Aalto University **Problem set 6B**
Department of Mathematics and Systems Analysis
MS-C1300 — Complex Analysis, 2025-2026/II K Kytölä

Exercise sessions: Thu-Fri, Nov 27-28, 2025

Topic: isolated singularities, residue calculus

The first two exercises (marked with symbol ♮) are to be solved before the exercise sessions so that you are ready to present your solution on the blackboard. The last two exercises are to be discussed and solved in exercise sessions.

♮ Blackboard exercise 1.

Let $a > 0$. Calculate the real integral

$$\int_0^\infty \frac{\cos(ax)}{1+x^2}\,\mathrm{d}x \;:=\; \lim_{R\to\infty}\int_0^R \frac{\cos(ax)}{1+x^2}\,\mathrm{d}x\,.$$

Hint: *To reduce the calculation of this real integral to a complex contour integral, a suitable contour is the boundary of a large semi-disk. One should choose a suitable analytic function to integrate, and the choice may not be the obvious-looking one. Furthermore, one needs to take into account also symmetry considerations to get to the desired result.*

♮ Blackboard exercise 2.

Calculate

$$\oint_{\partial\mathcal{B}(e;2)} \frac{1}{(z-1)\,\mathrm{Log}(z)}\,\mathrm{d}z,$$

where the circle $\partial\mathcal{B}\,(e;2)$ of radius 2 around $e = \exp(1) \in \mathbb{C}$ is positively oriented (counterclockwise), and Log is the principal branch of the complex logarithm function.

Exercise (in class) 3.
Calculate the real integral

$$\int_0^{2\pi} \frac{1}{1 + 8\,\sin^2(t)}\,\mathrm{d}t.$$

Hint: *Parametrize the unit circle as usual by the formula* $\gamma(t) = \mathrm{e}^{\mathrm{i}t}$ *and consider the contour integral* $\oint f(z)\,\mathrm{d}z$ *of a suitably chosen rational function* $f(z)$ *along the unit circle.*

Exercise (in class) 4.
Calculate the real integral

$$\int_{-\infty}^{\infty} \frac{1}{x^4+1}\,\mathrm{d}x \;:=\; \lim_{R\to\infty} \int_{-R}^{R} \frac{1}{x^4+1}\,\mathrm{d}x\,.$$

using residue calculus.
Hint: *Figuring out a suitable analytic function to contour integrate is the first step. A suitable contour here is the boundary of a semi-disk of a large radius* R. *One should analyze the contributions to the contour integral from the different parts of it.*