Version 29/8 2011

NumIntro: Introduction to Numerical Analysis, 2011

Come prepared:

- Buy "Kincaid & Cheney, Numerical Analysis: Mathematics of Scientific Computing"
- Get a laptop computer (at least one per study group is highly recommended).
- Print out this file and read it.

What's happening in NumIntro?

NumIntro is a course of the type that makes Copenhagen University what it is:
Strong on theoretical foundation.
KU graduates not only are able to apply the newest methods – they also understand what lies behind so that they can participate in creating the methods of the future.

We do see some "real life" applications as introduction to the new topics, but once we enter the topics we try to throw away the applications and focus on the theoretical content of the methods - **comparing and evaluating various methods for the same problems with respect to for example cost and precision**.

NumIntro contains the **basic fundamental topics within classical numerical analysis**.
You learn the notions and the methods that all else is built on top of.
These are methods in practical use today but more often they are the foundations of the state of the art methods being used and researched today.

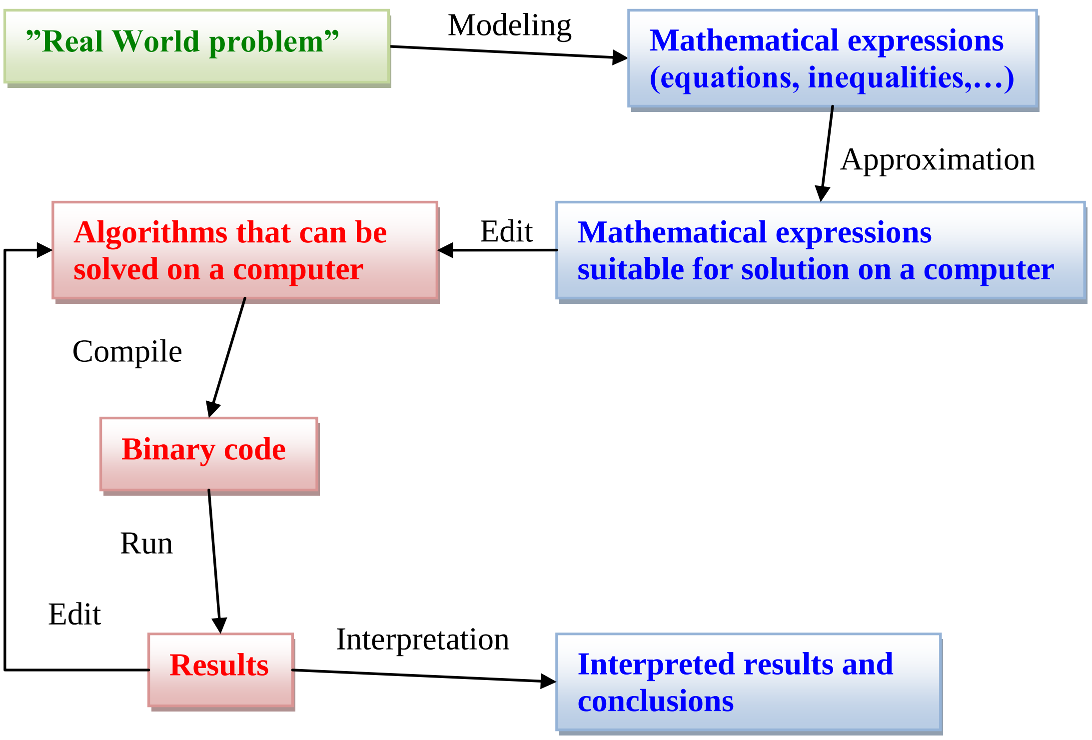
In NumIntro we also try out the methods - **writing and running code**.
We focus on "toy-problems" small enough that it is possible to focus on the methods and not get caught in "size-problems".
A follow up course to NumIntro could be named "Numerical Analysis - Size matters"

Why is Numerical Analysis important?

You learn programming in practice.
You learn the basic foundation of Numerical Analysis. This is what everything else builds upon.
Applications:

1. **During your studies:** Within *insurance, finance, operations research, physics, chemistry, biology, and you name it* -- you often need to solve your models on a computer. This will, one way or the other, involve the methods learned in this course. The methods from the course may be directly involved as for example in the solution of a large equation system, or they may be indirectly involved like in a problem involving differential equation models (typical in insurance contracts, finance, physics and chemistry) which are in turn reduced to linear equation systems.
2. **After your studies:**
    a. If you become a *researcher* you will need to solve your own models or help other solve theirs.
    b. If you become a *high school teacher* you will need the programming and numerical skills in interdisciplinary projects.
    c. If you go to the *industry*, you will need a training period. In this period you will typically "earn your salary" as a programming help for a project group.

What is Numerical Analysis?



Focus in NumIntro:

1. **A little bit of “modeling”.**
   The lectures starting a new chapter starts with an example of modeling.
2. **A lot of “Approximation”** in the cases where the mathematical expressions involve
   a. Linear equation systems or nonlinear equations.
   b. Approximation of functions or data sets.
   c. Differentiation or integration.
3. **Quite a few “Edit-Compile-Run-Error correct”-cycles** on expressions of the types above. **Maple** is used.
4. **Quite some “Interpretation” of results**, in the weekly assignments.

Lecturer:
Jens Hugger hugger@math.ku.dk, Office HCØ-04.4.10, Tel 28 75 06 84.
You are welcome in my office -- it is a good idea to make an appointment first.

TA's:
Section 1 - Akt: Jan Ulrich Lauridsen ulrich.lauridsen@gmail.com - Mailbox outside library 4.1.x
Section 2 - Mat-Øk: Attila Yilmaz attila.yilmaz@gmail.com - Mailbox outside library 4.1.x
Section 3 - NatIT: Raffaele Rani alta1@ymail.com - Mailbox outside library 4.1.x
Section 4 - Elective: Sima Mashayekhi s-mashayekhi@araku.ac.ir - Mailbox in 4.1.3

Course contents:
The goal of the course is that you as a student learn some of the basic tools needed for solving mathematical problems on a computer:

1. You learn the necessary mathematics. In NumIntro we touch upon most of everything that most people will need - apart from differential equations covered in the course NumDiff. If you are to make a living out of numerical analysis, then you will need more advanced courses.
2. You learn how to "Program", that is how to make a computer run the mathematical algorithms that you learn to construct. The "Programming language" that we use is Maple.

The topics are as follows:

Math-topics:
1. Background stuff from **mathematics**
2. Numerical solution of **nonlinear equations**
3. Numerical solution of **linear equation systems**
4. Numerical methods for **eigenvalue problems and other**
5. Numerical methods for **interpolation**
6. Numerical **differentiation and integration**

Maple-topics:
1. Basic Maple
2. Flow control
3. Procedures
4. Input/output

Reading material:
Jens Hugger, Lecture notes (free on Absalon).
Kincaid & Cheney, Numerical Analysis, 3rd ed. Published by AMS-undergraduate series (buy it).
Maple 15, Manuals (free on Absalon/KUnet->Softwarebiblioteket->Maple).

Confrontation and home work:
Listening and reading:
Each week holds 2 double lectures covering the math-topics – **Attend these and read the lecture notes and/or the book if you want to pass!**
Each week holds additionally 0, 1 or 2 single lectures focused on Maple – **Attend these and read the lecture notes and/or the Maple manuals if you are not a fluent programmer!**
Listening and reading is not enough to learn the subjects. You must also experiment:
Each week holds 5 hours of exercise sections where you can get help from a teaching assistant (TA).
For each double lecture there are a number of compulsory exercises pertaining to the topics covered. Typically one exercise is given for each major topic. If this is not enough for you, please find more voluntary exercises in the book. **The compulsory exercises must be handed in for grading by the instructors.**
Each week holds half an hour of "trial exam" – **Attend these and volunteer for a presentation**.

Weekly plan for NumIntro, Block 1A, 2011:

Monday		Tuesday		Wed	Thursday					Fri	Sat	Sun
		8:30-9:00	**Trial exam (Aud 4)**		8:15-9:00	**Problem solve with consultation**						
						HCØ A106[1]	**HCØ A102**[2]	**HCØ A111**[3]	**HCØ D315**[4]			
		9:15-11:00	**Lecture1 (Aud 4)**		9:15-11:00	**Lecture2 (Aud 4)**						
13:15	**Hand in last weeks homework**	11:15-12:00	**Maple1 (Aud 4)**		11:15-12:00	**Maple2 (Aud 4)**						
			Study Lec1 (after) + Study Lec2 (before)		12:15-13:00	**Selfstudy**						
						HCØ A106[1]	**HCØ A102**[2]	**HCØ A110**[3]	**NBI RF071**[4]			
						Problem solve with consultation						
					13:15-15:00	**HCØ A106**[1]	**HCØ A102**[2]	**HCØ A107**[3]	**NBI RF062**[4]			
					15:15-17:00	**DIKU 1-0-18**[1]	**DIKU 1-0-14**[2]	**DIKU 1-0-10**[3]	**NBI RF062**[4]			
						Finish exercises + Prepare and exercise exam questions						

[1/2/3/4] Section numbers

Workflow:

Monday:

No later than at 13:15 (Starting Monday in week 2), the homework assignment from last week must be handed in to the instructors in their **mailboxes** (ask them where they are located). For week 7, the assignments must be handed in no later than Friday at 17:00 in the same 7th week.

Tuesday:

8:30-9:00 in Aud. 4 there is a trial exam with the exam questions from last week. This is to help the students getting ready for the final oral exam. The trial exam is NOT part of the grade. Approximately 3-4 students will be given the chance each week with the lecturer, but not everybody in the class will get to try out. For the first week of classes the trial exam is replaced by an introduction to the course in the same room.

9:15-11:00 in Aud. 4 the first half of the week's material will be addressed. The format is 4 times 20 minutes with breaks:

- 9:15-9:35 Lecture
- 9:35-9:40 Small break, where everybody stay in the room
- 9:40-10:00 Lecture
- 10:00-10:15 Big break, where everybody may leave the room
- 10:15-10:35 Lecture
- 10:35-10:40 Small break, where everybody stay in the room
- 10:40-11:00 Lecture

11:15-12:00 in Aud. 4 there are Maple lectures, at least in the beginning of the block. Later they may be cancelled and/or replaced by problem solving in the lecture hall.

Tuesday from 12:00 and all of Wednesday wherever and whenever you may find a suitable place there is preparation / self-study / group study of lecture 1 and 2. Lecture 1 is studied AFTER the lecture; lecture 2 is studied BEFORE the lecture. If you prefer to study both parts before the lecture, you may move the study of lecture 1 up till Monday or even earlier. This mid-week part is estimated to take 4-6 hours for the average student.

Thursday:

08:15-9:00 in the class rooms: Solve the homework problem of the week in groups of 3-4 students. The instructors are available for consulting. Eventually the instructors may hold class sessions to talk about common problems in the home work.

9:15-11:00 in Aud. 6 the second half of the week's material will be addressed in the same format as the first lecture.

11:15-12:00 in Aud. 6 there are Maple lectures, at least in the beginning of the block.

13:15-14:00 in the classrooms: The home work handed in Monday is returned after having been graded (pass/fail) by the instructors. If the home work fail, there is until the following Monday at 13:15 to hand in a revised version and get a second chance to obtain a pass. Also the home work for the current week is discussed.

14:15-17:00 in the class rooms: Solve the homework problem of the week in groups of 3-4 students. The instructors are available for consulting. Eventually the instructors may hold class sessions to talk about common problems in the home work.

Thursday from 17:00 and all of Friday, Saturday and Sunday wherever and whenever you may find a suitable place there is preparation / self-study / group study with two items on the agenda: (1) Finish the homework assignment that is to be handed in no later than Monday at 13:15. (2) Prepare the exam questions of the week. This is likely to involve repeated study of the material in the text book. The preparation should end up in a small personal note including the main points relating to the question. This personal note may be brought both at the trial examinations and at the final exam. Once prepared, each exercise group presents the questions for each other and discusses the performance. This end of the week part is estimated to take 6-8 hours for the average student.

Since no computer rooms are provided, it is highly recommended that each study group has available at least one portable computer to work on the homework assignments in class and hence take advantage of the consultants.

Course work load:

The average student is expected to dedicate **12 hours with instruction** in block 1A and further **10-14 hours of self-study** without instruction in week 1-7 of the course. Week 8-9 of the course is then preparation for and participating in the final oral exam.

How to pass the weekly assignments:

1. **Do all exercises** – There must be a "serious attempt" at all exercises.
2. **Test your results whenever possible**. For example: $x^3-12x^2+35x=0$ has the solutions x=0, 5 and 7 since a cubic polynomial has exactly 3 roots and plugging in it is easily verified that 0, 5 and 7 solves the equation.
3. **Comment on all computational (Maple) results**. In Numerical Analysis we do not normally grade computer programs – we grade the results. So when you have spent 2 days making your program work, spend also the little additional time to document that it is working by getting some output and writing down what the output is showing. The program alone will not pass you.
4. **Make your programs work**.
   a. Write and test small pieces at a time.
   b. Put in print-statements to get intermediate results.
   c. Run simple cases where you can hand-check the results – Like 2x2-matrices even though you need a 40x40 matrix for the final run.
   d. Always hunt the first error first. The following errors may be caused by the first.
   e. Comment your program - What is this piece of code supposed to do? – this makes it easier to help you.

Study program – NumIntro: Lectures and exercises

Week	Time	Name	Lec note	Textbook	Topic/Problems
36[1]	Tu 8^{30}-11	Lec 1*	Lec1.mw	KC 1.0-1.1 = 10 p.	Mathematics: 1.1.5, 1.1.34, JH-1, cp1.2.1, JH-2
	Tu 11-12	Maple 1	Maple1.mw	KC cp1.2.1, JH-3	
	Th 9-11	Lec 2	Lec2.mw	KC 1.2-1.3 = 16½ p.	Mathematics: 1.2.4, 1.2.8, 1.3.1, 1.3.11a+b, JH-3
	Th 11-12	Mtheory 1	Mtheory1.mw	PG ch.1-4	Basic Maple
37[2]	Tu 8^{30}-11	Lec 3*	Lec3.mw	KC 3.0-3.3 = 20½ p.	Nonlinear eqn.: 3.2.9, 3.2.23a, cp3.2.1
	Tu 11-12	Mtheory 2	Mtheory2.mw	PG ch.5	Flow control
	Th 9-11	Lec 4	Lec4.mw	KC 3.4-3.6 ex p.117-128 = 21 p.	Nonlinear eqn.: 3.4.7, 3.5.1, 3.6.1
	Th 11-12	Maple 2	Maple2.mw	KC cp4.2.2	
38[3]	Tu 8^{30}-11	Lec 5*	Lec5.mw	KC 4.0-4.2 = 18 p.	Linear eqn.: 4.2.30, 4.2.41, cp4.2.2
	Tu 11-12	Mtheory 3	Mtheory3.mw	PG ch. 6	Procedures
	Th 9-11	Lec 6	Lec6.mw	KC 4.3-4.5 = 31 p.	Linear eqn.: 4.3.1 in Maple, 4.4.33, 4.4.40 in Maple with LinearAlgebra[ConditionNumber]
	Th 11-12	Maple 3	Maple3.mw	KC 4.3.1, 4.4.40	
39[4]	Tu 8^{30}-11	Lec 7*	Lec7.mw	KC 4.6 ex p.221½-229, 4.7 = 27 p.	Linear eqn.: cp4.6.1, cp4.6.2, 4.7.13
	Tu 11-12	Maple 4	Maple4.mw	KC cp4.6.1-2, 5.1.23, cp5.1.1	
	Th 9-11	Lec 8	Lec8.mw	KC 5.0-5.2 ex p. 260 = 14 p.	Eigenval.prob.: 5.1.17, 5.1.23, cp5.1.1, 5.2.37a
	Th 11-12	Mtheory 4	Mtheory4.mw	PG ch. 9	Input/output
40[5]	Tu 8^{30}-11	Lec 9*	Lec9.mw	KC 5.3-5.5 = 27 p.	Eigenval.prob. : 5.3.16, 5.4.2, cp5.4.1 In LinearAlgebra use **SingularValues(A,output=[U,S,Vt]);** but only to test use **MatrixInverse(A, method=pseudo);** Find the pseudoinverse from § 5.4 example 1+3 and 2+4 and compare it with Maples pseudoinverse. Also find the minimal solution from § 5.4 example 5.
	selfstudy	Mtheory 5	--	PG ch. 7^	Numerical programming
	Th 9-11	Lec 10	Lec10.mw	KC 6.0-6.1 ex p.315-323, 6.2 ex p. 334-335, 6.3 ex p.341½-348 = 14 p.	Interpolation: 6.1.1a,b,c, 6.2.4, 6.2.17
	selfstudy	Mtheory 6	--	PG ch. 13^	Combining Maple with other languages
41[6]	Tu 8^{30}-11	Lec 11*	Lec11.mw	KC 6.1 ex p.321-323, 6.4 ex p.358-361 = 15 p.	Interpolation: 6.1.1e, 6.1.22, cp6.4.1 Use Maple to create and plot (1) $(\sin(\pi t),\cos(\pi t))$, t=0..1 and (2) $(t^2\cos(\pi t), t^2\sin(\pi t))$, t=0..4. Do linear AND cubic splines using CurveFitting[Spline]
	Tu 11-12	Maple 6	Maple6.mw	KC cp6.4.1-2	
	Th 9-11	Lec 12	Lec12.mw	KC 6.7, 6.12-6.13 excl. p. 457-458 = 14 p.	Interpolation: 6.4.9, 6.4.19, cp6.4.2 test with the data sets from exe 6.4.19, 6.4.13 and 6.4.7
	selfstudy	Mtheory 7	--	PG ch. 11^	Graphics
43[7]	Tu 8^{30}-11	Lec 13*	Lec13.mw	KC 7.1 excl. p.470-471 = 10 p.	Diff and int: 7.1.6, 7.1.16, cp7.1.1
		Maple 7	Maple7.mw	KC cp7.1.1	
	Th 9-11	Lec 14	Lec14.mw	KC 7.2 excl. p.480 and p.486-488, 7.3 = 13 p.	Diff and int: 7.2.2, 7.3.3, 7.3.7a, 7.3.22a
	selfstudy	Mtheory 8	--	PG ch. 12^	Programming interactive elements

* 8:30-9:00: Trial exam. 9:15-11:00: Lecture

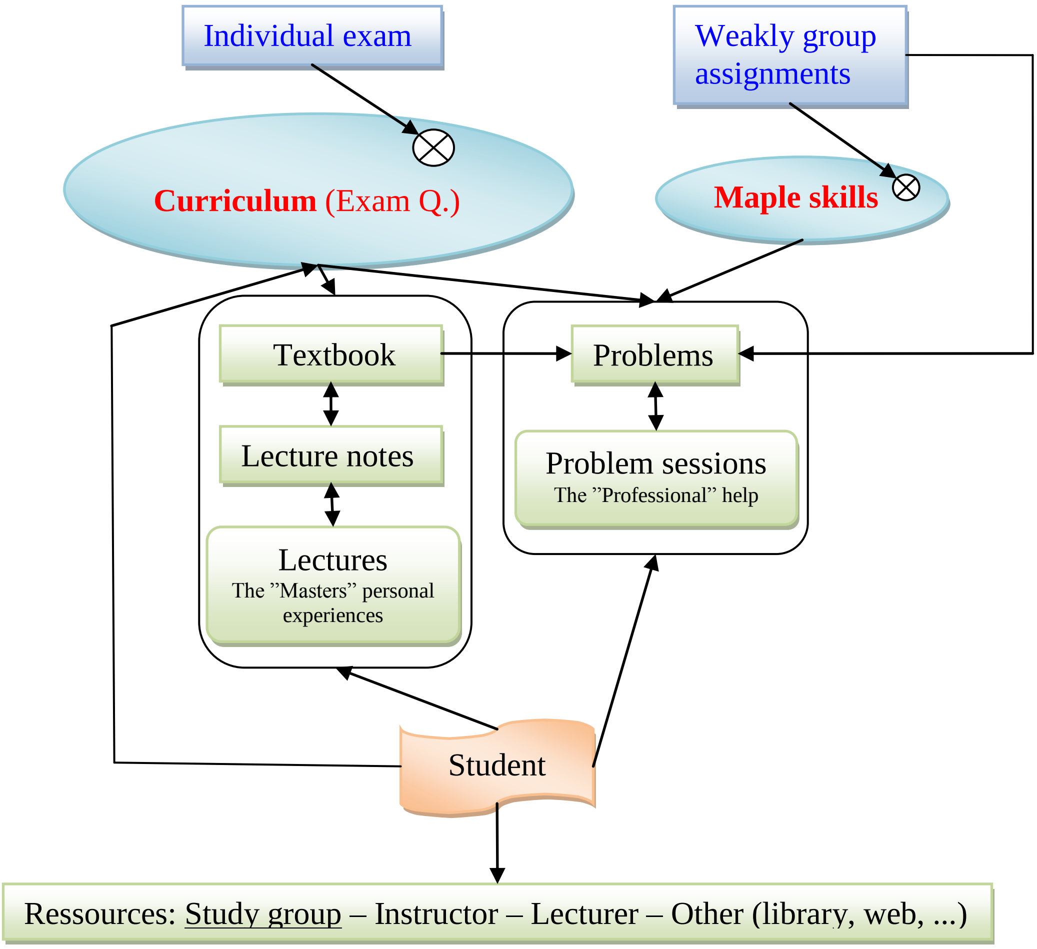
KC = Kincaid & Cheney (ex = excluding)

PG = Maple 15, Programming guide

Every week: Exercises Th 8:15-9:00 and 13:00-17:00

^ Benficial but not part of curriculum.

Learning plan



Year 1 is strongly controlled. At year 2 you must start taking control of your own learning process. The teacher is ***not*** "learning you the stuff". The teacher is "teaching the stuff", that is providing various options for you to take or leave. ***You* are learning the stuff by selecting amongst the means available to you.**

A word of advice:
In my experience, in order to "learn the stuff" and pass the course, you should during the course

1. Show up for all lectures.
2. Read the lecture notes and/or the book.
3. Make one page notes for each exam question.
4. Show up for all exercise sections.
5. Work in exercise groups to learn from your fellow students, and to reduce the amount of time needed to complete the exercises.
6. Prepare an exam question and present it to the class to test yourself.

In week 8 (after classes but before the exam) you should

1. Prepare and rehearse all exam questions with your exercise group, based on your one page notes.
2. Read the lecture notes and/or the book and look at your exercises specifically regarding the exam questions.

Spend 20-25 hours per week in week 1-7 and in average the same in week 8-9.

Requirements to participate in the final exam:

- 6 out of 7 weekly, written group assignments must be passed.
- Problem set N must be handed in no later than Monday at 13:15 in week N+1 at the location designated by your instructor (typically the secretary's office 04.1.03 or outside the library).
- If you fail at first attempt, you get an extended weekend to hand in a corrected version.
- If you have problems with the deadlines, please contact your instructor.

Final exam:

5-10 minute final oral exam.
The exam questions are known before the exam.
As you enter the examination room, you identify yourself to the censor using your Student ID [Studiekort], so
Remember to have your student ID in your hand as you enter the examination room.
You draw one question and get 30 seconds to collect your thoughts.
You are allowed to bring notes into the examination room and use them in the 30 seconds preparation time. After that, the notes are put away or handed to the censor.
Advice: Write approx. one page with details about what you will cover *and* a 10 line abstract on each question. If there is more, it will not help you in the 30 seconds of preparation, and censor will not be able to use the notes to help you, if you get stuck.
Then you start talking about a central part of the exam question.
(If you do not start, or get stuck, the examiner or censor will ask questions).
If you fail, you will be offered a second attempt in the exam period (if time permits).
Next reexamination opportunity is found in SIS (it is currently set at the 3rd of March).
On Tuesdays 8:30-9:00 there will be "trial exams" where a study group can try out the exam questions that they have prepared. These trial exams will be open to other students of the course. At request closed trial exams may be arranged.

Exam questions:

1. Taylor expansion (Lec 1 / § 1.1)
2. Difference equations (Lec 2 / § 1.3)
3. Solution of nonlinear equations and equation systems: Bisection, Secant and Newtons method (Lec 3 / § 3.0-3.3)
4. Computing roots of polynomials (Lec 4 / § 3.5)
5. Homotopy methods and linear programming (Lec 4 / § 3.6)
6. Gaussian elimination and LU-factorization (Lec 5 / § 4.2)
7. Matrix norms and condition number (Lec 6 / § 4.4)
8. Iterative solution of equation systems – Matrix splitting: Richardson, Jacobi, Gauss-Seidel, SOR, SSOR (Lec 7 / § 4.6)
9. Iterative solution of equation systems – Descent methods: Steepest descent, Conjugate gradient and preconditioned conjugate gradient methods (Lec 7 / § 4.7)
10. The matrix eigenvalue problem: Power methods (Lec 8 / § 5.1)
11. Gershgorin's theorems (Lec 8 / § 5.2)
12. The least squares problem and the minimal solution (Lec 9 / § 5.3-4)
13. The QR factorization and the QR algorithm (Lec9 / § 5.3+5.5)
14. Polynomial interpolation – Newton form and divided differences (Lec 10 / § 6.1-2)
15. Polynomial interpolation – Lagrange form (Lec 10 / § 6.1)
16. Piecewise polynomial interpolation and splines (Lec 11 / § 6.4 and lecture notes)
17. The Taylor series method for numerical differentiation (Lec 13 / § 7.1)
18. Richardson extrapolation (Lec 13 / § 7.1)
19. Numerical integration: Newton-Cotes quadrature (Lec 14 / § 7.2)
20. Numerical integration: Gaussian quadrature (Lec 14 / § 7.3)