A Factor Model Based Allocation

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1. Introduction

In this project, we build an investment strategy that maximizes the return of the portfolio subject to a constraint of target beta, which is the usual single factor market risk measure. We reallocate the portfolio on the first trading day each week from August 2007 to November 2019. We evaluate performance of each strategy on cumulative return, Conditional VaR, volatility, and Sharpe ratio to select the optimal strategy.

2. Purpose

The purpose of this project is to understand, analyze and compare the behavior of the Long/Short Macro Strategy, using the French Fama 3-Factor model, with factors momentum, value, and size. It is aimed to evaluate its sensitivity to different lengths of look-back periods and target beta during several historical periods: before the subprime (2008) crisis, during that crisis, and after the crisis. The length of the look-back period is considered in the following three cases: a long look-back period (>120 days), a short look-back period (<40 days) and a medium look-back period defining a term-structure for the covariance and expected return.

3. Construction

3.1 Portfolio Components

For practical considerations, the project assumes that the universe of investment is a set of ETFs large enough to represent the World global economy. The benchmark in this project is S&P 500 Index, which is represented by SPY. The ETFs are listed as below:

1. CurrencyShares Euro Trust (FXE)
2. iShares MSCI Japan Index (EWJ)
3. SPDR GOLD Trust (GLD)
4. Powershares NASDAQ-100 Trust (QQQ)
5. SPDR S&P 500 (SPY)
6. iShares Lehman Short Treasury Bond (SHV)
7. PowerShares DB Agriculture Fund (DBA)
8. United States Oil Fund LP (USO)
9. SPDR S&P Biotech (XBI)
10. iShares S&P Latin America 40 Index (ILF)
11. iShares MSCI Pacific ex-Japan Index Fund (EPP)
12. SPDR DJ Euro Stoxx 50 (FEZ)

3.1.1 Financial Crisis Periods

Before Crisis	08/2007 – 06/2008
During Crisis	07/2008 – 06/2009
After Crisis	07/2009 – 11/2019

3.1.2 Term Structure

Long-Term (LT)	120 Days
Mid-Term (ST)	90 Days
Short-Term (ST)	40 Days

3.1.3 Notations

Notation	Explanation
$\boldsymbol{r_i}$	Daily Return of Security i
$\boldsymbol{r_f}$	Daily Risk-free Rate
$\boldsymbol{\beta_i^3}$	Regression Coefficient for Market Factor
$\boldsymbol{\beta_i^s}$	Regression Coefficient for SMB Factor
$\boldsymbol{r_M}$	Daily Market Return
$\boldsymbol{\rho_{SMB}}$	Daily Return of SMB Factor
$\boldsymbol{\beta_i^v}$	Regression Coefficient for HML Factor
$\boldsymbol{\varepsilon_i}$	Residual Term of Regression
$\boldsymbol{\beta_i^m}$	Beta of Security i
$\boldsymbol{\rho}$	Vector of Average Returns of All Securities
$\boldsymbol{w}$	Optimal Weight of Portfolio for Current Week
$\boldsymbol{w_p}$	Optimal Weight of Portfolio for Previous Week
$\boldsymbol{\beta}$	Vector of Betas of All Securities
$\boldsymbol{\beta_T^m}$	Target Beta of Portfolio
$\boldsymbol{\Sigma}$	Covariance of Average Returns of All Securities
$\boldsymbol{e}$	Identity Vector
$\boldsymbol{\lambda}$	Turnover Limitation Factor
$\boldsymbol{\rho_{HML}}$	Daily Return of HML Factor

3.2 Strategy

3.2.1 Strategy Description

We first divide the whole period into three phases: before crisis, during crisis and after crisis. We build a factor model to find the optimized weight for the portfolio and rebalance it each week. There are three important parameters in our strategy: the time estimator for expected return, the time estimator for covariance and target beta of portfolio. The values of them will affect the result of optimal weights. With different combination of these three factors, we can build different strategy and check the performance of our portfolio. For example, a strategy $S_{60}^{40}(0.5)$ means at each week, we will look back for 4o days to do the regression of returns and get the expected return ρ for the portfolio. Afterwards we will look back for 60 days to do the regression of returns and get the covariance of expected return Σ, and 0.5 is the target beta of our portfolio. Furthermore, we use these values to solve the quadratic optimization problem. Finally, we record the daily return for the three phases and calculate the required financial statistics to analyze the performance of different strategies.

3.2.2 Functions

i) Regression

The first step in the model is to do the regression. The regression does the following three important tasks:

Firstly, it predicts the return of a security based on the predefined term-structure estimator. According to the French-Fama data, the independent variables consist of market factor, SMB factor and HML factor. The dependent variable is real security return minus risk-free rate, which show as the function below:

$$(r_i - r_f) = \beta_i^3 * (r_M - r_f) + \beta_i^s * \rho_{SMB} + \beta_i^v * \rho_{HML} + \varepsilon_i$$

Based on this function, we can forecast the return of security i. And then compute the expected return of this security.

Secondly, regression predicts the sigma of a security based on another time estimator in our strategy. Again from the above function, this difference is the look- back period we used. In same process, we can produce the expected return of this security.

Thirdly, we use regression to compute the beta between market and security i. In French-Fama table, we obtain columns of market return and risk-free rate. Thus, we can produce the daily market volatility. After that, we will use the following formula to compute the beta. The length of data is same as the covariance time estimator.

$$\beta_i^m = \frac{cov(r_i, r_M)}{\sigma^2(r_M)}$$

We will do this regression for each security on each week. Then we can get two list of expected return, the first is rho. In addition, the second list of expected return is used to compute the covariance. In the end, we can get the inputs required by quadratic solvers.

ii) Portfolio optimization model

The portfolio optimization problem is to find the optimal weights of the stocks to maximize the utility function given as follows. ρ is expected returns and Σ is covariance matrix from previous French- Fama factor model.

$$max\ \rho^T w - \lambda(w - w_p)^T \Sigma (w - w_p)$$

$$s.t. \quad \beta^T w = \beta_T^m$$

$$w^T e = 1, -2 \le w_i \le 2$$

This is a quadratic problem on w and we can transform this function into a neat format as

$$max\ -\lambda \mathrm{w}^T \Sigma w + (\rho^T + 2\lambda w_p^T \Sigma) + \mathrm{w_p}^T \Sigma w_p$$

$$s.t. \quad \beta^T w = \beta_T^m$$

$$w^T e = 1, -2 \le w_i \le 2$$

We utilize the quadratic problem solver from a library called *cvxopt* to find optimal solution after transforming the problem into a convex function and minimize it. We ignore the last constraint because a constant will not affect our solution. Therefore, we rewrite the problem as below:

$$min\ \ \lambda w^T \Sigma w - (\rho^T + 2\lambda w_p^T \Sigma) w$$

$$s.t. \quad \beta^T w = \beta_T^m$$

$$w^T e = 1, -2 \le w_i \le 2$$

iii) Rebalance

We rebalance our portfolio on the first trading day every week and get optimal weights for each strategy. On the first day of each trading week, we used the optimal weight to buy instruments and hold the same shares for a week. Then we multiply shares by prices to obtain portfolio's capital for each strategy, and compute daily return.

4 Backtesting

In back testing part, we go through all the steps described in function section. For each strategy, we obtain three list of daily return for the three phases in order to do the further analysis. The table below summarizes all the strategy in use:

Strategy	Estimator for Return	Estimator for Covariance	Target Beta
S_{40}^{90}	40	90	-0.5
			0.5
			1.5
S_{40}^{120}	40	120	-0.5
			0.5
			1.5
S_{90}^{120}	90	120	-0.5
			0.5
			1.5

In total, we have 9 different strategies.

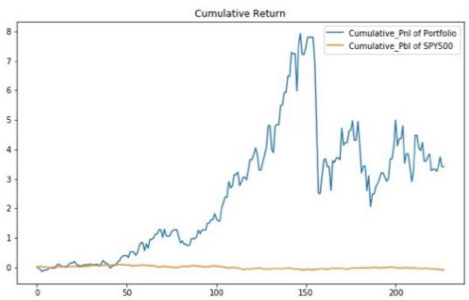
5 Analysis

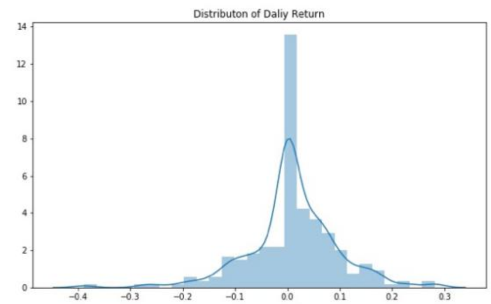
5.1Before crisis:

	S_{90}^{120}			S_{40}^{90}			S_{40}^{120}			SPY
Beta	-0.500	0.500	1.500	-0.500	0.500	1.500	-0.500	0.500	1.500	0.424
Conditional_VaR	3.286	3.235	3.218	2.877	2.825	2.858	2.894	2.837	2.867	-0.112
Cumulative_Return	3.737	4.204	5.465	2.174	2.503	2.550	2.333	2.480	2.522	-0.099
Daily_Mean_Arith	2.612	2.676	2.918	1.935	2.032	2.063	2.014	2.054	2.090	-0.119
Daily_Mean_Geom	1.632	1.734	1.969	1.201	1.307	1.321	1.253	1.300	1.312	0.693
Kurtosis	2.714	2.479	2.233	3.338	3.598	3.486	3.064	3.225	3.137	0.077
Max_Drawdown	5.302	5.526	6.227	2.208	2.493	2.613	2.227	2.454	2.599	-0.032

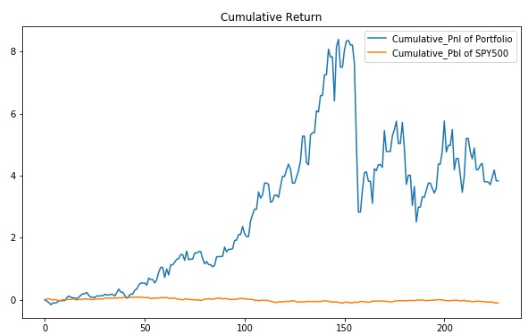
Min_Daily_Return	-0.389	-0.365	-0.346	-0.363	-0.361	-0.360	-0.363	-0.361	-0.360	0.076
Modified_Var	3.639	3.711	3.983	2.760	2.831	2.888	2.880	2.902	2.961	-0.557
Sharpe_Ratio	1.893	1.975	2.144	1.613	1.707	1.716	1.649	1.689	1.695	0.171
Skewness	-0.429	-0.428	-0.408	-0.376	-0.440	-0.497	-0.361	-0.376	-0.405	0.196
Volatility	1.375	1.350	1.356	1.194	1.184	1.196	1.215	1.210	1.227	0.424

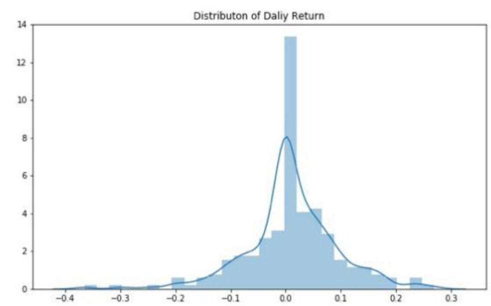
S_{90}^{120} $Beta = -0.5$



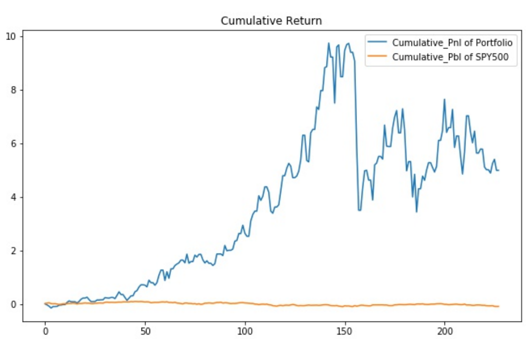


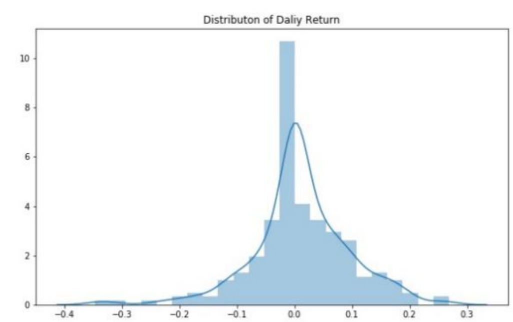
S_{90}^{120} $Beta = 1$



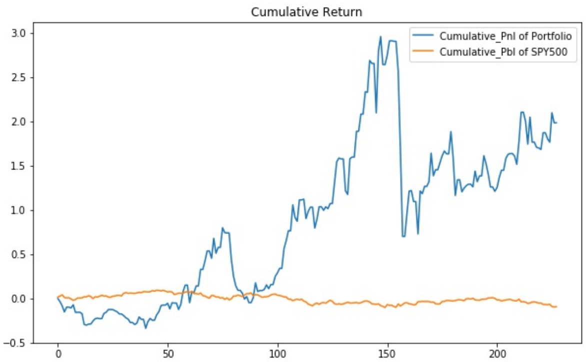


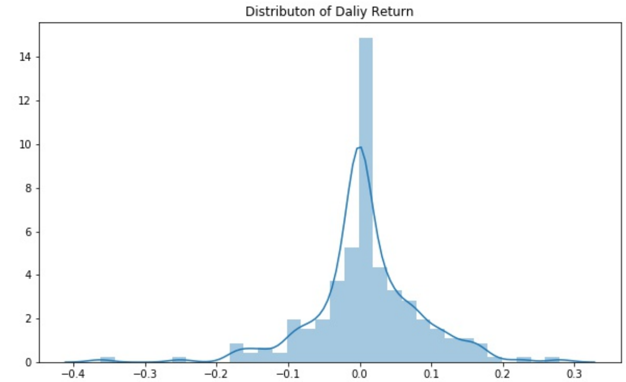
S_{90}^{120} $Beta = 1.5$



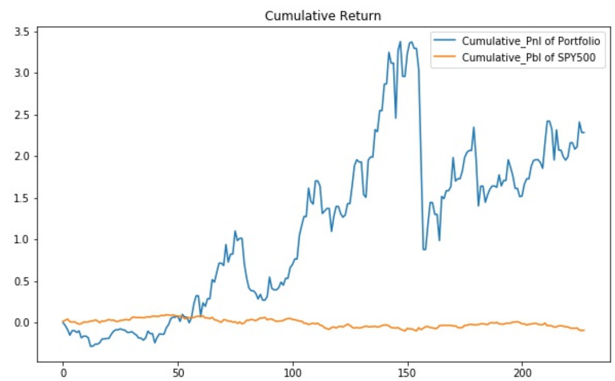


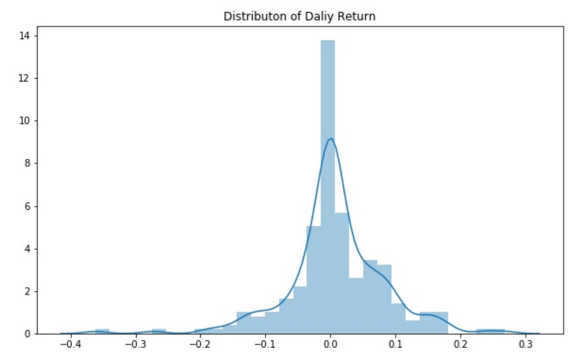
S_{40}^{90} $Beta = -0.5$



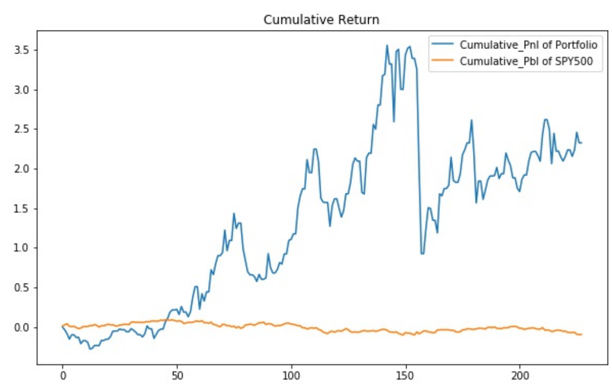


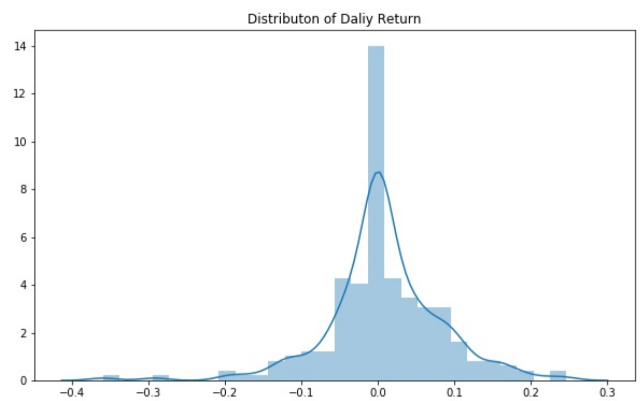
$$S_{40}^{90}\ Beta = 1$$



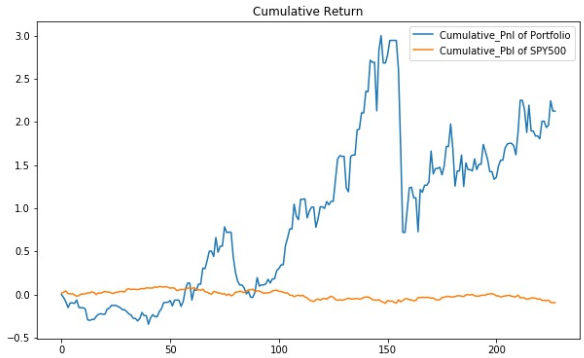


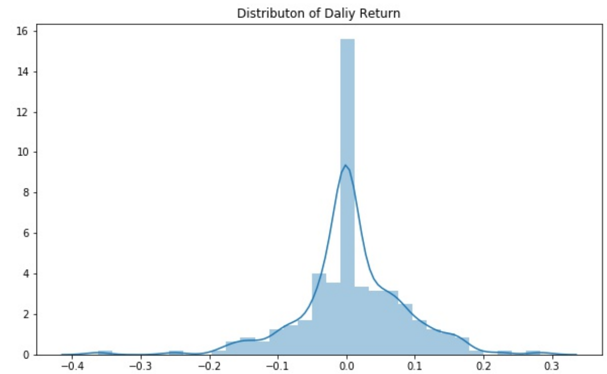
$$S_{40}^{90}\ Beta = 1.5$$



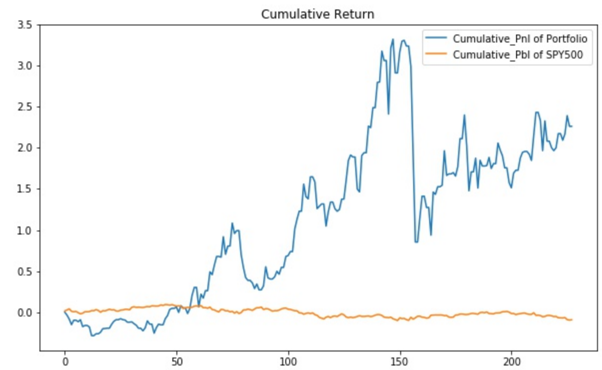


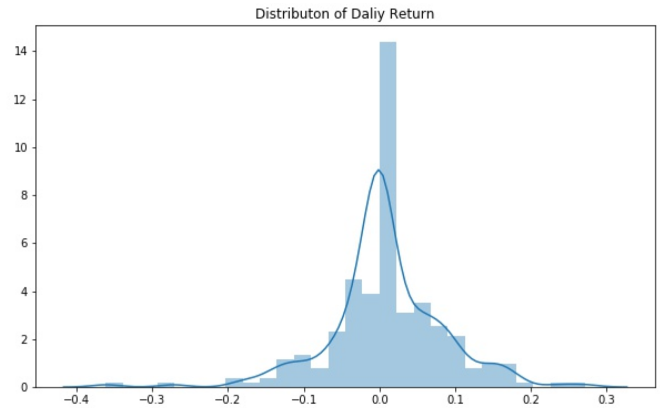
$$S_{40}^{120}\ Beta = -0.5$$



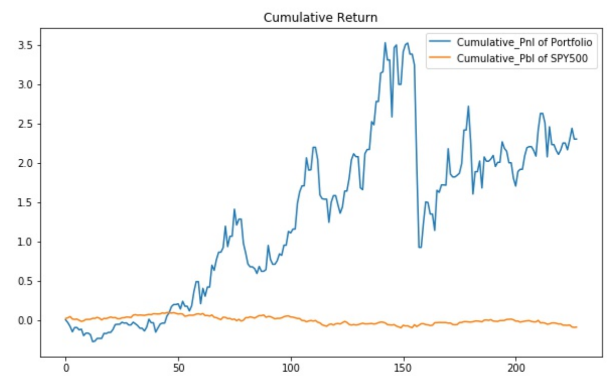


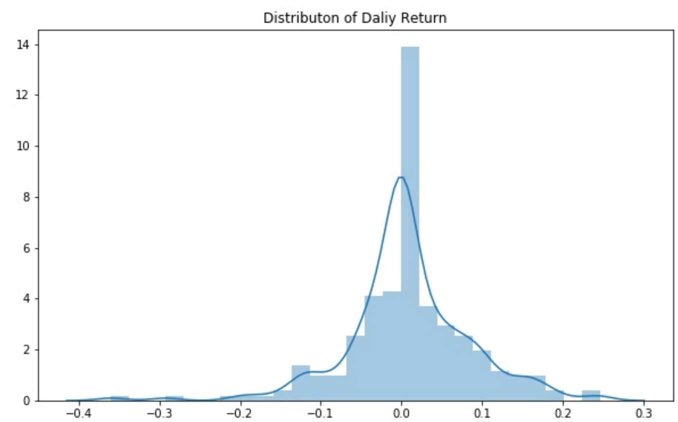
S_{40}^{120} $Beta = 1$





S_{40}^{120} $Beta = 1.5$





The geometric return and mean return of SPY500 are negative, while under each time structure they are all positive. This reflects that our strategies before crisis are profitable. From the Cumulative Pnl figure, our portfolio strategies all perform better than SPY500. It demonstrates that our strategies are efficient before cirsis. And with larger beta, we heavily short our portfolio will produce better return.

The sharpe ratio of SPY500 before crisis is positive, while the sharpe ratio under each time structure is all greater than SPY500. Under S_{90}^{120}, the sharpe ratio performs better than other strategies, and with a beta of 1.5, it performs the best result.

The volatility, VaR and Maxdrop-Down of our strategy are much bigger than those of the market because it is riskier. And with short-term return prediction we have smaller volatilities. Therefore, when considering the historical return, the nearest period is more valuable.

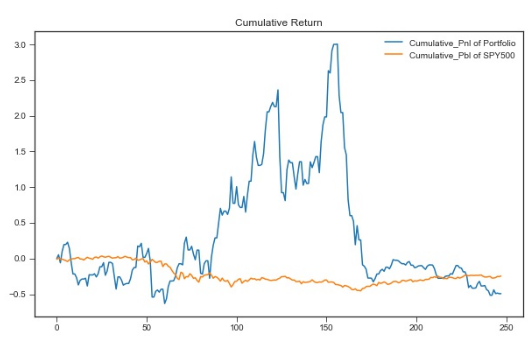
The distributions of daily return under each time structure are similar. They all have negative skewness thus a long-left tail. The returns are more likely to be less than mean return. If the strategy has high kurtosis and short-time variance, it more likely to have heavy tails and outliers. As a result, it’s better to use long-time strategy to calculate variance.

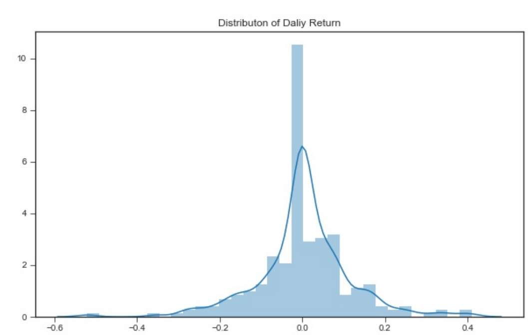
5.2During Crisis

	S_{40}^{90}			S_{40}^{120}			S_{90}^{120}			SP500
	-0.5	0.5	1.5	-0.5	0.5	1.5	-0.5	0.5	1.5	1
Conditional VaR	4.338	3.977	3.784	4.500	4.106	3.755	4.072	3.454	3.145	1.038
Cumulative Return	-0.496	-0.315	-0.137	-0.668	-0.541	-0.238	2.663	5.583	8.847	-0.281
Daily Mean Arith	1.042	1.112	1.258	0.694	0.767	1.065	2.777	3.075	3.314	-0.228
Daily Mean Geom	-0.683	-0.377	-0.147	-1.093	-0.774	-0.272	1.306	1.900	2.309	-0.330
Kurtosis	2.983	2.919	3.211	3.527	3.505	3.596	3.705	2.994	1.710	3.736
Max Drawdown	2.484	2.471	2.186	1.800	1.847	2.139	9.058	10.445	10.536	0.240
Min Daily Return	-0.522	-0.520	-0.519	-0.520	-0.519	-0.450	-0.385	-0.361	-0.323	-0.098
Modified Var	2.322	2.321	2.396	1.910	1.902	2.115	3.882	4.151	4.457	0.066
Sharpe Ratio	0.569	0.656	0.765	0.369	0.441	0.655	1.615	1.999	2.343	-0.524
Skewness	-0.138	-0.300	-0.336	0.015	-0.123	0.014	0.294	0.198	-0.114	0.425
Volatility	1.813	1.679	1.630	1.851	1.714	1.612	1.713	1.533	1.410	0.454

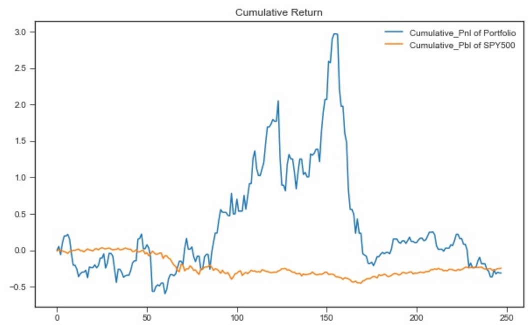
Term structure 40,90

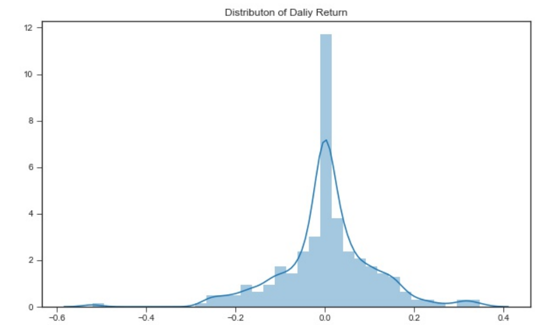
$S_{40}^{90}\ beta = -0.5$



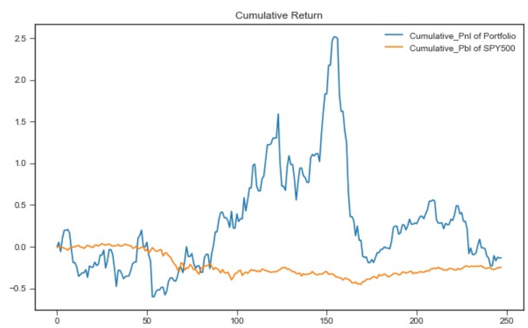


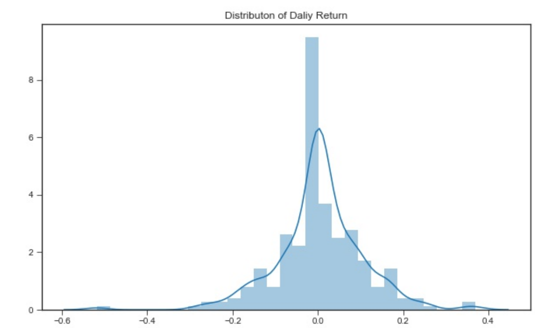
$S_{40}^{90}\ beta = 0.5$



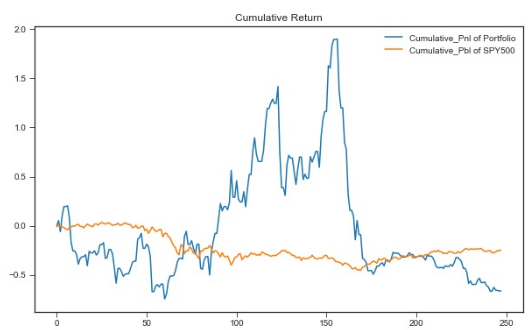


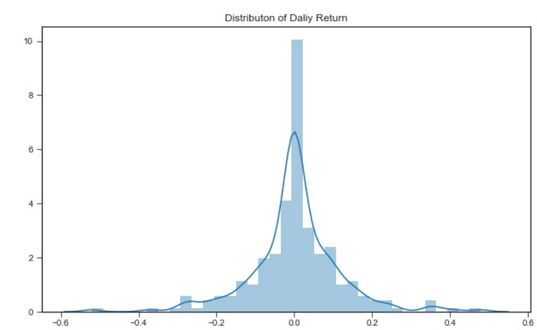
S_{40}^{90} $beta = 01.5$



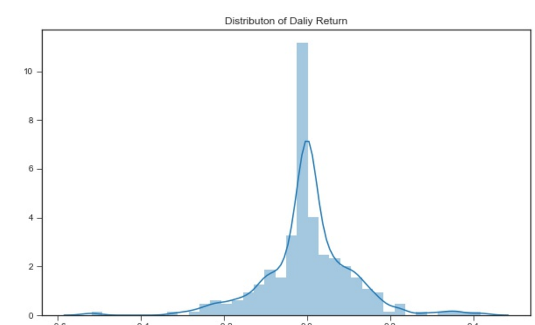


S_{40}^{120} $beta = -0.5$

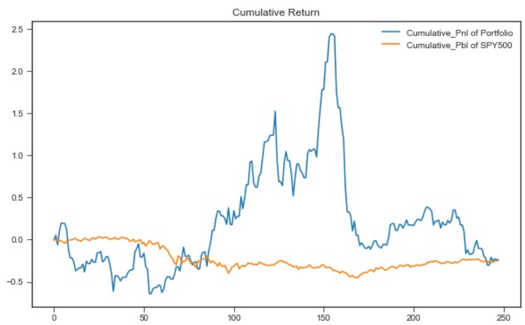


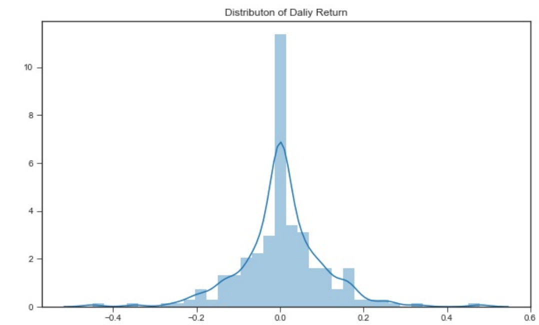


S_{40}^{120} $beta = 0.5$

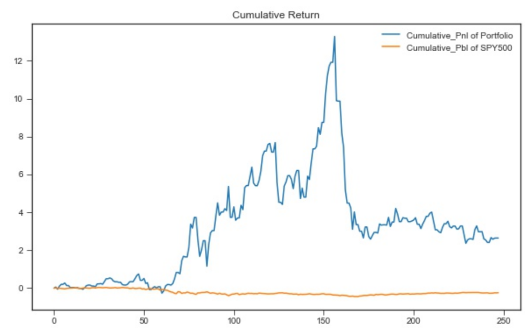


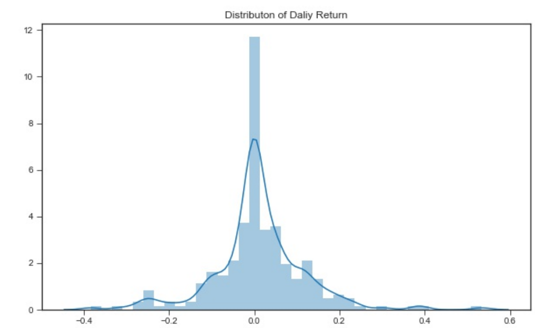
S_{40}^{120} $beta = 1.5$



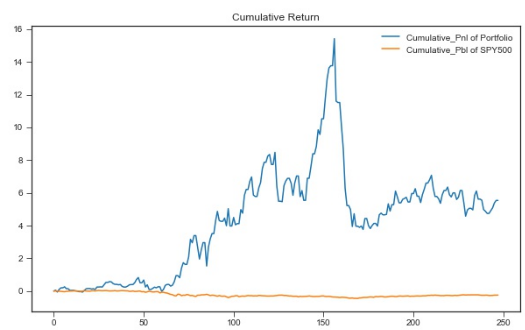


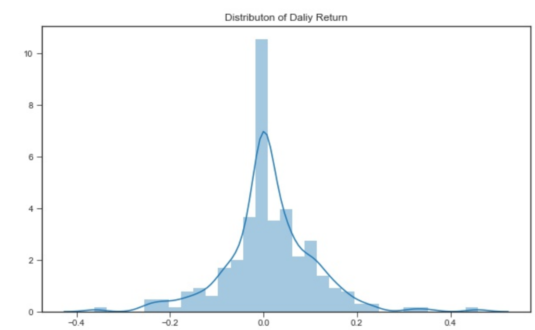
$S_{90}^{1200}\ beta = -0.5$



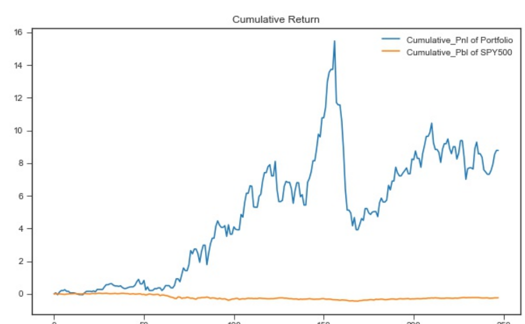


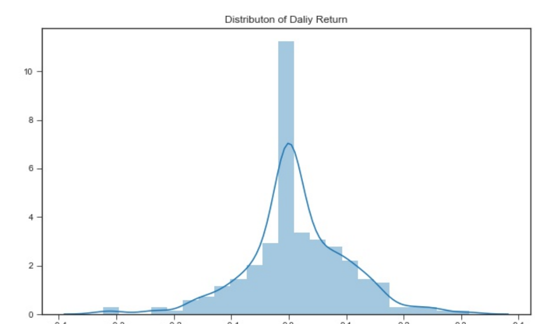
$S_{90}^{120}\ beta = 0.5$





$S_{90}^{120}\ beta = 1.5$





The optimal parameters is $S_{90}^{120}(\ \beta_T = 1.5)$ with highest sharp ratio of 2.343 during crisis. Sharp ratio is higher as target beta increasing. Sharp ratio is determined by average return and volatility. Target beta and volatility have a negative correlation. Meanwhile, there is a positive correlation between target beta and average return.

During crisis, the cumulative for SP 500 is -0.281. Though cumulative return of S_{40}^{90} and S_{40}^{120} are higher than SP500, the cumulative returns are still negative. Cumulative turns to be positive when the time structures extend to be S_{90}^{120}, that is, longer estimator for mean and covariance outperform during crisis.

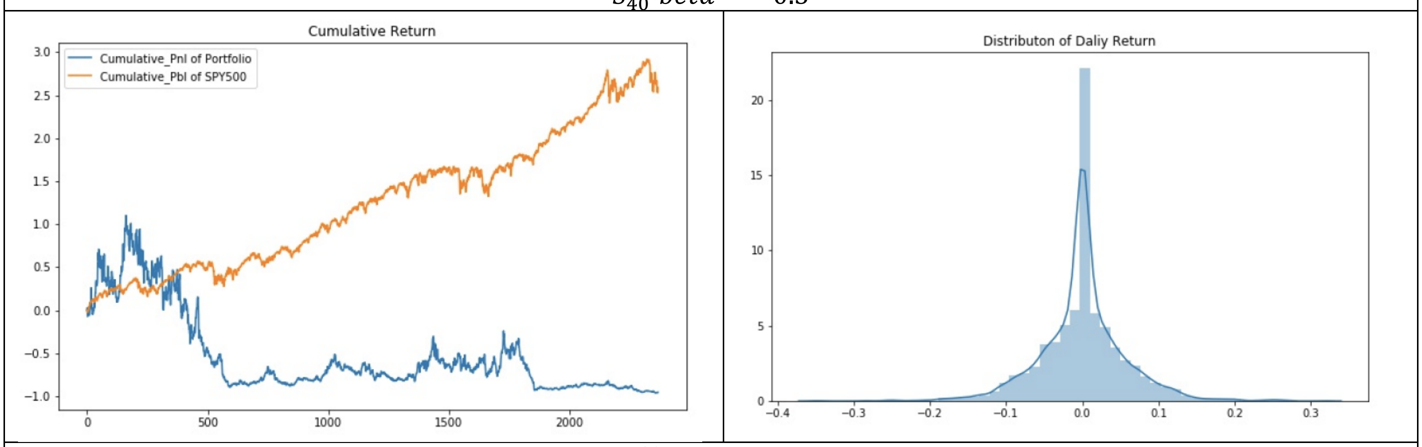
Regarding the risk, conditional VaR is highest when beta target is -0.5 and modified VaR is highest when target beta is 1.5. Under term structure of S_{40}^{120}, the kurtosis is higher, indicating more outliers.

5.3After Crisis

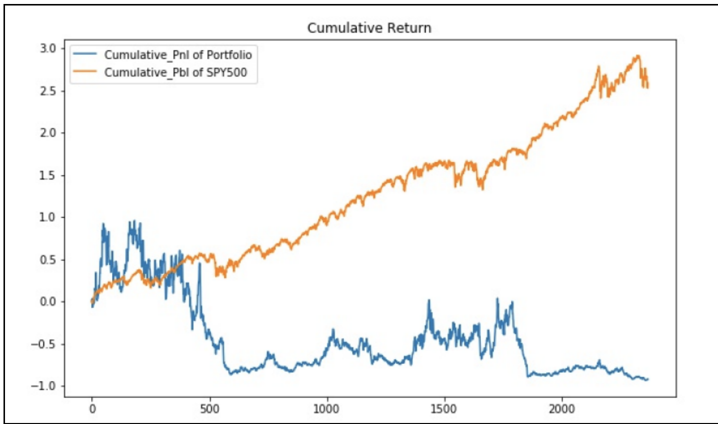
	S_{90}^{120}			S_{40}^{90}			S_{40}^{120}			SPY
Beta	-0.500	0.500	1.500	-0.500	0.500	1.500	-0.500	0.500	1.500	1
Conditional_VaR	2.156	2.146	2.168	2.088	2.096	2.157	2.044	2.046	2.110	0.363
Cumulative_Return	-0.105	-0.104	-0.102	-0.101	-0.098	-0.091	-0.091	-0.086	-0.077	0.289
Daily_Mean_Arith	-0.118	-0.022	0.080	0.087	0.135	0.215	0.195	0.216	0.271	0.150
Daily_Mean_Geom	-0.526	-0.430	-0.345	-0.322	-0.273	-0.212	-0.207	-0.178	-0.137	0.139
Kurtosis	3.300	3.180	3.043	3.471	3.469	3.741	3.323	3.400	3.550	4.050
Max_Drawdown	0.390	0.388	0.547	0.627	0.651	1.011	1.098	1.001	1.291	0.383
Min_Daily_Return	-0.341	-0.365	-0.378	-0.348	-0.352	-0.356	-0.348	-0.352	-0.357	-0.065
Modified_Var	0.491	0.596	0.721	0.682	0.731	0.804	0.797	0.806	0.862	0.244
Sharpe_Ratio	-0.143	-0.036	0.077	0.085	0.139	0.223	0.207	0.234	0.291	0.950
Skewness	-0.129	-0.121	-0.100	0.034	-0.028	-0.120	0.029	-0.037	-0.135	-0.437
Volatility	0.896	0.897	0.916	0.901	0.899	0.917	0.893	0.883	0.896	0.148

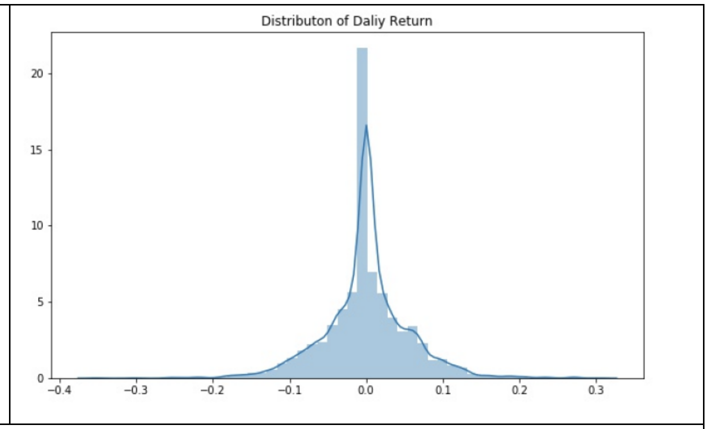
Term structure 40,90

S_{40}^{90} $beta = -0.5$

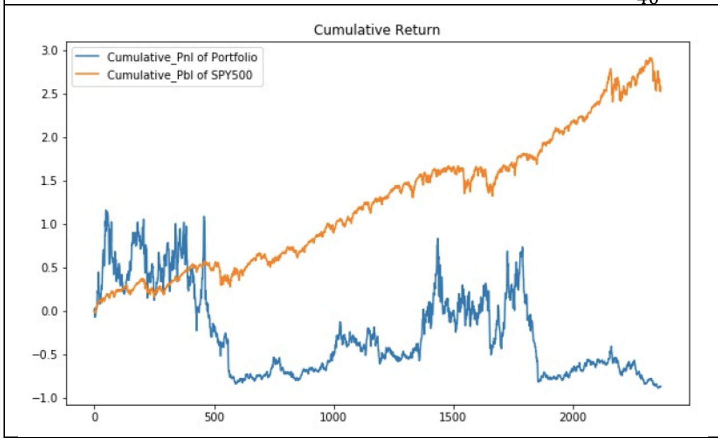


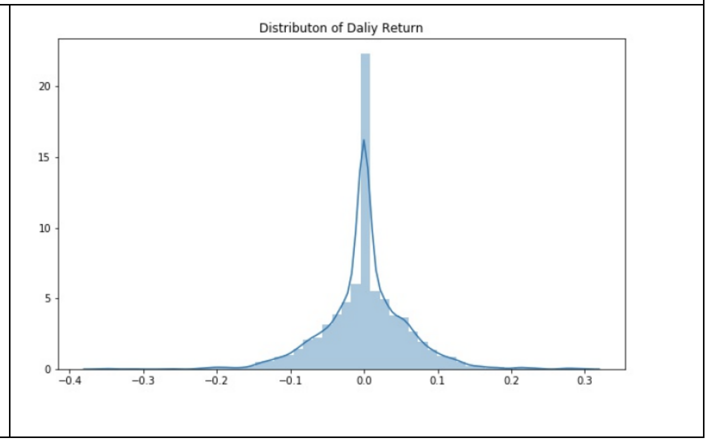
S_{40}^{90} $beta = 0.5$





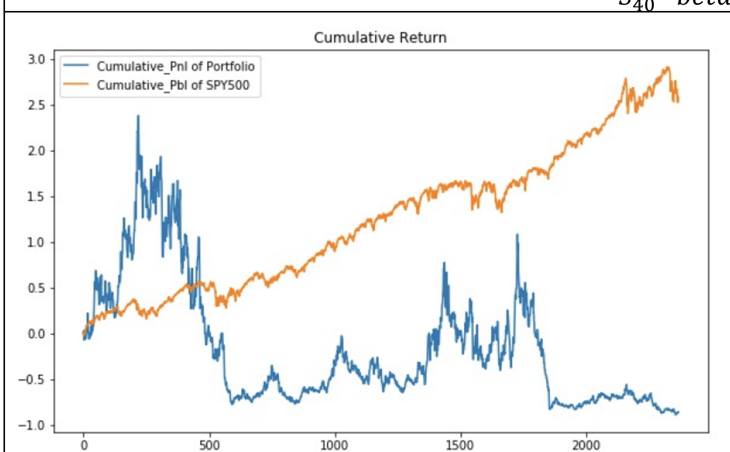
$S_{40}^{90}\ beta = 1.5$

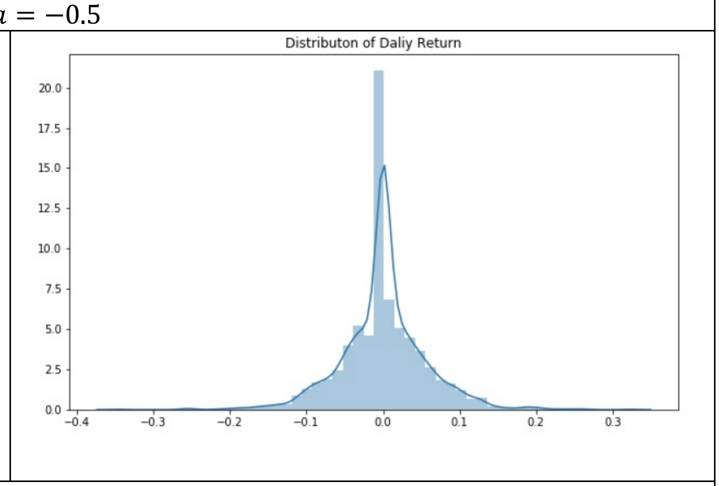




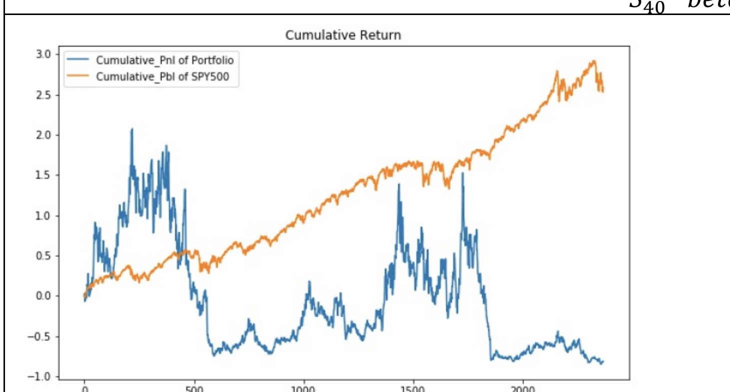
Term structure 40,120

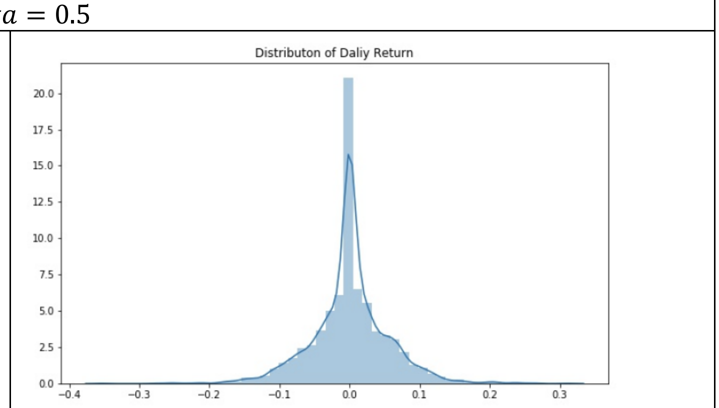
$S_{40}^{120}\ beta = -0.5$



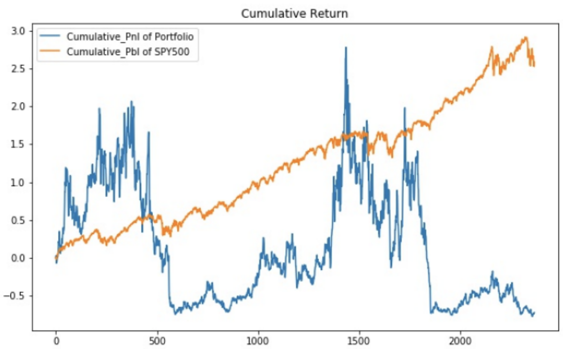


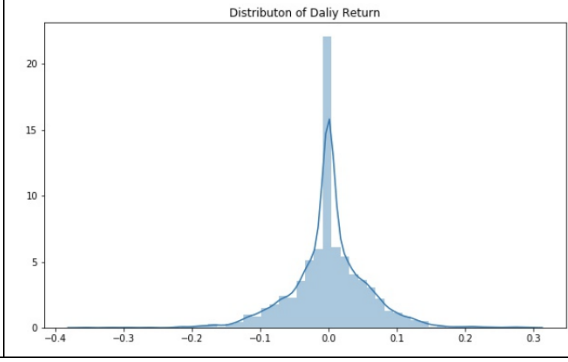
$S_{40}^{120}\ beta = 0.5$





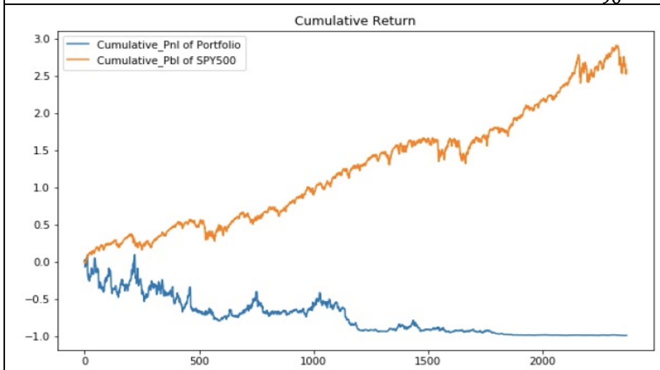
$S_{40}^{120}\ beta = 1.5$

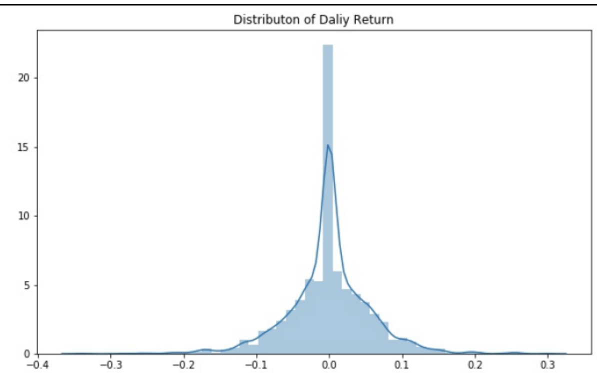




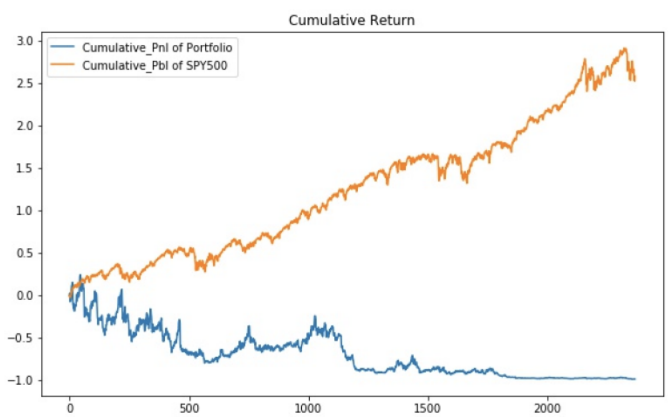
Term structure 90,120

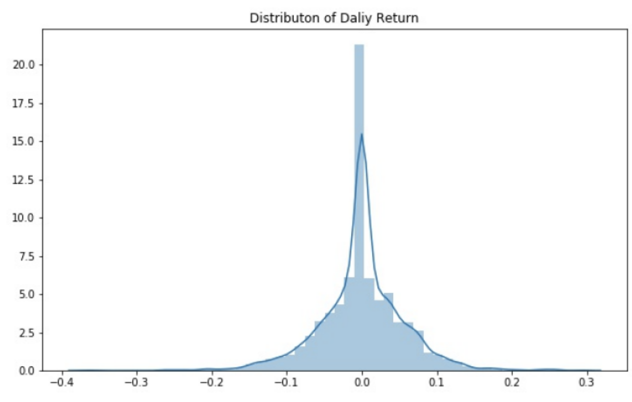
S_{90}^{1200} $beta = -0.5$



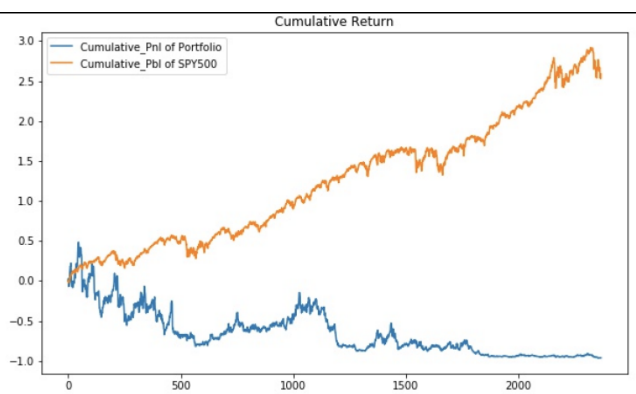


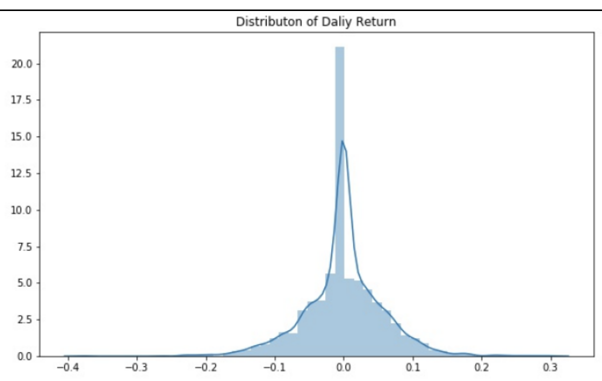
S_{90}^{120} $beta = 0.5$





S_{90}^{120} $beta = 1.5$





After crisis, the geometric return, cumulative return and mean return of SPY500 are all positive, meanwhile the portfolio returns of most strategies are negative, which means our strategies cannot beat the market after crisis.

The sharpe ratio of SPY500 after crisis is 0.95, while the sharpe ratios of all strategies are relatively smaller. Sharp ratio goes higher as target beta increases. And under the term structure S_{40}^{120} and beta of 1.5, the sharpe ratio is the highest among others. Therefore, this strategy is the best one.

The volatility,VaR and maxdrop-down of our strategies are much bigger than those of the market,which reflects that the risk of our strategies is larger than the market. Strategy S_{40}^{120} has the lowest volatility and the lowest conditional VaR.

The distributions of Daily Return under each time structure are similar. Under the term structure of S_{40}^{120} and beta of 1.5, the kurtosis is the highest and skewness is lowest. The distribution of this strategy is the closest to the distribution of SPY and this strategy has the highest Sharpe ratio.

6 Conclusion

We select the optimal strategy based on the risk-adjusted return, which is the sharpe ratio. The below table summarizes the optimal strategy for each interval.

optimal portfolio		portfolio sharpe ratio	market sharp ratio
before crisis	S_{90}^{120} , beta 1.5	2.144	0.171
during crisis	S_{90}^{120} , beta 1.5	2.343	-0.524
after crisis	S_{40}^{120} , beta 1.5	0.291	0.95

$S_{90}^{120}(\beta_T = 1.5)$ is the optimal strategy for period from 2007 August to 2009 June. This strategy produces much better result than market, especially during the financial crisis. While post crisis, the market has changed. None of our nine strategies can beat market return, and S_{40}^{120} (β=1.5) performs relatively better than other strategies.