Aalto University
Department of Mathematics and Systems Analysis
MS-C1300 — Complex Analysis, 2024-2025/II

**Problem set 1A**

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**Exercise sessions: 22.-23.10.** **Hand-in due: Wed 23.10.2024 at 23:59**

*Topic: calculation with and geometric interpretation of complex numbers*

*The first two exercises are to be discussed and solved in exercise sessions. The last two exercises are homework (marked with symbol ✍): written solutions to them are to be returned in MyCourses. Each exercise is graded on a scale 0–3. The deadline for returning solutions to problem set 1A is Wed 23.10.2024 at 23:59.*

**Exercise (in class) 1.**

(a) Let $z \in \mathbb{C} \setminus \{1\}$ and let $n \in \mathbb{N}$. Prove that

$$\sum_{j=0}^{n} z^j \;=\; \frac{1 - z^{n+1}}{1 - z} \,.$$

(b) Let $n \in \mathbb{N}$, $n \geq 2$. Find all solutions $z \in \mathbb{C}$ to the equation

$$z^{n-1} + z^{n-2} + \cdots + z + 1 \;=\; 0 \,.$$

**Exercise (in class) 2.**
Visualize and describe in geometric terms the following subsets of the complex plane:

(a)

$$A \;=\; \Big\{ z \in \mathbb{C} \;\Big|\; |z - 3\,\mathrm{i}| \,<\, 2 \Big\} \;;$$

(b)

$$B \;=\; \Big\{ z \in \mathbb{C} \;\Big|\; |z + 1| \,=\, |z + \mathrm{i}| \Big\} \;;$$

(c)

$$C \;=\; \Big\{ z \in \mathbb{C} \;\Big|\; |z| \,=\, 2\,|z - 1| \Big\} \;;$$

(d)

$$D \;=\; \Big\{ z \in \mathbb{C} \;\Big|\; z^2 \,=\, \overline{z}^2 \Big\} \,.$$

✍ **Homework exercise 3.**
Calculate:

(a) all complex square roots of $-1 + \mathbb{i}\sqrt{3}$ ;
(b) all complex cube roots of $-8$ .

Express the answers in Cartesian coordinates (i.e., as $z = x + \mathbb{i}y$ with $x, y \in \mathbb{R}$) and in polar coordinates (i.e., as $z = r\,e^{\mathbb{i}\theta} = r\big(\cos(\theta) + \mathbb{i}\,\sin(\theta)\big)$ with $r \geq 0$ and $\theta \in \mathbb{R}$).

✍ **Homework exercise 4.**
Show geometrically (recommended) or by a calculation (also possible) that if $z \in \mathbb{C}$ is such that $|z = 1|$ and $z \neq 1$, then

$$\Im\mathfrak{m}\left(\frac{z}{(z+1)^2}\right) \;=\; 0\,.$$

Which other points $z \in \mathbb{C}$ satisfy the above equation?