Actor-Critic Methods

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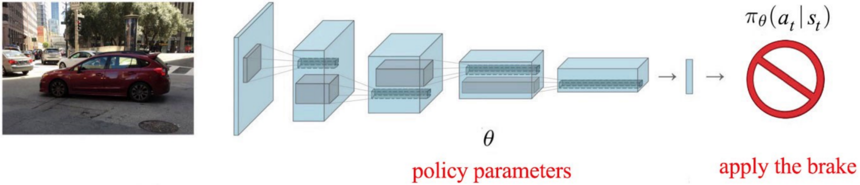
Recap: Parameterizing Policy

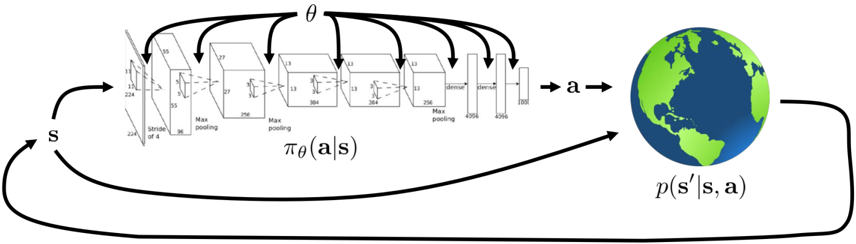


Q) How can we find a good $\pi(a|s)$, which is a **function**?

Idea:

- Parameterize policy by a parameter vector $\theta \in \mathbb{R}^\ell$: $\pi_\theta(a|s)$
- Find an optimal θ

Recap: How to find optimal parameters θ?



- Let $\tau := (s_0, a_0, \dots, s_T, a_T)$ denote the state-action trajectory
- By Markov property,

$$p_\theta(\tau) = p(s_0) \prod_{t=0}^{T} \pi_\theta(a_t|s_t) p(s_{t+1}|s_t, a_t)$$

- Approximate MDP problem:

$$\max_\theta \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\sum_t r(s_t, a_t) \right] =: J(\theta)$$

Recap: Policy Gradient Theorem & REINFORCE

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\left(\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) \right) \left(\sum_{t=0}^{T} r(s_t, a_t) \right) \right]$$

Initialize θ;

1. Sample $\{\tau^i\}_{i=1}^N := \{(s_0^i, a_0^i, \ldots, s_T^i, a_T^i)\}_{i=1}^N$ using the current policy $\pi_\theta(a_t|s_t)$
2. Estimate the gradient

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i|s_t^i) \right) \left(\sum_{t=0}^{T} r(s_t^i, a_t^i) \right)$$

3. Perform gradient ascent:

$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta);$$

Recap: Policy Gradient with Baselines

- Policy gradient

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) [Q^{\pi_\theta}(s_t^i, a_t^i) - b]$$

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- Good baseline: $b = v^{\pi_\theta}(s_t, \cancel{a_t})$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) A^{\pi_\theta}(s_t^i, a_t^i),$$

where $A^{\pi_\theta}(s_t^i, a_t^i) := Q^{\pi_\theta}(s_t^i, a_t^i) - v^{\pi_\theta}(s_t^i)$ is called the advantage function

How can we compute a good gradient?

Good gradient:

bias - variance trade - off 있음

1. Unbiased (Ok!)
2. Low variance (How?)

How can we compute a good gradient?

Good gradient:

1. Unbiased (Ok!)
2. Low variance (How?)
   - Use baseline $b = v^{\pi_\theta}(s_t, \cancel{a_t})$:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \underbrace{(Q^{\pi_\theta}(s_t^i, a_t^i) - v^{\pi_\theta}(s_t^i))}_{A^{\pi_\theta}(s_t^i, a_t^i)}$$

   - Need to accurately estimate v^π, Q^π or A^π (**Policy evaluation**)

Policy evaluation

Q) How to evaluate $v^\pi(s_t)$?

Policy evaluation

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- Monte Carlo:

$$v^\pi(s_t) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t'=t}^{T} r_{t'}^{i}$$

Policy evaluation with function approximation

Want to fit the value function by

$$v^\pi(s_t) \approx v^\pi_\phi(s_t) \quad \phi : \text{parameter vector}$$

Q) What are the training data?

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- $\{(\underbrace{s_t^i}_{s^i}, \underbrace{\sum_{t'=t}^{T} r_{t'}^i}_{y^i})\} =: \{(s^i, y^i)\}$

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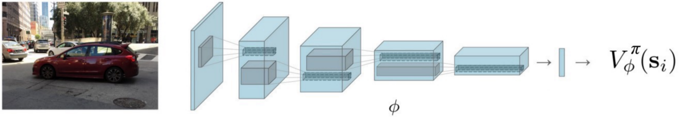
- (supervised) regression:

$$\min_{\phi} \mathcal{L}(\phi) := \frac{1}{2} \sum_i \|v^{\pi}_{\phi}(s_i) - y_i\|^2$$



Better version?

Recap) Monte Carlo target: $y_i := \sum_{t'=t}^{T} r_{t'}^i$

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$$y_i := r(s_t^i, a_t^i) + \underbrace{v_\phi^\pi(s_{t+1}^i)}_{\text{previous fitted value ftn}}$$

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- Training data:
  - $\{(\underbrace{s_t^i}_{s^i}, \underbrace{r_t^i + v_\phi^\pi(s_{t+1}^i)}_{y^i})\} =: \{(s^i, y^i)\}$

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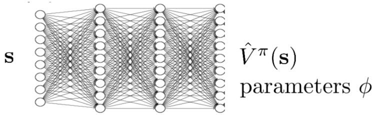
- Regression:

$$\min_\phi \mathcal{L}(\phi) := \frac{1}{2}\sum_i \|v_\phi^\pi(s_i) - y_i\|^2$$



Actor-Critic algorithm

Batch version:

1. Sample $\{(s_t^i, a_t^i, s_{t+1}^i, r_t^i)\}$ using $\pi_\theta(a|s)$;
2. Fit $v_\phi^\pi(s)$ by solving the regression problem $\min_\phi \; \mathcal{L}(\phi) := \frac{1}{2}\sum_i \|v_\phi^\pi(s_i) - y_i\|^2$;
3. Evaluate Advantage $A^\pi(s_t^i, a_t^i) = \underbrace{r_t^i + v_\phi^\pi(s_{t+1}^i)}_{Q} - v_\phi^\pi(s_t^i)$;
4. Estimate SG $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(a_t^i|s_t^i) A^\pi(s_t^i, a_t^i)$;
5. Update $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$;

평가

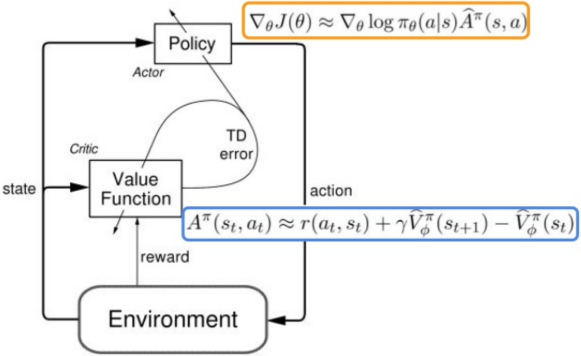
업데
이트

Note:

- Critic: Step 2, 3
- Actor: Step 4, 5

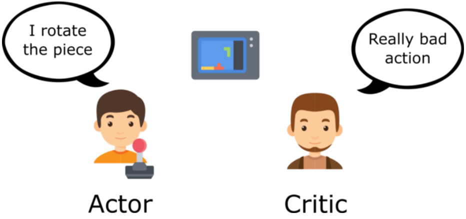
PI의 deep neural 버전

Intuition behind actor-critic



$$\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(a|s) \widehat{A}^\pi(s,a)$$

$$A^\pi(s_t, a_t) \approx r(a_t, s_t) + \gamma \widehat{V}^\pi_\phi(s_{t+1}) - \widehat{V}^\pi_\phi(s_t)$$



Actor-Critic algorithm with discount factor

With discount factor γ:

1. Sample $\{(s_t^i, a_t^i, s_{t+1}^i, r_t^i)\}$ using $\pi_\theta(a|s)$;
2. Fit $v_\phi^\pi(s)$ by solving the regression problem $\min_\phi \ \mathcal{L}(\phi) := \frac{1}{2}\sum_i \|v_\phi^\pi(s_i) - y_i\|^2$;
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5. Update $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$;

Online Actor-Critic algorithm

Fully incremental online version:

1. Take action $a \sim \pi_\theta(a|s)$, and observe (s, a, s', r);
2. Fit $v_\phi^\pi(s)$ using target $r + \gamma v_\phi^\pi(s')$;
3. Estimate Advantage $A^\pi(s, a) = r + \gamma v_\phi^\pi(s') - v_\phi^\pi(s)$;
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- On-policy: sample inefficient

Off-Policy Actor-Critic

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Key idea: Use behavior policy $\beta(a|s) \neq \pi_\theta(a|s)$

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- Changes in objective: $J(\theta) \to J_\beta(\theta)$, where

$$J_\beta(\theta) := \mathbb{E}_{\tau \sim p^\beta}\left[\sum_t r(s_t, a_t)\right] = \int p^\beta(\tau) r(\tau) d\tau$$

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$$J_\beta(\theta) := \mathbb{E}_{\tau \sim p^\beta}\left[\sum_t r(s_t, a_t)\right] = \int p^\beta(\tau) r(\tau) d\tau$$

- Approximate gradient:

$$\nabla_\theta J_\beta(\theta) \approx \mathbb{E}_{\tau \sim p^\beta}\left[\frac{\pi_\theta(a|s)}{\beta(a|s)} \nabla_\theta \log \pi_\theta(a|s) r(\tau)\right]$$

Advantages and Disadvantages

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- Lower variance (thanks to critic)
- (Often) Fast and stable convergence

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Disadvantages:

- Not unbiased (because the critic is not perfect)
- Training two networks required (for actor and critic)

Will learn actor-critic deep RL methods

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HIGH-DIMENSIONAL CONTINUOUS CONTROL USING GENERALIZED ADVANTAGE ESTIMATION

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