MS-EV0029 Project Contribution Report

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Project overview

As a part of the course MS-EV0029, I contributed to the project on the formalization of the Fischer–Tippett–Gnedenko theorem characterizing univariate extreme value distributions:

Extreme value distribution project
Webpage: https://kkytola.github.io/ExtremeValueProject/
Github repository: https://github.com/kkytola/ExtremeValueProject

The main goal of the project is to classify the possible nondegenerate limits in distribution of maxima of an increasing number of independent and identically distributed random variables, up to shift and rescaling. A more elementary equivalent definition of these limit objects is:

Definition 1 (Extreme value distribution) *A cumulative distribution function G is an **extreme value distribution** if G is nondegenerate and if for some cumulative distribution function F and some real-number sequences $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ with $a_n > 0$, we have $\lim_{n\to\infty} F(a_n x + b_n)^n = G(x)$ at all points x where G is continuous.*

The classification result is:

Theorem 1 (Fischer–Tippett–Gnedenko theorem) *A cumulative distribution function G is an extreme value distribution if and only if it is of the form $G(x) = F_\gamma(ax+b)$ for some $\gamma \in \mathbb{R}$ and $a > 0$ and $b \in \mathbb{R}$, where F_γ is a certain explicit cumulative distribution function ($\gamma = 0$ Gumbel, $\gamma < 0$ Weibull, $\gamma > 0$ Fréchet).*

Main contributions

Below is a list of my most significant contributions made during the project:
(Preferably identify contributions with pull-requet numbers or commit hashes.)

1: commit 82f362d
Wrote the (informal) statements and proofs of the characterization of convergence in distribution functions in terms of cumulative distribution functions in the blueprint, Theorem 4.8 *(convergence-in-distribution-with-cdf)* and Lemma 4.9 *(cdf-convergence-from-convergence-in-distribution)*.

2: commit d753d68
Proved that left/right-continuous pseudoinverses `lcInv`/`rcInv` are indeed order-theoretically left/right-continuous. The main result of the commit is `leftOrdContinuous_lcInv`.

3: commit c6bfcf9
Stated and proved `tendsto_smul_apply_smul_deriv_of_tendsto_atTop_of_tendsto_smul_apply_smul_deri` and auxiliary results towards it. This is Lemma 2.11 *(modify-limit-taylor)* in the blueprint.

Notes

Collaboration:

The project was a collaboration with a number of participants: `https://github.com/kkytola/ExtremeValueProject/graphs/contributors`.

Additional technical note:

This report is a template for actual course participants' reports. Your report does not have to be very detailed, as long as it contains a clear link to the project and roughly indicates which parts were your own work.