Aalto University   **Problem set 5B**
Department of Mathematics and Systems Analysis
MS-C1300 — Complex Analysis, 2024-2025/II   K Kytölä & A Vavilov

Exercise sessions: 28.-29.11.   **Hand-in due: Sun 1.12.2024 at 23:59**

Topic: isolated singularities, residue calculus

The first two exercises are to be discussed and solved in exercise sessions. The last two exercises are homework (marked with symbol ✍): written solutions to them are to be returned in MyCourses. Each exercise is graded on a scale 0–3. The deadline for returning solutions to problem set 5B is Sun 1.12.2024 at 23:59.

Exercise (in class) 1.
Calculate

$$\oint_{\partial \mathcal{B}(e;2)} \frac{1}{(z-1)\operatorname{Log}(z)}\,\mathrm{d}z,$$

where the circle $\partial \mathcal{B}(e;2)$ of radius 2 around $e = \exp(1) \in \mathbb{C}$ is positively oriented (counterclockwise), and Log is the principal branch of the complex logarithm function.

Exercise (in class) 2.
Calculate the real integral

$$\int_{-\infty}^{\infty} \frac{1}{x^4+1}\,\mathrm{d}x \;:=\; \lim_{R\to\infty} \int_{-R}^{R} \frac{1}{x^4+1}\,\mathrm{d}x\,.$$

using residue calculus.

<u>Hint</u>: *Figuring out a suitable analytic function to contour integrate is the first step. A suitable contour here is the boundary of a semi-disk of a large radius R. One should analyze the contributions to the contour integral from the different parts of it.*

✏ **Homework exercise 3.**
Calculate the integral

$$\oint_\gamma \frac{e^z}{z^4 + 5z^3} \, dz,$$

where γ parametrizes the circle $\partial \mathcal{B}(0; 3)$ of radius 3 around the origin in the *negative(!)* orientation (*clockwise*).

✏ **Homework exercise 4.**
Let $a > 0$. Calculate the real integral

$$\int_0^\infty \frac{\cos(ax)}{1 + x^2} \, dx \; := \; \lim_{R \to \infty} \int_0^R \frac{\cos(ax)}{1 + x^2} \, dx \, .$$

Hint: *A suitable contour here is again the boundary of a large semi-disk, and one should find a suitable analytic function to integrate. Furthermore, one needs to take into account also symmetry considerations to get to the desired result.*