Definition – Permanence

Let $P : \mathrm{Mor}(\mathbf{Sch}) \to \mathbf{Prop}$ be a predicate on morphisms of schemes. Then we say :

- P is *stable under composition* when for all $X \to Y \to Z$ in **Sch**, $P(X \to Y)$ and $P(Y \to Z)$ implies $P(X \to Z)$.
- P is *stable under base change* when for all pullback diagrams

$$\begin{array}{ccc} X & \longrightarrow & S \\ \uparrow & & \uparrow \\ X \times_S Y & \longrightarrow & Y \end{array}$$

  $P(X \to S)$ implies $P(X \times_S Y \to Y)$.
- P is *stable under fiber product* when for all pullback diagrams as the above, $P(X \to S)$ and $P(Y \to S)$ implies $P(X \times_S Y \to S)$.