Ugeopgave 4 genaflevering af Mads Schiøler Tingsgård og Klaes Rasmussen.

Opgave cp 4.6.1

Gauss-Seidel iteration

```
> restart;
> gaussSeidel :=proc(n, A :: Matrix, b, x, M)
  local k, i, j, temp1, temp2, x1;
  x1 := copy(x);
  temp1 := 0;
  for k from 1 to M do
     for i from 1 to n do
     temp1 := 0;
        for j from 1 to n do
           if j ≠ i then
              temp1 := (temp1 + ( A[i, j] ·x1[j]));
           end if;
        end do;

        x1[i] := ((b[i] - temp1) / A[i, i]);
     end do;
     print(k, evalf(x1));
  end do;
  return;
  end proc:
```

> $a1 := Matrix(3, [3, 1, 1, 1, 3, -1, 3, 1, -5])$

$$a1 := \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -5 \end{bmatrix} \quad (1.1)$$

> $b1 := Vector([5, 3, -1]);$

$$b1 := \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix} \quad (1.2)$$

> $X := Vector([0, 0, 0]);$

$$X := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (1.3)$$

> $gaussSeidel(3, a1, b1, X, 10)$

$$1, \begin{bmatrix} 1.666666667 \\ 0.4444444444 \\ 1.288888889 \end{bmatrix}$$

$$2, \begin{bmatrix} 1.088888889 \\ 1.066666667 \\ 1.066666667 \end{bmatrix}$$

$$3, \begin{bmatrix} 0.9555555556 \\ 1.037037037 \\ 0.9807407407 \end{bmatrix}$$

$$4, \begin{bmatrix} 0.9940740741 \\ 0.9955555556 \\ 0.9955555556 \end{bmatrix}$$

$$5, \begin{bmatrix} 1.002962963 \\ 0.9975308642 \\ 1.001283951 \end{bmatrix}$$

$$6, \begin{bmatrix} 1.000395062 \\ 1.000296296 \\ 1.000296296 \end{bmatrix}$$

$$7, \begin{bmatrix} 0.9998024691 \\ 1.000164609 \\ 0.9999144033 \end{bmatrix}$$

$$8, \begin{bmatrix} 0.9999736626 \\ 0.9999802469 \\ 0.9999802469 \end{bmatrix}$$

$$9, \begin{bmatrix} 1.000013169 \\ 0.9999890261 \\ 1.000005706 \end{bmatrix}$$

$$10, \begin{bmatrix} 1.000001756 \\ 1.000001317 \\ 1.000001317 \end{bmatrix} \quad \textbf{(1.4)}$$

> $a2 := Matrix(3, [3, 1, 1, 3, 1, -5, 1, 3, -1])$

(1.5)

$$a2 := \begin{bmatrix} 3 & 1 & 1 \\ 3 & 1 & -5 \\ 1 & 3 & -1 \end{bmatrix} \quad \textbf{(1.5)}$$

> $b2 := Vector(3, [5, -1, 3])$

$$b2 := \begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix} \quad \textbf{(1.6)}$$

> $gaussSeidel(3, a2, b2, X, 10)$

$$1, \begin{bmatrix} 1.666666667 \\ -6. \\ -19.33333333 \end{bmatrix}$$

$$2, \begin{bmatrix} 10.11111111 \\ -128. \\ -376.8888889 \end{bmatrix}$$

$$3, \begin{bmatrix} 169.9629630 \\ -2395.333333 \\ -7019.037037 \end{bmatrix}$$

$$4, \begin{bmatrix} 3139.790123 \\ -44515.55556 \\ -130409.8765 \end{bmatrix}$$

$$5, \begin{bmatrix} 58310.14403 \\ -826980.8148 \\ -2.422635300\ 10^{6} \end{bmatrix}$$

$$6, \begin{bmatrix} 1.083207038\ 10^{6} \\ -1.536279862\ 10^{7} \\ -4.500519181\ 10^{7} \end{bmatrix}$$

$$7, \begin{bmatrix} 2.012266514\ 10^{7} \\ -2.853939555\ 10^{8} \\ -8.360592044\ 10^{8} \end{bmatrix}$$

$$8, \begin{bmatrix} 3.738177216\ 10^{8} \\ -5.301749188\ 10^{9} \\ -1.553142984\ 10^{10} \end{bmatrix}$$

$$9, \begin{bmatrix} 6.944393012\ 10^{9} \\ -9.849032826\ 10^{10} \\ -2.885265918\ 10^{11} \end{bmatrix}$$

$$10, \begin{bmatrix} 1.290056400\ 10^{11} \\ -1.829649879\ 10^{12} \\ -5.359943997\ 10^{12} \end{bmatrix} \quad (1.7)$$

Ved normal gaussisk elimination finder vi de første værdier, dvs $x_1 = x_2 = x_3 = 1$. Den iterative metode virker kun første gang da denne matrix er diagonal dominant. I anden omgang bliver anden og tredje række byttet om. Derfor er matricen ikke længere diagonal dominant og Gauss-Seigel konvergerer aldrig i dette tilfælde.

Opgave cp 4.6.2

Vi skal også bruge gauss-seidel på det næste system:

> $A3 := Matrix(2, [0.96326, 0.81321, 0.81321, 0.68654])$

$$A3 := \begin{bmatrix} 0.96326 & 0.81321 \\ 0.81321 & 0.68654 \end{bmatrix} \quad (2.1)$$

> $B3 := Vector([0.88824, 0.74988])$

$$B3 := \begin{bmatrix} 0.88824 \\ 0.74988 \end{bmatrix} \quad (2.2)$$

> $X3 := Vector([0.33116, 0.7])$

$$X3 := \begin{bmatrix} 0.33116 \\ 0.7 \end{bmatrix} \quad (2.3)$$

> $gaussSeidel(2, A3, B3, X3, 10)$

$$1, \begin{bmatrix} 0.3311598115 \\ 0.6999993150 \end{bmatrix}$$

$$2, \begin{bmatrix} 0.3311603897 \\ 0.6999986301 \end{bmatrix}$$

$$3, \begin{bmatrix} 0.3311609680 \\ 0.6999979451 \end{bmatrix}$$

$$4, \begin{bmatrix} 0.3311615463 \\ 0.6999972600 \end{bmatrix}$$

$$5, \begin{bmatrix} 0.3311621247 \\ 0.6999965750 \end{bmatrix}$$

$$6, \begin{bmatrix} 0.3311627029 \\ 0.6999958901 \end{bmatrix}$$

$$7, \begin{bmatrix} 0.3311632811 \\ 0.6999952052 \end{bmatrix}$$

$$8, \begin{bmatrix} 0.3311638594 \\ 0.6999945202 \end{bmatrix}$$

$$9, \begin{bmatrix} 0.3311644376 \\ 0.6999938353 \end{bmatrix}$$

$$10, \begin{bmatrix} 0.3311650159 \\ 0.6999931503 \end{bmatrix} \qquad (2.4)$$

x_1 værdien er stigende siden den første iteration, hvorimod x_2 værdien er faldende, dog går det meget langsom for begge værdier. Vi har ingen konvergens da matricen ikke er diagonal dominant.

Opgave 4.7.13

Jacobi iteration

```
> jacobi :=proc(n, A :: Matrix, b, x, M)
  local k, i, j, temp1, x1, u;
  x1 := copy(x);
  u := Vector(n);
  temp1 := 0;
  for k from 1 to M do
    for i from 1 to n do
    temp1 := 0;
      for j from 1 to n do
        if j ≠ i then
          temp1 := evalf(temp1 + (A[i,j] · x1[j]));
        end if;
      end do;

      u[i] := evalf((b[i] - temp1) / A[i,i]);
    end do;
    x1 := copy(u);
    print(k, x1);
  end do;
  return;
  end proc:
```

Conjugate gradient iteration

```
> with(ArrayTools) :
> conjugateGradient :=proc(X :: Vector, A :: Matrix, B :: Vector, M, e, s)
```

```
local r, v, c, k, z, t, x, d, i, n;
x := copy(X);
n := Size(A, 1);
r := B - A.x;
v := r;
c := r.r;
for k from 1 to M do
if (evalf(sqrt(v.v)) < s) then
print("Stopping since sqrt v.v < s");
break;
end if;
z := A.v;
t := c/(v.z);
x := x + t.v;
r := r - t.z;
d := r.r;
if d < e then
print("Stopping since d<e");
break;
end if;
v := r + (d/c)·v;
c := d;
print( k, evalf(x), evalf(r));
end do;
return;
end proc:
```

> A := Matrix(3, [2, 0, -1, -2, -10, 0, -1, -1, 4])

$$A := \begin{bmatrix} 2 & 0 & -1 \\ -2 & -10 & 0 \\ -1 & -1 & 4 \end{bmatrix} \quad \text{(3.1)}$$

> B := Vector([1, -12, 2])

$$B := \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix} \quad \text{(3.2)}$$

> X := Vector([0, 0, 0])

$$X := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(3.3)}$$

> jacobi(3, A, B, X, 2)

$$1, \begin{bmatrix} 0.5000000000 \\ 1.200000000 \\ 0.5000000000 \end{bmatrix}$$

$$2, \begin{bmatrix} 0.7500000000 \\ 1.100000000 \\ 0.9250000000 \end{bmatrix} \tag{3.4}$$

> $gaussSeidel(3, A, B, X, 2)$

$$1, \begin{bmatrix} 0.5000000000 \\ 1.100000000 \\ 0.9000000000 \end{bmatrix}$$

$$2, \begin{bmatrix} 0.9500000000 \\ 1.010000000 \\ 0.9900000000 \end{bmatrix} \tag{3.5}$$

> $conjugateGradient(X, A, B, 2, 0, 0);$

$$1, \begin{bmatrix} -0.1081277213 \\ 1.297532656 \\ -0.2162554427 \end{bmatrix}, \begin{bmatrix} 1. \\ 0.7590711176 \\ 4.054426705 \end{bmatrix}$$

$$2, \begin{bmatrix} 0.1957514198 \\ 1.109993383 \\ 0.9484621449 \end{bmatrix}, \begin{bmatrix} 1.556959305 \\ -0.5085633258 \\ -0.4881037762 \end{bmatrix} \tag{3.6}$$

Vi ser at Gauss-Seidel konvergerer hurtigere end Jacobi, da Jacobi først opdaterer hver x-værdi efter en hel iteration over alle værdier. Conjugate Gradient metoden er som man kan se meget upræcis til at begynde med. Selvom alle metoder konvergerer mod [1,1,1] ved nok iterationer.

Opgave 5.1.17

> $restart;$

> $with(LinearAlgebra):$

> $A := Matrix(2, [1, 1, 0, 1])$

$$A := \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{4.1}$$

> $X := Vector([x, y])$

$$X := \begin{bmatrix} x \\ y \end{bmatrix} \tag{4.2}$$

> $A1 := Matrix(2, [1 - l, 1, 0, 1 - l])$

$$A1 := \begin{bmatrix} 1 - l & 1 \\ 0 & 1 - l \end{bmatrix} \tag{4.3}$$

> $det(A1) = 0$

$$(1 - l)^2 = 0 \tag{4.4}$$

> $solve(det(A1) = 0, l)$

$$1, 1 \tag{4.5}$$

> $A.X = X$

$$\begin{bmatrix} x+y \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \tag{4.6}$$

> $x + y = x; y = y; y = 0;$

$$x + y = x$$
$$y = y$$
$$y = 0 \tag{4.7}$$

Vi har altså kun 1 lineær afhængig eigenvektor:

> $x.Vector([1, 0])$

$$x \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{4.8}$$

Men vi skal bruge 2 uafhængige eigenvektorer for at udspænde rummet $\mathbb{R}^2$. Derfor er matricen A defektiv.

Opgave 5.1.23

```
> restart; with(LinearAlgebra) :
> powerMethod := proc(n, A, x, M, t)
  local y, r, x1, k, yMax;
  x1 := copy(x);
  print(0, x1);
  for k from 1 to M do
  y := A.x1;
  yMax := max(abs(y));
  r := yMax / max(x1);
  if t then
  x1 := y / yMax;
  else
  x1 := evalf(y / yMax);
  end if;
  if t then
  print(k, x1, r);
  else
  print(k, evalf(x1), evalf(r));
  end if;
  end do;
  return;
  end proc:
> A := Matrix(3, [2, 0, -1, -2, -10, 0, -1, -1, 4])
```

(5.1)

$$A := \begin{bmatrix} 2 & 0 & -1 \\ -2 & -10 & 0 \\ -1 & -1 & 4 \end{bmatrix} \tag{5.1}$$

> $X := Vector([1, 1, 1])$

$$X := \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{5.2}$$

> $powerMethod(3, A, X, 2, true);$

$$0, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$1, \begin{bmatrix} \frac{1}{12} \\ -1 \\ \frac{1}{6} \end{bmatrix}, 12$$

$$2, \begin{bmatrix} 0 \\ 1 \\ \frac{19}{118} \end{bmatrix}, 59 \tag{5.3}$$

Opgave cp 5.1.1

> $Digits := 10$

$$Digits := 10 \tag{6.1}$$

> $A2 := Matrix(3, [6, 5, -5, 2, 6, -2, 2, 5, -1])$

$$A2 := \begin{bmatrix} 6 & 5 & -5 \\ 2 & 6 & -2 \\ 2 & 5 & -1 \end{bmatrix} \tag{6.2}$$

> $X2 := Vector([1, 2, 3])$

$$X2 := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \tag{6.3}$$

> $powerMethod(3, A2, X2, 100, false);$

$$0, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$1, \begin{bmatrix} 0.1111111111 \\ 0.8888888889 \\ 1. \end{bmatrix}, 3.$$

$$2, \begin{bmatrix} 0.0303030303 \\ 0.9696969696 \\ 0.9999999999 \end{bmatrix}, 3.666666667$$

$$3, \begin{bmatrix} 0.007751937984 \\ 0.992248062 \\ 0.9999999999 \end{bmatrix}, 3.909090909$$

$$4, \begin{bmatrix} 0.001949317844 \\ 0.9980506823 \\ 1.0 \end{bmatrix}, 3.976744186$$

$$5, \begin{bmatrix} 0.0004880431544 \\ 0.9995119571 \\ 1.0 \end{bmatrix}, 3.994152047$$

$$6, \begin{bmatrix} 0.0001220557829 \\ 0.9998779445 \\ 0.9999999999 \end{bmatrix}, 3.998535872$$

$$7, \begin{bmatrix} 0.00003051721795 \\ 0.9999694835 \\ 1.0 \end{bmatrix}, 3.999633834$$

$$8, \begin{bmatrix} 0.000007630376562 \\ 0.9999923708 \\ 1.0 \end{bmatrix}, 3.999908452$$

$$9, \begin{bmatrix} 0.000001909075765 \\ 0.9999980925 \\ 0.9999999999 \end{bmatrix}, 3.999977115$$

$$10, \begin{bmatrix} 4.79364333\ 10^{-7} \\ 0.9999995229 \\ 0.9999999998 \end{bmatrix}, 3.999994281$$

$$11, \begin{bmatrix} 1.229215435\ 10^{-7} \\ 0.9999998809 \\ 1.0 \end{bmatrix}, 3.999998574$$

$$12, \begin{bmatrix} 3.550731842\ 10^{-8} \\ 0.9999999704 \\ 1.0 \end{bmatrix}, 3.999999650$$

$$13, \begin{bmatrix} 1.626097799\ 10^{-8} \\ 0.9999999926 \\ 1.0 \end{bmatrix}, 3.999999923$$

$$14, \begin{bmatrix} 1.514146698\ 10^{-8} \\ 0.9999999978 \\ 0.9999999997 \end{bmatrix}, 3.999999996$$

$$15, \begin{bmatrix} 2.033720049\ 10^{-8} \\ 0.9999999996 \\ 1.0 \end{bmatrix}, 4.000000021$$

$$16, \begin{bmatrix} 3.000580044\ 10^{-8} \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000000039$$

$$17, \begin{bmatrix} 4.500870008\ 10^{-8} \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 4.000000060$$

$$18, \begin{bmatrix} 6.751304853\ 10^{-8} \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.000000090$$

$$19, \begin{bmatrix} 1.012695693\ 10^{-7} \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000000135$$

$$20, \begin{bmatrix} 1.519043463\ 10^{-7} \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 4.000000203$$

$$21, \begin{bmatrix} 2.278565022\ 10^{-7} \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000000304$$

$$22, \begin{bmatrix} 3.417847144\ 10^{-7} \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.000000456$$

$$23, \begin{bmatrix} 5.126769841\ 10^{-7} \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000000683$$

$$24, \begin{bmatrix} 7.69015279\ 10^{-7} \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.000001025$$

$$25, \begin{bmatrix} 0.000001153522475 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000001538$$

$$26, \begin{bmatrix} 0.000001730282715 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000002307$$

$$27, \begin{bmatrix} 0.000002595421827 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.000003461$$

$$28, \begin{bmatrix} 0.000003893127689 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000005190$$

$$29, \begin{bmatrix} 0.000005839680167 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000007786$$

$$30, \begin{bmatrix} 0.000008759494676 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000011679$$

$$31, \begin{bmatrix} 0.00001313918447 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000017519$$

$$32, \begin{bmatrix} 0.00001970864723 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.000026278$$

$$33, \begin{bmatrix} 0.00002956267953 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000039417$$

$$34, \begin{bmatrix} 0.00004434336384 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.000059125$$

$$35, \begin{bmatrix} 0.00006651357104 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.000088686$$

$$36, \begin{bmatrix} 0.00009976703864 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000133027$$

$$37, \begin{bmatrix} 0.0001496430932 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 4.000199534$$

$$38, \begin{bmatrix} 0.0002244478463 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000299286$$

$$39, \begin{bmatrix} 0.000336633991 \\ 0.9999999997 \\ 0.9999999997 \end{bmatrix}, 4.000448896$$

$$40, \begin{bmatrix} 0.0005048660091 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.000673268$$

$$41, \begin{bmatrix} 0.0007571078947 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.001009732$$

$$42, \begin{bmatrix} 0.001135232095 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.001514216$$

$$43, \begin{bmatrix} 0.001701882127 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.002270464$$

$$44, \begin{bmatrix} 0.002550652735 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.003403764$$

$$45, \begin{bmatrix} 0.003821105945 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.005101305$$

$$46, \begin{bmatrix} 0.005720729162 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.007642211$$

$$47, \begin{bmatrix} 0.008556618696 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.011441458$$

$$48, \begin{bmatrix} 0.01278025018 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.017113237$$

$$49, \begin{bmatrix} 0.01904865201 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.025560500$$

$$50, \begin{bmatrix} 0.02830340713 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 4.038097304$$

$$51, \begin{bmatrix} 0.04186268243 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.056606814$$

$$52, \begin{bmatrix} 0.06150660784 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 4.083725365$$

$$53, \begin{bmatrix} 0.08950726758 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.123013216$$

$$54, \begin{bmatrix} 0.1285096285 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.179014535$$

$$55, \begin{bmatrix} 0.1811262116 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.257019257$$

$$56, \begin{bmatrix} 0.2491275525 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 4.362252423$$

$$57, \begin{bmatrix} 0.3322989205 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.498255105$$

$$58, \begin{bmatrix} 0.4274309578 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 4.664597841$$

$$59, \begin{bmatrix} 0.5282510176 \\ 1.0 \\ 1.0 \end{bmatrix}, 4.854861915$$

$$60, \begin{bmatrix} 0.6268179236 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.056502035$$

$$61, \begin{bmatrix} 0.7158675729 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.253635847$$

$$62, \begin{bmatrix} 0.7907612066 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.431735146$$

$$63, \begin{bmatrix} 0.8500489451 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.581522413$$

$$64, \begin{bmatrix} 0.8947729966 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 5.700097890$$

$$65, \begin{bmatrix} 0.9272986146 \\ 0.9999999997 \\ 0.9999999997 \end{bmatrix}, 5.789545994$$

$$66, \begin{bmatrix} 0.9503286853 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 5.854597230$$

$$67, \begin{bmatrix} 0.9663282841 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 5.900657371$$

$$68, \begin{bmatrix} 0.9772973774 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.932656569$$

$$69, \begin{bmatrix} 0.9847495097 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.954594755$$

$$70, \begin{bmatrix} 0.9897810584 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 5.969499019$$

$$71, \begin{bmatrix} 0.9931640871 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.979562117$$

$$72, \begin{bmatrix} 0.995432317 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.986328174$$

$$73, \begin{bmatrix} 0.9969502346 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.990864634$$

$$74, \begin{bmatrix} 0.9979647541 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.993900469$$

$$75, \begin{bmatrix} 0.998642248 \\ 0.9999999997 \\ 0.9999999997 \end{bmatrix}, 5.995929508$$

$$76, \begin{bmatrix} 0.9990944222 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 5.997284497$$

$$77, \begin{bmatrix} 0.999396099 \\ 0.9999999997 \\ 0.9999999997 \end{bmatrix}, 5.998188845$$

$$78, \begin{bmatrix} 0.9995973184 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 5.998792199$$

$$79, \begin{bmatrix} 0.9997315097 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999194637$$

$$80, \begin{bmatrix} 0.9998209903 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 5.999463019$$

$$81, \begin{bmatrix} 0.9998806534 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999641981$$

$$82, \begin{bmatrix} 0.9999204325 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999761307$$

$$83, \begin{bmatrix} 0.9999469536 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999840865$$

$$84, \begin{bmatrix} 0.9999646349 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 5.999893907$$

$$85, \begin{bmatrix} 0.9999764229 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 5.999929270$$

$$86, \begin{bmatrix} 0.9999842817 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 5.999952846$$

$$87, \begin{bmatrix} 0.999989521 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 5.999968564$$

$$88, \begin{bmatrix} 0.9999930144 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999979042$$

$$89, \begin{bmatrix} 0.9999953432 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999986029$$

$$90, \begin{bmatrix} 0.9999968956 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999990686$$

$$91, \begin{bmatrix} 0.9999979302 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 5.999993791$$

$$92, \begin{bmatrix} 0.9999986204 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999995861$$

$$93, \begin{bmatrix} 0.9999990802 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 5.999997241$$

$$94, \begin{bmatrix} 0.999999387 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999998161$$

$$95, \begin{bmatrix} 0.9999995912 \\ 0.9999999999 \\ 0.9999999999 \end{bmatrix}, 5.999998774$$

$$96, \begin{bmatrix} 0.9999997276 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999999183$$

$$97, \begin{bmatrix} 0.9999998184 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999999455$$

$$98, \begin{bmatrix} 0.9999998792 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999999637$$

$$99, \begin{bmatrix} 0.9999999196 \\ 1.0 \\ 1.0 \end{bmatrix}, 5.999999758$$

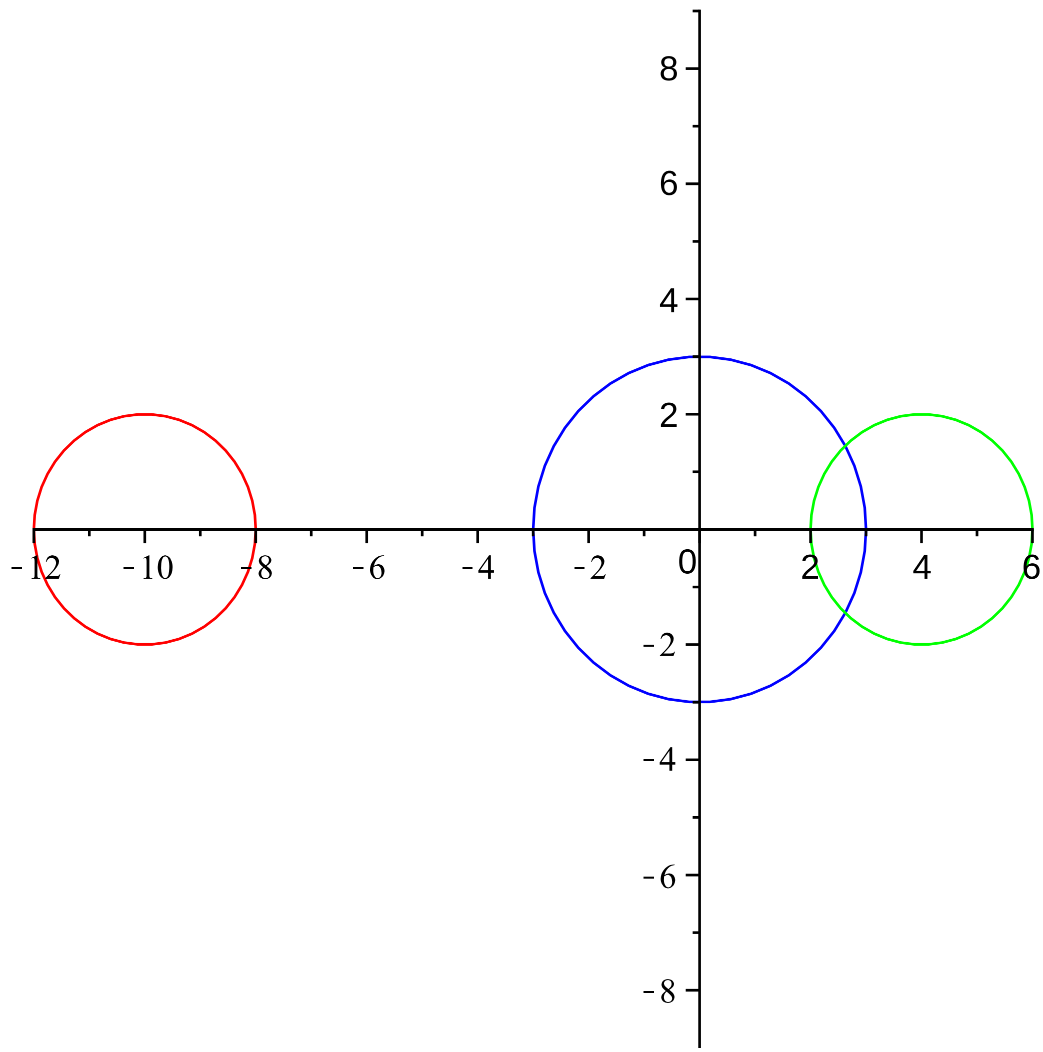
(6.4)

$$100, \begin{bmatrix} 0.9999999462 \\ 0.9999999998 \\ 0.9999999998 \end{bmatrix}, 5.999999839 \quad \textbf{(6.4)}$$

Vi fandt først at denne værdi af r kun konvergerede mod 4. Derefter indsatte vi en $evalf(\ )$ tidligere i udregningerne, for at få dårligere præcision så vi kunne finde det andet konvergens tal, 6. Derudover synes vi opgaven er dårligt formuleret, i og med at man kun kan få de ønskede resultater ved at introducere afrundingsfejl i udregningerne.

Opgave 5.2.37a

```
> restart;
> with(plottools) : with(plots) :
> c1 := circle([0, 0], 3, color = blue) :
> c2 := circle([-10, 0], 2, color = red) :
> c3 := circle([4, 0], 2, color = green) :
> display(c1, c2, c3, view = [-12..6, -9..9]);
```



Bound for spectral radius is: $0 \leq |\lambda| \leq 12$