

# **Exercise 3**

Module 1 - Introduction to Cryptography and Data Security

Knut Lucas Andersen  
Master Applied Information Security  
isits AG International School of IT Security

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## 1 Addition in $\text{GF}(2^8)$

- a)  $A(x) = x^7 + x^5 + x^3 + x^2 + 1, B(x) = x^4 + x^3 + 1$
- b)  $A(x) = x^5 + x^3 + x^2, B(x) = x^6 + x^4 + x^3 + 1$
- c)  $A(x) = x^6 + x^5 + x^3 + x + 1, B(x) = x^7 + x^6 + x^4 + x^2 + 1$
- d) Which effect has the reduction polynome in general on the result of an addition?

## 2 Multiplication in $\text{GF}(5^4)$

Consider the finite field  $\text{F}(5^4)$  with the irreducible reduction polynome  $P(x) = x^4 + x^2 + 2x + 2$ .

- a) Compute the addition table for this field
- b) Compute the multiplication table for this field.
- c) Compute  $x^4 \bmod P(x)$ ,  $x^5 \bmod P(x)$  and  $x^6 \bmod P(x)$ .
- d) Calculate  $A(x) \times B(x) \bmod P(x)$  for  $A(x) = x^4 + x^1 + 2$ ,  $B(x) = 2x^3 + 2x^2 + 1$

### 3 Multiplication in $GF(2^8)$

- a) Compute  $A(x) \times B(x) \bmod P(x)$  for the following values and give the result in HEX
- b) With which operation is it possible to realise both these multiplications  $B_1(x) = x$ ,  $B_2(x) = x+1$  efficiently

### 4 Avalanche effect in AES

- a) Calculate the respective Output to the Input W after the first round of AES! use the round-keys  $K_0, \dots, K_1$ !
- b) Compute all the output bytes for the case that all the input bytes are zero (solution only in HEX)
- c) How many outputbytes have changed now? (We consider just one round of AES)

### 5 Keygeneration in AES

- a) Given is a main key K, consisting of zeros. Find the sub-key  $K_1$  after the first round of key-generation.
- b) Given is the main key  $K = (0x00000008; 0x00000004; 0x00000002; 0x00000001)$ . Find the sub-key  $K_1$  after the first round of key-generation.

### 6 Solution template for Avalanche effect in AES

- a) After conversion of the Input in matrix-form
- b) After conversion of the Input in matrix-form