

Opgave CP 4.6.1

Gauss-Seidel iteration

```
> restart;
> gaussSeidel := proc(n, A :: Matrix, b, x, M)
  local k, i, j, temp1, temp2, x1;
  x1 := copy(x);
  for k from 1 to M do
    for i from 1 to n do
      temp1 := 0;
      for j from 1 to n do
        if j ≠ i then
          temp2 := temp1 + Ai,j · x1j;
          temp1 := temp2;
        end if;
      end do;

      
$$x1_i := \frac{(b_i - temp1)}{A_{i,i}};$$


    end do;
    print(k, x1);
  end do;
  return;
end proc;
```

```
> a1 := Matrix(3, [3, 1, 1, 1, 3, -1, 3, 1, -5])
```

$$a1 := \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -5 \end{bmatrix}$$

(1.1.1)

```
> b1 := Vector([5, 3, -1]);
```

$$b1 := \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$$

(1.1.2)

```
> X := Vector([x, y, z]);
```

$$X := \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(1.1.3)

```
> gaussSeidel(3, a1, b1, X, 3)
```

$$\begin{aligned}
& 1, \left[\begin{array}{c} \frac{5}{3} + -\frac{y}{3}, -\frac{2}{3} - \frac{z}{3}, -1 \\ 1 + -\frac{5}{9} + \frac{y}{9}, \frac{2}{9} + \frac{z}{9}, \frac{1}{3}, -\frac{1}{3} + \frac{z}{3}, -1 \\ \frac{1}{5} + 1 + -\frac{y}{5} + \frac{y}{45}, \frac{2}{45} + \frac{z}{45}, -\frac{1}{3} - \frac{z}{5} - \frac{1}{9}, -\frac{2}{3} + \frac{z}{15}, 0 + \frac{1}{5}, \frac{2}{5} \end{array} \right] \\
& 2, \left[\left[\frac{5}{3} + -\frac{2}{5} + \frac{2}{9} + -\frac{2y}{45}, -\frac{4}{45} - \frac{2z}{45} + \frac{y}{15}, 0 + \frac{z}{15}, \frac{1}{3} - \frac{2z}{15} - \frac{1}{3}, \frac{1}{3}, \right. \right. \\
& \quad \left. \left. -\frac{4}{5} - \frac{1}{15}, -1 \right], \right. \\
& \quad \left[1 + -\frac{22}{45} + \frac{13}{135} + -\frac{2}{27} + \frac{2y}{135}, \frac{4}{135} + \frac{2z}{135} - \frac{y}{45}, 0 - \frac{z}{45} + \frac{y}{135}, \right. \\
& \quad \left. -\frac{13}{135} + \frac{7z}{135} + \frac{1}{9} - \frac{y}{15}, -\frac{2}{9} - \frac{z}{15}, \frac{2}{45} + \frac{16}{45} + \frac{z}{45}, \frac{1}{3}, -\frac{1}{5} + \frac{1}{15}, -1 \right. \\
& \quad \left. \right], \\
& \quad \left[\frac{1}{5} + \frac{76}{75} + -\frac{38}{225} + \frac{7}{45} + -\frac{7y}{225} + \frac{2y}{675}, \frac{4}{675} + \frac{2z}{675}, -\frac{4}{75} - \frac{7z}{225} \right. \\
& \quad \left. + \frac{28y}{675} - \frac{2}{135}, -\frac{13}{675} + \frac{34z}{675} - \frac{y}{75}, \frac{7}{45} - \frac{7z}{75} - \frac{122}{675}, \frac{47}{225} + \frac{z}{225}, -\frac{31}{75} \right. \\
& \quad \left. -\frac{31}{225}, -\frac{16}{25}, 0 + \frac{1}{5}, \frac{2}{5} \right] \Bigg] \\
& 3, \left[\left[\frac{5}{3} + -\frac{2}{5} + \frac{47}{225} + \frac{19}{675} + \frac{4}{135} + -\frac{4y}{675}, -\frac{8}{675} - \frac{4z}{675} + \frac{4y}{225}, \frac{4}{225} \right. \right. \\
& \quad \left. \left. + \frac{4z}{225} - \frac{11y}{675}, \frac{26}{675} - \frac{23z}{675} - \frac{4}{45} + \frac{2y}{75}, \frac{1}{45} + \frac{4z}{75}, -\frac{19}{225} - \frac{14}{225} \right. \right. \\
& \quad \left. \left. -\frac{2z}{225}, \frac{2}{75}, \frac{7}{25} - \frac{9}{25}, \frac{1}{3}, -\frac{4}{5} - \frac{1}{15}, -1 \right], \right. \\
& \quad \left[1 + -\frac{22}{45} + \frac{59}{675} + -\frac{263}{2025} + -\frac{29}{2025} + -\frac{4}{405} + \frac{4y}{2025}, \frac{8}{2025} + \frac{4z}{2025} \right. \\
& \quad \left. -\frac{4y}{675}, -\frac{4}{675} - \frac{4z}{675} + \frac{13y}{2025}, -\frac{22}{2025} + \frac{z}{81} + \frac{4}{135} - \frac{13y}{675}, -\frac{17}{675} \right. \\
& \quad \left. -\frac{19z}{675} + \frac{28y}{2025}, \frac{44}{2025} + \frac{49}{675} + \frac{8z}{405} - \frac{y}{225}, \frac{29}{675} - \frac{7z}{225}, -\frac{16}{675} + \frac{43}{675} \right. \\
& \quad \left. + \frac{z}{675}, -\frac{56}{225}, \frac{4}{75} + \frac{9}{25}, \frac{1}{3}, -\frac{1}{5} + \frac{1}{15}, -1 \right], \\
& \quad \left[\frac{1}{5} + \frac{76}{75} + -\frac{21}{125} + \frac{466}{3375} + \frac{106}{3375} + \frac{16}{675} + -\frac{16y}{3375} + \frac{4y}{10125}, \frac{8}{10125} \right. \\
& \quad \left. + \frac{4z}{10125}, -\frac{28}{3375} - \frac{16z}{3375} + \frac{121y}{10125} - \frac{4}{2025}, \frac{86}{10125} + \frac{133z}{10125} - \frac{46y}{3375}, \right. \\
& \quad \left. \frac{61}{3375} - \frac{88z}{3375} - \frac{569}{10125} + \frac{38y}{2025}, \frac{179}{10125} + \frac{364z}{10125} - \frac{y}{1125}, -\frac{142}{3375} \right. \\
& \quad \left. \right] \Bigg] \tag{1.1.4}
\end{aligned}$$

$$\left[-\frac{641}{10125} - \frac{13z}{1125}, \frac{38}{3375} + \frac{z}{3375}, \frac{133}{1125} - \frac{134}{675}, \frac{79}{375}, -\frac{31}{75} - \frac{31}{225}, -\frac{16}{25}, 0 + \frac{1}{5}, \frac{2}{5} \right]$$

> a2 := Matrix(3, [3, 1, 1, 3, 1, -5, 1, 3, -1])

$$a2 := \begin{bmatrix} 3 & 1 & 1 \\ 3 & 1 & -5 \\ 1 & 3 & -1 \end{bmatrix} \quad (1.1.5)$$

> b2 := Vector(3, [5, -1, 3])

$$b2 := \begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix} \quad (1.1.6)$$

> gaussSeidel(3, a2, b2, X, 3)

$$\begin{aligned} & 1, \begin{bmatrix} \frac{5}{3} + -\frac{y}{3}, -\frac{2}{3} - \frac{z}{3}, -1 \\ -1 + -5 + y, 2 + z, 3, -1 + 5z, -3 \\ -3 + \frac{5}{3} + -\frac{y}{3} + 3y, 6 + 3z, \frac{25}{3} - \frac{z}{3} - 15, -4 + 15z, -8 - 3, 2 \end{bmatrix} \\ & 2, \left[\left[\frac{5}{3} + \frac{4}{3} + \frac{20}{3} + -\frac{4y}{3}, -\frac{8}{3} - \frac{4z}{3} + \frac{y}{9}, -\frac{34}{9} + \frac{z}{9}, \frac{5}{3} - \frac{20z}{3} - \frac{5}{9}, \frac{11}{3}, -\frac{4}{3} + 1, -1 \right], \right. \\ & \quad \left[-1 + -20 + -79 + -20 + 4y, 8 + 4z - \frac{y}{3}, \frac{34}{3} - \frac{z}{3} + 15y, 25 + 35z + \frac{5}{3} - \frac{5y}{3}, \frac{92}{3} - \frac{5z}{3}, -16 + \frac{16}{3} + 75z, -37, 9 - 15, -3 \right], \\ & \quad \left[-3 + -\frac{130}{3} + \frac{52}{3} + \frac{35}{3} + -\frac{7y}{3} + 12y, 24 + 12z, \frac{94}{3} - \frac{7z}{3} + \frac{406y}{9} - 60, \frac{641}{9} + \frac{946z}{9} - 5y, \frac{281}{3} - \frac{35z}{3} - \frac{2138}{9}, -\frac{133}{3} + 225z, -\frac{337}{3} - 59, \right. \\ & \quad \left. 26, -8 - 3, 2 \right] \Big] \\ & 3, \left[\left[\frac{5}{3} + \frac{4}{3} + \frac{79}{3} + \frac{2849}{27} + \frac{80}{3} + -\frac{16y}{3}, -\frac{32}{3} - \frac{16z}{3} + \frac{8y}{9}, -\frac{128}{9} + \frac{8z}{9} - \frac{541y}{27}, -\frac{866}{27} - \frac{1261z}{27} - \frac{40}{9} + \frac{20y}{9}, -\frac{373}{9} + \frac{40z}{9}, \frac{181}{9} - \frac{68}{9} - 100z, \frac{448}{9}, -\frac{35}{3} + \frac{175}{9}, \frac{11}{3}, -\frac{4}{3} + 1, -1 \right], \right. \\ & \quad \left[-1 + -20 + -299 + -\frac{11401}{9} + -\frac{5549}{9} + -80 + 16y, 32 + 16z - \frac{8y}{3}, \frac{128}{3} \right] \end{aligned} \quad (1.1.7)$$

$$\begin{aligned}
& -\frac{8z}{3} + \frac{1081y}{9}, \frac{1946}{9} + \frac{1801z}{9} + \frac{40}{3} - \frac{55y}{3}, 281 - 25z + \frac{2030y}{9}, \frac{2662}{9} \\
& + 81 + \frac{7430z}{9} - 25y, 319 - \frac{175z}{3}, -\frac{560}{3} + \frac{85}{3} + 1125z, -\frac{1718}{3}, 134 \\
& -\frac{659}{3}, -37, 9 - 15, -3 \Big], \\
& \left[-3 + -\frac{130}{3} + -\frac{1973}{3} + \frac{334}{3} + \frac{9410}{27} + \frac{200}{3} + -\frac{40y}{3} + 48y, 96 + 48z, \right. \\
& \frac{352}{3} - \frac{40z}{3} + \frac{3251y}{9} - 240, \frac{5710}{9} + \frac{5411z}{9} - \frac{2026y}{27}, \frac{21895}{27} - \frac{3286z}{27} \\
& -\frac{16687}{9} + \frac{6110y}{9}, \frac{7613}{9} + \frac{22330z}{9} - 75y, \frac{8794}{9} - \frac{34271}{9} - 275z, \\
& \left. -\frac{4592}{9} + 3375z, -\frac{5189}{3} - \frac{7898}{9}, \frac{1217}{3}, -\frac{337}{3} - 59, 26, -8 - 3, 2 \right] \Big]
\end{aligned}$$