Politechnika Wrocławska

Faculty of Pure and Applied Mathematics
Field of study: Applied Mathematics
Specialty: Computational Mathematics

Master Thesis

APPLICATIONS OF WAVELETS IN DATA MINING

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keywords:
wavelets
statistics
learning and adaptive systems

Short summary:
The aim of this thesis is an analysis and comparison of statistical methods employing wavelets. Special attention will be paid to data mining and machine learning methods.

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- *a)* *Category A (perpetual files)*
- *b)* *Category BE 50 (subject to expertise after 50 years)*

** Delete as appropriate*

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Wrocław, 2018

Contents

Introduction

Few words about my thesis...
Generally what is a data mining, what are the wavelets, what is the main purpose - edge detection.

Chapter 1

A wavelet literally means a small wave. The term says a lot about wavelets nature. Wavelets are a family of functions which oscillates like wave and should be compactly supported. Additionally, the wavelet has zero mean, an amplitude begins at 0, increases and decreases to 0 again.

Definition 1.1. Wavelets are created by scaling and shifting of the, so called, mother wavelet $\psi(t)$. The child wavelets are defined as

$$\psi^{(a,b)}(t) = |a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right). \tag{1.1}$$

There are plenty of different mother wavelets, for example

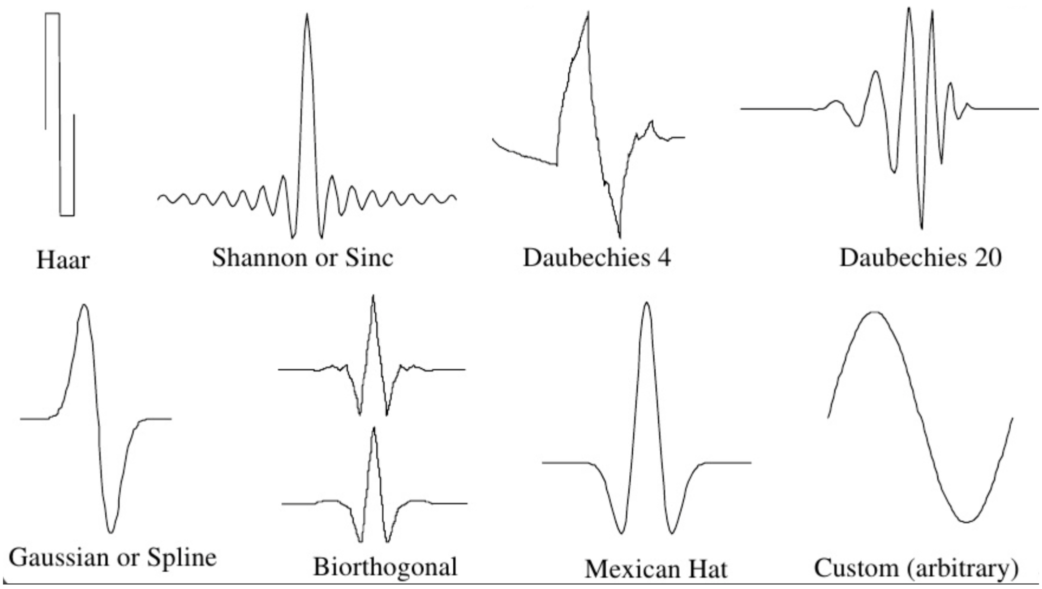

Figure 1.1: Different types of wavelets.

*** Add more about wavelets properties ***

1.1. Wavelet Transform

*** Add some short introduction ***

Definition 1.2. Wavelet transform

$$W(a,b) = \int_{-\infty}^{\infty} y(t)a^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right)dt, \tag{1.2}$$

Wavelet transform	Fourier transform
Suitable for stationary and non-stationary signals	Suitable for stationary signals
High time and frequency resolution	Zero time resolution and very high frequency resolution
Very suitable for studying the local behaviours of the signal	No suitable
Sine and cosine waves	Scaled and translated mother wavelets

where a is scale parameter, b translation parameter and $y(t)$ original signal.

1.1.1. Wavelet transform vs Fourier transform

Definition 1.3. Fourier transform

$$Y(f) = \int_{-\infty}^{\infty} y(t)e^{-i\omega t}dt, \tag{1.3}$$

where $y(t)$ is time domain signal and $Y(f)$ is frequency domain signal.

Wavelet transform is similar to the Fourier transform but with the different functions. In Fourier case there are sine and cosine functions, wherein wavelet transform uses wavelets.
Why use Wavelet transform? Sine function oscillates on the whole real axis, thus it cannot represent abrupt changes. On the other hand, the Wavelet transform is localized in space and time, so it can be used to detect sudden changes in signals and images. Moreover, wide range of wavelet functions is a main advantage of wavelet analysis.

1.2. Discrete Wavelet transform

There are two types of the wavelet transform:
— Continuous Wavelet Transform (CWT),
— Discrete Wavelet Transform (DWT).
DWT is used to denoising and compression of signals and images. Also, DWT allows to detect smooth regions interrupted by edges or abrupt changes in contrast of images.
Scale and translation parameters are defined as

$$a = 2^j \text{ and } b = 2^j k,\ \ j, k = 1, 2, \ldots. \tag{1.4}$$

to avoid redundancy in coefficients.
The figure 1.2 on a page 9 shows how DWT works. Discrete Wavelet Transform splits signal with two filters: $h(n)$ - high pass filter (HPF) and $g(n)$ - low pass filter (LPF). The HPF captures a part with bigger frequencies which is the main signal. Whereas, the LPF captures smaller frequencies - a noise of the signal. Subsequently, both parts are down sampled by a factor of 2.

This decomposition can be repeated on the HPF part of the signal. Hence, the next levels of DWT coefficients.

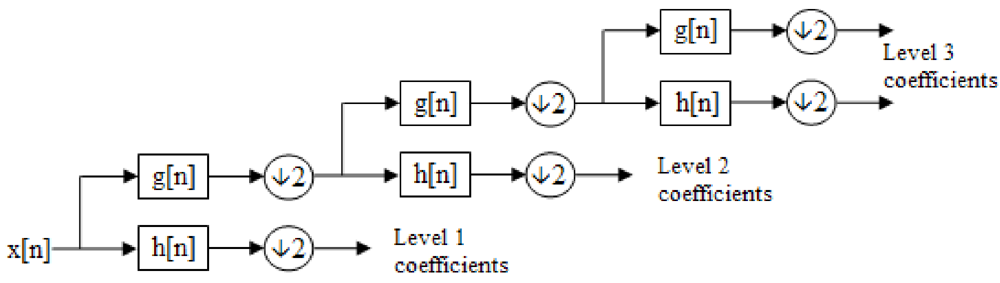

Figure 1.2: Discrete Wavelet transform on a signal $x(n)$.

1.3. 2-D Discrete Wavelet transform

*** Add introduction that 2D transform is for images and how it works ***
2-D Descrete Wavelet Transform works the same way as 1-D except that one level of the decomposition contains filtering columns and rows.

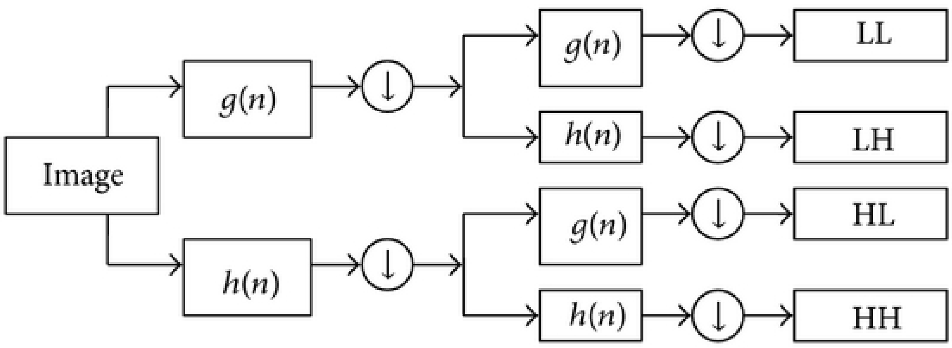

Figure 1.3: 2-D Discrete Wavelet transform on an image.

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Bibliography