Soft Actor-Critic (SAC)

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Review: Max-entropy stochastic control

$$\max_{\pi} \ \mathbb{E}^{\pi}\left[\sum_t [r(s_t, u_t) + \alpha \underbrace{H(\pi_t(\cdot|s_t))}_{\text{entropy of } \pi}]\right]$$

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- Bellman equation:

$$\begin{aligned} Q^*(s,a) &= r(s,a) + \gamma \mathbb{E}_{s'}[V^*(s')] \\ V^*(s) &= \alpha \log \int \exp\left(\frac{1}{\alpha} Q^*(s,a)\right) da \end{aligned}$$

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$$V^*(s) = \alpha \log \int \exp\left(\frac{1}{\alpha} Q^*(s,a)\right) da$$

- Optimal policy:

$$\pi^*(a|s) = \exp\left(\frac{1}{\alpha}[Q^*(s,a) - V^*(s)]\right)$$

Desired features in RL

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Q) Can we use max-entropy method in RL to have all the desired features?

Soft Actor-Critic (SAC)

Soft Actor-Critic:
Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor

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Idea:

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Advantages:

- Exploration
- Sample efficiency
- Stable convergence
- Little hyperparameter tuning

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- It's just a max-entropy variant of standard PI

Soft policy evaluation

- Modified Bellman operator:

$$T^\pi Q(s,a) := r(s,a) + \gamma \mathbb{E}_{s'}[V(s')],$$

where $V(s) := \mathbb{E}_{a\sim\pi(\cdot|s)}\big[Q(s,a) \quad \underbrace{-\log\pi(a|s)}_{\text{takes into account entropy}} \quad \big]$

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- **Result: Q_k converges to Q^π!**

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- If no constraint on policies,

$$\pi_{new}(a|s) := \exp\left(\frac{1}{\alpha}[Q^{\pi_{old}}(s,a) - V^{\pi_{old}}(s)]\right)$$

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Information projection:

$$\pi_{new}(\cdot|s) \in \underset{\pi'(\cdot|s)\in\Pi}{\arg\min}\, D_{KL}\left(\pi'(\cdot|s) \parallel \underbrace{\exp\left(\frac{1}{\alpha}[Q^{\pi_{old}}(s,\cdot) - V^{\pi_{old}}(s)]\right)}_{\text{target policy}}\right)$$

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- **Result: Monotonic improvement on policy!**
  (π_{new} better than π_{old})

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  - Soft actor:
    improves max-entropy policy using critic's evaluation

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$$\min_\phi \; J_Q(\phi) := \mathbb{E}_{(s_t,a_t)}\left[\frac{1}{2}\left(Q_\phi(s_t,a_t) - \underbrace{\left[r(s_t,a_t) + \gamma \mathbb{E}_{s_{t+1}}[V_{\phi^-}(s_{t+1})]\right]}_{=: y_t^- \text{ target}}\right)^2\right]$$

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- Stochastic gradient:

$$\hat{\nabla}_\phi \ J_Q(\phi) = \nabla_\phi Q_\phi(s_t,a_t) \times$$
$$\Big[Q_\phi(s_t,a_t) - [r(s_t,a_t) + \gamma \underbrace{\left(Q_{\phi^-}(s_{t+1},a_{t+1}) - \alpha \log(\pi_\theta(a_{t+1}|s_{t+1})\right)}_{\text{sample estimate of } \mathbb{E}_{s_{t+1}}[V_{\phi^-}(s_{t+1})]}\Big]$$

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Idea: Policy gradient + Max-entropy:

- Minimizing the expected KL-divergence:

$$\min_{\theta} D_{KL}\left(\pi_\theta(\cdot|s_t) \parallel \underbrace{\exp\left(\frac{1}{\alpha}[Q_\phi(s_t,\cdot) - V_\phi(s_t)]\right)}_{\text{target policy}}\right),$$

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  which is equivalent to

$$\min_\theta \; J_\pi(\theta) := \mathbb{E}_{s_t}\left[\mathbb{E}_{a_t\sim\pi_\theta(\cdot|s_t)}[\alpha \log \pi_\theta(a_t|s_t) - Q_\phi(s_t, a_t)]\right]$$

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- Reparameterize the policy as

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- Rewrite the policy optimization problem as

$$\min_\theta \; J_\pi(\theta) := \mathbb{E}_{s_t,\epsilon_t}\left[\alpha \log \pi_\theta(f_\theta(\epsilon_t; s_t)|s_t) - Q_\phi(s_t, f_\theta(\epsilon_t; s_t))\right]$$

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- Stochastic gradient:

$$\begin{aligned}\hat{\nabla}_\theta J_\pi(\theta) &= \nabla_\theta \alpha \log \pi_\theta(a_t|s_t) \\ &\quad + [\nabla_a \alpha \log \pi_\phi(a_t|s_t) - \nabla_a Q_\phi(s_t, a_t)]\nabla_\theta f_\theta(\epsilon_t; s_t),\end{aligned}$$

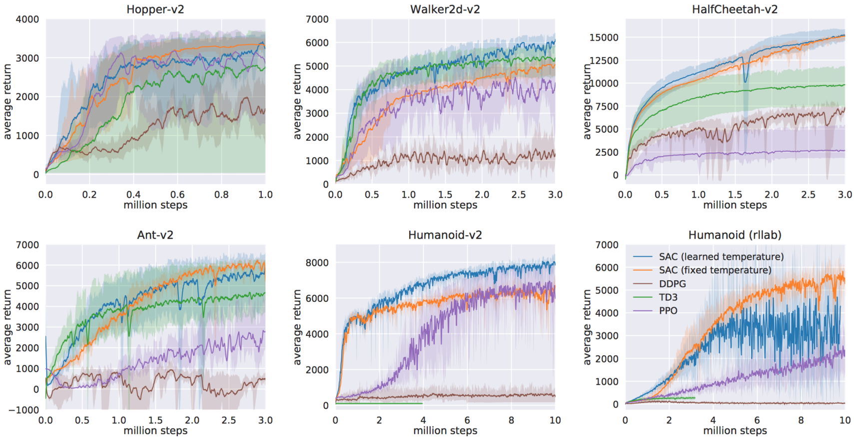
where a_t is evaluated at $f_\theta(\epsilon_t; s_t)$

Putting everything together: Soft Actor-Critic (SAC)

Algorithm 1 Soft Actor-Critic

Input: θ_1, θ_2, ϕ ▷ Initial parameters
  $\bar{\theta}_1 \leftarrow \theta_1, \bar{\theta}_2 \leftarrow \theta_2$ ▷ Initialize target network weights
  $\mathcal{D} \leftarrow \emptyset$ ▷ Initialize an empty replay pool
  for each iteration **do**
    for each environment step **do**
      $\mathbf{a}_t \sim \pi_\phi(\mathbf{a}_t|\mathbf{s}_t)$ ▷ Sample action from the policy
      $\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$ ▷ Sample transition from the environment
      $\mathcal{D} \leftarrow \mathcal{D} \cup \{(\mathbf{s}_t, \mathbf{a}_t, r(\mathbf{s}_t, \mathbf{a}_t), \mathbf{s}_{t+1})\}$ ▷ Store the transition in the replay pool
    end for
    for each gradient step **do**
      $\theta_i \leftarrow \theta_i - \lambda_Q \hat{\nabla}_{\theta_i} J_Q(\theta_i)$ for $i \in \{1, 2\}$ ▷ Update the Q-function parameters
      $\phi \leftarrow \phi - \lambda_\pi \hat{\nabla}_\phi J_\pi(\phi)$ ▷ Update policy weights
      $\alpha \leftarrow \alpha - \lambda \hat{\nabla}_\alpha J(\alpha)$ ▷ Adjust temperature
      $\bar{\theta}_i \leftarrow \tau \theta_i + (1 - \tau)\bar{\theta}_i$ for $i \in \{1, 2\}$ ▷ Update target network weights
    end for
  end for
Output: θ_1, θ_2, ϕ ▷ Optimized parameters

Results



- Soft actor-critic (blue and yellow) performs **consistently** across all tasks and **outperforming** both on-policy and off-policy methods in the most challenging tasks.

Dynamixel Claw task from vision

- The robot must rotate the valve so that the colored peg faces the right.
- The video embedded in the bottom right corner shows the frames as seen by the policy

Testing robustness of the learned policy against visual perturbations

- The robot must rotate the valve so that the colored peg faces the right.

Train the Minitaure robot to walk in 2 hours

Which algorithm should I use?

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- Guideline from Sergey Levine (UC Berkeley)