Deep Deterministic Policy Gradient (DDPG)

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Stochastic vs. Deterministic Policy

- Stochastic (randomized) policy:

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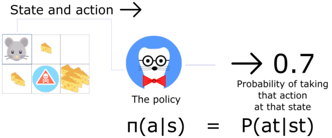

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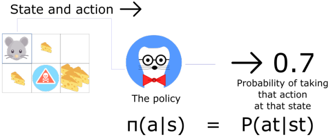

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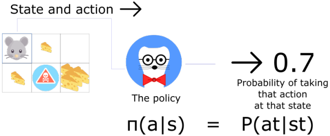

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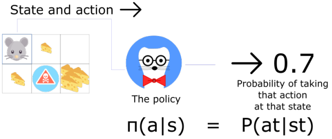

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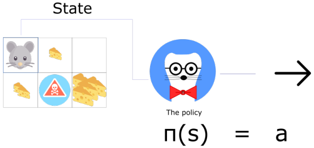

Deterministic policy

So far we considered stochastic policy gradient

Stochastic policy gradient:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_i \left(\sum_{t=0}^{T} \nabla_\theta \log \underbrace{\pi_\theta(a_t^i | s_t^i)}_{\text{stochastic policy}} \right) \left(\sum_{t=0}^{T} r(s_t^i, a_t^i) \right)$$

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(Online) Actor-critic algorithm:

1. Take action $a \sim \pi_\theta(a|s)$, and observe (s, a, s', r);
2. Fit $v_\phi^\pi(s)$ using target $r + \gamma v_\phi^\pi(s')$;
3. Evaluate Advantage $A^\pi(s, a) = r + \gamma v_\phi^\pi(s') - v_\phi^\pi(s)$;
4. Estimate SG $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(a|s) A^\pi(s, a)$;
5. Update $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$;

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- Yes, Deterministic policy gradient (DPG)

Deterministic Policy Gradient Algorithms

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$$\theta \leftarrow \theta + \alpha \mathbb{E}_{s \sim \rho^{\mu^k}} \left[\nabla_\theta Q^{\mu^k}(s, \mu_\theta(s)) \right],$$

where

$$\rho^{\mu}(s') := \int \sum_{t=1}^{\infty} \gamma^t p_0(s) p(s \to s', t, \mu) ds$$

denotes the discounted state visitation distribution

Applying chain rule

$$\theta \leftarrow \theta + \alpha \mathbb{E}_{s\sim\rho^{\mu^k}}\left[\nabla_\theta Q^{\mu^k}(s, \underbrace{\mu_\theta(s)}_{\text{deterministic policy}})\right]$$

$$= \theta + \alpha \mathbb{E}_{s\sim\rho^{\mu^k}}\left[\underbrace{\nabla_\theta \mu_\theta(s) \nabla_a Q^{\mu^k}(s,a)|_{a=\mu_\theta(s)}}_{\text{chain rule}}\right]$$

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Q) Does it work? Is it valid?

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- Deterministic policy gradient:

$$\nabla_\theta J(\theta) = \mathbb{E}_{s\sim\rho^{\mu}}\left[\nabla_\theta \mu_\theta(s) \nabla_a Q^\mu(s,a)|_{a=\mu_\theta(s)}\right]$$

Deterministic Actor-Critic algorithm

Initialize $\underbrace{\theta}_{\text{actor net}}$, $\underbrace{\phi}_{\text{critic net}}$;

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2. Fit $Q^\mu_\phi(s, a)$ using target $r + \gamma Q^\mu_\phi(s', \mu_\theta(s'))$;
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- On-policy: sample inefficient

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- Approximation (drop the second term):

$$\begin{aligned} \nabla_\theta J_\beta(\theta) &\approx \int \rho^\beta(s) \nabla_\theta \mu_\theta(s) \nabla_a Q^{\mu_\theta}(s,a)|_{a=\mu_\theta(s)} ds \\ &= \mathbb{E}_{s \sim \rho^\beta} \big[\nabla_\theta \mu_\theta(s) \nabla_a Q^{\mu_\theta}(s,a)|_{a=\mu_\theta(s)} \big] \end{aligned}$$

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Initialize θ, ϕ;

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- Advantage: Sample efficiency
- Disadvantage: Bias

Can we combine it with DQN?

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Why not? Deep deterministic policy gradient (DDPG)

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CONTINUOUS CONTROL WITH DEEP REINFORCEMENT LEARNING

Timothy P. Lillicrap,* Jonathan J. Hunt,* Alexander Pritzel, Nicolas Heess,
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ABSTRACT

We adapt the ideas underlying the success of Deep Q-Learning to the continuous action domain. We present an actor-critic, model-free algorithm based on the deterministic policy gradient that can operate over continuous action spaces. Using the same learning algorithm, network architecture and hyper-parameters, our algorithm robustly solves more than 20 simulated physics tasks, including classic problems such as cartpole swing-up, dexterous manipulation, legged locomotion and car driving. Our algorithm is able to find policies whose performance is competitive with those found by a planning algorithm with full access to the dynamics of the domain and its derivatives. We further demonstrate that for many of the tasks the algorithm can learn policies "end-to-end": directly from raw pixel inputs.

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- Actor: Deterministic policy gradient
  1. Simple
  2. Continuous control

Best of Both Worlds

- Actor: Deterministic policy gradient
  1. Simple
  2. Continuous control
- Critic: DQN
  1. Off-policy (sample efficient)
  2. Experience replay (minimize correlations between samples)
  3. Target network (consistency)

Why not stochastic policy gradient + DQN?

- DQN is computationally inefficient to use with stochastic policies

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  When computing target, it requires integral over action:

$$y_j^- := r_j + \gamma \int \pi_\theta(a|s'_j) Q^\pi_{\phi^-}(s'_j, a) da$$

DDPG algorithm

- Initialize critic network Q_ϕ and actor network μ_θ with ϕ and θ;
- Initialize target networks Q_{ϕ^-}, μ_{θ^-} with $\phi^- \leftarrow \phi$ and $\theta^- \leftarrow \theta$;
- for episode $= 1 : M$
  - Initialize a random process $\mathcal{N}$ for exploration;
  - Receive initial state s_0;
  - for $t = 1 : T$
    1. Execute action $a_t = \mu_\theta(s_t) + \mathcal{N}_t$ and store (s_t, a_t, s_{t+1}, r_t) in **Buffer**;
    2. Sample a minibatch $\{(s_i, a_i, s_{i+1}, r_i)\}$ from **Buffer**;
    3. Set target $y_i^- := r_i + \gamma Q_{\phi^-}(s_{i+1}, \mu_{\theta^-}(s_{i+1}))$;
    4. Update the critic network by minimizing $L(\phi) := \frac{1}{N}\sum_i (Q_\phi(s_i, a_i) - y_i)^2$;
    5. Update the actor network by using deterministic policy gradient: $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_i \nabla_\theta \mu_\theta(s_i) \nabla_a Q_\phi(s_i, a)|_{a=\mu_\theta(s_i)}$
    6. Update the target networks: $\phi^- \leftarrow \tau\phi + (1-\tau)\phi^-$, and $\theta^- \leftarrow \tau\theta + (1-\tau)\theta^-$ with small τ;

Advantages and Disadvantages

Advantages:

- Can handle continuous spaces (policy gradient)
- Sample efficiency (off-policy)
- Minimize correlations between samples (experience replay)
- Consistency (slowly changing target networks)

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Disadvantages:

- Exploration

DDPG results