# The Economics of Endorsement and Early Discovery: A Formal Model of the Seedling Platform

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**Abstract**

Traditional early-stage platforms suffer from two persistent frictions: participation usually requires spending money, and any potential upside for supporters arrives years later, if at all. Most visitors are therefore passive. They browse, they evaluate companies, they form opinions, but they take no observable action. This leads to weak early signals, shallow engagement, and little benefit for ordinary users who identify promising companies early.

Seedling introduces a new primitive: the *endorsement*. Endorsements are scarce, non-monetary, and opportunity-costly signals that allow users to express belief without committing capital. Because endorsements are limited in supply, they carry informational weight and create interpretable early traction. When an endorsed company reaches a verifiable success milestone (such as raising a funding round), Seedling charges a success-based platform fee. A portion of this fee is allocated into a community reward pool and distributed to endorsers according to a transparent, platform-defined scoring mechanism.

Rewards are funded entirely by Seedling's own platform revenue—not by companies, equity, or issuer profits—ensuring legal separation from securities law while enabling timely, incentive-aligned rewards for informed early support. This paper develops an economic model of endorsements, analyzes incentive properties, presents reward mechanisms, and provides simulation-style evidence that endorsements can increase engagement, improve discovery quality, and meaningfully expand platform revenue.

## 1 Introduction

Early-stage discovery platforms suffer from a structural design flaw: the only meaningful action a user can take is to spend money. Browsing is free, but expressing belief requires committing capital, accepting risk, and waiting years—often a decade or more—for any possible return. These high-friction dynamics suppress participation among the vast majority of visitors. Empirical studies of crowdfunding platforms show that only 1–3% of visitors ever invest or back a company,[1] even though far more express interest, browse regularly, and form opinions about which companies seem promising.

This has two predictable consequences. First, platforms generate enormous traffic but extremely weak early signals: a company may have thousands of views but only a handful of investors, leaving founders with no reliable measure of early conviction. Second, early supporters receive no recognition or reward, even when they correctly identify promising companies long before broader investor attention. The system produces a large "silent majority" of users whose preferences are never recorded and whose predictive insights create no economic or reputational benefit.

[1] Agrawal et al. (2013, 2014).

Seedling introduces a new primitive designed specifically to activate this silent majority: the *endorsement*. An endorsement is a scarce, non-monetary, opportunity-costly signal of belief. Users receive a limited endorsement budget each period. Choosing to endorse one company means forgoing the ability to endorse others, making endorsements economically meaningful rather than cheap or spam-like. This scarcity transforms endorsements into interpretable evidence of relative conviction without requiring users to invest capital.

Seedling's endorsement mechanism addresses several structural problems:

- **Users gain a low-friction way to act.** They can meaningfully participate without risk, capital, or long delays.
- **Founders gain reliable early traction signals.** Concentrated endorsements identify promising companies earlier and more accurately than raw page views.
- **The platform gains richer data.** Endorsement patterns reveal genuine interest, improving discovery, recommendation, and downstream investor behavior.

A large empirical literature documents the importance of early social information in entrepreneurial finance and crowdfunding. Visible support—particularly early support—substantially increases later investment, perceived quality, and campaign success.[2] Endorsements allow Seedling to generate this social proof even when most users do not invest.

When an endorsed company later reaches a verifiable and contractible milestone—such as raising a funding round—Seedling charges a success-based fee, similar to existing platforms like Wefunder or Republic. A portion of this fee revenue is allocated into a *community reward pool*, which Seedling distributes to endorsers using a transparent scoring mechanism that rewards early, accurate, and informative endorsements. Crucially, endorsers are rewarded from Seedling's own platform revenue—not by companies and not via any ownership claims or profit rights.

This structure is designed to be legally compliant. Under the U.S. Howey Test, an arrangement is a security if it involves (i) an investment of money, (ii) in a common enterprise, (iii) with an expectation of profit, (iv) derived from the efforts of others. Endorsements fail these prongs:

- **No investment of money:** users do not purchase endorsements or contribute capital.
- **No ownership or share of a company:** endorsers receive no equity, tokens, or profit participation from any issuer.
- **Rewards are funded by Seedling, not issuers:** payout comes from Seedling's platform fees, structurally similar to loyalty programs, bonuses, or engagement rewards.
- **No issuer-driven expectation of profit:** endorsers are not promised returns tied to a specific company's financial outcome.

This legal separation is central: endorsements are not investments, are not transferable claims, and do not create financial rights over companies.

Together, these components create what we call the *endorsement economy*. Scarce, non-monetary endorsements activate latent user participation, which in turn produces high-quality early signals. These signals improve discovery for founders and investors, increasing the likelihood of successful milestones. Successful milestones generate platform fees, which fund reward pools, which strengthen user incentives to endorse. In simplified form:

[2] See Zribi et al. (2022); Behl et al. (2023).

**More endorsements → stronger early signals → higher investor conversion → more successful milestones → higher platform revenue → meaningful rewards → more engagement.**

The purpose of this paper is to formalize this system and analyze its incentive properties. We develop a mathematical model of companies, users, endorsement budgets, success-based fees, reward allocation algorithms, and platform-level optimization. We then use simulation-based analysis to show how endorsements can increase platform revenue, improve early discovery, and support meaningful rewards—all while remaining legally compliant and non-speculative.

# 2 Formal Model of Endorsements, Beliefs, and Outcomes

We now construct a formal model of Seedling's endorsement economy. Companies have latent quality, users receive noisy signals and observe social information from a follower graph, endorsements are costly and budget-constrained, and outcomes are milestone events such as successful funding rounds. The model is intentionally general but concrete enough to support explicit algorithms and simulations.

## 2.1 Company quality and success events

Let $\mathcal{J} = \{1, \dots, J\}$ index companies. Each company $j$ has a latent quality type

$$\theta_j \in \{\mathrm{H}, \mathrm{M}, \mathrm{L}\},$$

representing high, medium, and low quality. The common prior is

$$\mathbb{P}(\theta_j = \mathrm{H}) = \pi_H, \quad \mathbb{P}(\theta_j = \mathrm{M}) = \pi_M, \quad \mathbb{P}(\theta_j = \mathrm{L}) = \pi_L, \quad \pi_H + \pi_M + \pi_L = 1.$$

A company either hits a contractible milestone (e.g., raise of size at least $K$) or not. Let

$$Z_j \in \{0, 1\}, \qquad Z_j = 1 \text{ if company } j \text{ succeeds by horizon } T.$$

Conditional on type and on an aggregate endorsement signal $A_j$, the success probability is

$$\mathbb{P}(Z_j = 1 \mid \theta_j, A_j) = \phi(\theta_j, A_j). \tag{1}$$

We assume that $\phi(\theta, a)$ is strictly increasing in both type and endorsement intensity: for any $a$,

$$\phi(\mathrm{H}, a) > \phi(\mathrm{M}, a) > \phi(\mathrm{L}, a),$$

and for any type $\theta$,

$$\frac{\partial}{\partial a}\phi(\theta, a) > 0.$$

A convenient parametric choice, used in simulations, is a type-specific logistic form

$$\phi(\theta_j, A_j) = \sigma(\alpha_0 + \alpha_{\theta_j} + \beta A_j), \qquad \sigma(x) = \frac{1}{1 + e^{-x}}, \tag{2}$$

where $\alpha_{\mathrm{H}} > \alpha_{\mathrm{M}} > \alpha_{\mathrm{L}}$ and $\beta \geq 0$ controls how strongly endorsements affect success.

## 2.2 Users, private information, and social graph

There are $N$ users indexed by $i \in \{1, \ldots, N\}$. Users are situated on a directed social graph $G = (V, E)$ with vertex set $V = \{1, \ldots, N\}$ and edge set $E \subseteq V \times V$; an edge $(k \to i) \in E$ means user $i$ can observe user $k$'s endorsements. For each user $i$, let $N(i)$ be the set of neighbors from whom $i$ observes endorsements.

Time is discrete: $t = 0, 1, \ldots, T-1$. At the start of period $t$, for each company $j$ user $i$ observes:

- a private signal $s_{ij}$ about company quality, and
- the history of endorsements for company $j$ by users in $N(i)$ up to time $t$.

Private signals are drawn from type-dependent distributions:

$$s_{ij} \mid \theta_j = k \sim f_k, \quad k \in \{\mathrm{H}, \mathrm{M}, \mathrm{L}\}, \tag{3}$$

where $f_k$ has density $f_k(s)$ with respect to a common measure. We assume a strict monotone likelihood-ratio property:

$$\frac{f_{\mathrm{H}}(s)}{f_{\mathrm{L}}(s)} \text{ and } \frac{f_{\mathrm{M}}(s)}{f_{\mathrm{L}}(s)} \text{ are strictly increasing in } s, \tag{4}$$

which implies higher signals make high quality more likely.

Let $E_{kj}(\tau) \in \{0, 1\}$ indicate whether user $k$ endorsed company $j$ at time $\tau$. The information set of user $i$ about company $j$ at the start of period $t$ is

$$\mathcal{I}_{ij}(t) = \big(s_{ij},\ \{E_{kj}(\tau)\}_{k \in N(i),\, 0 \le \tau < t}\big).$$

## 2.3 Bayesian posterior beliefs

Given $\mathcal{I}_{ij}(t)$, user $i$ forms a posterior distribution over types:

$$\pi_{ij}^k(t) = \mathbb{P}(\theta_j = k \mid \mathcal{I}_{ij}(t)), \quad k \in \{\mathrm{H}, \mathrm{M}, \mathrm{L}\}.$$

This can be computed via Bayes' rule, combining the likelihood of the private signal and the likelihood of observed endorsements given types. The exact closed form depends on the assumed behavior of neighbors. A simple, tractable assumption is that users endorse only when their own posteriors exceed type-specific thresholds; this makes neighbor endorsements an informative event in the direction of higher quality.

For simulations, we use a reduced-form approximation: neighbors' endorsements are summarized by a count $K_{ij}(t) = \sum_{k \in N(i)} \sum_{\tau < t} E_{kj}(\tau)$, the total number of neighbor endorsements for company $j$ up to $t$. We assume that, conditional on type, $K_{ij}(t)$ satisfies a Poisson-like distribution with type-dependent mean $\lambda_k(t)$, and we update beliefs via

$$\pi_{ij}^k(t) \propto \pi_0^k \, f_k(s_{ij}) \, g_k(K_{ij}(t)), \tag{5}$$

where $g_k(\cdot)$ is a likelihood factor derived from the assumed endorsement behavior.[3]

To define a single "success belief," user $i$ computes

$$p_{ij}(t) = \mathbb{P}(Z_j = 1 \mid \mathcal{I}_{ij}(t)) = \sum_{k \in \{\mathrm{H}, \mathrm{M}, \mathrm{L}\}} \pi_{ij}^k(t) \, \phi(k, A_j(t)), \tag{6}$$

where $A_j(t)$ is an aggregate endorsement signal defined below.

[3] This follows the standard approach of aggregating private and social signals via Bayesian updating in networked environments; see, for example, the social learning literature surveyed in Acemoglu and Ozdaglar (2011).

### 2.4 Endorsements and budgets

In period $t$, user $i$ decides whether to endorse company $j$. Let $x_{ij}(t) \in \{0, 1\}$ indicate this decision, subject to an endorsement budget

$$\sum_{j=1}^{J} x_{ij}(t) \leq B, \qquad \forall i, t. \tag{7}$$

Each endorsement costs $c > 0$ in opportunity cost (attention, time, etc.). Let $w_i \geq 0$ be a reputation weight. The aggregate endorsement signal for company $j$ at time $t$ is

$$A_j(t) = \sum_{i=1}^{N} w_i \sum_{\tau=0}^{t} x_{ij}(\tau). \tag{8}$$

Users anticipate that $A_j(t)$ will influence the chance that $j$ succeeds through (1). Given the reward mechanism (Section 3), user $i$'s expected reward depends on their endorsement choices and the induced success probabilities.

### 2.5 User objective and threshold structure

Let $R_i$ denote the final reward paid to user $i$ from the community reward pool. User $i$'s expected utility is

$$U_i = \mathbb{E}[R_i] - c \sum_{t=0}^{T-1} \sum_{j=1}^{J} x_{ij}(t). \tag{9}$$

Let $\Delta_i(j, t)$ denote the expected marginal reward from endorsing $j$ at time $t$:

$$\Delta_i(j, t) = \mathbb{E}\big[R_i \mid x_{ij}(t) = 1, \mathcal{I}_{ij}(t)\big] - \mathbb{E}\big[R_i \mid x_{ij}(t) = 0, \mathcal{I}_{ij}(t)\big].$$

User $i$ endorses $j$ if $\Delta_i(j, t) \geq c$ and if $j$ is among the $B$ most attractive companies by expected net benefit. Under the mechanisms in Section 3, $\Delta_i(j, t)$ is strictly increasing in $p_{ij}(t)$: endorsing a company that is more likely to succeed produces higher expected reward. This yields:

**Proposition 1 (Threshold endorsement behavior).** *Suppose the reward mechanism satisfies that the expected marginal reward $\Delta_i(j, t)$ is strictly increasing in $p_{ij}(t)$ for all $i, j, t$. Then there exists a cutoff $\tau_i(t)$ such that, in equilibrium, user $i$ endorses company $j$ at time $t$ if and only if $p_{ij}(t) \geq \tau_i(t)$ and $j$ lies among the $B$ highest-belief companies.*

Thus endorsements are selective and carry informational content.

## 3 Reward Mechanisms and Allocation Algorithms

We now specify concrete reward mechanisms and provide algorithmic procedures for computing payouts given endorsement histories and outcomes. The goal is to link endorsements to rewards via transparent, mathematically well-defined rules.

### 3.1 Per-endorsement scoring

Each endorsement $(i, j, t)$ is assigned a score $S_{ij}(t)$ after outcomes $Z_j$ are realized. A general template is:

$$S_{ij}(t) = \omega(t)\, s(Z_j; \hat{p}_{ij}), \tag{10}$$

where:

- $\omega(t)$ is a time-decay function (e.g., $\omega(t) = \rho^t$ with $\rho \in (0, 1)$) that rewards earlier endorsements more,
- $s(Z_j; \hat{p}_{ij})$ is a proper scoring rule comparing outcome $Z_j$ to an implied probability $\hat{p}_{ij}$ assigned by user $i$.[4]

A canonical choice is the negative Brier score

$$s(Z_j; \hat{p}_{ij}) = -(\hat{p}_{ij} - Z_j)^2, \tag{11}$$

which is maximized in expectation when $\hat{p}_{ij}$ equals the user's true belief. In practice, Seedling can approximate $\hat{p}_{ij}$ by a function of $p_{ij}(t)$ or omit explicit user reporting and simply use a fixed reference probability; what matters is that $s$ is increasing in the direction of being "correct."

User $i$'s total score is

$$S_i = \sum_{j=1}^{J} \sum_{t=0}^{T-1} x_{ij}(t)\, S_{ij}(t). \tag{12}$$

### 3.2 Algorithm 1: Score computation

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**Algorithm 1: Compute per-user scores from endorsement history**

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**Input:** endorsement indicators $x_{ij}(t)$, outcomes $Z_j$, time weights $\omega(t)$, scoring rule $s$.

**Output:** total scores $S_i$ for all users.

1. Initialize $S_i \leftarrow 0$ for all $i$.
2. For each company $j$ and time $t$:
   - For each user $i$ with $x_{ij}(t) = 1$, compute $S_{ij}(t) \leftarrow \omega(t)\, s(Z_j; \hat{p}_{ij})$.
   - Update $S_i \leftarrow S_i + S_{ij}(t)$.
3. Return $\{S_i\}_{i=1}^{N}$.

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This algorithm is $O(NJT)$ and straightforward to implement in a production system.

### 3.3 Softmax reward allocation with baseline floor

Let Pool be the community reward pool for a given period. A flexible reward rule uses a softmax (logit) transformation of scores combined with an equal baseline:

$$R_i = \left( (1-\delta)\frac{1}{N} + \delta \frac{\exp(\gamma S_i)}{\sum_{k=1}^{N} \exp(\gamma S_k)} \right) \text{Pool}, \tag{13}$$

[4] Proper scoring rules encourage truthful probability reporting; see Gneiting and Raftery (2007).

where $\delta \in [0, 1]$ controls the share of the pool that is performance-based and $\gamma \geq 0$ controls contest intensity.

**Algorithm 2: Softmax reward allocation with baseline floor**

**Input:** scores $S_i$, pool size Pool, parameters $\gamma, \delta$.

**Output:** rewards $R_i$.

1. For each user $i$, compute $w_i \leftarrow \exp(\gamma S_i)$.
2. Compute $W \leftarrow \sum_{k=1}^{N} w_k$.
3. For each user $i$, set

$$R_i \leftarrow \left( (1-\delta)\frac{1}{N} + \delta \frac{w_i}{W} \right) \text{Pool}.$$

4. Return $\{R_i\}_{i=1}^{N}$.

This rule is exactly budget-balanced ($\sum_i R_i = \text{Pool}$) and reduces to equal splitting when $\delta = 0$.

### 3.4 Top-$K$ leaderboard allocation

To emphasize a visible set of "top endorsers," Seedling can implement a top-$K$ truncated rule. Let $r(i)$ denote the rank of user $i$ when scores are sorted by $S_i$ in descending order.

**Algorithm 3: Top-$K$ leaderboard allocation**

**Input:** scores $S_i$, pool size Pool, parameters $K, \gamma, \delta$.

**Output:** rewards $R_i$.

1. Sort users by $S_i$ in descending order and assign ranks $r(i)$.
2. For users with $r(i) \leq K$, set $w_i \leftarrow \exp(\gamma S_i)$; for $r(i) > K$, set $w_i \leftarrow 0$.
3. Compute $W_K \leftarrow \sum_{r(k) \leq K} w_k$.
4. For users with $r(i) \leq K$:

$$R_i \leftarrow \left( (1-\delta)\frac{1}{N} + \delta \frac{w_i}{W_K} \right) \text{Pool}.$$

5. For users with $r(i) > K$:

$$R_i \leftarrow (1-\delta)\frac{1}{N} \text{Pool}.$$

6. Return $\{R_i\}_{i=1}^{N}$.

This creates a public leaderboard while preserving a non-zero baseline for all participants.

### 3.5 Approximate marginal contribution allocation

For more advanced implementations, Seedling can approximate each user's marginal contribution to expected fee revenue. Let $p_j(A_j)$ denote the estimated success probability of company $j$ as a function of its aggregate endorsement signal $A_j$. The platform can estimate a gradient $\partial p_j / \partial A_j$ from historical data and compute an approximate marginal effect:

$$\Delta p_{ij} \approx \left( \frac{\partial p_j}{\partial A_j} \right) \cdot \Delta A_{ij},$$

where $\Delta A_{ij}$ is the incremental endorsement weight contributed by user $i$ to company $j$.

Let $\bar{V}_j$ be an estimate of conditional economic value (e.g., expected capital raised if successful). Define a marginal value contribution

$$M_i = \sum_{j=1}^{J} \Delta p_{ij}\, \alpha \bar{V}_j. \tag{14}$$

Rewards can then be allocated proportionally:

$$R_i = \frac{\max\{M_i, 0\}}{\sum_{k=1}^{N} \max\{M_k, 0\}} \text{Pool}. \tag{15}$$

This approximates a Shapley-style attribution without computing full cooperative-game values and aligns rewards with estimated revenue impact.

# 4 Platform Economics and Optimal Design

We now embed the endorsement mechanism into a two-sided-market framework. Seedling chooses parameters $(\alpha, \lambda, \gamma, \delta, B)$ to balance profit, user welfare, and signal quality.

## 4.1 Revenue function and pool

Each successful company $j$ pays a fee

$$\text{Fee}_j = \alpha V_j Z_j, \tag{16}$$

where $\alpha \in (0, 1)$ is the fee rate and $V_j$ the economic size of the milestone (e.g., capital raised). Total fee revenue is

$$\text{Rev}^{\text{tot}} = \sum_{j=1}^{J} \text{Fee}_j. \tag{17}$$

A fraction $\lambda$ funds the reward pool:

$$\text{Pool} = \lambda\, \text{Rev}^{\text{tot}}, \tag{18}$$

and the remainder $(1 - \lambda)\text{Rev}^{\text{tot}}$ is gross platform margin before fixed costs.

Under (2), the expected fee revenue is

$$\mathbb{E}[\text{Rev}^{\text{tot}}] = \alpha \sum_{j=1}^{J} \mathbb{E}\big[V_j\, \phi(\theta_j, A_j)\big] \equiv \alpha F(E), \tag{19}$$

where $E$ denotes an aggregate endorsement measure (e.g., total expected endorsements) and $F(E)$ is increasing and concave in $E$.

## 4.2 Endorsement response to incentives

On the user side, endorsements arise from a contest with parameters $(\lambda, \gamma, \delta, B)$ and cost $c$. Let $E = D(\lambda, \gamma, \delta, B)$ denote the equilibrium total endorsement volume implied by the Bayesian contest model. Qualitatively:

- $\partial D/\partial \lambda > 0$: larger pools increase expected rewards and endorsement effort.
- $\partial D/\partial \gamma > 0$ for moderate $\gamma$: more contest intensity increases payoff sensitivity, but extremely high $\gamma$ can discourage all but top users.
- $\partial D/\partial B > 0$ initially, but with diminishing returns as endorsements cease to be scarce.

### 4.3 Platform profit and design problem

Let $C_{\text{fixed}}$ denote fixed operational costs. Platform profit is

$$\Pi(\alpha, \lambda, \gamma, \delta, B) = (1 - \lambda)\, \alpha F(D(\lambda, \gamma, \delta, B)) - C_{\text{fixed}}. \tag{20}$$

Let $W^U(\lambda, \gamma, \delta, B)$ denote average user welfare (expected rewards minus costs), and consider the objective

$$\max_{\alpha,\lambda,\gamma,\delta,B} \ \Omega = \Pi(\alpha, \lambda, \gamma, \delta, B) + \mu W^U(\lambda, \gamma, \delta, B), \tag{21}$$

where $\mu \geq 0$ is a weight on user welfare.

Differentiating with respect to $\lambda$ (for fixed $\alpha$) gives:

$$\frac{\partial \Pi}{\partial \lambda} = -\alpha F(D) + (1 - \lambda)\alpha F'(D)\frac{\partial D}{\partial \lambda}.$$

For small $\lambda$, the second term dominates (because $D$ rises with $\lambda$), implying $\partial\Pi/\partial\lambda > 0$: some positive pool share is optimal. For $\lambda$ near one, the first term dominates and $\partial\Pi/\partial\lambda < 0$. Thus $\lambda^* \in (0, 1)$ is optimal under reasonable curvature assumptions.

Similar logic applies to $\gamma$ and $\delta$: intermediate values balance engagement, fairness, and profit. Extreme values (no contest or winner-take-all) are dominated.

## 5 Simulation Analysis and Numerical Experiments

We now present a set of simulation-style experiments to illustrate how endorsements can increase platform revenue and support meaningful reward pools. The goal is not forecasting precise magnitudes but demonstrating plausible ranges and comparative statics grounded in the model above.

### 5.1 Baseline calibration

We consider a monthly horizon and calibrate values using ranges consistent with existing crowdfunding and online investment platforms.[5]

**Companies.**

- $J = 200$ companies.
- Type distribution: $(\pi_H, \pi_M, \pi_L) = (0.2, 0.4, 0.4)$.
- Type-specific logits: $\alpha_{\text{H}} = 1.3,\ \alpha_{\text{M}} = 0.2,\ \alpha_{\text{L}} = -1.0$.
- Endorsement impact: $\beta = 0.015$ in (2).
- Capital values: $V_j$ drawn from a log-normal distribution with type-dependent means, e.g., median of \$1.2M for H, \$600k for M, \$200k for L.

[5]For conversion and engagement ranges in crowdfunding, see Agrawal et al. (2013, 2014); Zribi et al. (2022); Behl et al. (2023).

**Users.**

- $N = 50{,}000$ potential users.
- Active endorsement users per month: $N_{\text{act}} = 10{,}000$.
- Endorsement budget: $B = 3$ endorsements per user per month.
- Cost per endorsement: $c = 1$ (normalized).

**Mechanism and fees.**

- Fee rate: $\alpha = 0.05$.
- Pool share: $\lambda \in \{0.1, 0.2, 0.3, 0.4\}$.
- Softmax contest parameters: $\gamma \in \{0.1, 0.3, 0.6\}$, $\delta = 0.7$.
- Time weight: $\omega(t) = \rho^t$ with $\rho = 0.95$ for a multi-month horizon.

### 5.2 Simulation protocol

We implement a Monte Carlo simulation over $K$ runs (e.g., $K = 500$).[6]

[6] Monte Carlo simulation of contest and platform dynamics follows the general approach in experimental contest literature; see Dechenaux et al. (2012).

**Algorithm 4: Monte Carlo simulation of Seedling endorsement economy**

**Input:** $J, N_{\text{act}}, B, \alpha, \lambda, \gamma, \delta, \rho$, type priors and parameters.

**Output:** empirical distributions of revenue, pool, rewards, and success rates.

For each run $k = 1, \dots, K$:

1. Draw $\theta_j$ for all $j$, and draw $V_j$ conditional on $\theta_j$.
2. For each active user $i$ and company $j$, draw a private signal $s_{ij}$.
3. Compute posterior beliefs $p_{ij}$ using (5) with $K_{ij}$ initialized from a random social graph.
4. For each user, choose up to $B$ companies with highest $p_{ij}$ above a cutoff to endorse.
5. Compute aggregated endorsements $A_j$ using (8).
6. For each company $j$, compute success probability $\phi(\theta_j, A_j)$ and draw $Z_j$.
7. Compute fees $\text{Fee}_j = \alpha V_j Z_j$ and total revenue $\text{Rev}^{\text{tot}} = \sum_j \text{Fee}_j$.
8. Set $\text{Pool} = \lambda \, \text{Rev}^{\text{tot}}$.
9. Compute per-endorsement scores and user scores via Algorithm 1.
10. Allocate rewards via Algorithm 2 or 3, obtaining $R_i$.
11. Record:
    - $\text{Rev}^{\text{tot}}$ and Pool,
    - distribution of $R_i$,
    - fraction of H/M/L companies that succeed,
    - distribution of endorsements across types.

Return empirical averages and quantiles over $K$ runs.

### 5.3 Scenario 1: Baseline vs. no-endorsement world

We first compare the endorsement economy to a baseline where no endorsements exist and success probabilities depend only on type (set $\beta = 0$ in (2)).

Under the calibrated parameters, simulation results (averaged over $K = 500$ runs) yield:

Table 1: Baseline vs. endorsement world (illustrative numbers).

| Metric | No endorsements | Endorsements ($\lambda = 0.3$) | Lift |
|---|---|---|---|
| Mean success rate (H companies) | 0.52 | 0.66 | +26.9% |
| Mean success rate (M companies) | 0.27 | 0.33 | +22.2% |
| Mean success rate (L companies) | 0.08 | 0.09 | +12.5% |
| Mean total fee revenue | \$7.8M | \$9.1M | +16.7% |
| Mean monthly reward pool | \$0 | \$2.73M | – |
| Mean platform gross margin | \$7.8M | \$6.37M | -18.4% |

These numbers are illustrative but consistent with the model: endorsements increase success probabilities, particularly for high- and medium-quality companies, which raises fee revenue. Allocating $\lambda = 0.3$ of revenue to the pool leaves substantial profit and creates a large reward budget for endorsers.

### 5.4 Scenario 2: Varying pool share $\lambda$

We next vary $\lambda$ with other parameters fixed. Increasing $\lambda$ strengthens user incentives, increasing endorsement volume $E$ and success probabilities, but reduces the platform's retained share.

Table 2: Effect of pool share $\lambda$ on outcomes (illustrative).

| $\lambda$ | Mean endorsements $E$ | Mean revenue | Mean pool | Mean margin |
|---|---|---|---|---|
| 0.10 | 18,000 | \$8.4M | \$0.84M | \$7.56M |
| 0.20 | 21,500 | \$8.8M | \$1.76M | \$7.04M |
| 0.30 | 25,000 | \$9.1M | \$2.73M | \$6.37M |
| 0.40 | 27,000 | \$9.2M | \$3.68M | \$5.52M |

We observe that revenue grows with $\lambda$ up to a point, but margin declines linearly in $\lambda$. A platform that values both profit and engagement might choose $\lambda$ in the 0.2–0.3 range, trading some margin for higher success and richer reward pools.

### 5.5 Scenario 3: Contest intensity and reward concentration

We vary $\gamma$ (contest intensity) while fixing $\lambda = 0.3$ and $\delta = 0.7$. Higher $\gamma$ makes rewards more sensitive to score differences.

Table 3: Effect of $\gamma$ on reward concentration (Gini coefficient).

| $\gamma$ | Mean endorsements $E$ | Revenue | Gini of $R_i$ |
|---|---|---|---|
| 0.10 | 22,000 | \$8.7M | 0.22 |
| 0.30 | 25,000 | \$9.1M | 0.35 |
| 0.60 | 26,500 | \$9.3M | 0.52 |

As $\gamma$ rises, endorsement effort and revenue both increase modestly, but the distribution of rewards becomes more unequal. This aligns with contest theory: stronger contests elicit more effort from high-skill participants but create more dispersion in payoffs.[7]

### 5.6 Scenario 4: Endorsement budget $B$ and signal quality

Finally, we vary the endorsement budget $B$ per user while keeping $\lambda = 0.3$ and $\gamma = 0.3$.

Table 4: Effect of endorsement budget $B$ (illustrative).

| $B$ | Mean endorsements $E$ | Mean success rate (H) | Mean revenue |
|---|---|---|---|
| 1 | 9,000 | 0.62 | \$8.5M |
| 3 | 25,000 | 0.66 | \$9.1M |
| 5 | 40,000 | 0.67 | \$9.2M |
| 10 | 70,000 | 0.67 | \$9.1M |

As $B$ increases from 1 to 3, both endorsement volume and success probabilities rise substantially. Beyond $B = 5$, marginal improvements diminish: endorsements become less scarce, and additional

[7] On the tradeoff between effort and equality in contests, see Dechenaux et al. (2012).

endorsements add little new information. This suggests an optimal range for $B$ in which endorsements are informative but not overly constrained.

### 5.7 Summary of simulation insights

Across these scenarios, the simulations show that:

- Endorsements can significantly increase success rates for high- and medium-quality companies.
- This translates into higher fee revenue even after allocating a substantial share to reward pools.
- Moderate pool shares ($\lambda \approx 0.2$–$0.3$) appear to balance margin and engagement.
- Contest intensity and endorsement budgets have non-linear effects, with interior optima.

These findings are qualitatively consistent with both contest theory and empirical work on social proof: rewarding early, accurate visibility can simultaneously improve platform economics and create meaningful upside for informed supporters, without tying rewards to issuer equity or securities.

## A Appendix A: Mathematical Derivations

This appendix provides derivations and supporting results for the Bayesian belief model, endorsement behavior, contest mechanism, and platform-level optimization.

### A.1 A.1 Bayesian posterior with private and social signals

Recall that company $j$ has type $\theta_j \in \{\mathrm{H}, \mathrm{M}, \mathrm{L}\}$ with prior probabilities $(\pi_H, \pi_M, \pi_L)$. User $i$ observes a private signal $s_{ij}$ and a summary of social information $K_{ij}(t)$, the number (or weighted count) of neighbor endorsements for company $j$ up to time $t$.

Conditional on type, private signals satisfy

$$s_{ij} \mid \theta_j = k \sim f_k(s), \qquad k \in \{\mathrm{H}, \mathrm{M}, \mathrm{L}\},$$

and social information satisfies

$$K_{ij}(t) \mid \theta_j = k \sim h_{k,t}(k_{ij}),$$

where $h_{k,t}$ is a type-dependent mass function capturing how likely we are to see a given number of endorsements from neighbors when the company is of type $k$. In simulations, we often take $h_{k,t}$ to be Poisson or binomial with type-specific parameters.

By Bayes' rule, the posterior probability that $\theta_j = k$ conditional on $\mathcal{I}_{ij}(t) = (s_{ij}, K_{ij}(t))$ is

$$\pi_{ij}^k(t) = \mathbb{P}(\theta_j = k \mid s_{ij}, K_{ij}(t)) = \frac{\pi_k f_k(s_{ij}) h_{k,t}(K_{ij}(t))}{\sum_{k' \in \{\mathrm{H},\mathrm{M},\mathrm{L}\}} \pi_{k'} f_{k'}(s_{ij}) h_{k',t}(K_{ij}(t))}. \tag{22}$$

This corresponds to the reduced-form expression (5) in the main text, with

$$g_k(K_{ij}(t)) \equiv h_{k,t}(K_{ij}(t)).$$

Conditional on $\theta_j$, the success probability is $\phi(\theta_j, A_j)$, where $A_j$ is aggregate endorsement intensity. The posterior success belief $p_{ij}(t)$ is:

$$p_{ij}(t) = \mathbb{P}(Z_j = 1 \mid s_{ij}, K_{ij}(t)) = \sum_{k \in \{\mathrm{H},\mathrm{M},\mathrm{L}\}} \pi_{ij}^k(t)\, \phi(k, A_j(t)). \tag{23}$$

## A.2 A.2 Monotonicity of posteriors in private and social signals

Assume the monotone likelihood-ratio property (MLRP):

$$\frac{f_{\mathrm{H}}(s)}{f_{\mathrm{L}}(s)} \text{ is strictly increasing in } s, \qquad \frac{f_{\mathrm{M}}(s)}{f_{\mathrm{L}}(s)} \text{ is strictly increasing in } s,$$

and suppose that for the social term:

$$\frac{h_{\mathrm{H},t}(k)}{h_{\mathrm{L},t}(k)} \text{ is strictly increasing in } k, \qquad \frac{h_{\mathrm{M},t}(k)}{h_{\mathrm{L},t}(k)} \text{ is strictly increasing in } k,$$

so that both higher signals and more neighbor endorsements are "good news" in the sense of higher likelihood for high-quality types. Under these conditions, the posterior probability $\pi_{ij}^{\mathrm{H}}(t)$ is increasing in both $s_{ij}$ and $K_{ij}(t)$ in the usual sense of Bayesian monotonicity.[8]

To see this, fix $K_{ij}(t)$ and consider the odds ratio

$$\frac{\pi_{ij}^{\mathrm{H}}(t)}{\pi_{ij}^{\mathrm{L}}(t)} = \frac{\pi_H f_{\mathrm{H}}(s_{ij}) h_{\mathrm{H},t}(K_{ij})}{\pi_L f_{\mathrm{L}}(s_{ij}) h_{\mathrm{L},t}(K_{ij})} = \frac{\pi_H}{\pi_L} \frac{f_{\mathrm{H}}(s_{ij})}{f_{\mathrm{L}}(s_{ij})} \frac{h_{\mathrm{H},t}(K_{ij})}{h_{\mathrm{L},t}(K_{ij})}.$$

Holding $K_{ij}$ fixed, the term $\frac{h_{\mathrm{H},t}(K_{ij})}{h_{\mathrm{L},t}(K_{ij})}$ is constant, and the MLRP in $f_{\mathrm{H}}/f_{\mathrm{L}}$ implies that the odds ratio is strictly increasing in $s_{ij}$. Similarly, holding $s_{ij}$ fixed, monotonicity of $h_{\mathrm{H},t}/h_{\mathrm{L},t}$ implies that the odds ratio is strictly increasing in $K_{ij}$. These monotone odds translate into monotone posteriors for $\pi_{ij}^{\mathrm{H}}(t)$ and, by linearity of (23), into monotone success beliefs $p_{ij}(t)$.

## A.3 A.3 Marginal effect of endorsements on expected revenue

Expected platform revenue is

$$\mathbb{E}[\mathrm{Rev}^{\mathrm{tot}}] = \alpha \sum_{j=1}^{J} \mathbb{E}\big[V_j \phi(\theta_j, A_j)\big]. \tag{24}$$

Assume $V_j$ is independent of $\theta_j$, and write $V_j$ as $V_j = \mathbb{E}[V_j] + \varepsilon_j$ with $\mathbb{E}[\varepsilon_j \mid \theta_j, A_j] = 0$. Then

$$\mathbb{E}[\mathrm{Rev}^{\mathrm{tot}}] = \alpha \sum_{j=1}^{J} \mathbb{E}[V_j] \cdot \mathbb{E}[\phi(\theta_j, A_j)].$$

The derivative with respect to $A_j$ is

$$\begin{aligned} \frac{\partial}{\partial A_j} \mathbb{E}[\mathrm{Rev}^{\mathrm{tot}}] &= \alpha \mathbb{E}[V_j] \cdot \frac{\partial}{\partial A_j} \mathbb{E}[\phi(\theta_j, A_j)] \\ &= \alpha \mathbb{E}[V_j] \cdot \mathbb{E}\left[\frac{\partial \phi(\theta_j, A_j)}{\partial A_j}\right]. \end{aligned} \tag{25}$$

Under (2), we have

$$\phi(\theta_j, A_j) = \sigma(\alpha_0 + \alpha_{\theta_j} + \beta A_j),$$

and

$$\frac{\partial \phi}{\partial A_j} = \beta \sigma(\cdot)(1 - \sigma(\cdot)) > 0.$$

[8] See Acemoglu and Ozdaglar (2011) for formal results on social learning and monotone likelihood ratios in network settings.

Thus $\frac{\partial}{\partial A_j}\mathbb{E}[\mathrm{Rev}^{\mathrm{tot}}] > 0$ as long as $\beta > 0$ and $\mathbb{E}[V_j] > 0$. Aggregating over companies, an increase in endorsement intensity $A_j$ strictly increases expected fee revenue.

If we define a summary variable $E$ measuring aggregate endorsement volume (e.g., $E = \sum_j A_j$ or $E = \mathbb{E}[\sum_j A_j]$), we can express

$$\mathbb{E}[\mathrm{Rev}^{\mathrm{tot}}] = \alpha F(E),$$

with $F'(E) > 0$ and (under logistic saturation) $F''(E) < 0$, capturing diminishing marginal returns.

## A.4 A.4 Threshold endorsement behavior

User $i$'s expected utility is

$$U_i = \mathbb{E}[R_i] - c\sum_{t,j} x_{ij}(t).$$

Consider the effect of toggling $x_{ij}(t)$ from 0 to 1 for a single company and time, holding other decisions fixed. Let $H$ denote the full history and $H_{ij}^{+}$ the history with $x_{ij}(t) = 1$ instead of 0. Then

$$\Delta_i(j,t) = \mathbb{E}[R_i \mid H_{ij}^{+}] - \mathbb{E}[R_i \mid H].$$

Suppose the reward rule is such that $R_i$ is strictly increasing in user $i$'s score $S_i$, and $S_i$ is strictly increasing in $Z_j$ conditional on endorsing $j$. Then, for fixed $A_j(t)$ and reward parameters, the expected marginal reward from endorsing $j$ increases with $p_{ij}(t)$. Formally, if $R_i = f(S_1, \dots, S_N)$ with $\partial f/\partial S_i > 0$ and $S_i = S_i(H) + x_{ij}(t)\tilde{S}_{ij}(Z_j)$ for some $\tilde{S}_{ij}$ increasing in $Z_j$, we can write

$$\Delta_i(j,t) = \mathbb{E}[f(S_i + \tilde{S}_{ij}(Z_j), S_{-i}) - f(S_i, S_{-i})]$$

and, taking expectations over $Z_j$, obtain a function that is monotone in $p_{ij}(t)$ because the distribution of $Z_j$ shifts monotonically in first-order stochastic dominance as $p_{ij}(t)$ increases.

Given this monotonicity, user $i$'s decision problem in period $t$ is a knapsack with identical costs and monotone values:

$$\max_{x_{ij}(t)\in\{0,1\}} \sum_j x_{ij}(t)(\Delta_i(j,t) - c) \quad \text{s.t.} \quad \sum_j x_{ij}(t) \le B.$$

The optimal solution is to endorse the $B$ companies with the largest $\Delta_i(j,t)$ among those with $\Delta_i(j,t) \ge c$, which is equivalent to endorsing companies with beliefs $p_{ij}(t) \ge \tau_i(t)$ for some cutoff $\tau_i(t)$, and taking the top $B$ among them.

## A.5 A.5 Contest equilibrium and comparative statics

Consider the simplified effort-based contest where each user $i$ chooses a scalar effort $e_i \ge 0$ that proxies for total endorsement activity. The score $S_i$ is random and depends on both effort and user skill $\theta_i$; for tractability, let

$$S_i = \theta_i e_i + \varepsilon_i,$$

where $\varepsilon_i$ is zero-mean noise independent across users. In a softmax contest with parameter $\gamma$, user $i$'s expected share of the pool is approximately

$$a_i(e,\theta) \approx \frac{\exp(\gamma\theta_i e_i)}{\sum_k \exp(\gamma\theta_k e_k)}.$$

Let Pool be fixed for this derivation. User $i$ chooses $e_i$ to maximize

$$U_i(e_i; e_{-i}, \theta) = a_i(e,\theta)\cdot \mathrm{Pool} - ce_i.$$

The first-order condition (ignoring corner solutions) is

$$\frac{\partial a_i}{\partial e_i} \cdot \text{Pool} = c.$$

Differentiating the softmax:

$$\frac{\partial a_i}{\partial e_i} = a_i \gamma \theta_i (1 - a_i),$$

so the condition is

$$a_i \gamma \theta_i (1 - a_i) \cdot \text{Pool} = c.$$

For interior $a_i \in (0, 1)$, this defines an implicit relationship between $a_i$ and $\theta_i$: higher $\theta_i$ (better skill) allows a larger $a_i$ at the same effort cost $c/\text{Pool}$, yielding higher equilibrium effort $e_i^*$ and score $S_i^*$. This provides a formal underpinning for the statement that better-informed users exert more effort and capture a larger share of rewards in equilibrium.[9]

## A.6 A.6 Platform optimality conditions

Recall platform profit:

$$\Pi(\alpha, \lambda, \gamma, \delta, B) = (1 - \lambda)\, \alpha F(D(\lambda, \gamma, \delta, B)) - C_{\text{fixed}},$$

where $D(\lambda, \gamma, \delta, B)$ is equilibrium endorsement volume and $F(E)$ is expected revenue as a function of $E$.

Assume:

- $F'(E) > 0$, $F''(E) < 0$ (diminishing returns),
- $\frac{\partial D}{\partial \lambda} > 0$ and $\frac{\partial^2 D}{\partial \lambda^2} \leq 0$ (pool share increases endorsements with decreasing sensitivity),
- similar monotone–concave behavior in $(\gamma, \delta, B)$ locally.

Then, for fixed $(\alpha, \gamma, \delta, B)$, the first derivative of $\Pi$ with respect to $\lambda$ is

$$\frac{\partial \Pi}{\partial \lambda} = -\alpha F(D(\lambda, \cdot)) + (1 - \lambda) \alpha F'(D) \frac{\partial D}{\partial \lambda}. \tag{26}$$

At $\lambda = 0$, the slope is

$$\left.\frac{\partial \Pi}{\partial \lambda}\right|_{\lambda=0} = -\alpha F(D(0)) + \alpha F'(D(0)) \left.\frac{\partial D}{\partial \lambda}\right|_{\lambda=0}.$$

If $F'(D(0)) \frac{\partial D}{\partial \lambda}|_{\lambda=0} > F(D(0))$, then the derivative is positive and $\Pi$ initially increases with $\lambda$; that is, allocating a small portion of revenue to rewards increases profit via its effect on endorsements and success.

As $\lambda \to 1$, the prefactor $(1 - \lambda)$ forces $\Pi \to 0 - C_{\text{fixed}}$, so $\frac{\partial \Pi}{\partial \lambda} < 0$ near 1. By continuity, there is an interior maximizer $\lambda^* \in (0, 1)$. Similar reasoning applies to $\gamma$ and $\delta$, leading to interior optimal contest intensity and performance-share parameters under reasonable concavity conditions.

[9] All-pay contest models generally deliver monotone effort in ability under regularity conditions; see Dechenaux et al. (2012) for a survey.

# B Appendix B: Simulation and Estimation Protocols

This appendix specifies the simulation and estimation protocols used for the numerical experiments in Section 5. We separate (i) Monte Carlo simulation of the endorsement economy, (ii) grid-based sensitivity analysis, and (iii) Bayesian parameter estimation via MCMC.

## B.1 B.1 State representation and data structures

We simulate the platform over a finite horizon with the following core objects:

- **Company state:**

$$\theta_j \in \{\mathrm{H}, \mathrm{M}, \mathrm{L}\}, \quad V_j \in \mathbb{R}_+, \quad A_j(t) \in \mathbb{R}_+, \quad Z_j \in \{0, 1\}.$$

- **User state:** private signals $\{s_{ij}\}$, neighborhood sets $N(i)$, endorsement decisions $x_{ij}(t)$, scores $S_i$, and rewards $R_i$.
- **Global parameters:** $(\alpha_0, \alpha_{\mathrm{H}}, \alpha_{\mathrm{M}}, \alpha_{\mathrm{L}}, \beta)$ for $\phi(\theta, A)$; fee rate $\alpha$; pool share $\lambda$; contest parameters $(\gamma, \delta)$; endorsement budget $B$; and cost $c$.

All simulations are implemented as discrete-time updates over $t = 0, \ldots, T-1$, with final outcome realization at $T$.

## B.2 B.2 Full Monte Carlo simulation protocol

We expand the simulation algorithm sketched in Section 5 into a more detailed multi-step protocol.

**Algorithm B.1: Full Monte Carlo simulation of the endorsement economy**

**Input:** $J, N_{\text{act}}, B, T, \alpha, \lambda, \gamma, \delta, \rho$; distributions for $\theta_j$, $V_j$, $s_{ij}$.

**Output:** empirical distributions of success rates, revenue, pool, rewards, and endorsement patterns.

**For** run $k = 1$ to $K$ **do:**

1. *Company initialization:*
   - Draw $\theta_j \sim \{\pi_H, \pi_M, \pi_L\}$ for each $j$.
   - Draw $V_j$ from a log-normal distribution conditional on $\theta_j$.
   - Set $A_j(0) \leftarrow 0$ for all $j$.
2. *User initialization:*
   - Sample $N_{\text{act}}$ active users from $\{1, \dots, N\}$.
   - Generate a random social graph $G$ over active users with specified expected degree.[a]
   - For each active user $i$ and company $j$, draw a private signal $s_{ij}$ from $f_{\theta_j}$.
3. *Endorsement phase:* For $t = 0$ to $T - 1$:
   (a) For each user $i$ and company $j$, construct $K_{ij}(t)$ as the number of endorsements for $j$ by neighbors $N(i)$ in periods $< t$.
   (b) Update posteriors $\pi^k_{ij}(t)$ using (22); compute success beliefs $p_{ij}(t)$ via (23).
   (c) For each user $i$, select up to $B$ companies with the highest $p_{ij}(t)$ above a cutoff $\tau_i(t)$ and set $x_{ij}(t) = 1$ for those, 0 otherwise.
   (d) Update aggregate endorsements:
   $$A_j(t+1) \leftarrow A_j(t) + \sum_i w_i x_{ij}(t).$$
4. *Outcome phase:*
   - For each company $j$, compute success probability $p^{\text{succ}}_j = \phi(\theta_j, A_j(T))$ using (2).
   - Draw $Z_j \sim \text{Bernoulli}(p^{\text{succ}}_j)$.
   - Compute fee revenue: $\text{Fee}_j = \alpha V_j Z_j$ and $\text{Rev}^{\text{tot}} = \sum_j \text{Fee}_j$.
   - Set $\text{Pool} = \lambda\, \text{Rev}^{\text{tot}}$.
5. *Scoring and rewards:*
   - Compute per-endorsement scores $S_{ij}(t)$ and total scores $S_i$ using Algorithm 1.
   - Allocate rewards $R_i$ via Algorithm 2 or 3.
6. *Record statistics:*
   - Store success rates for H/M/L types, total $\text{Rev}^{\text{tot}}$, pool Pool, margin $(1-\lambda)\text{Rev}^{\text{tot}}$, empirical distribution of $R_i$, and endorsement patterns (e.g., endorsements per company by type).

**End For**

Aggregate results across $K$ runs to obtain mean values and confidence intervals.

[a] Random graphs may follow Erdős–Rényi, configuration, or small-world models depending on desired clustering; see standard texts on random networks.

This Monte Carlo procedure allows us to generate the illustrative tables in Section 5 and to examine sensitivity to parameters.

## B.3 B.3 Grid-based sensitivity analysis

Beyond fixed scenarios, Seedling can conduct high-dimensional grid searches over design parameters to characterize tradeoffs between revenue, reward pools, and inequality in rewards.

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**Algorithm B.2: Grid search for design sensitivity**

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**Input:** grids $\Lambda, \Gamma, \Delta, \mathcal{B}$ for $(\lambda, \gamma, \delta, B)$, fixed $\alpha$.

**Output:** response surface of key metrics as a function of parameters.

**For each** $\lambda \in \Lambda$
**For each** $\gamma \in \Gamma$
**For each** $\delta \in \Delta$
**For each** $B \in \mathcal{B}$
Run Algorithm B.1 for $K$ simulations with parameters $(\lambda, \gamma, \delta, B)$.
Compute:

- mean fee revenue $\overline{\mathrm{Rev}}$,
- mean margin $\overline{\Pi} = (1 - \lambda)\overline{\mathrm{Rev}} - C_{\mathrm{fixed}}$,
- mean reward pool $\overline{\mathrm{Pool}}$,
- inequality measure for rewards (e.g., Gini coefficient of $\{R_i\}$),
- endorsement volume $\overline{E}$,
- success rates by type.

Store these in a table indexed by $(\lambda, \gamma, \delta, B)$.
**End For**
**End For**
**End For**
**End For**

---

This grid produces a rich response surface which can be visualized as heatmaps or slices showing, for example, the tradeoff between revenue and inequality as $\gamma$ and $\delta$ vary.

## B.4 B.4 Bayesian parameter estimation via MCMC

We now outline how Seedling could use observed data to estimate model parameters such as $(\alpha_0, \alpha_{\mathrm{H}}, \alpha_{\mathrm{M}}, \alpha_{\mathrm{L}}, \beta)$ governing $\phi(\theta, A)$. Suppose we observe, for a set of companies indexed by $j$, the triplets $(\hat{\theta}_j, A_j, Z_j)$ where $\hat{\theta}_j$ is a platform-side estimate of quality (e.g., based on founder characteristics or external ratings), $A_j$ is realized endorsement intensity, and $Z_j$ is a binary milestone outcome.

We posit a logistic regression–style model:

$$\mathbb{P}(Z_j = 1 \mid \hat{\theta}_j, A_j, \boldsymbol{\beta}) = \sigma\big(\beta_0 + \beta_1 \mathbf{1}\{\hat{\theta}_j = \mathrm{H}\} + \beta_2 \mathbf{1}\{\hat{\theta}_j = \mathrm{M}\} + \beta_3 A_j\big), \tag{27}$$

where $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3)$.

### Likelihood and prior

Given data $\mathcal{D} = \{(\hat{\theta}_j, A_j, Z_j)\}_{j=1}^{J}$, the likelihood is

$$L(\boldsymbol{\beta} \mid \mathcal{D}) = \prod_{j=1}^{J} \sigma(\eta_j)^{Z_j} \big(1 - \sigma(\eta_j)\big)^{1-Z_j},$$

where

$$\eta_j = \beta_0 + \beta_1 \mathbf{1}\{\hat{\theta}_j = \mathrm{H}\} + \beta_2 \mathbf{1}\{\hat{\theta}_j = \mathrm{M}\} + \beta_3 A_j.$$

We place a multivariate normal prior,

$$\boldsymbol{\beta} \sim \mathcal{N}(\mu_0, \Sigma_0),$$

with mean $\mu_0$ and covariance $\Sigma_0$ reflecting prior beliefs about effect sizes.

### Metropolis–Hastings algorithm

We use a Metropolis–Hastings (MH) sampler to approximate the posterior $p(\boldsymbol{\beta} \mid \mathcal{D})$.[10]

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**Algorithm B.3: Metropolis–Hastings for platform success model**

---

**Input:** data $\mathcal{D}$, prior $(\mu_0, \Sigma_0)$, proposal covariance $\Sigma_q$, iterations $M$.

**Output:** posterior sample $\{\boldsymbol{\beta}^{(m)}\}_{m=1}^{M}$.

1. Initialize $\boldsymbol{\beta}^{(0)}$ (e.g., at MLE or $\mu_0$).
2. For $m = 1$ to $M$:
   1. Propose $\boldsymbol{\beta}' \sim \mathcal{N}(\boldsymbol{\beta}^{(m-1)}, \Sigma_q)$.
   2. Compute log-likelihood ratio:
   $$\log r_L = \log L(\boldsymbol{\beta}' \mid \mathcal{D}) - \log L(\boldsymbol{\beta}^{(m-1)} \mid \mathcal{D}).$$
   3. Compute log-prior ratio:
   $$\log r_P = \log p(\boldsymbol{\beta}') - \log p(\boldsymbol{\beta}^{(m-1)}).$$
   4. Set $\log r = \log r_L + \log r_P$ and acceptance probability $a = \min\{1, \exp(\log r)\}$.
   5. Draw $u \sim \mathrm{Uniform}(0,1)$. If $u < a$, set $\boldsymbol{\beta}^{(m)} \leftarrow \boldsymbol{\beta}'$, else set $\boldsymbol{\beta}^{(m)} \leftarrow \boldsymbol{\beta}^{(m-1)}$.
3. Discard burn-in and thin the chain as needed.

---

Given posterior samples of $\boldsymbol{\beta}$, Seedling can generate posterior predictive distributions of success probabilities as a function of $(\hat{\theta}, A)$ and feed these into both simulation and reward-design analysis.

## B.5 B.5 Joint simulation–estimation loop

Seedling can combine simulation and MCMC estimation in a "posterior predictive planning" loop:

- Use observed historical data to estimate $\boldsymbol{\beta}$ via Algorithm B.3.
- Condition on posterior draws of $\boldsymbol{\beta}$ to define $\phi(\theta, A)$.

[10] Standard references for Bayesian logistic regression via MCMC include Gelman et al. (2013).

- Run Algorithm B.1 using these $\phi$ functions to simulate the impact of design choices $(\lambda, \gamma, \delta, B)$.
- Use Algorithm B.2 to explore the parameter space and select mechanisms that optimize a composite objective like (21).

This constitutes a Bayesian decision-theoretic approach: platform policy is chosen not under a single point estimate but under a posterior distribution over model parameters, improving robustness.

### B.6 B.6 Implementation complexity

The computational cost of the full pipeline is dominated by:

- Monte Carlo simulation: $O(K\,T\,N_{\text{act}}\,\bar{d})$, where $\bar{d}$ is average degree in the social graph.
- MCMC estimation: $O(MJ)$ for $M$ MH iterations and $J$ companies.
- Grid search: multiplicative in the size of $(\Lambda, \Gamma, \Delta, \mathcal{B})$.

In practice, Seedling can use parallelization across runs $k$ and across grid points to maintain reasonable runtimes.

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