Home assignment-2

1) Given matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

* compose the transpose determinant & inverse of matrix A
* matrix the eigen & corresponding eigen vector matrix A
* vector that A·v true for eigen values & eigen vector pair

A) Given matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

1) a) Transpose $A^T = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

b) determinat $\det[A] = (4)(3) - (2)(1)$

$= 12 - 2$

$\det A = 10$

c) Inverse $A^{-1} = \frac{1}{\det A} \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$

$\frac{1}{10} \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$

2) Eigen values & eigen vectors

a) equation $\det[A - \lambda I] = 0$

$\begin{bmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix} = 0$

$(4-\lambda)(3-\lambda) - 2(1) = 0$

$\lambda^2 - 7\lambda + 10 = 0$

$\lambda^2 - 5\lambda - 2\lambda + 10 = 0$

$\lambda_1 = 5 \quad \lambda_2 = 2$

Eigen vectors $\lambda_1 = 5$

$[A - 5\lambda] V = 0$

$\begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-x + 2y = 0 \Rightarrow x = 2y$

$V_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\lambda_2 = 2$

$[A - 2I] V = 0$

$$\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$2x + 2y = 0$$
$$x + y = 0$$
$$x = -y$$
$$V_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

3) verification

$$\boxed{A \cdot V = \lambda \cdot V}$$

Take $\lambda = 5$ $V_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$AV_1 = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4(2) + 2(1) \\ 1(2) + 3(1) \end{bmatrix} = \begin{bmatrix} 8+2 \\ 2+3 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\lambda_1 \cdot V_1 = 5\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

$$\boxed{A \cdot V_1 = \lambda_1 \cdot V_1}$$

$A \cdot V_1 = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ $\lambda_1 \cdot V_1 = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ Hence proved $A \cdot V = \lambda \cdot V$

2) Given data about size of a house & its corresponding Price use linear regression predict price of a data structure

house size	Price in Lakhs
1000	50
1500	65
2000	80
2500	95
3000	110

Tasks

1) fit a simple linear regression model
2) Derive the regression eq^n
3) predict the price house 220ft size

linear regression

$\bar{y} = a + bx$

$\Sigma x = 1000 + 1500 + 2000 + 25000 + 3000 = 10{,}000$

$\Sigma_1 = 50 + 65 + 80 + 95 + 110 = 600$

$\Sigma xy = 1000(50) + 1500(65) + 2000(80) + 2500\,0(0) + 3000(110)$

$= 875000$

$\Sigma x^2 = 1000^2 + 1500^2 + 2000^2 + 2500^2 + 3000^2$

$= 22500000$

To find b :-

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - [\Sigma x]^2}$$

$$= \frac{(5)[875000] - [10000][400]}{(5)[22500000] - [10000]^2}$$

$$= \frac{4375000 - 400000}{112500000 - 100000000} = \frac{375000}{12500000}$$

$b = 0.3$

To find a

$$a = \frac{\Sigma y - b\Sigma x}{n} = \frac{400 - (6.3)(1000)}{5}$$

$$= \frac{400 - 300}{5} = \frac{100}{5} = 20$$

$\boxed{a = 20}$

3) Predict for 2200 sq.ft ($x = 2200$)

$\bar{y} = a + bx$

$= 20 + (0.3)(2200)$

$= 20 + 66$

$\bar{y} = 86$

predect price for 2200 sq.ft house = ₹ 86