Aalto University
Department of Mathematics and Systems Analysis
MS-C1300 — Complex Analysis, 2024-2025/II

Problem set 5B

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Exercise sessions: 28.-29.11. **Hand-in due: Sun 1.12.2024 at 23:59**

Topic: isolated singularities, residue calculus

The first two exercises are to be discussed and solved in exercise sessions. The last two exercises are homework (marked with symbol ✍): written solutions to them are to be returned in MyCourses. Each exercise is graded on a scale 0–3. The deadline for returning solutions to problem set 5B is Sun 1.12.2024 at 23:59.

Exercise (in class) 1.
Calculate

$$\oint_{\partial\mathcal{B}(e;2)} \frac{1}{(z-1)\,\mathrm{Log}(z)}\,\mathrm{d}z,$$

where the circle $\mathcal{B}(e;2)$ of radius 2 around $e = \exp(1) \in \mathbb{C}$ is positively oriented (counterclockwise).

Exercise (in class) 2.
Calculate the real integral

$$\int_{-\infty}^{\infty} \frac{1}{x^4+1}\,\mathrm{d}x \;:=\; \lim_{R\to\infty} \int_{-R}^{R} \frac{1}{x^4+1}\,\mathrm{d}x\,.$$

using residue calculus.

<u>Hint</u>: *Figuring out a suitable analytic function to contour integrate is the first step. A suitable contour here is the boundary of a semi-disk of a large radius R. One should analyze the contributions to the contour integral from the different parts of it.*

✍ **Homework exercise 3.**
Calculate the integral

$$\oint_\gamma \frac{e^z}{z^4 + 5z^3} \, \mathrm{d}z,$$

where γ parametrizes the circle $\partial\mathcal{B}(0;3)$ of radius 3 around the origin in the *negative(!)* orientation (*clockwise*).

✍ **Homework exercise 4.**
Let $a > 0$. Calculate the real integral

$$\int_0^\infty \frac{\cos(ax)}{1+x^2} \, \mathrm{d}x \;:=\; \lim_{R\to\infty} \int_0^R \frac{\cos(ax)}{1+x^2} \, \mathrm{d}x \,.$$

Hint: *A suitable contour here is again the boundary of a large semi-disk, and one should find a suitable analytic function to integrate. Furthermore, one needs to take into account also symmetry considerations to get to the desired result.*