Aalto University **Problem set 3A**
Department of Mathematics and Systems Analysis
MS-C1300 — Complex Analysis, 2025-2026/II K Kytölä

Exercise sessions: Tue-Wed, Nov 4-5, 2025

Topic: complex contour integration, paths, primitives

The first three exercises are to be discussed and solved in exercise sessions. The last exercise (marked with symbol ✍) is a quiz for which you will write a solution in an exam-like setup (no materials or extra equipment allowed) during the last 20min of the exercise session.

Exercise (in class) 1.
Find functions $F\colon \mathbb{C} \to \mathbb{C}$ such that $F'(z) = f(z)$ for all $z \in \mathbb{C}$ when

(a) $f(z) = z\, e^{z^2}$

(b) $f(z) = z\, e^{z}$

(c) $f(z) = z^2 \sin(4z)$.

Exercise (in class) 2.
Consider the path $\gamma\colon [-1,1] \to \mathbb{C}$ given by

$$\gamma(t) = \frac{1-t^2}{1+t^2} + \mathbb{i}\,\frac{2t}{1+t^2}.$$

Show that the formula

$$\phi(s) = \tan(s/2)$$

defines a smooth increasing function $\phi\colon [-\pi/2,\, \pi/2] \to [-1,1]$. Consider the reparametrization

$$\eta = \gamma \circ \phi\colon [-\pi/2,\, \pi/2] \to \mathbb{C},$$

of γ by ϕ, and simplify the formula for $\eta(s) = \gamma\big(\phi(s)\big)$.

Hint: *The simplified answer should make it apparent that η (and thus also γ) is a path that parametrizes a certain semicircle.*

Exercise (in class) 3.
Prove the following estimates for the contour integrals:

(a)
$$\left| \int_\gamma \frac{\mathrm{d}z}{z^2 - \mathbb{i}} \right| \le \frac{3\pi}{4}$$
when γ parametrizes the circle $|z| = 3$ once in the positive direction;

(b)
$$\left| \int_\gamma \frac{e^{3z}}{1 + e^z} \,\mathrm{d}z \right| \le \frac{2\pi e^{3R}}{e^R - 1}$$
when γ parametrizes a line segment from $z = R > 0$ to $z = R + 2\pi\mathbb{i}$;

(c)
$$\left| \int_\gamma e^{\sin(z)} \,\mathrm{d}z \right| \le 1$$
when γ parametrizes a line segment from $z = 0$ to $z = \mathbb{i}$.

Hint: *"Triangle inequality for integrals"*.

✍ **Quiz 4.**
Let $\gamma\colon [0, \pi] \to \mathbb{C}$ be given by $\gamma(t) = t\, e^{\mathbb{i}t}$. Calculate the following integrals:

(a) $$\int_\gamma \overline{z} \,\mathrm{d}z$$

(b) $$\int_\gamma |z| \,|\mathrm{d}z|$$

(c) $$\int_\gamma z \,\mathrm{d}z.$$