Department of Mathematics and Systems Analysis MS-C1300 — Complex Analysis, 2025-2026/II

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Exercise sessions: Tue-Wed, Oct 28-29, 2025

Topic: complex differentiability, functions as mappings

The first three exercises are to be discussed and solved in exercise sessions. The last exercise (marked with symbol \triangle) is a quiz for which you will write a solution in an exam-like setup (no materials or extra equipment allowed) during the last 20min of the exercise session.

Exercise (in class) 1.

Consider the function $f: \mathbb{C} \to \mathbb{C}$ defined by

$$f(x+\mathrm{i}y) = \begin{cases} \frac{xy(x+\mathrm{i}y)}{x^2+y^2} & \text{for } x,y \in \mathbb{R} \text{ such that } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0). \end{cases}$$

- (a) Is f continuous at the origin?
- (b) Do the Cauchy-Riemann equations hold for f at the origin?
- (c) Is f complex-differentiable at the origin?
- (d) Is f analytic at the origin?

Exercise (in class) 2.

Let $A \subset \mathbb{C}$ be the part of the plane that lies to the right of the hyperbola $x^2 - y^2 = 1$. Show that the function f defined by the formula

$$f(z) = z^2$$

maps the region A bijectively onto the half-plane $R = \{w \in \mathbb{C} \mid \Re \mathfrak{e}(w) > 1\}$. Sketch the curves in A which map onto the vertical lines $\Re \mathfrak{e}(w) = \text{const.}$, and the curves which map onto horizontal half-lines $\Im \mathfrak{m}(w) = \text{const.}$, $\Re \mathfrak{e}(w) > 1$.

Exercise (in class) 3.

Define $f: \mathbb{C} \to \mathbb{C}$ by

$$f(x + iy) = 3 - 2y + x^2 + i(2x + 4xy + 2y^2 + x^2)$$
 for $x, y \in \mathbb{R}$.

Locate all points z = x + iy at which f is complex-differentiable, and determine the complex derivative f'(z) for each such point.

💪 Quiz 4.

Consider the disk

$$D = \left\{ z \in \mathbb{C} \; \middle| \; |z - \mathbf{i}| < 1 \right\}.$$

Show that the function f defined by the formula

$$f(z) = \frac{-2}{z}$$

maps the disk D bijectively onto the half-plane $U=\big\{w\in\mathbb{C}\,\big|\,\,\Im\mathfrak{m}(w)>1\big\}.$