Aalto University
Department of Mathematics and Systems Analysis
MS-C1300 — Complex Analysis, 2025-2026/II

Problem set 5A

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Exercise sessions: Tue-Wed, Nov 18-19, 2025

Topic: series of analytic functions, power series, uniform convergence

The first three exercises are to be discussed and solved in exercise sessions. The last exercise (marked with symbol ✍) is a quiz for which you will write a solution in an exam-like setup (no materials or extra equipment allowed) during the last 20min of the exercise session.

Exercise (in class) 1.
Determine the disks of convergence of the following power series in a complex variable z:

$$\text{(i)} \qquad \sum_{n=1}^{\infty} \sqrt[n]{n}\,(z-1)^n$$

$$\text{(ii)} \qquad \sum_{n=1}^{\infty} n!\, z^n$$

$$\text{(iii)} \qquad \sum_{n=1}^{\infty} n^4\,(z+3\mathrm{i})^{2^n}\,.$$

Exercise (in class) 2.
Determine the radius of convergence of the following power series in a complex variable z, and find simplified formulas for the analytic functions determined by the series:

$$\text{(a):} \quad \sum_{n=1}^{\infty} z^n \qquad \text{(b):} \quad \sum_{n=1}^{\infty} n\, z^n \qquad \text{(c):} \quad \sum_{n=1}^{\infty} n^2\, z^n \qquad \text{(d):} \quad \sum_{n=1}^{\infty} n^3\, z^n\,.$$

Exercise (in class) 3.
Let $A \subset \mathbb{C}$ be a subset of the complex plane, and let $(f_n)_{n\in\mathbb{N}}$ and $(g_n)_{n\in\mathbb{N}}$ be two sequences[1] of complex-valued functions on A which converge uniformly to limit functions $f\colon A \to \mathbb{C}$ and $g\colon A \to \mathbb{C}$, respectively.

(i) Show that the sequence $(f_n + g_n)_{n\in\mathbb{N}}$ of the pointwise sum functions $f_n + g_n\colon A \to \mathbb{C}$ converges uniformly on A to the limit function which is the pointwise sum $f+g$ of the limits (i.e., the function $z \mapsto f(z)+g(z)$).

(ii) Assuming furthermore that both sequences of functions are uniformly bounded, show that the sequence $(f_n\, g_n)_{n\in\mathbb{N}}$ of their pointwise product functions $f_n\, g_n\colon A \to \mathbb{C}$ converges uniformly on A to the limit function which is the pointwise product fg of the limits (i.e., the function $z \mapsto f(z)\,g(z)$).

Remark: Here, e.g., a sequence $(f_n)_{n\in\mathbb{N}}$ of functions $f\colon A \to \mathbb{C}$ is said to be uniformly bounded if there exists a constant $c < \infty$ such that $|f_n(z)| \le c$ for all $n \in \mathbb{N}$ and all $z \in A$.

[1] Yes — sequences, not series, in exercise 3.

✍ **Quiz 4.**
Determine the disks of convergence of the following power series in a complex variable z:

$$\text{(i)} \qquad \sum_{n=1}^{\infty} \frac{\mathbb{i}^n}{n} (z-1)^n$$

$$\text{(ii)} \qquad \sum_{n=1}^{\infty} \frac{(2n)!}{n!\,(n+1)!} z^n$$

$$\text{(iii)} \qquad \sum_{n=1}^{\infty} n!\, z^{n!}\,.$$