda}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} \frac{e^{\lambda x}}{\lambda} dx$$

$$0 - \left[\frac{e^{\lambda x}}{\lambda^2}\right]_{-\infty}^{0}$$

$$= -\left[\frac{1}{\lambda^2}\right] \times \frac{\lambda}{2}$$

$u = x \quad dv = e^{-\lambda x} dx$

$v = \frac{e^{-\lambda x}}{-\lambda}$

$$= 0 - \int_{0}^{\infty} \frac{e^{-\lambda x}}{-\lambda} dx$$

$$= +\left[\frac{1}{\lambda^2}\right] \lambda/2$$

$$\Rightarrow E[x] = \frac{-\lambda}{\lambda^2 2} + \frac{\lambda}{2\lambda^2} \Rightarrow 0/\tfrac{\lambda^2}{2}.$$

Mean $= 0/\lambda^2 = 0$

$Var[x] = E[x^2] - (E[x])^2$.

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(x) dx.$$

$$= \int_{-\infty}^{0} \frac{x^2}{2} \lambda e^{-\lambda x} dx$$

$$= \frac{\lambda}{2} \int_{-\infty}^{0} x^2 \frac{e^{-\lambda x}}{-\lambda} - \int_{-\infty}^{0} 2x \frac{e^{-\lambda x}}{-\lambda} dx$$ ④

$$= \frac{2\lambda}{2\lambda} \left[0 + \int_{-\infty}^{0} x e^{-\lambda x} dx \right]$$

$$= 1 \left[x \frac{e^{-\lambda x}}{-\lambda} - \int_{-\infty}^{0} e^{-\lambda x} dx \right]$$

$$= 1 \left[0 + \left[\frac{e^{-\lambda x}}{+\lambda} \right]_{-\infty}^{0} \right]$$

$$= \frac{1}{\lambda} \left[\frac{1}{+\lambda} \right] = 1/\lambda^2$$

$$\int_0^\infty \frac{x^2}{2} \lambda e^{+\lambda x} dx \Rightarrow \frac{\lambda}{2} \int_0^\infty x^2 \lambda e^{+\lambda x} dx.$$

$$= \frac{\lambda}{2} \left[x^2 \frac{e^{\lambda x}}{\lambda} \right]_0^\infty - \int_0^\infty 2x \frac{e^{\lambda x}}{\lambda} dx.$$

$$= \frac{2\lambda}{2\lambda} \left[0 - \int_0^\infty x e^{\lambda x} dx \right]$$

$$= \left[\frac{x e^{\lambda x}}{\lambda} \right]_0^\infty - \int_0^\infty dx \frac{e^{\lambda x}}{\lambda}$$

$$= \frac{1}{\lambda} \left[\frac{-e^{\lambda x}}{\lambda} \right]_0^\infty$$

$$= \frac{1}{\lambda} \times \frac{1}{\lambda} = \frac{1}{\lambda^2}$$

$$\therefore E[x^2] = \frac{1}{\lambda^2} + 1/\lambda^2 = 2/\lambda^2$$

$$\text{Var}[x] = E[x^2] = 2/\lambda^2.$$

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Q.7 We know that the Q-function is probability that a Gaussian R.V. will obtain a value larger than $(x \times S.D) + mean$.

& hence $Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \, dy.$

(ie) $P(X \geq x_0) = \int_{x_0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$

now $y = \frac{x-\mu}{\sigma}$

$\therefore \int_{\frac{x_0-\mu}{\sigma}}^{\infty} \frac{\sigma}{\sqrt{2\pi}\sigma} e^{-y^2/2} \, dy$

$= \int_{\frac{x_0-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \, dy.$

This is termed as the Q-function.

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-p^2/2} \, dp.$$

replace $t = p/(\sqrt{2}) \Rightarrow dt = dp/(\sqrt{2})$

$$= \frac{1}{\sqrt{\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-t^2} \, dt$$

Now multiply & Divide by 2.

$$= \frac{1}{2} \underbrace{\left[\frac{2}{\sqrt{\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-t^2} \, dt \right]}_{} \rightarrow erfc$$

$$\therefore \quad Q(x) = \frac{1}{2} \, erfc\left(\frac{x}{\sqrt{2}}\right).$$

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b) $\operatorname{erfc}(x) = 2Q(\sqrt{2}x)$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt.$$

Now $Q(x) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_x^\infty e^{-t^2/2}\, dt.$

$$\therefore 2 \times Q(\sqrt{2}x) = \frac{2}{\sqrt{2\pi}} \int_{\sqrt{2}x}^\infty e^{-t^2/2}\, dt.$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}} \int_{x\sqrt{2}}^\infty e^{-t^2/2}\, dt.$$

Let ~~$p = t$~~ $t = p\sqrt{2}$

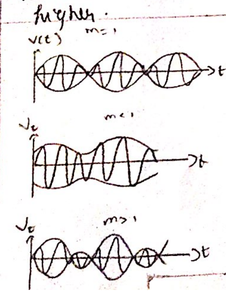
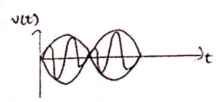
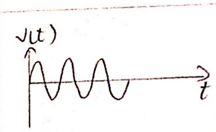
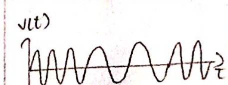
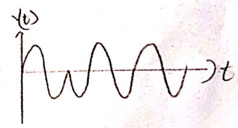
$dt = dp\sqrt{2}$

If $t = x\sqrt{2}$ then $x = p$

$$= \frac{2}{\sqrt{\pi}} \int_{p}^\infty e^{-p^2}\, dp$$

$= \operatorname{erfc}(p)$. Hence proved.

Section 2
Q.1

	AM	DSB-SC	SSB	FM	PM
Wave form	$[1 + V_m \cos(2\pi f_m t + \phi)]$ $V_c \sin(2\pi f_c t)$	$\frac{V_m V_c}{2}[\cos(w_m + w_c)t + \cos(w_m - w_c)t]$	$m(t)\cos(2\pi f_c t) \mp \hat{m}(t)\sin(2\pi f_c t)$	$V_c \cos\left(2\pi f_c t - \frac{\Delta f}{f_m}\cos(2\pi f_m t)\right)$	$V_c \sin(w_c t + m(t) + \phi_c)$
Band width	$2 f_m$ modulation index $m = \frac{V_c}{V_m}$	$2 f_m$	f_m	$2(\beta + 1) f_m$ where $\beta = \frac{k_f * a}{f_m}$	$2(k_p a + 1) f_m$.
Power	$P_u = \frac{V_c^2}{2} P_m$ where $P_m = 1 + a^2 P_{m_n}$	$P_u = \frac{V_c^2}{2} P_m$ $P_{us} = \frac{V_c^2 a^2}{4} = P_{ks}$	$P_u = \frac{V_c^2}{4}$.	$P_c = \frac{V_c^2}{2}$	$\frac{V_c^2}{2}$
Advantage & Disadvantage	Useful in case of short range. Working decreases as the B.W is higher.	Similar to A.M. but with better power efficiency due to absence of Carrier.	Reduced B.W & increased power efficiency	Efficient than F.M. but complexity increases with increase in B.W. requirement.	Better performance than F.M. owing to easier generation & efficiency.
					

Section 2
Q.2

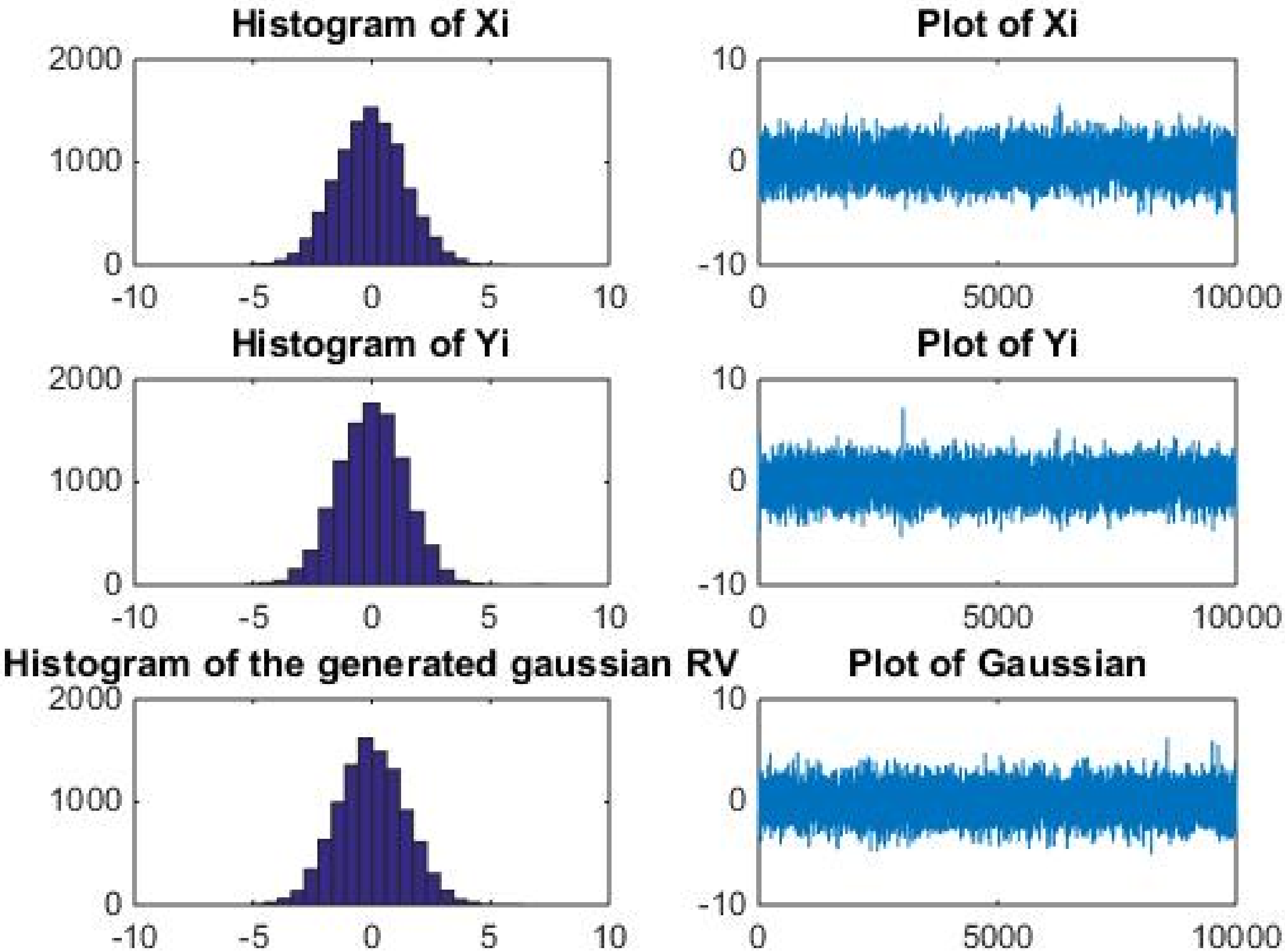
	AM	DSB-SC	SSB	FM	PM
SNR	$\frac{V_c^2 a^2 P_m}{2N_oW}$	$\frac{A_c^2 P_m}{2WN_o}$	$\frac{A_c^2 P_m}{WN_o}$	$\frac{3K_f^2 A_c^2}{2W^2} \frac{P_m}{N_oW}$	$\frac{K_p^2 A_c^2}{2} \frac{P_m}{N_oW}$

Section 3

Question 1

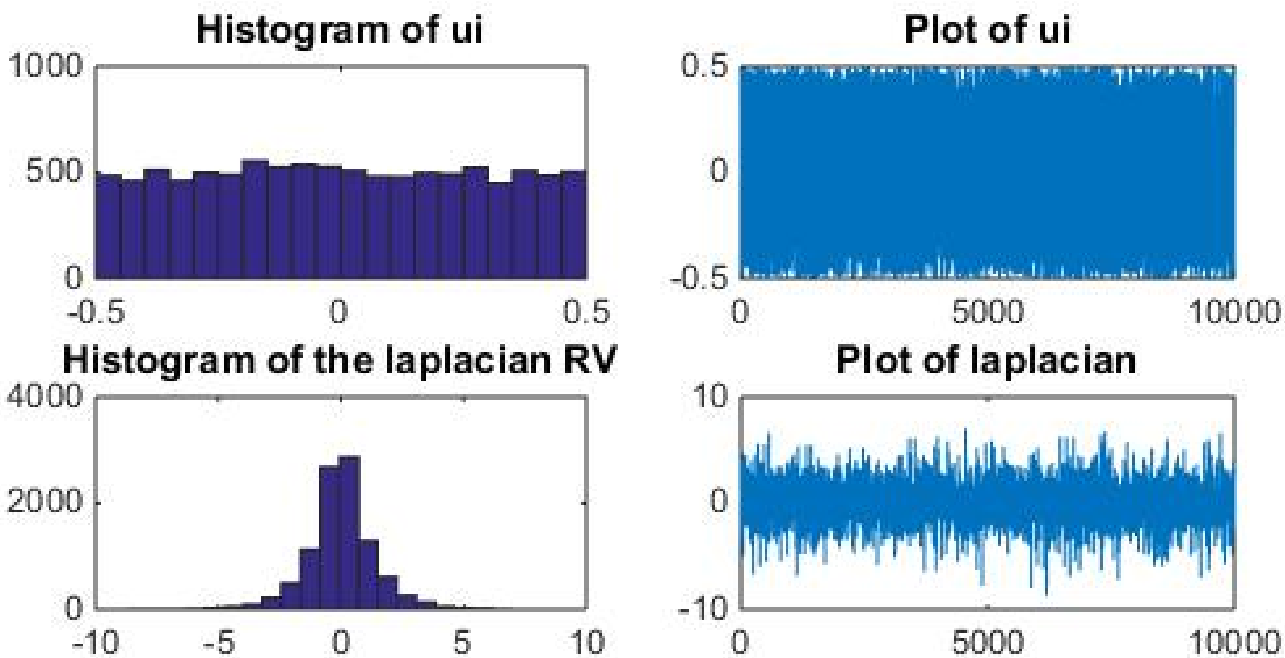
```
%generation of theta
theta=(2*pi)*rand(10000,1);
%generation of uniform random variable
ui=rand(10000,1);
prompt='Enter the variance';
var=input(prompt);
r=sqrt(2*var*(log(1./(1-ui))));
xi=r.*cos(theta);
yi=r.*sin(theta);
%generation of gaussian R.V
gaussian=normrnd(0,sqrt(var),[10000 1]);
subplot(3,2,1);
hist(xi,20);
title('Histogram of Xi');
subplot(3,2,2);
plot(xi);
title('Plot of Xi');
subplot(3,2,3);
hist(yi,20);
title('Histogram of Yi');
subplot(3,2,4);
plot(yi);
title('Plot of Yi');
subplot(3,2,5);
hist(gaussian,20);
title('Histogram of the generated gaussian RV');
subplot(3,2,6);
plot(gaussian);
title('Plot of Gaussian');
Xi=hist(xi,20);
Yi=hist(yi,20);
gauss=hist(gaussian,20);
sumxi=0;
sumyi=0;
for i=1:20
    sumxi=Xi(i)+sumxi;
    sumyi=Yi(i)+sumyi;
    fprintf('The number of samples in %d column of Xi is %d and Yi is %d
\n',i,Xi(i),Yi(i));
end
```

```
>> Q1
Enter the variance 2
The number of samples in 1 column of Xi is 9 and Yi is 5
The number of samples in 2 column of Xi is 11 and Yi is 17
The number of samples in 3 column of Xi is 47 and Yi is 41
The number of samples in 4 column of Xi is 109 and Yi is 153
The number of samples in 5 column of Xi is 253 and Yi is 333
The number of samples in 6 column of Xi is 509 and Yi is 740
The number of samples in 7 column of Xi is 816 and Yi is 1199
The number of samples in 8 column of Xi is 1116 and Yi is 1571
The number of samples in 9 column of Xi is 1392 and Yi is 1766
The number of samples in 10 column of Xi is 1530 and Yi is 1658
The number of samples in 11 column of Xi is 1372 and Yi is 1230
The number of samples in 12 column of Xi is 1168 and Yi is 710
The number of samples in 13 column of Xi is 740 and Yi is 380
The number of samples in 14 column of Xi is 467 and Yi is 141
The number of samples in 15 column of Xi is 265 and Yi is 40
The number of samples in 16 column of Xi is 123 and Yi is 13
The number of samples in 17 column of Xi is 54 and Yi is 2
The number of samples in 18 column of Xi is 12 and Yi is 0
The number of samples in 19 column of Xi is 6 and Yi is 0
The number of samples in 20 column of Xi is 1 and Yi is 1
fx >>
```


Histogram of Xi
Plot of Xi
Histogram of Yi
Plot of Yi
Histogram of the generated gaussian RV
Plot of Gaussian
2000
1000
0
-10
-5
0
5
10
10
0
-10
0
5000
10000

Section 3
Question 2

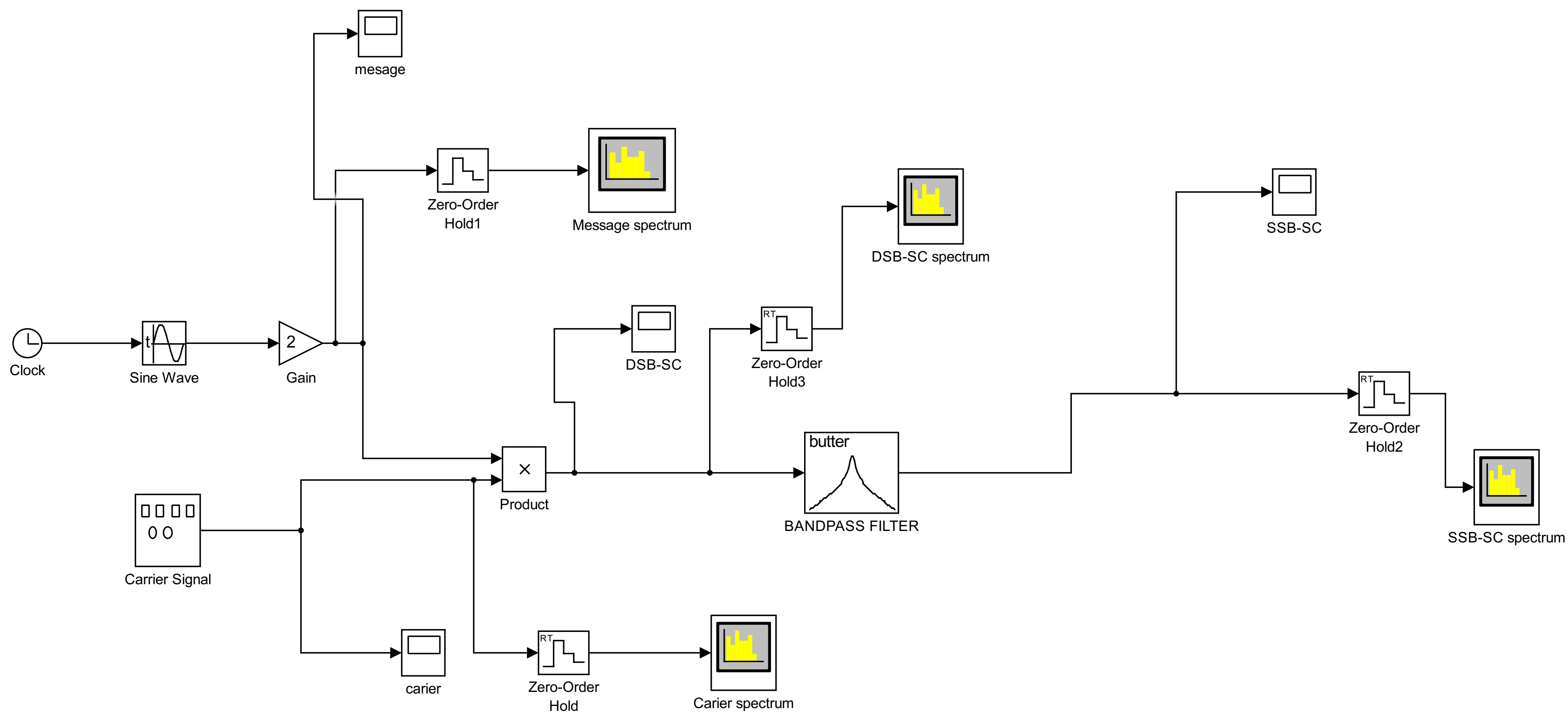
```
%generating the uniform random variable
ui=rand(10000,1)-0.5;
b=1;
mu=0;
r= mu - b*sign(ui).*log(1-2*abs(ui));
subplot(3,2,1);
hist(ui,20);
title('Histogram of ui');
%histogram plot of Ui
subplot(3,2,2);
plot(ui);
title('Plot of ui');
%histogram plot of Laplacian R.V.
subplot(3,2,3);
hist(r,20);
title('Histogram of the laplacian RV');
subplot(3,2,4);
plot(r);
title('Plot of laplacian');
```

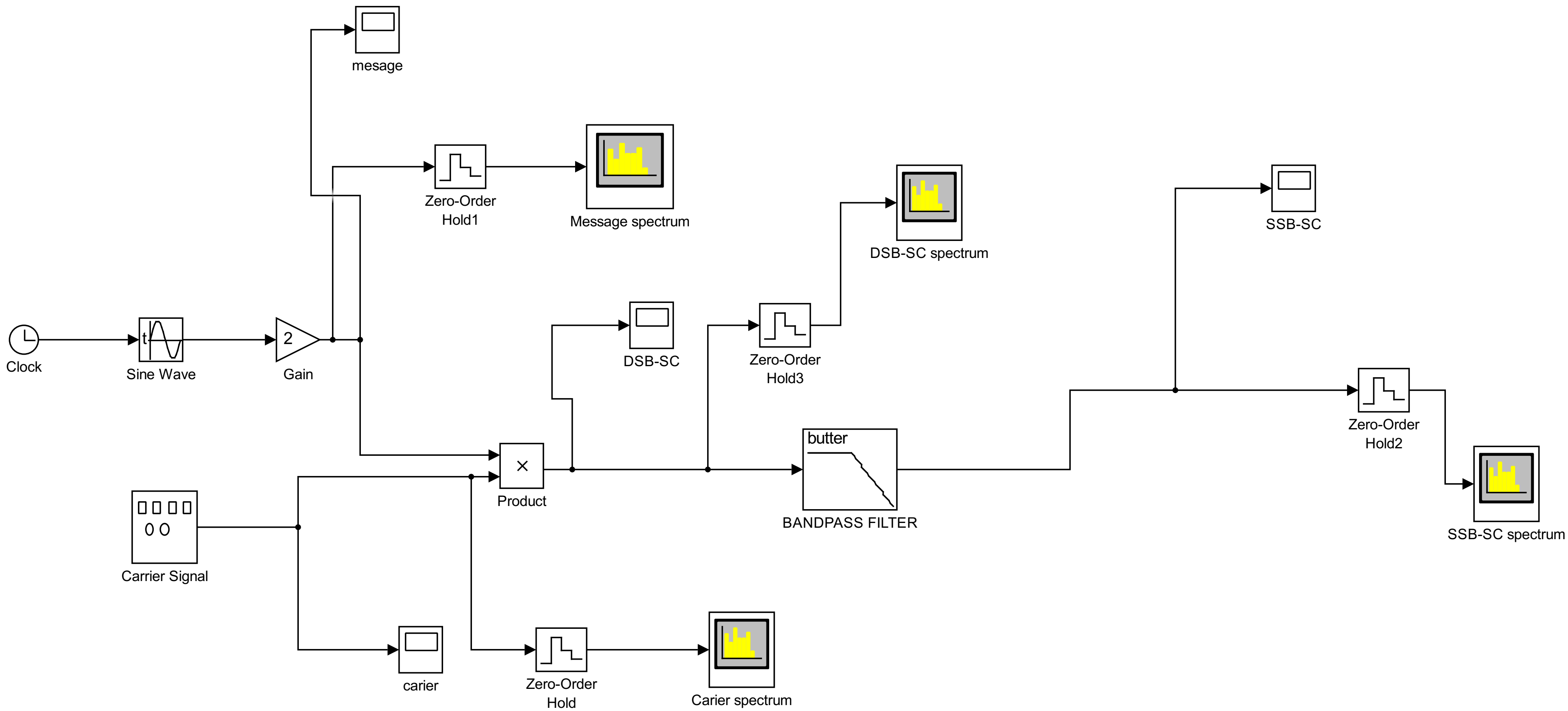


Histogram of ui
1000
500
0
-0.5
0
0.5
Plot of ui
0.5
0
-0.5
0
5000
10000
Histogram of the laplacian RV
4000
2000
0
-10
-5
0
5
10
Plot of laplacian
10
0
-10
0
5000
10000

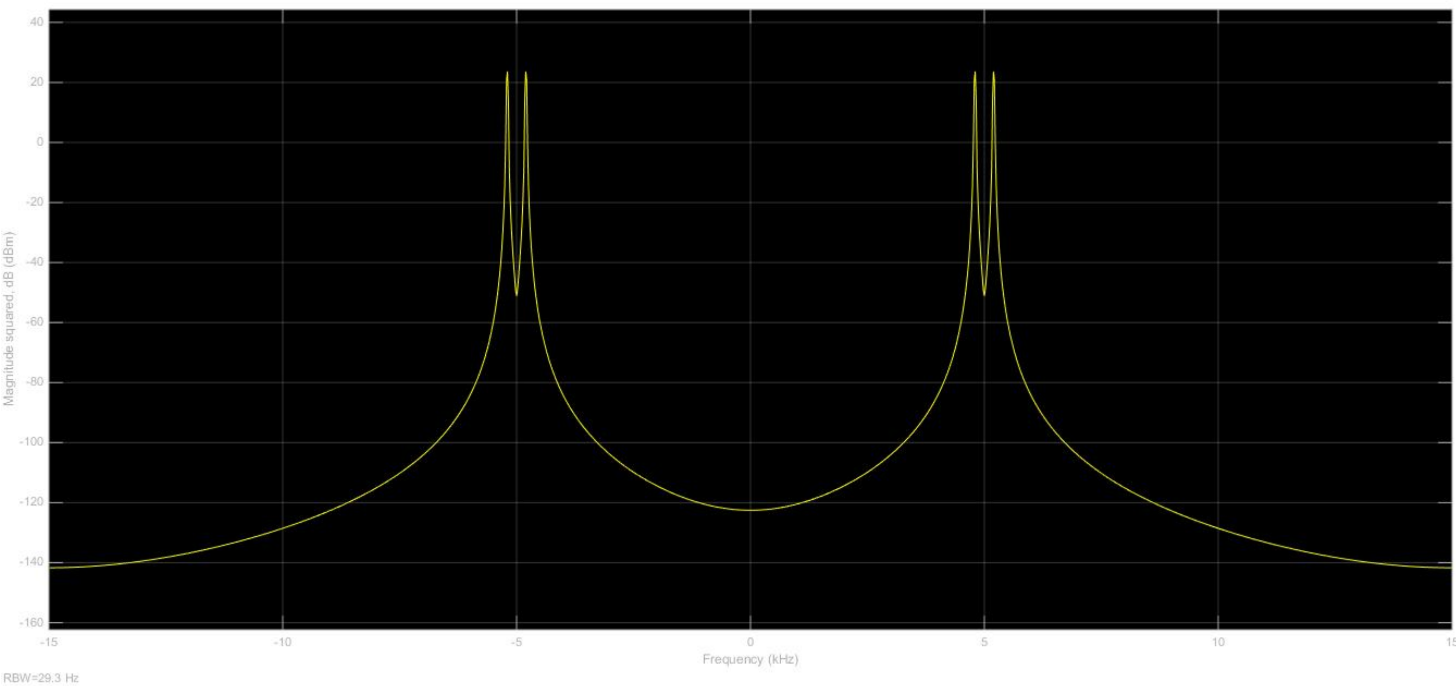
Section 3

Question 3

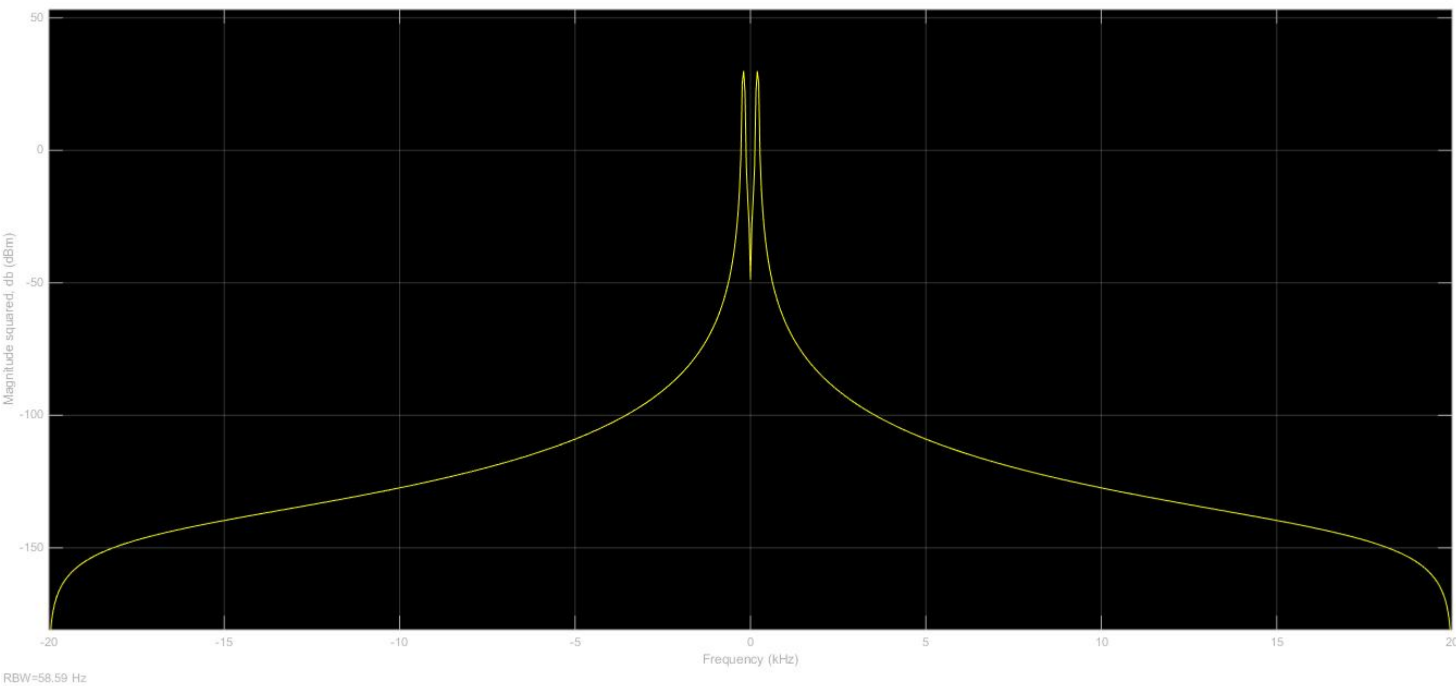



mesage
Zero-Order Hold1
Message spectrum
DSB-SC spectrum
SSB-SC
Clock
Sine Wave
2
Gain
DSB-SC
Zero-Order Hold3
butter
Zero-Order Hold2
Product
BANDPASS FILTER
SSB-SC spectrum
Carrier Signal
carier
Zero-Order Hold
Carier spectrum

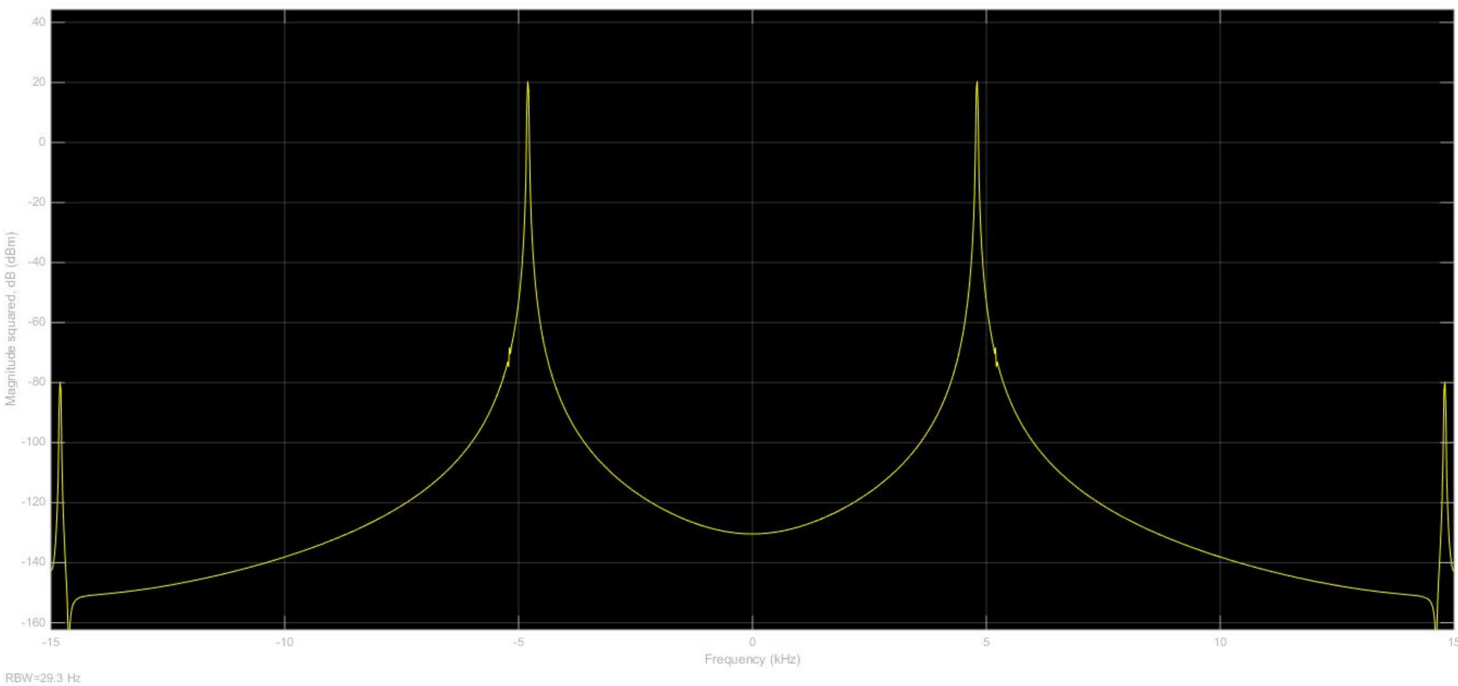
DSB-SC Spectrum



Message Spectrum

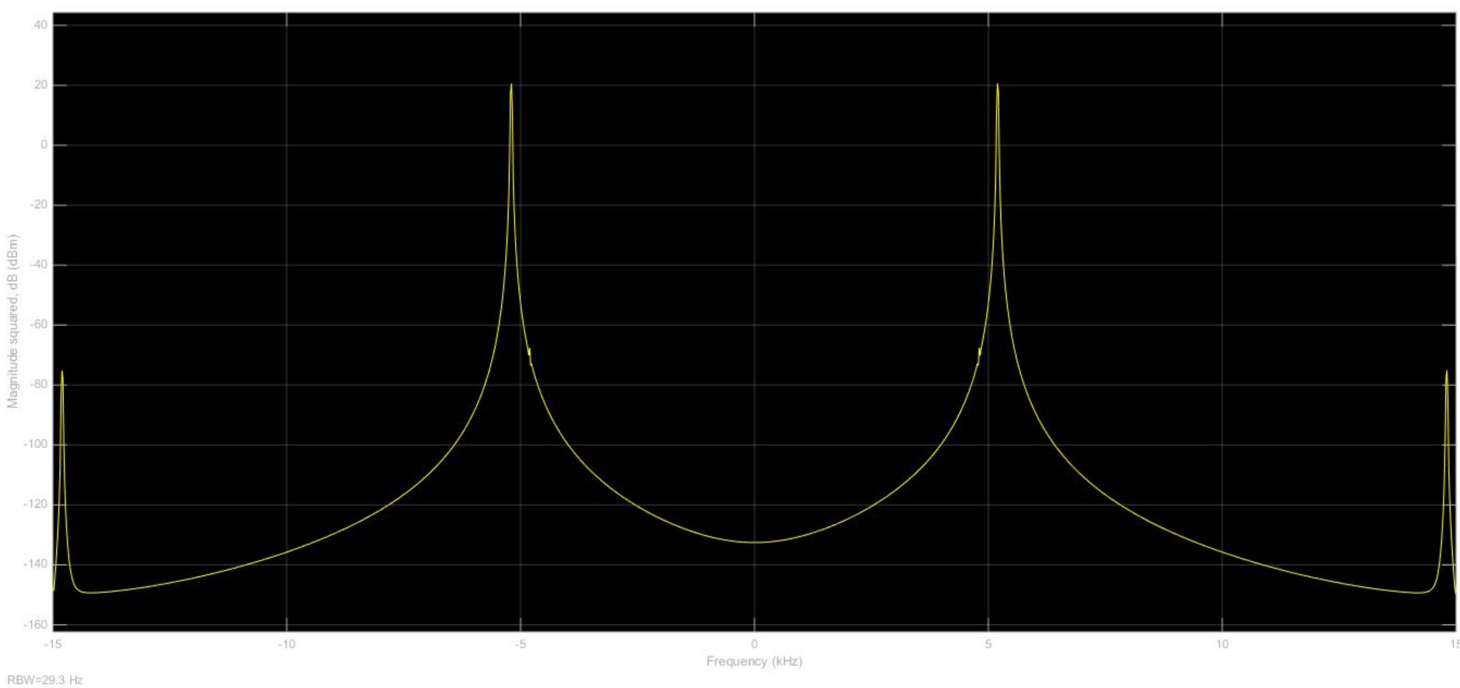


Lower SSB



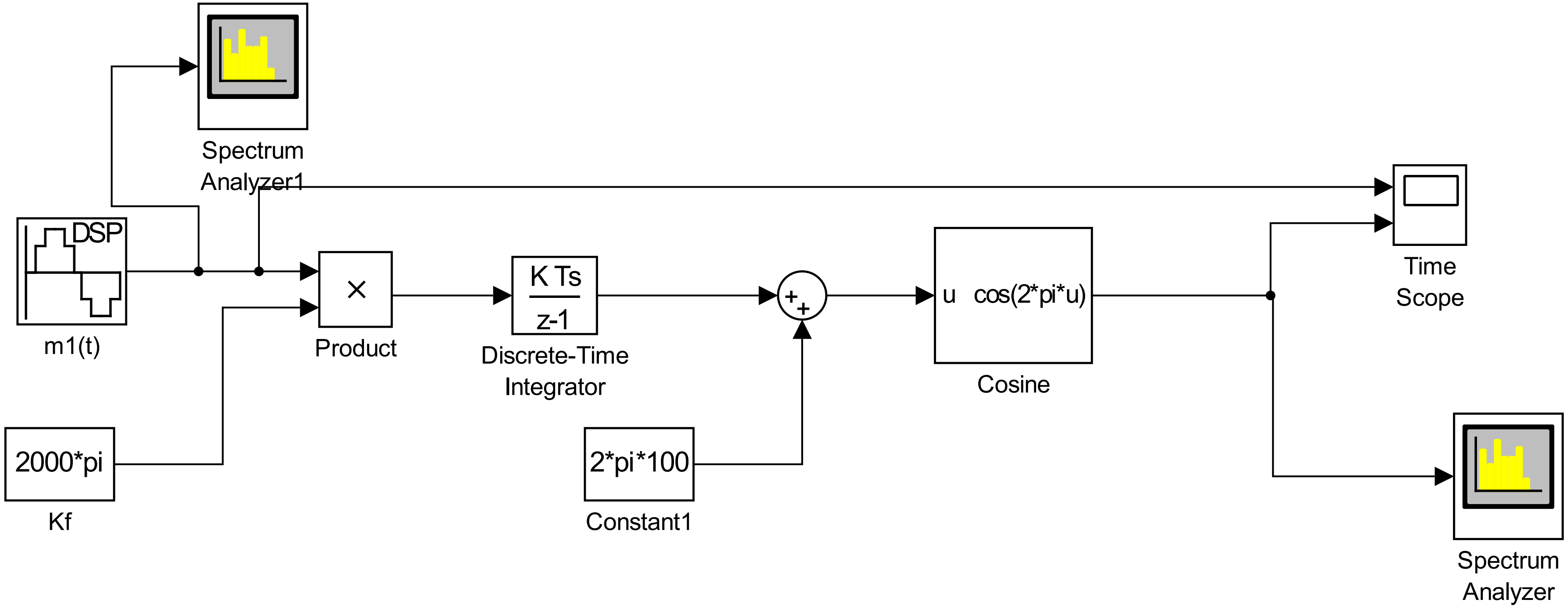
Magnitude squared, dB (dBm)
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-15
-10
-5
0
5
10
15
Frequency (kHz)
RBW=29.3 Hz

Upper SSB

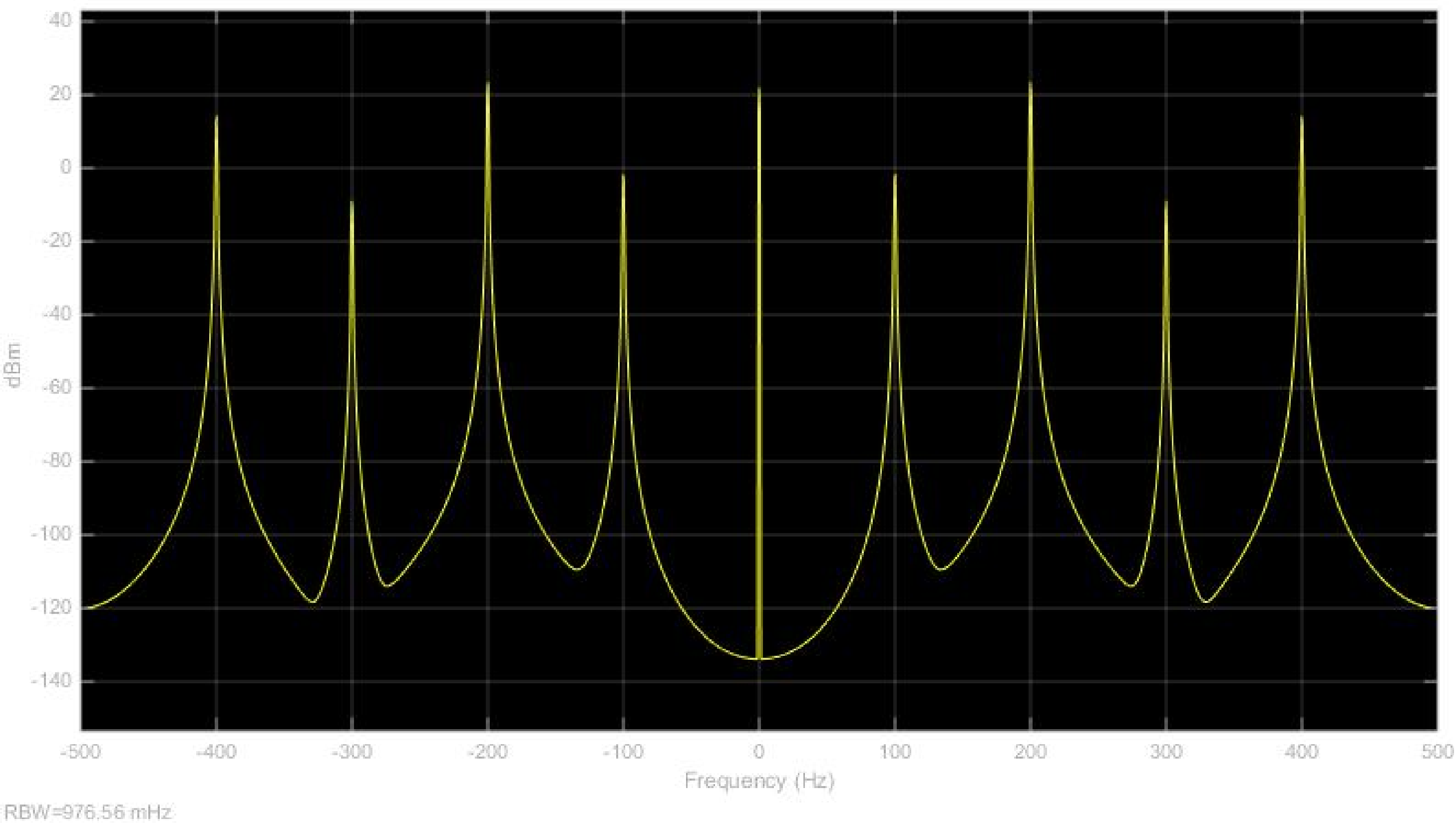


40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
Magnitude squared, dB (dBm)
-15
-10
-5
0
5
10
15
Frequency (kHz)
RBW=29.3 Hz

Section 3
Question 4



Frequency Modulation Spectrum.

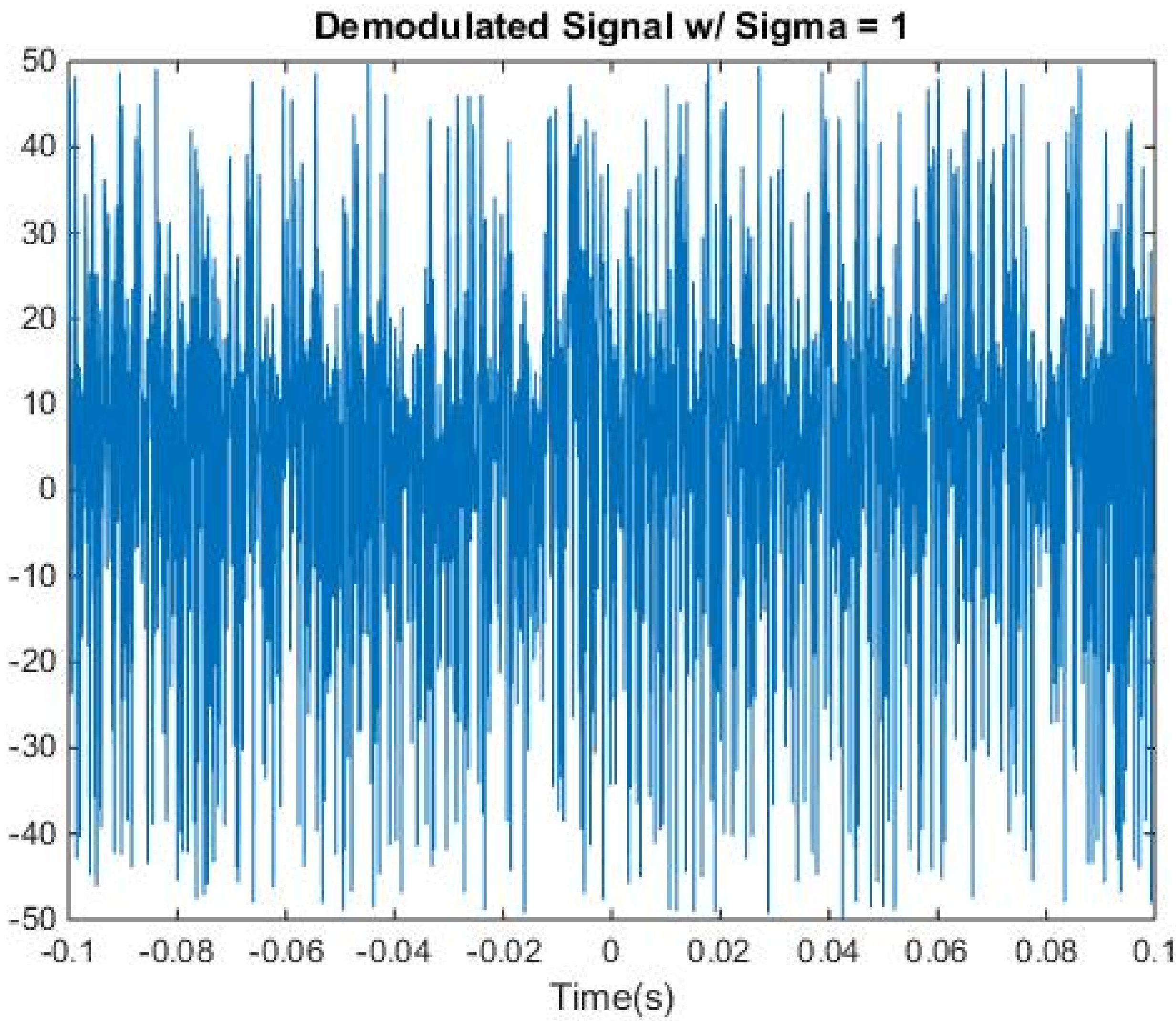


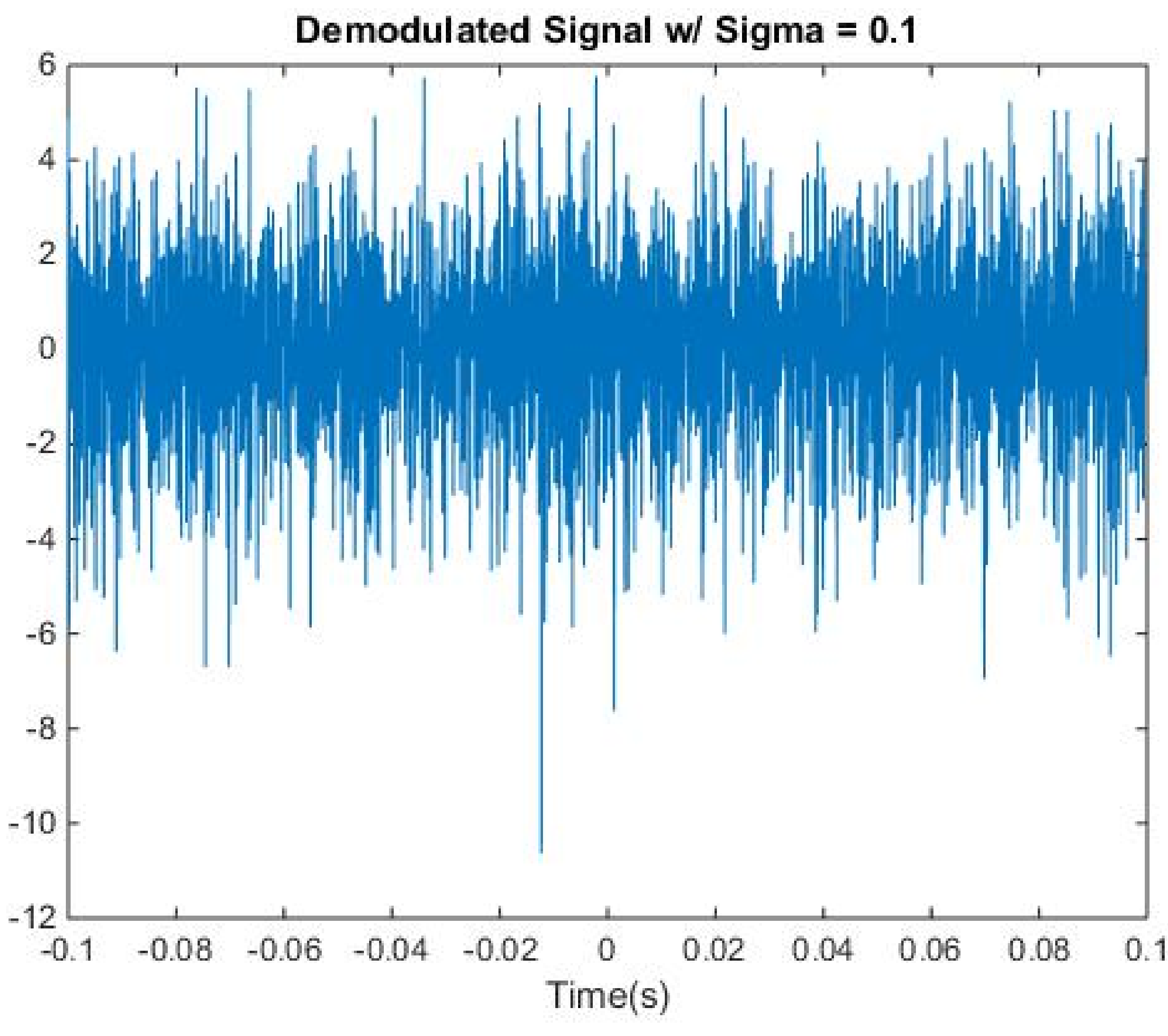
Question 5

```
clear all
clc
%% Part 1
%Instantiating the signal parameters:
t0 = 0.1;
ts = 0.0001;
simulationTime = 0.2;
fs = 1/ts;
t = [0:ts:simulationTime-ts];
%Creating the original message signal:
m(1) = 0;
for i = 2:length(t)
m = [m m(i-1)+ts];
end
%Computing the integral of the message signal:
m_integral(1)=0;
for i = 1:length(t)-1
m_integral(i+1) = m_integral(i) + (m(i)*ts);
end
clear i;
fc = 250;
kf = 100;
%Plotting both m(t) and the integral of m(t):
plot(t,m)
title('Original Message Signal')
xlabel('Time(s)')
ylabel('m(t)')
figure();
plot(t,m_integral)
title('Integral of Message Signal')
xlabel('Time(s)')
%% Part 2
%In order to compute the bandwidth of the modulated signal, the spectrum
%of u(t) must first be determined:
u = cos(2*pi*fc*t + (2*pi*kf*m_integral));
length_u_fft = 2^(nextpow2(length(u)));
U = fft(u,length_u_fft);
U = U/fs;
U = fftshift(U);
f = [-length(U)/2:length(U)/2 - 1];
figure();
plot(f,abs(U))
title('Spectrum of the Modulated Signal')
xlabel('Frequency')
%Using the spectrum of u(t), the bandwidth may now be determined by
%calculating the range of frequencies that contain the majority of the
%modulated signal. This is done by simply analyzing the plot. The
%bandwidth is calculated in this way because Carson's rule cannot be
%applied in the context of this problem.
%% Part 3
%The modulated signal is recalculated for completeness. It is also
%resampled, with only 1,999 samples being taken. A reduced number of
%samples were performed in order to make the computation in the next part
%more straightforward.
t = [-t0:ts:t0-(2*ts)];
m_integral = m_integral(1:1999);
u = cos(2*pi*fc*t + (2*pi*kf*m_integral));
```

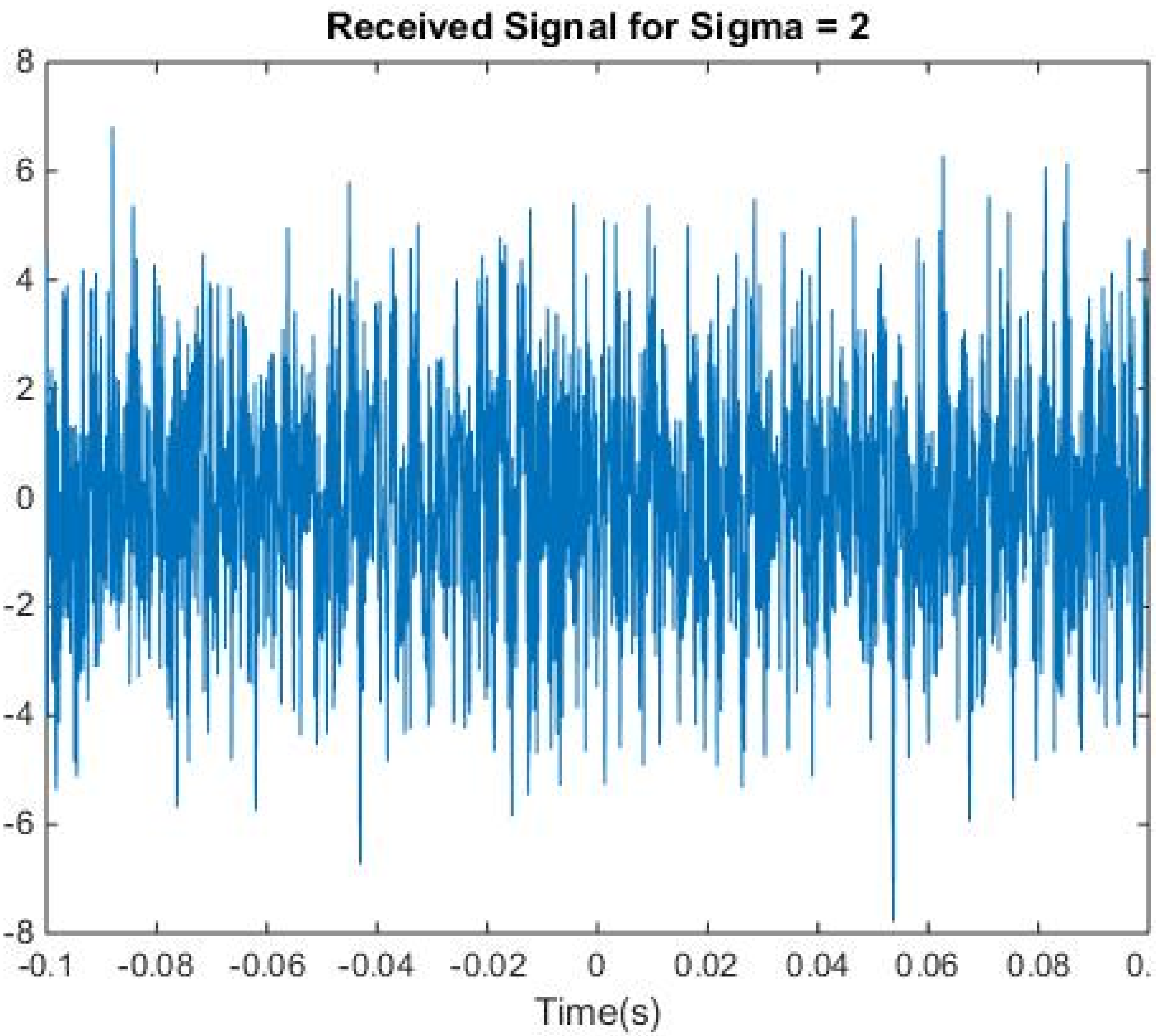
```
figure();
plot(t,u)
title('Modulated Message Signal')
xlabel('Time(s)')
ylabel('u(t)')
%% Part 4
%First computing the spectrum of the original message signal, m(t):
length_m_fft = 2^(nextpow2(length(m)));
M = fft(m,length_m_fft);
M = M/fs;
M = fftshift(M);
df = fs/length_m_fft;
f = [0:df:df*(length(M)-1)]-fs/2;
figure()
plot(f,abs(M))
title('Spectrum of the Message Signal')
xlabel('Frequency')
%Now computing the spectrum of the modulated signal, u(t):
length_u_fft = 2^(nextpow2(length(u)));
U = fft(u,length_u_fft);
U = U/fs;
U = fftshift(U);
df = fs/length_u_fft;
f = [0:df:df*(length(U)-1)]-fs/2;
figure();
plot(f,abs(U))
title('Spectrum of the Modulated Signal')
xlabel('Frequency')
%% Part 5
%Initializing the variables required for modeling the received signal
%sequences with noise applied:
sigma01 = 0.1;
sigma1 = 1;
sigma2 = 2;
wc = randn(1,1999);
ws = randn(1,1999);
n = [-999:1:999];
%Applying noise to each received signal sequence for each value of sigma:
r_sigma01 = u + sigma01*(wc.*cos(2*pi*fc*t) - ws.*sin(2*pi*fc*t));
r_sigma1 = u + sigma1*(wc.*cos(2*pi*fc*t) - ws.*sin(2*pi*fc*t));
r_sigma2 = u + sigma2*(wc.*cos(2*pi*fc*t) - ws.*sin(2*pi*fc*t));
%Plotting the received signal for each value of sigma:
figure();
plot(t,r_sigma01)
title('Received Signal for Sigma = 0.1')
xlabel('Time(s)')
figure();
plot(t,r_sigma1)
title('Received Signal for Sigma = 1')
xlabel('Time(s)')
figure();
plot(t,r_sigma2)
title('Received Signal for Sigma = 2')
xlabel('Time(s)')
```

```
%% Part 6
%First, the Hilbert function of the received signal is taken for each value
%of sigma. Then, the result is multiplied by an exponential in order to
%remove the influence of the carrier frequency.
r_sigma01 = hilbert(r_sigma01).*exp(-j*2*pi*fc*t);
r_sigma1 = hilbert(r_sigma1).*exp(-j*2*pi*fc*t);
r_sigma2 = hilbert(r_sigma2).*exp(-j*2*pi*fc*t);
%Recall that the original m(t) is contained within the PHASE of the
%received signal. Therefore, the phase is calculated for each received
%signal sequence:
r_sigma01_angles = unwrap(angle(r_sigma01));
r_sigma1_angles = unwrap(angle(r_sigma1));
r_sigma2_angles = unwrap(angle(r_sigma2));
%The original message signal may be obtained by differentiating the
%phase and then removing the 2*pi*kf term:
r_sigma01 = (diff(r_sigma01_angles)/ts)/(2*pi*kf);
r_sigma1 = (diff(r_sigma1_angles)/ts)/(2*pi*kf);
r_sigma2 = (diff(r_sigma2_angles)/ts)/(2*pi*kf);
%Plotting the results:
t = t(1:1998);
figure();
plot(t,r_sigma01);
title('Demodulated Signal w/ Sigma = 0.1');
xlabel('Time(s)')
figure();
plot(t,r_sigma1);
title('Demodulated Signal w/ Sigma = 1');
xlabel('Time(s)')
figure();
plot(t,r_sigma2);
title('Demodulated Signal w/ Sigma = 2');
xlabel('Time(s)')
```

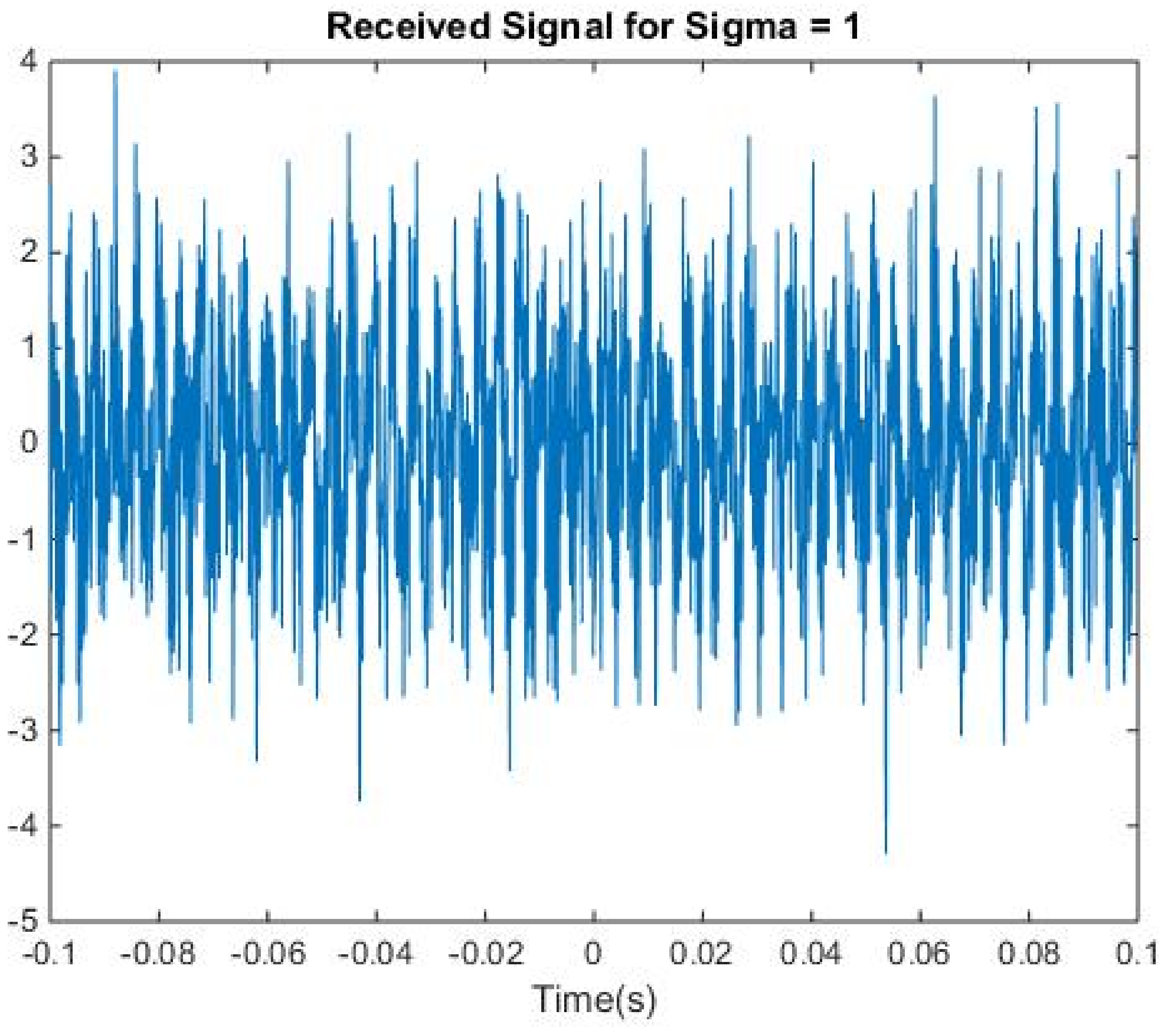

Demodulated Signal w/ Sigma = 1
50
40
30
20
10
0
-10
-20
-30
-40
-50
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
Time(s)

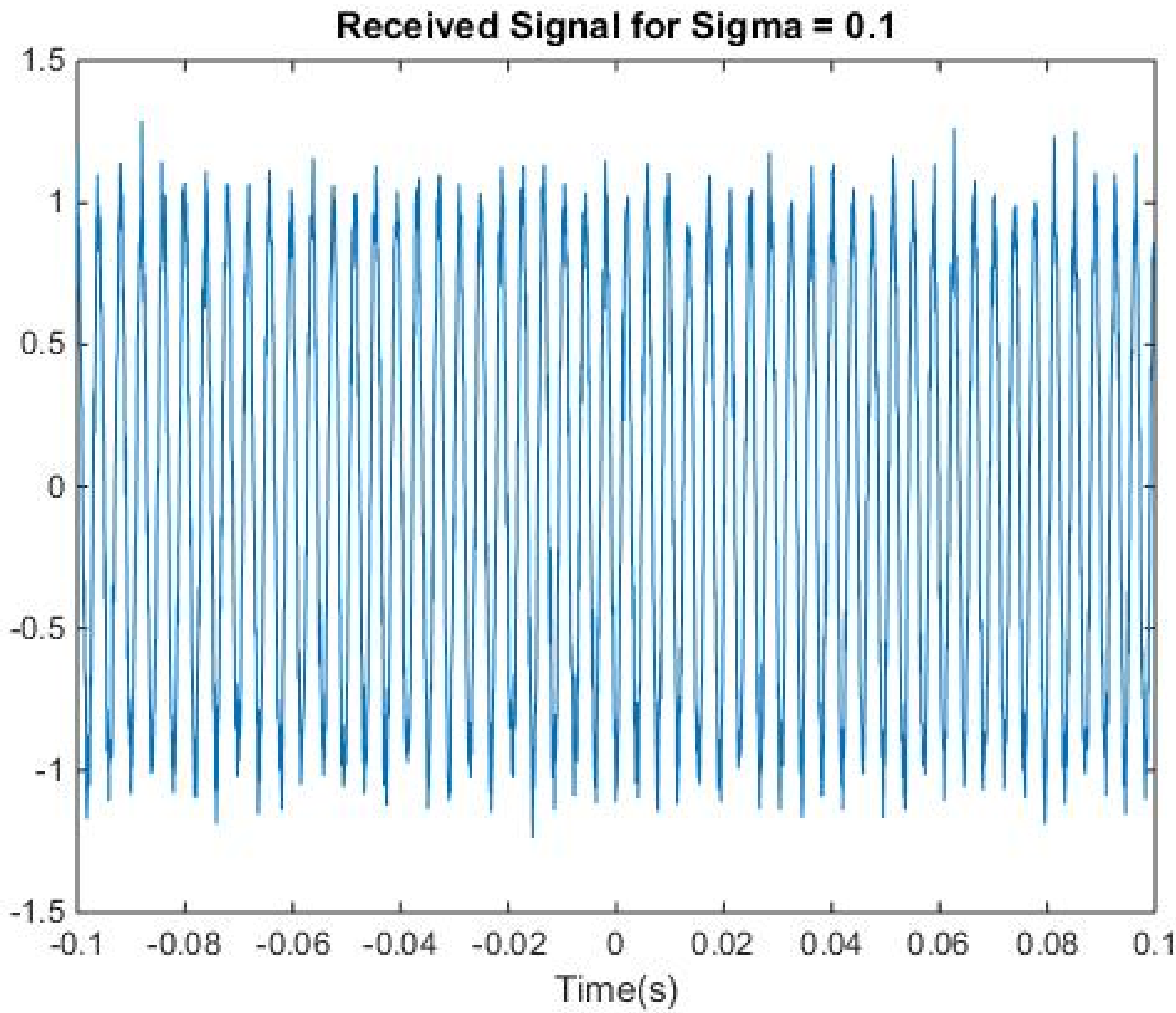


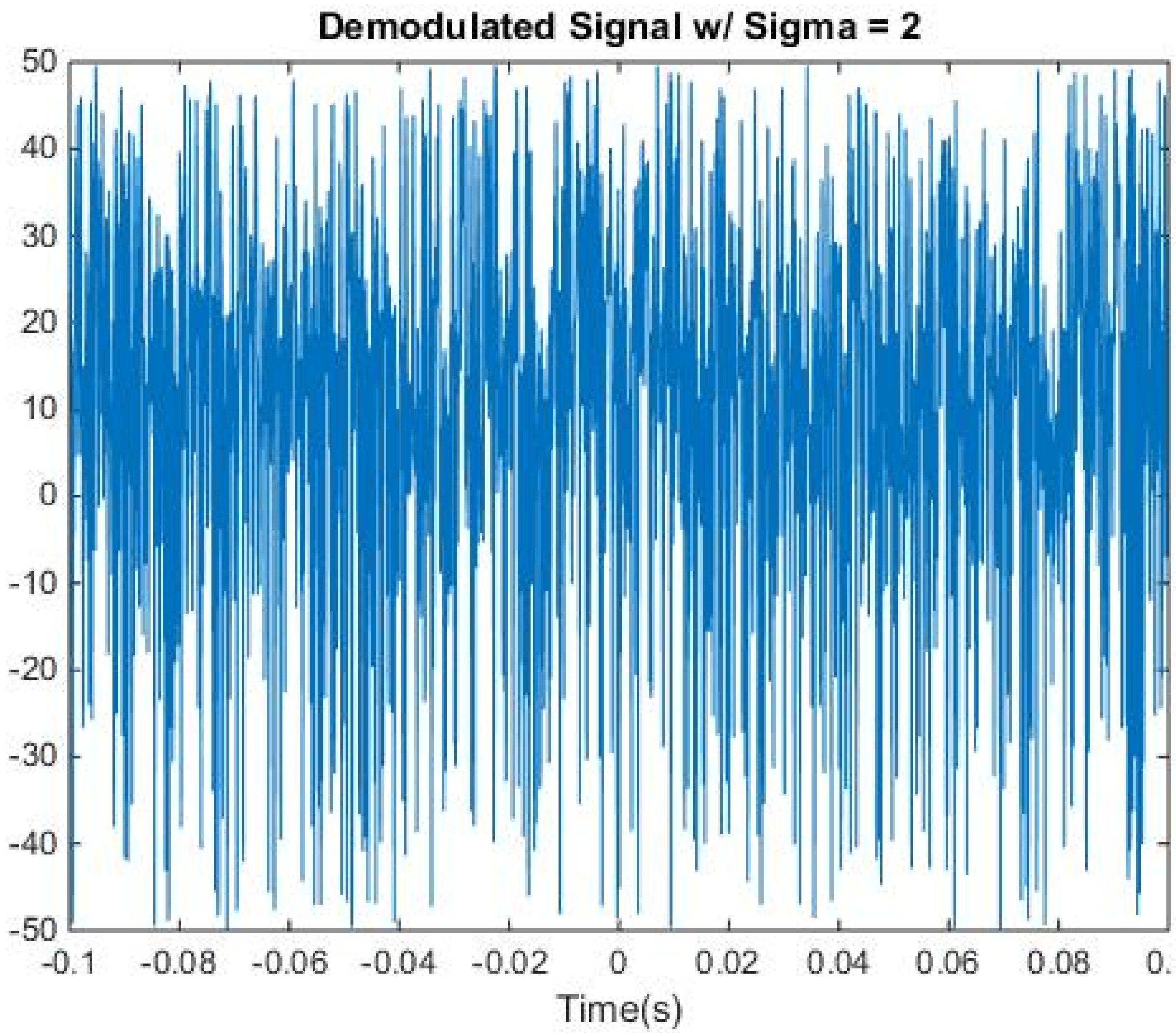
Demodulated Signal w/ Sigma = 0.1
6
4
2
0
-2
-4
-6
-8
-10
-12
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
Time(s)



Received Signal for Sigma = 2
8
6
4
2
0
-2
-4
-6
-8
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
Time(s)


Received Signal for Sigma = 1
4
3
2
1
0
-1
-2
-3
-4
-5
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
Time(s)


Received Signal for Sigma = 0.1
1.5
1
0.5
0
-0.5
-1
-1.5
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
Time(s)


Demodulated Signal w/ Sigma = 2
50
40
30
20
10
0
-10
-20
-30
-40
-50
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
Time(s)

Section - 3

Q.1 The histogram of the generated uniform R.V. matches the generated Gaussian function ~~a~~ with high degree of correlation.

The Gaussian & the generated Uniform R.V. look ~~to~~ highly resemble each other, with less degree of mis-match.

Q.2. The generated uniform R.V. is mapped to laplacian by the transformation

laplacian = mean - ~~Var~~ $b * \text{sign}(u_i) * \log(1 - 2|u_i|)$.

where $u_i \rightarrow$ Uniform R.V.

Q.4. $\beta = \frac{2000\pi}{200\pi} = 10.$

$BW = 2(\beta+1) f_m = 22 \times 10$

$= 220$ Hz.

As $\beta \gg 1$, it is a WBFM.

Q.5 The demodulated signal resembles the ~~re~~ input signal for the value of sigma = 1 with a very high degree