Aalto University
Department of Mathematics and Systems Analysis
MS-C1300 — Complex Analysis, 2025-2026/II

Problem set 1A

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Exercise sessions: Tue-Wed, Oct 21-22, 2025

Topic: complex number calculations and geometric interpretation

The first three exercises are to be discussed and solved in exercise sessions. The last exercise (marked with symbol ✍) is a quiz for which you will write a solution in an exam-like setup (no materials or extra equipment allowed) during the last 20min of the exercise session.

Exercise 1.
Calculate:

(a) all complex square roots of $-1 + \mathrm{i}\sqrt{3}$;
(b) all complex cube roots of -8 .

Express the answers in Cartesian coordinates (i.e., as $z = x + \mathrm{i}y$ with $x, y \in \mathbb{R}$) and in polar coordinates (i.e., as $z = r\,e^{\mathrm{i}\theta} = r\big(\cos(\theta) + \mathrm{i}\,\sin(\theta)\big)$ with $r \geq 0$ and $\theta \in \mathbb{R}$).

Exercise 2.

(a) Let $z \in \mathbb{C} \setminus \{1\}$ and let $n \in \mathbb{N}$. Prove that

$$\sum_{j=0}^{n} z^j \;=\; \frac{1 - z^{n+1}}{1 - z} \;.$$

(b) Let $n \in \mathbb{N}$, $n \geq 2$. Find all solutions $z \in \mathbb{C}$ to the equation

$$z^{n-1} + z^{n-2} + \cdots + z + 1 \;=\; 0 \;.$$

Exercise 3.
Show geometrically (recommended) or by a calculation (also possible) that if $z \in \mathbb{C}$ is such that $|z| = 1$ and $z \neq -1$, then

$$\mathfrak{Im}\left(\frac{z}{(z+1)^2}\right) \;=\; 0 \;.$$

Which other points $z \in \mathbb{C}$ satisfy the above equation?

✍ **Quiz 4.**
Visualize and describe in geometric terms the following subsets of the complex plane:

(a)
$$A = \left\{ z \in \mathbb{C} \;\middle|\; |z - 3\,\mathbb{i}| < 2 \right\} ;$$

(b)
$$B = \left\{ z \in \mathbb{C} \;\middle|\; |z + 1| = |z + \mathbb{i}| \right\} ;$$

(c)
$$C = \left\{ z \in \mathbb{C} \;\middle|\; |z| = 2\,|z - 1| \right\} ;$$

(d)
$$D = \left\{ z \in \mathbb{C} \;\middle|\; z^2 = \overline{z}^2 \right\} .$$

Justify concisely the main features of your descriptions and visualizations starting from the definitions of the sets.