Perceptrons and Structured Perceptrons

CS114B Lab 8

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Perceptrons

- Documents are characterized by features
  - No independence assumptions
- For each feature j:
  - Value x_j
  - Weight w_j
- Bias term b
- "Score" $z = \left(\sum_{j=1}^{n} w_j x_j \right) + b = \mathbf{w} \cdot \mathbf{x} + b$

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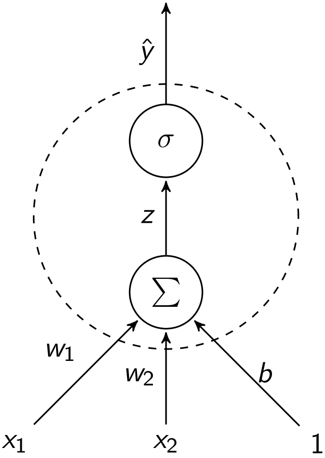
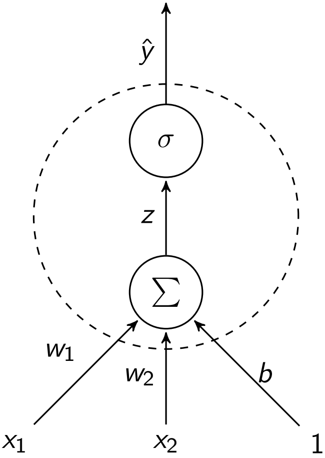
- Does this look familiar?

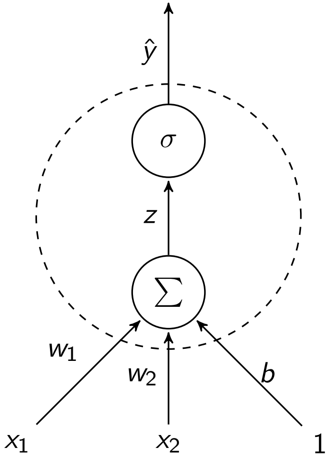
Graphical Representation of Logistic Regression



Graphical Representation of a Neuron



Graphical Representation of a Perceptron



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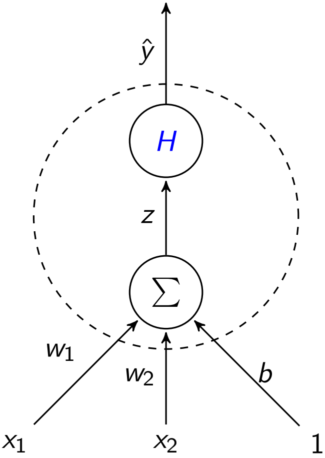

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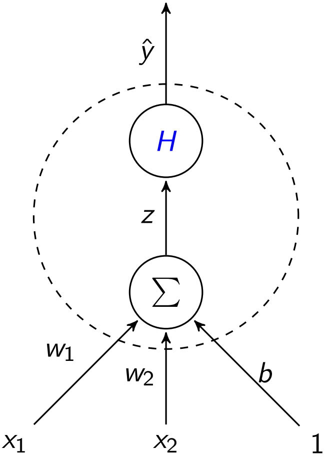

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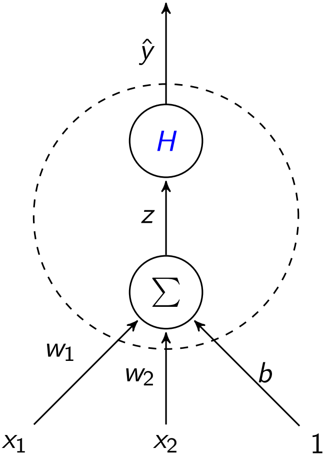

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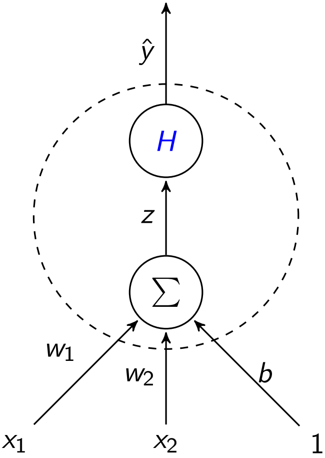

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    - Set by convention (1, 0, or 1/2)

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  - Problem: $\frac{\partial \hat{y}}{\partial z} = 0$ almost everywhere
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  - Solution: consider $\frac{\partial L}{\partial z}$ directly

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Gradients in Perceptrons

- $\dfrac{\partial L}{\partial w_j} = \dfrac{\partial L}{\partial z} x_j$
  - $\dfrac{\partial z}{\partial w_j} = x_j$

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- $\frac{\partial L}{\partial b} = \hat{y} - y$
  - $\frac{\partial z}{\partial b} = 1$
  - ...

Gradient Descent

- Initialize parameters $\theta = \mathbf{w}, b$ (randomly or $\mathbf{0}$)
- At each time step t:
  - Compute gradient ∇L
  - Move in direction of negative gradient
- $\theta_{t+1} = \theta_t - \eta \nabla L$

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- If $\hat{y} = y$, then $\hat{y} - y = 0$
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- If $\hat{y} = y$, then $\hat{y} - y = 0$
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- If $\hat{y} = 0$ and $y = 1$, then $\hat{y} - y = -1$
  - $\nabla L = -\mathbf{x}$
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  - Increment weights

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- If $\hat{y} = 1$ and $y = 0$, then $\hat{y} - y = 1$
  - $\nabla L = \mathbf{x}$
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  - Decrement weights

Multi-class Perceptrons

- Separate weights and bias terms for each class c

$$z_c = \left(\sum_{j=1}^{n} w_{jc} x_j \right) + b_c = \mathbf{w}_c \cdot \mathbf{x} + b_c$$

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- Hinge loss $L(\hat{y}, y) = z_{\hat{y}} - z_y$

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  - For other classes, do nothing

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- $\hat{Y} = \underset{k \in K^T}{\operatorname{argmax}} Z_k$

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Structured Perceptrons

- Score Z decomposes into a sum of local parts
  - At each time step i, for each possible combination of current tag y_i and previous tag y_{i-1}, compute a local score $z(y_i, y_{i-1})$
  - Use the Viterbi algorithm to combine the local scores across the sequence, and find the argmax

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  - Using $\mathbf{f}$ instead of $\mathbf{x}$, because features can include more than just the input

Structured Perceptrons

- We can arrange our weight matrix $\mathbf{\Theta}$ as follows:

$$\begin{bmatrix} \rule{2cm}{0.4pt} \\ \\ \\ \rule{2cm}{0.4pt} \\ \\ \\ \end{bmatrix}$$

Structured Perceptrons

- We can arrange our weight matrix $\boldsymbol{\Theta}$ as follows:

$$\left[\begin{array}{c} \pi \\ \hline \\ \\ \\ \hline \\ \\ \\ \end{array}\right]$$

- Initial features
  - $y_{i-1} = \text{<S>}, y_i = \ldots$

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Structured Perceptrons

- We can arrange our weight matrix $\mathbf{\Theta}$ as follows:

$$\left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array}\right]$$

- Initial features
  - $y_{i-1} = \texttt{<S>}, y_i = \ldots$
- Transition features
  - $y_{i-1} = \ldots, y_i = \ldots$
- Emission features
  - $x_i = \ldots, y_i = \ldots$

Initialization Step

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Initialization Step

$$\mathbf{z}_1 = \mathbf{f}(X, y_1, \texttt{<S>}, 1) \cdot \mathbf{\Theta}$$

$$= \left[\quad \middle| \qquad \middle| \qquad\qquad \right] \cdot \begin{bmatrix} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{bmatrix}$$

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$$\mathbf{z}_1 = \mathbf{f}(X, y_1, \texttt{<S>}, 1) \cdot \boldsymbol{\Theta}$$

$$= \left[\ 1 \;\middle|\; \qquad \;\middle|\; \qquad \right] \cdot \begin{bmatrix} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{bmatrix}$$

- We know that $y_{i-1} = \texttt{<S>}$

Initialization Step

$$\mathbf{z}_1 = \mathbf{f}(X, y_1, \texttt{<S>}, 1) \cdot \mathbf{\Theta}$$

$$= \left[\begin{array}{c|c|c} 1 & \mathbf{0} & \quad\quad\quad\quad \end{array}\right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array}\right]$$

- We know that y_{i-1} cannot be any other tag

Initialization Step

$$\mathbf{z}_1 = \mathbf{f}(X, y_1, \texttt{<S>}, 1) \cdot \boldsymbol{\Theta}$$

$$= \left[\begin{array}{c|c|c} 1 & \mathbf{0} & \mathbf{1}\{x_1 = o_1\} \end{array}\right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array}\right]$$

- One-hot vector of the first word

Initialization Step

$$
\begin{aligned}
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&= \left[\begin{array}{c|c|c} 1 & \mathbf{0} & \mathbf{1}\{x_1 = o_1\} \end{array}\right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array}\right] \\
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$$

- These local scores go into the first column of the Viterbi trellis

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    - Form a feature matrix $\mathbf{F}_i$
  - Compute $\mathbf{Z}_i = \mathbf{F}_i \cdot \mathbf{\Theta}$

Recursion Step

$$\mathbf{Z}_i = \mathbf{F}_i \cdot \boldsymbol{\Theta}$$

$$= \left[\begin{array}{c|c|c} \quad & \quad\quad & \quad\quad\quad\quad \end{array} \right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array} \right]$$

Recursion Step

$$\mathbf{Z}_i = \mathbf{F}_i \cdot \boldsymbol{\Theta}$$

$$= \left[\begin{array}{c|c|c} \mathbf{0} & \quad & \qquad \end{array} \right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array} \right]$$

- We know that $y_{i-1} \neq$ <S>

Recursion Step

$$\mathbf{Z}_i = \mathbf{F}_i \cdot \boldsymbol{\Theta}$$

$$= \left[\begin{array}{c|c|c} \mathbf{0} & \mathbf{I} & \quad\quad\quad\quad\quad \end{array}\right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array}\right]$$

- Identity matrix!

Recursion Step

$$\mathbf{Z}_i = \mathbf{F}_i \cdot \boldsymbol{\Theta}$$

$$= \left[\begin{array}{c|c|c} \mathbf{0} & \mathbf{I} & \mathbf{1}\{x_i = o_i\} \end{array}\right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array}\right]$$

- Stack of one-hot vectors

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\begin{aligned}
\mathbf{Z}_i &= \mathbf{F}_i \cdot \boldsymbol{\Theta} \\
&= \left[\begin{array}{c|c|c} \mathbf{0} & \mathbf{I} & \mathbf{1}\{x_i = o_i\} \end{array}\right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array}\right] \\
&= 0 \cdot \pi + \mathbf{I} \cdot \mathbf{A} + \mathbf{1}\{x_i = o_i\} \cdot \mathbf{B} \\
&= \mathbf{A} + \mathbf{b}(o_i)
\end{aligned}
$$

Recursion Step

$$
\begin{aligned}
\mathbf{Z}_i &= \mathbf{F}_i \cdot \boldsymbol{\Theta} \\
&= \left[\begin{array}{c|c|c} \mathbf{0} & \mathbf{I} & \mathbf{1}\{x_i = o_i\} \end{array}\right] \cdot \left[\begin{array}{c} \pi \\ \hline \mathbf{A} \\ \hline \mathbf{B} \end{array}\right] \\
&= 0 \cdot \pi + \mathbf{I} \cdot \mathbf{A} + \mathbf{1}\{x_i = o_i\} \cdot \mathbf{B} \\
&= \mathbf{A} + \mathbf{b}(o_i)
\end{aligned}
$$

- Use the Viterbi algorithm to combine these local scores with scores from the rest of the sequence

Structured Perceptron Learning Algorithm

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  - In other words, for each time step i:
    - Increment weights for features in y_i, decrement weights for features in $\hat{y}_i$

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- Use the Viterbi algorithm to compute the best tag sequence
- If $\hat{Y} = Y$, then do nothing
- Else:
  - Increment weights for features in Y, decrement weights for features in $\hat{Y}$
  - In other words, for each time step i:
    - Increment weights for features in y_i, decrement weights for features in $\hat{y}_i$
  - Nothing fancy; no Numpy tricks needed