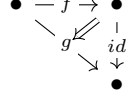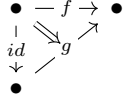Lemma (Lifting against Inner 3-Horns). Let $X \in \mathbf{sSet}$ *lift against* $\{I^n \to \Delta^n\}_{n \leq 3}$ *and* $\{\Lambda_1^3 \to \Delta^3, \Lambda_2^3 \to \Delta^3\}$. *Then we have the following :*

1. *For* $x, y : \Delta^0 \to X$, *the following two relations on* $X(x, y)$ *are the same :*

   – $f \sim g$ *when there exists a triangle in* X *of the following form :*

$$
\begin{array}{ccc}
\bullet & \!\!-f\rightarrow\!\! & \bullet \\
 & \searrow_{g}\ \Swarrow & \ \ \big\downarrow{\scriptstyle id} \\
 & & \bullet
\end{array}
$$

   – $f \sim g$ *when there exists a triangle in* X *of the following form :*

$$
\begin{array}{ccc}
\bullet & \!\!-f\rightarrow\!\! & \bullet \\
{\scriptstyle id}\big\downarrow & \searrow_{g}\ \nearrow & \\
\bullet & &
\end{array}
$$

2. *The relation in the previous point is in fact an equivalence relation.*

3. *For* $I^2 \to X$ *representing two composable edges* f, g, *define* composites *of* f, g *to be extensions of* $I^2 \to X$ *to all of* Δ^2. *Then for any pair of composable edges in* X, *a composite exists and is unique up to the equivalence relation in* (1).

Proof. (1)

$\square$