Maximum Entropy Stochastic Control and Reinforcement Learning

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So far...

- DQN, Double-DQN
- Policy gradient
- Actor-critic
- DDPG
- TRPO

Common issue: exploration

- Is there a value of exploring unknown regions of the environment?
- Which action should we try to explore unknown regions of the environment?

Tradeoff between exploration and exploitation

- Exploitation: Make the best decision given current information
- Exploration: Gather more information

Q) Why dilemma?

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Q) Why dilemma?

- The best long-term strategy may involve short-term sacrifices
- Need to gather enough information to make the best overall decisions

Examples

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4. Game Playing
   - Exploitation: Play the move you believe is best
   - Exploration: Play an experimental move

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  5. Information theoretic exploration

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Disadvantages:

1. Fine tuning ϵ
2. Not quite systematic: No focus on unexplored regions
3. Inefficient

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2. Computation intensive

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Method IV: Noisy networks

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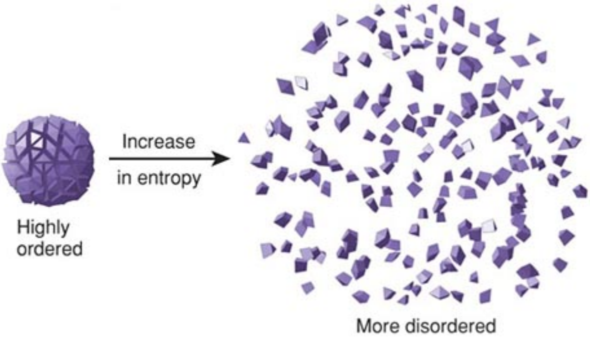
- More generally, entropy refers to *disorder*



What's the benefit of high entropy policy?

Entropy of policy:

$$H(\pi(\cdot|s)) := -\sum_{a} \pi(a|s) \log \pi(a|s) = \mathbb{E}_{a\sim\pi(\cdot|s)}[-\log \pi(a|s)]$$

- Higher disorder in π
- Try new risky behaviors: Potentially explore unexplored regions

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- Maximum entropy MDP problem:

$$\max_{\pi} \; \mathbb{E}^{\pi}\left[\sum_t [r(s_t, u_t) + \alpha H(\pi_t(\cdot|s_t))]\right],$$

where α is called the temperature

Soft value functions

- Soft Q-function of policy π:

$$Q^\pi(s_t, a_t) := r(s_t, a_t) + \mathbb{E}^\pi\left[\sum_{l=1}^{\infty} \gamma^l [r(s_{t+l}, a_{t+l}) + \alpha H(\pi(\cdot|s_{t+l}))]\right]$$

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- Optimal value functions:

$$Q^*(s_t, a_t) := \max_\pi Q^\pi(s_t, a_t)$$
$$V^*(s_t) := \alpha \log \int \exp\left(\frac{1}{\alpha} Q^*(s_t, a)\right) da$$

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$$\pi_{new}(a|s) := \exp\left(\frac{1}{\alpha}[Q^{\pi_{old}}(s,a) - V^{\pi_{old}}(s)]\right)$$

- Then, we have

$$Q^{\pi_{old}}(s,a) \leq Q^{\pi_{new}}(s,a) \quad \forall (s,a)$$

$\implies$ **Monotonic policy improvement**

What is the optimal policy?

Idea: Keep updating using the soft policy improvement theorem

$$\pi_{old}(a|s) \to \pi_{new}(a|s) := \exp\left(\frac{1}{\alpha}[Q^{\pi_{old}}(s,a) - V^{\pi_{old}}(s)]\right)$$

Result:

- Optimal policy

$$\pi^*(a|s) = \exp\left(\frac{1}{\alpha}[Q^*(s,a) - V^*(s)]\right)$$

Soft Bellman equation

$$
\begin{aligned}
Q^*(s,a) &= r(s,a) + \gamma \mathbb{E}_{s'}[V^*(s')] \\
&= r(s,a) + \gamma \sum_{s'} p(s'|s,a) V^*(s')
\end{aligned}
$$

Q) Looks familiar?

Comparison to standard MDP: Bellman equation

- Standard:

$$Q^*(s,a) = r(s,a) + \gamma \mathbb{E}_{s'}[V^*(s')]$$
$$V^*(s) = \max_a Q^*(s,a)$$

- Max-Entropy:

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$$\alpha \log \int \exp\left(\frac{1}{\alpha} Q^*(s,a)\right) da \to \max_a Q^*(s,a) \quad \text{as } \alpha \to 0$$

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- Max-Entropy:

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$\Longrightarrow$ stochastic policy

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- Exploration:
  High entropy policy
- Structural similarity:
  Can combine it with many RL methods for standard MDP