Applications of Wavelets in Data Mining

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Motivation

Data Mining is a process of automatically extracting novel, useful, and understandable patterns from a large collection of data.

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- provide data presentations that enable efficient and accurate mining process,
- be incorporated at the kernel for many algorithms...
- especially, in edge detection.

The main goal - Edge detection

What is a wavelet?

The term **wavelet** means a **small wave**:

- wave - the function is oscillatory,
- smallness - the function is of finite length or compactly supported.

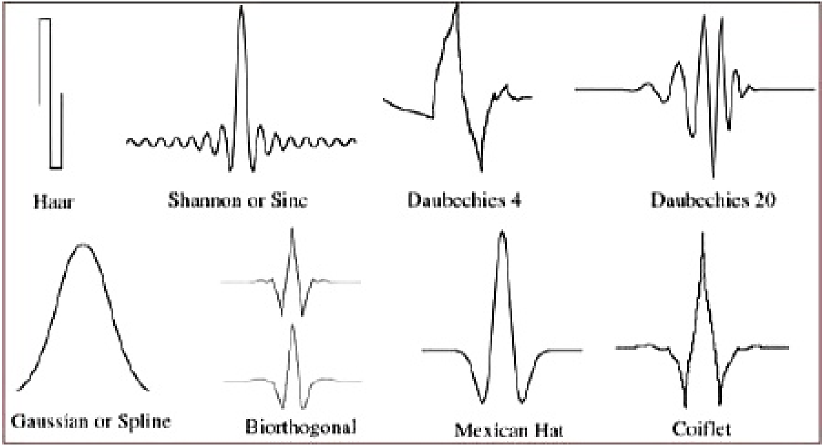
A **mother wavelet** is a function $\psi(x)$ such that

$$\{\psi(2^j x - k),\ i, k \in Z\}$$

is an orthonormal basis of $L^2(R)$.

The term mother implies that the functions with different regions of support that are used in the transformation process are derived by dilation and translation of the mother wavelet.

The most popular mother wavelets



Dilation Equation - How to find the wavelets?

The key idea is self-similarity. Start with a function $\phi(x)$ that is made up of smaller version of itself. This is the dilation equation

$$\phi(x) = \sum_{k=-\infty}^{\infty} a_k \phi(2x - k),$$

where a_k's are called filter coefficients or masks. The function $\phi(x)$ is called **the scaling function (or father wavelet)**.
Under certain conditions, the formula below gives a wavelet.

$$\psi(x) = \sum_{k=-\infty}^{\infty} (-1)^k a_k \phi(2x - k)$$

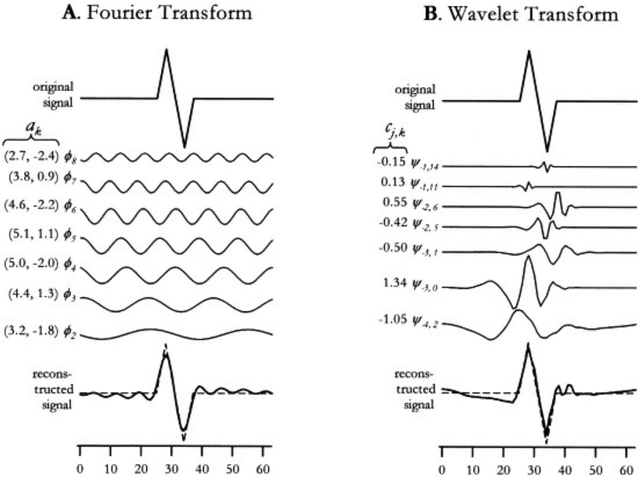
Wavelet transform

Why bother to have wavelets if they are very much the same as Fourier transforms except they have different bases?

Wavelet transform is capable of providing **time and frequency localizations simultaneously** while Fourier transforms could only provide frequency representations. Fourier transforms are designed for stationary signals because they are expanded as sine and cosine waves which extend in time forever.

Fourier transform is not suitable for non-stationary signal with time varying frequency.

Fourier transform vs Wavelet transform



Properties of Wavelets

- **Computational Complexity**: Fast wavelet transform only needs $O(N)$ multiplications. The space complexity is also linear. To compare, discrete Fourier transform requires $O(N^2)$ multiplications and fast Fourier transform also needs $O(Nlog(N))$ multiplications.

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- **Computational Complexity**: Fast wavelet transform only needs $O(N)$ multiplications. The space complexity is also linear. To compare, discrete Fourier transform requires $O(N^2)$ multiplications and fast Fourier transform also needs $O(Nlog(N))$ multiplications.
- **Vanishing Moments**: A function $f(x)$ which is supported in bounded region w is called to have n-vanishing moments if it satisfies the following equation: $\int_w f(x)x^j dx = 0, \ j = 0, 1, \ldots, n$. For example, Haar wavelet has 1-vanishing moment and $db2$ has 2-vanishing moment. The intuition of vanishing moments of wavelets is the **oscillatory nature** which can thought to be the characterization of difference or details between a neighborhood of the data.

Properties of Wavelets

- **Compact Support**: Each wavelet basis function is supported on a finite interval. Compact support guarantees the localization of wavelets - processing a region of data with wavelet does not affect the data out of this region.

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- There are also some other favorable properties of wavelets such as the **symmetry** of scaling and wavelet functions, **smoothness** and the availability of **many different wavelet basis functions**.

Edge detection

Edges can be considered as transients in a signal or mathematically defined as local singularities. In practice, edges are points in an image where brightness changes suddenly.

Edge detection refers to the process of identifying and locating sharp discontinuities in an image. In wavelet edge detection technique, the is used **Discrete Wavelet Transform** (DWT) and the filter is one which searches for the local maxima in a wavelet domain.

2-D Discrete Wavelet Transform

The 2D algorithm is based on separate variables leading to prioritizing of x and directions. The scaling function is defined by

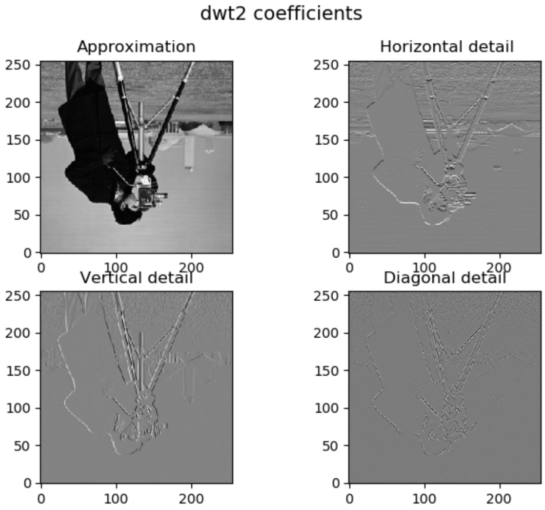
$$\phi(x, y) = \phi(x)\phi(y).$$

The detail signal is obtained from three wavelets:

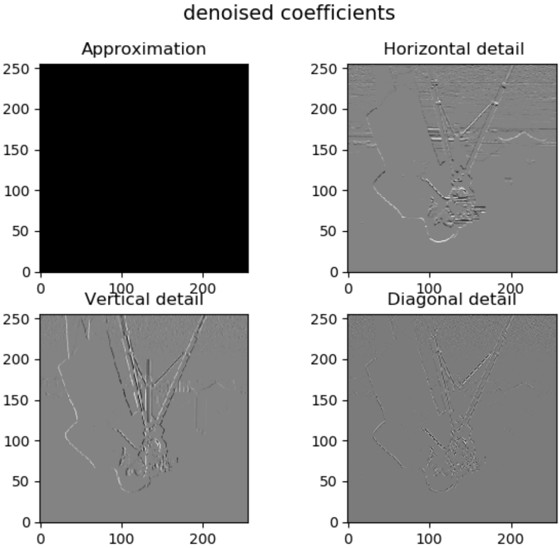
- a vertical wavelet: $\psi^1(x, y) = \phi(x)\psi(y)$,
- a horizontal wavelet: $\psi^2(x, y) = \psi(x)\phi(y)$,
- a diagonal wavelet: $\psi^3(x, y) = \psi(x)\psi(y)$.

which leads to three sub-images in each of the decomposition levels.

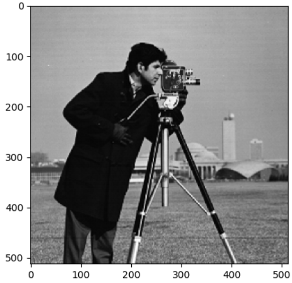
2-D DWT Coefficients



Thresholding



Inverse DWT



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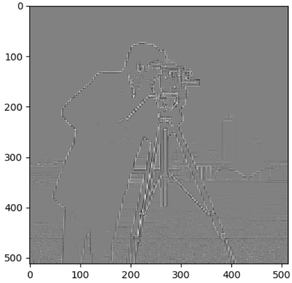

(b)

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