Politechnika Wrocławska

Faculty of Pure and Applied Mathematics

Field of study: Applied Mathematics
Specialty: Computational Mathematics

Master Thesis

APPLICATIONS OF WAVELETS IN DATA MINING

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keywords:
wavelets
statistics
learning and adaptive systems

Short summary:
*** To be changed ***
The aim of this thesis is an analysis and comparison of statistical methods employing wavelets. Special attention will be paid to data mining and machine learning methods.

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  a) *Category A (perpetual files)*
  b) *Category BE 50 (subject to expertise after 50 years)*
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Wrocław, 2018

Contents

Introduction

*** Few words about my thesis... Generally what is a data mining, what are the wavelets, what is the main purpose - edge detection. ***

Chapter 1

What exactly is a wavelet? A wavelet means a small wave. This term says a lot about it nature. Wavelets are a family of functions which oscillates like wave and should be compactly supported. Additionally, the wavelet has zero mean.

Definition 1.1. Wavelets are created by scaling and shifting of the, so called, mother wavelet $\psi(t)$. The child wavelets are defined as

$$\psi^{(a,b)}(t) = |a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right), \ a > 0. \tag{1.1}$$

There are plenty of different mother wavelets, for example

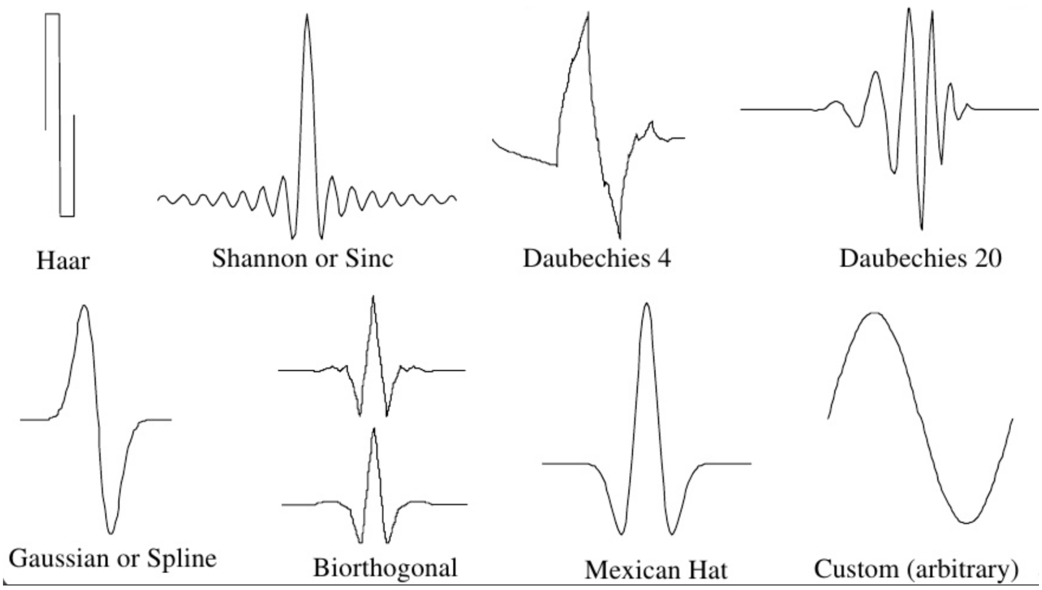

Figure 1.1: Different types of wavelets.

*** Add more about wavelets properties? ***

1.1. Daubechies wavelets

Each type of wavelet function is more suitable for different applications. The best for image analysis are the Daubechies wavelets.

Definition 1.2. Daubechies wavelets are collection of orthogonal and compactly supported functions. A denotation for those wavelets is dbN, where N means a maximal number of vanishing moments.

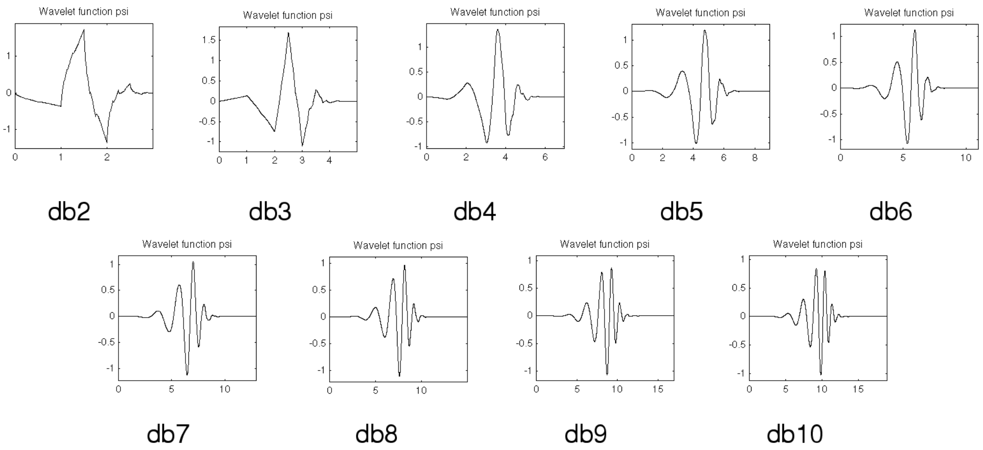

Figure 1.2: Daubechies wavelets.

1.2. Wavelet Transform

*** Add some short introduction? ***

Definition 1.3. Wavelet transform

$$W(a,b) = \int_{-\infty}^{\infty} y(t) a^{-\frac{1}{2}} \psi\left(\frac{t-b}{a}\right) dt, \tag{1.2}$$

where a is scale parameter, b translation parameter and $y(t)$ original signal.

1.2.1. Wavelet transform vs Fourier transform

Definition 1.4. Fourier transform

$$Y(f) = \int_{-\infty}^{\infty} y(t) e^{-i\omega t} dt, \tag{1.3}$$

where $y(t)$ is time domain signal and $Y(f)$ is frequency domain signal.

Wavelet transform	**Fourier transform**
Suitable for stationary and non--stationary signals	Suitable for stationary signals
High time and frequency resolution	Zero time resolution and very high frequency resolution
Very suitable for studying the local behaviours of the signal	No suitable
Sine and cosine waves	Scaled and translated mother wavelets

What differs both transformations is the type of function. In Fourier case there are sine and cosine functions, wherein wavelet transform uses wavelets.

Why use the Wavelet transform? Sine function oscillates on the whole real axis, thus it cannot represent abrupt changes. However, the Wavelet transform is localized in space and time, so it can be used to detect sudden changes in signals and images. Moreover, wide range of wavelet functions is a main advantage of wavelet analysis.

1.3. Discrete Wavelet transform

There are two types of the wavelet transform:

- Continuous Wavelet Transform (CWT),
- Discrete Wavelet Transform (DWT).

DWT is used to denoising and compression of signals and images. Also, DWT allows to detect smooth regions interrupted by edges or abrupt changes in contrast of images.
Scale and translation parameters are defined as

$$a = 2^j \text{ and } b = 2^j k, \;\; j, k = 1, 2, \ldots . \tag{1.4}$$

to avoid redundancy in coefficients.
The figure 1.3 on a page 9 shows how DWT works. Discrete Wavelet Transform splits signal with two filters: $h(n)$ - high pass filter (HPF) and $g(n)$ - low pass filter (LPF). The HPF captures a part with bigger frequencies which is the main signal. Whereas, the LPF captures smaller frequencies - a noise of the signal. Subsequently, both parts are down sampled by a factor of 2. This decomposition can be repeated on the HPF part of the signal. Hence, the next levels of DWT coefficients.

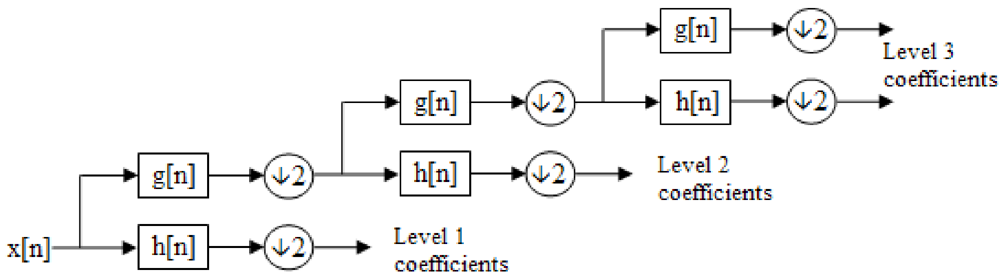

Figure 1.3: Discrete Wavelet transform on a signal $x(n)$.

1.4. 2-D Discrete Wavelet transform

2-D Discrete Wavelet Transform works similar way as 1-D with High Pass Filter and Low Pass Filter, except that one level of the decomposition includes double filtering, on columns and rows. The figure 1.4 shows image decomposition. Firstly, the DWT is applied on columns of the input image and then on the rows of the both outputs. Ultimately, there are four results:

- LL - result of LPF applied on both, columns and rows,

- LH - result of LPF applied on columns and HPF on rows,
- HL - result of HPF applied on columns and LPF on rows,
- HH - result of HPF applied on both, columns and rows.

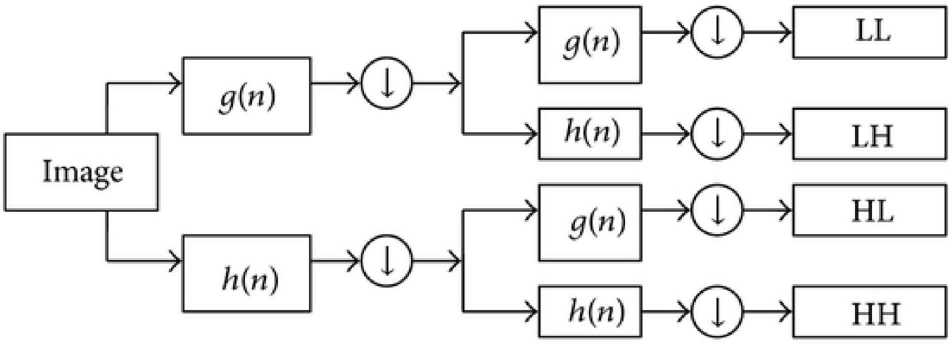

Figure 1.4: 2-D Discrete Wavelet transform on an image.

Now lets focus on what exactly each result represent. First one, the LL is just an approximation of the image. Next, the LH shows abrupt changes in a horizontal direction, whereas the HL part present similar issues but in a vertical direction. The HH shows sudden changes in a diagonal direction. In conclusion, the output of 2-D DWT gives us an approximation of the image and three parts with abrupt changes in different directions. What information gives as these sudden variations? Thanks to those we are able to find a places where two smooth regions meets. This kind of image anomaly could be interpreted as edges.

Chapter 2

*** Focus on the python algorithm and after processing - contrast etc. ***

Chapter 3

*** Results ***

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