Opgave 1.1.5

Opgave 1.1.5a

Deriving the Taylor series at 0 for the function
$$f(x) = \ln(x+1)$$

$$f := x \rightarrow \ln(x+1)$$

$$f := x \rightarrow \ln(x+1)$$
(1.1.1)

$$\rightarrow f'(x)$$

$$\frac{1}{x+1}$$
 (1.1.2)

$$- f''(x)$$

$$-\frac{1}{(x+1)^2}$$
 (1.1.3)

>
$$\ln(x+1) = \sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(0) (x-0)^k + E_n(x)$$

$$\rightarrow taylor(f(x), x, 10)$$

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} x^k$$

$$ln(x+1) = ln(x+1)$$
 (1.1.6)

 $\ln(x+1) = \ln(x+1)$ > k starter fra 1 fordi summeringen har k alene under brøkstregen. Vi kan se at maple også udregner summeringen til

>
$$E_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) (x-c)^{n+1}$$

$$E_n(x) = \frac{D^{(n+1)}(f)(\xi)(x-c)^{n+1}}{(n+1)!}$$
(1.1.7)

 $E_{n}(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) (x-c)^{n+1}$ $E_{n}(x) = \frac{D^{(n+1)}(f)(\xi) (x-c)^{n+1}}{(n+1)!}$ > Vi truncater vores serie til hhv 3 og 4 termer og giver herunder 2 udtryk for resten.

> $E_{3}(x) = \frac{1}{(3+1)!} \left(-\frac{6}{(x+1)^{4}} \right) (\xi) x^{4}$ $E_{3}(x) = -\frac{1}{4} \frac{x^{4}}{(x(\xi)+1)^{4}}$

>
$$E_3(x) = \frac{1}{(3+1)!} \left(-\frac{6}{(x+1)^4} \right) (\xi) x^4$$

$$E_3(x) = -\frac{1}{4} \frac{x^4}{(x(\xi) + 1)^4}$$
 (1.1.8)

$$E_4(x) = \frac{1}{(4+1)!} \left(\frac{24}{(x+1)^5} \right) (\xi) x^5$$

$$E_4(x) = \frac{1}{5} \frac{x^5}{(x(\xi) + 1)^5}$$
 (1.1.9)

Opgave 1.1.5b

- > Vi skal forsøge at finde $\ln(1.5)$ ved hjælp af en taylor serie. Vores fejl margen skal være mindre end 10^{-8} .
- > Vi ved at usikkerheden er opadtil begrænset af den sidste term i udviklingen. Samtidig gør $(-1)^{n-1}$ ingen forskel for størrelsen af fejl. Derfor har vi:

>
$$E := solve\left(\frac{0.5^n}{n} \le 10^{-8}, n\right);$$

 $E := RealRange(-\infty, Open(0.)), RealRange(Open(22.10887129), \infty)$ (1.2.1)

$$> \frac{0.5^{22}}{22}$$

$$1.083720814 \cdot 10^{-8}$$
 (1.2.2)

$$> \frac{0.5^{23}}{23}$$

> Her ser vi at der skal minimum 23 termer til at få fejlen ned på mindre end 10^{-8} .

Opgave 1.1.5c

> For at finde det mindste antal termer for $\ln(1.6)$ til en præcision på 10^{-10} tager vi altså:

>
$$E := solve\left(\frac{0.6^n}{n} \le 10^{-10}, n\right)$$

 $E := RealRange(-\infty, Open(0.)), RealRange(37.95697913, \infty)$ (1.3.1)

> Her skal der bruges 38 termer for at få den ønskede præcision.

Opgave 1.1.34

- > "Determine a function that can be termed the linearization of x^3-2 x at 2."
- > Man kan udregne linearization af en funktion ved at se på taylor ekapansionens første termer:

$$f := x \rightarrow x^3 - 2x$$

$$f := x \to x^3 - 2x$$
 (2.1)

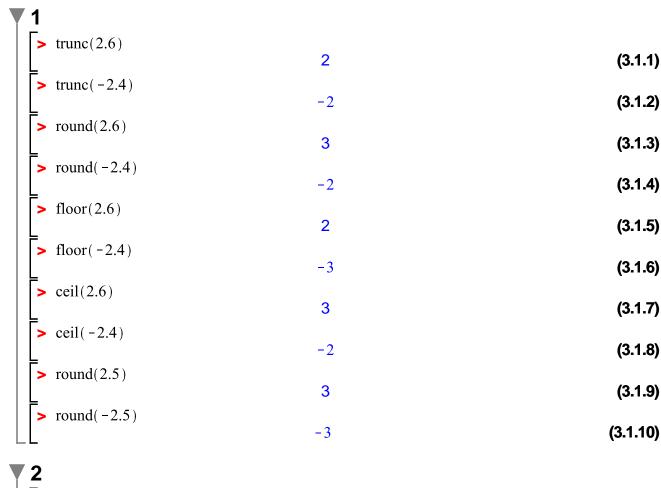
> taylor(f(x), x = 2, 2)

$$4 + 10(x-2) + O((x-2)^2)$$
 (2.2)

Derfor er linearization af:

$$\{f(x), x=2\} = 4 + 10(x-2)$$

Opgave JH-1



trunc(x) = floor(x) for x > 0

> trunc(x) runder x ned til nærmeste heltal for x > 0 og runder x op til nærmeste heltal for x

round(x) runder x til nærmeste heltal **for** alle x. floor(x) runder x ned til nærmeste heltal **for** alle x ceil(x) runder x op til nærmeste heltal **for** alle x

$$c := \frac{(1+\sqrt{5})}{2}; x_0 := 1; x_1 := c;$$

for n from 1 to 29 do

$$x_{n+1} := x_n + x_{n-1};$$

end do

$$c := \frac{1}{2} + \frac{1}{2}\sqrt{5}$$

$$x_0 := 1$$

$$x_1 := \frac{1}{2} + \frac{1}{2}\sqrt{5}$$

$$x_2 := \frac{3}{2} + \frac{1}{2}\sqrt{5}$$

$$x_3 := 2 + \sqrt{5}$$

$$x_4 := \frac{7}{2} + \frac{3}{2}\sqrt{5}$$

$$x_5 := \frac{11}{2} + \frac{5}{2}\sqrt{5}$$

$$x_6 := 9 + 4\sqrt{5}$$

$$x_7 := \frac{29}{2} + \frac{13}{2}\sqrt{5}$$

$$x_9 := 38 + 17\sqrt{5}$$

$$x_{10} := \frac{123}{2} + \frac{55}{2}\sqrt{5}$$

$$x_{11} := \frac{199}{2} + \frac{89}{2}\sqrt{5}$$

$$x_{12} := 161 + 72\sqrt{5}$$

$$x_{13} := \frac{521}{2} + \frac{233}{2}\sqrt{5}$$

$$x_{14} := \frac{843}{2} + \frac{377}{2}\sqrt{5}$$

$$x_{15} := 682 + 305\sqrt{5}$$

$$x_{16} := \frac{2207}{2} + \frac{987}{2}\sqrt{5}$$

$$x_{17} := \frac{3571}{2} + \frac{1597}{2}\sqrt{5}$$

$$x_{18} := 2889 + 1292\sqrt{5}$$

$$x_{19} := \frac{9349}{2} + \frac{4181}{2}\sqrt{5}$$

$$x_{20} := \frac{15127}{2} + \frac{6765}{2} \sqrt{5}$$

$$x_{21} := 12238 + 5473 \sqrt{5}$$

$$x_{22} := \frac{39603}{2} + \frac{17711}{2} \sqrt{5}$$

$$x_{23} := \frac{64079}{2} + \frac{28657}{2} \sqrt{5}$$

$$x_{24} := 51841 + 23184 \sqrt{5}$$

$$x_{25} := \frac{167761}{2} + \frac{75025}{2} \sqrt{5}$$

$$x_{26} := \frac{271443}{2} + \frac{121393}{2} \sqrt{5}$$

$$x_{27} := 219602 + 98209 \sqrt{5}$$

$$x_{28} := \frac{710647}{2} + \frac{317811}{2} \sqrt{5}$$

$$x_{29} := \frac{1149851}{2} + \frac{514229}{2} \sqrt{5}$$

$$x_{30} := 930249 + 416020 \sqrt{5}$$

(4.1.1)

> for n from 1 to 30 do $x_n := \left(\frac{1+\sqrt{5}}{2}\right)^n$; end do $x_1 := \frac{1}{2} + \frac{1}{2}\sqrt{5}$ $x_2 := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^2$ $x_3 := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^3$ $x_4 := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^4$ $x_5 := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^5$ $x_6 := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^6$ $x_7 := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^7$ $x_8 := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^8$ $x_9 := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^9$

$$x_{10} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{10}$$

$$x_{11} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{11}$$

$$x_{12} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{12}$$

$$x_{13} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{13}$$

$$x_{14} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{14}$$

$$x_{15} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{15}$$

$$x_{16} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{15}$$

$$x_{17} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{17}$$

$$x_{18} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{17}$$

$$x_{20} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{19}$$

$$x_{21} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{20}$$

$$x_{21} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{21}$$

$$x_{22} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{22}$$

$$x_{23} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{23}$$

$$x_{24} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{25}$$

$$x_{25} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{25}$$

$$x_{26} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{25}$$

$$x_{27} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{25}$$

$$x_{29} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{29}$$

$$x_{30} := \left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{30}$$
 (4.1.2)

h

$$c := \frac{\left(1 - \sqrt{5}\right)}{2}; x_0 := 1; x_1 := c;$$

for n from 1 to 29 do

$$x_{n+1} := x_n + x_{n-1};$$

end do

$$c := \frac{1}{2} - \frac{1}{2}\sqrt{5}$$

$$x_0 := 1$$

$$x_1 := \frac{1}{2} - \frac{1}{2}\sqrt{5}$$

$$x_2 := \frac{3}{2} - \frac{1}{2}\sqrt{5}$$

$$x_3 := 2 - \sqrt{5}$$

$$x_4 := \frac{7}{2} - \frac{3}{2}\sqrt{5}$$

$$x_5 := \frac{11}{2} - \frac{5}{2}\sqrt{5}$$

$$x_6 := 9 - 4\sqrt{5}$$

$$x_7 := \frac{29}{2} - \frac{13}{2}\sqrt{5}$$

$$x_8 := \frac{47}{2} - \frac{21}{2}\sqrt{5}$$

$$x_{10} := \frac{123}{2} - \frac{55}{2}\sqrt{5}$$

$$x_{11} := \frac{199}{2} - \frac{89}{2}\sqrt{5}$$

$$x_{12} := 161 - 72\sqrt{5}$$

$$x_{13} := \frac{521}{2} - \frac{233}{2}\sqrt{5}$$

$$x_{14} := \frac{843}{2} - \frac{377}{2}\sqrt{5}$$

$$x_{15} := 682 - 305\sqrt{5}$$

$$x_{16} := \frac{2207}{2} - \frac{987}{2}\sqrt{5}$$

$$x_{17} := \frac{3571}{2} - \frac{1597}{2} \sqrt{5}$$

$$x_{18} := 2889 - 1292 \sqrt{5}$$

$$x_{19} := \frac{9349}{2} - \frac{4181}{2} \sqrt{5}$$

$$x_{20} := \frac{15127}{2} - \frac{6765}{2} \sqrt{5}$$

$$x_{21} := 12238 - 5473 \sqrt{5}$$

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$$x_{24} := 51841 - 23184 \sqrt{5}$$

$$x_{25} := \frac{167761}{2} - \frac{75025}{2} \sqrt{5}$$

$$x_{26} := \frac{271443}{2} - \frac{121393}{2} \sqrt{5}$$

$$x_{27} := 219602 - 98209 \sqrt{5}$$

$$x_{28} := \frac{710647}{2} - \frac{317811}{2} \sqrt{5}$$

$$x_{29} := \frac{1149851}{2} - \frac{514229}{2} \sqrt{5}$$

$$x_{30} := 930249 - 416020 \sqrt{5}$$
(4.2.1)

For
$$n$$
 from 1 to 30 do $x_n := \left(\frac{1-\sqrt{5}}{2}\right)^n$; end do
$$x_1 := \frac{1}{2} - \frac{1}{2}\sqrt{5}$$

$$x_2 := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^2$$

$$x_3 := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^3$$

$$x_4 := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^4$$

$$x_5 := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^5$$

$$x_6 := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^6$$

$$x_7 := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^7$$

$$x_{8} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{8}$$

$$x_{9} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{9}$$

$$x_{10} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{10}$$

$$x_{11} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{11}$$

$$x_{12} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{12}$$

$$x_{13} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{13}$$

$$x_{14} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{14}$$

$$x_{15} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{15}$$

$$x_{16} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{15}$$

$$x_{17} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{15}$$

$$x_{18} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{17}$$

$$x_{20} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{19}$$

$$x_{21} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{20}$$

$$x_{22} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{20}$$

$$x_{23} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{25}$$

$$x_{24} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{25}$$

$$x_{25} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{25}$$

$$x_{26} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{25}$$

$$x_{27} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{27}$$

$$x_{28} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{28}$$

$$x_{29} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{29}$$

$$x_{30} := \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^{30}$$
(4.2.2)

C

>
$$c := 1; x_0 := 1; x_1 := c;$$

for n from 1 to 29 do
 $x_{n+1} := x_n + x_{n-1};$
end do

$$c := 1$$
 $x_0 := 1$
 $x_1 := 1$
 $x_2 := 2$
 $x_3 := 3$
 $x_4 := 5$
 $x_5 := 8$
 $x_6 := 13$
 $x_7 := 21$
 $x_8 := 34$
 $x_9 := 55$
 $x_{10} := 89$
 $x_{11} := 144$
 $x_{12} := 233$
 $x_{13} := 377$
 $x_{14} := 610$
 $x_{15} := 987$
 $x_{16} := 1597$
 $x_{17} := 2584$
 $x_{18} := 4181$
 $x_{19} := 6765$
 $x_{20} := 10946$
 $x_{21} := 17711$
 $x_{22} := 28657$

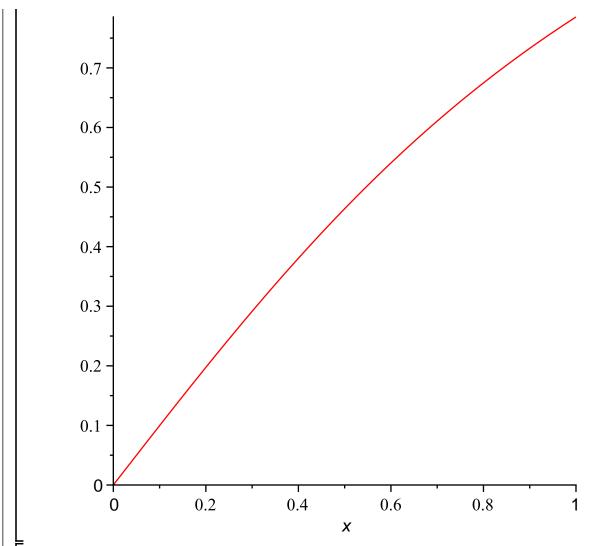
(4.3.1)

$$x_{24} := 75024.99912$$
 $x_{25} := 1.213929985 10^5$
 $x_{26} := 1.964179975 10^5$
 $x_{27} := 3.178109958 10^5$
 $x_{28} := 5.142289932 10^5$
 $x_{29} := 8.320399882 10^5$
 $x_{30} := 1.346268980 10^6$
(4.3.2)

> For hvert sæt af resultater med samme c-værdi giver x_n i realiteten samme værdi. Men grundet approximationer i udregningerne bliver de endelige resultater fra maple lidt forskellige. Dette ses tydligst hvor c=1 hvor fejlen bliver mere tydelig, jo længere vi kommer i fibbonachi sekvensen.

Opgave 1.2.1

- > Vi troede dette var problem 1.2.1 indtil vi opdagede at der fandtes "computer problems"
- > "Determine the best integer value of k in the equation $\arctan(x) = x + Oh(x^k)$ as $x \to 0$ "
- > $plot(\arctan(x), x = 0..1)$



> taylor(arctan(x), x, 7)

$$x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + O(x^7)$$
 (5.1)

 $\rightarrow taylor(arctan(x), x, 1)$

$$O(x) ag{5.2}$$

- > Som vi kan se, ligner taylorudviklingen af arctan(x) den funktion vi bliver bedt om at udregne den bedste k værdi for. Men ud fra vores plot ses det let, at x bliver tæt på lineær tæt på 0. Dette passer også med den første udvikling af vores taylorudvikling. For at finde den bedste k-værdi når $x \rightarrow 0$ skal $x^k \rightarrow 0$ for at $x + x^k \rightarrow \arctan(x)$.
- > Der findes ikke en enkelt bedste k-værdi, men for $1 \le x$ skal k=0.
- **>** For $0 \le x < 1$ skal $k \to \infty$ for at x^k har mindst mulig indflydelse.

Opgave JH-2

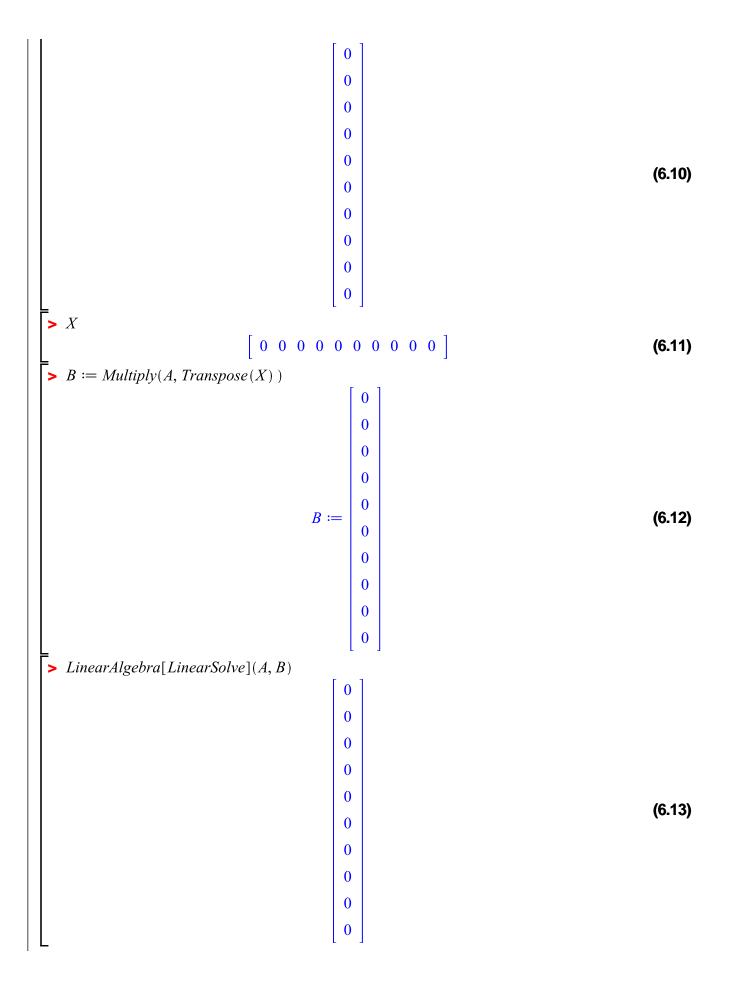
 \searrow with(linalg):

with(LinearAlgebra):

```
> fib := \mathbf{proc}(val :: integer, num :: integer) :: Array,
      local Values, c, n;
      Values := Array(0..(num + 1));
     c := val; Values[0] := 1; Values[1] := c;
     for n from 1 to num do
      Values[n+1] := Values[n] + Values[n-1];
     end do;
     return Values;
   end proc;
fib := \mathbf{proc}(val::integer, num::integer)::Array,
                                                                                                              (6.1)
     local Values, c, n;
     Values := Array(0..num + 1);
     c := val;
     Values[0] := 1;
     Values[1] := c;
     for n to num do Values[n+1] := Values[n] + Values[n-1] end do;
     return Values
end proc
                           X := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
                                                 N := 10
                                                                                                              (6.2)
                                                                                                              (6.3)
                               temp := \begin{bmatrix} 0 .. 11 \ Array \\ Data \ Type: anything \\ Storage: rectangular \\ Order: Fortran_order \end{bmatrix}
                                                                                                              (6.4)
> for n from 2 to N + 1 do X[1][n - 1] := temp[n]; end do
                                               (X_1)_1 := 1
                                               (X_1)_2 := 1
                                               (X_1)_3 := 2
                                               (X_1)_4 := 3
                                               (X_1)_5 := 5
                                               (X_1)_6 := 8
                                               (X_1)_7 := 13
                                               (X_1)_8 \coloneqq 21
                                               (X_1)_0 := 34
                                                                                                              (6.5)
```

 $(X_1)_{10} := 55$ (6.5)(6.6)0, 0, 0, 0 (6.7)(6.8) \rightarrow A := Matrix(N, shape = identity)(6.9)Transpose(X)

(6.10)



LL>

Opgave 1.2.4

