Context-Free Grammars and CKY Algorithm

CS114B Lab 10

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Formal Language Theory

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Formal Language Theory

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- How to define a language?

Formal Language Theory

- A string is in a language iff:

Formal Language Theory

- A string is in a language iff:
  - It is recognized by a machine for that language

Formal Language Theory

- A string is in a language iff:
  - It is recognized by a machine for that language
  - It is generated by a grammar for that language

The Chomsky Hierarchy

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  - How hard is it to decide membership in a language?
  - How much structure does the language define for its strings?

The Chomsky Hierarchy

Grammar	Language	Machine
Unrestricted (Type 0)	Recursively enumerable	Turing machine
Context-sensitive (Type 1)	Context-sensitive	Linear-bounded automaton
Context-free (Type 2)	Context-free	Pushdown automaton
Regular (Type 3)	Regular	Finite-state automaton

The Chomsky Hierarchy

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  - Example: implementing an HMM using FSTs

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- Regular languages: useful for describing linear structure
  - Example: implementing an HMM using FSTs
- Context-free languages: useful for describing hierarchical structure

Phrase structure grammars = context-free grammars

- G = (T, N, S, R)
  - T is set of terminals
  - N is set of nonterminals
    - For NLP, we usually distinguish out a set $P \subset N$ of preterminals, which always rewrite as terminals
    - S is the start symbol (one of the nonterminals)
    - R is rules/productions of the form $X \rightarrow \gamma$, where X is a nonterminal and γ is a sequence of terminals and nonterminals (possibly an empty sequence)
- A grammar G generates a language L.

Phrase structure grammars = context-free grammars

- $S \rightarrow aSb$
- $S \rightarrow \epsilon$

Phrase structure grammars = context-free grammars

Grammar	Lexicon
$S \rightarrow NP\ VP$	$Det \rightarrow that \mid this \mid the \mid a$
$S \rightarrow Aux\ NP\ VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$
$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$Proper\text{-}Noun \rightarrow Houston \mid NWA$
$NP \rightarrow Det\ Nominal$	$Aux \rightarrow does$
$Nominal \rightarrow Noun$	$Preposition \rightarrow from \mid to \mid on \mid near \mid through$
$Nominal \rightarrow Nominal\ Noun$	
$Nominal \rightarrow Nominal\ PP$	
$VP \rightarrow Verb$	
$VP \rightarrow Verb\ NP$	
$VP \rightarrow Verb\ NP\ PP$	
$VP \rightarrow Verb\ PP$	
$VP \rightarrow VP\ PP$	
$PP \rightarrow Preposition\ NP$	

Figure 13.1 The $\mathscr{L}_1$ miniature English grammar and lexicon.

Chomsky Normal Form

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- Every CFG is equivalent to a CFG in Chomsky normal form

Converting a CFG into Chomsky Normal Form

- Add a new start symbol

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- Add a new start symbol
- Remove ϵ-rules

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- Break up rules with more than 3 things on the right hand side

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- Replace terminals with nonterminals and add new rules as needed

Converting a CFG into Chomsky Normal Form

- Add a new start symbol
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- Remove unary rules
- Break up rules with more than 3 things on the right hand side
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  - We can modify CKY algorithm to handle unary rules

Probabilistic CFGs

- Augment each rule in R with a conditional probability

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- This allows us to use dynamic programming
- Does this look familiar?

Probabilistic CFGs

Grammar		Lexicon
$S \rightarrow NP\ VP$	[.80]	$Det \rightarrow that$ [.10] | a [.30] | the [.60]
$S \rightarrow Aux\ NP\ VP$	[.15]	$Noun \rightarrow book$ [.10] | $trip$ [.30]
$S \rightarrow VP$	[.05]	| $meal$ [.05] | $money$ [.05]
$NP \rightarrow Pronoun$	[.35]	| $flight$ [.40] | $dinner$ [.10]
$NP \rightarrow Proper\text{-}Noun$	[.30]	$Verb \rightarrow book$ [.30] | $include$ [.30]
$NP \rightarrow Det\ Nominal$	[.20]	| $prefer$ [.40]
$NP \rightarrow Nominal$	[.15]	$Pronoun \rightarrow I$ [.40] | she [.05]
$Nominal \rightarrow Noun$	[.75]	| me [.15] | you [.40]
$Nominal \rightarrow Nominal\ Noun$	[.20]	$Proper\text{-}Noun \rightarrow Houston$ [.60]
$Nominal \rightarrow Nominal\ PP$	[.05]	| NWA [.40]
$VP \rightarrow Verb$	[.35]	$Aux \rightarrow does$ [.60] | can [.40]
$VP \rightarrow Verb\ NP$	[.20]	$Preposition \rightarrow from$ [.30] | to [.30]
$VP \rightarrow Verb\ NP\ PP$	[.10]	| on [.20] | $near$ [.15]
$VP \rightarrow Verb\ PP$	[.15]	| $through$ [.05]
$VP \rightarrow Verb\ NP\ NP$	[.05]	
$VP \rightarrow VP\ PP$	[.15]	
$PP \rightarrow Preposition\ NP$	[1.0]	

Figure C.1 A PCFG that is a probabilistic augmentation of the $\mathscr{L}_1$ miniature English CFG grammar and lexicon of Fig. **??**. These probabilities were made up for pedagogical purposes and are not based on a corpus (any real corpus would have many more rules, so the true probabilities of each rule would be much smaller).

CKY Algorithm

- Given tables *table* and *back*:

CKY Algorithm

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  - Base case

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    - Fill in *table* cells $[i, i+1]$ with all possible nonterminals that can generate that word

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    - In *back*, fill in cell $[i, k]$ with backpointers (e.g. A: j, B, C)

Probabilistic CKY Algorithm

- Given tables *table* and *back*:
  - Base case
    - Fill in *table* cells $[i, i+1]$ with all possible nonterminals that can generate that word, and their probabilities
  - Recursive case
    - In *table*, if $A \rightarrow BC$ and B is in cell $[i, j]$ and C is in cell $[j, k]$, and $table[i, j, A] < P(A \rightarrow BC) \times table[i, j, B] \times table[j, k, C]$, fill in cell $[i, k]$ with A:
      $P(A \rightarrow BC) \times table[i, j, B] \times table[j, k, C]$
    - In *back*, fill in cell $[i, k]$ with backpointers (e.g. A: j, B, C)

Probabilistic CKY Algorithm

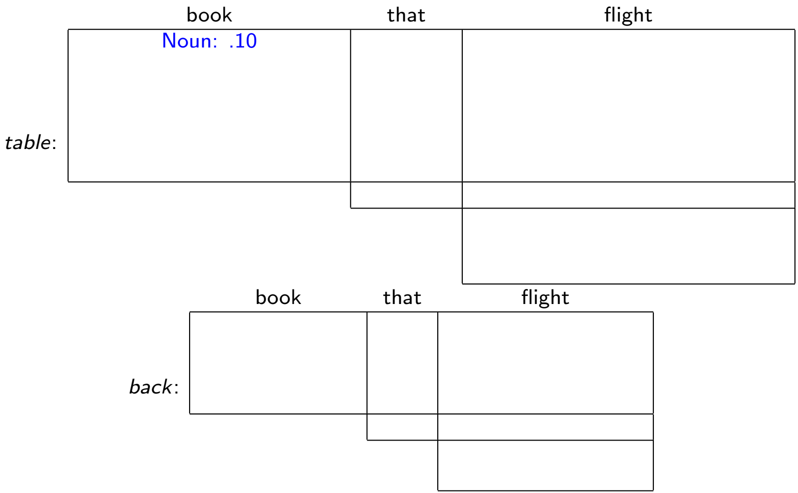
table:

book	that	flight

back:

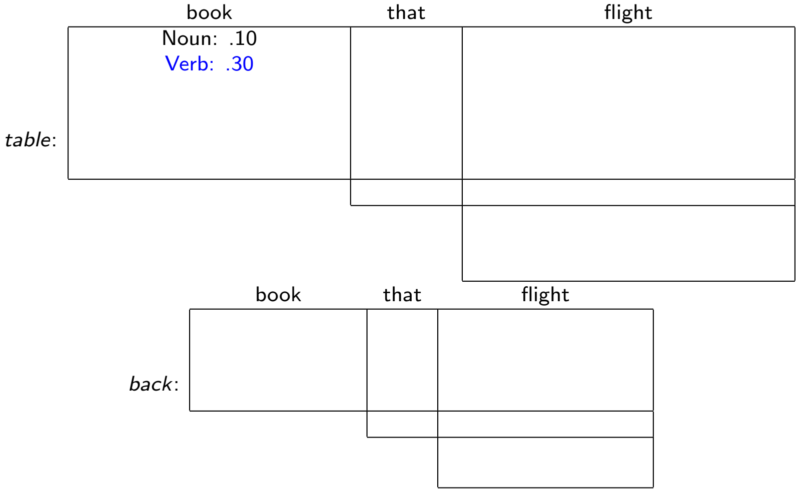
book	that	flight

Probabilistic CKY Algorithm



- Noun $\rightarrow$ book [.10]

Probabilistic CKY Algorithm



- Verb $\rightarrow$ book [.30]

Probabilistic CKY Algorithm

table:

	book	that	flight
	Noun: .10 Verb: .30		
		Det: .10	

back:

	book	that	flight

- Det $\rightarrow$ that [.10]

Probabilistic CKY Algorithm

table:

	book	that	flight
	Noun: .10 Verb: .30		
		Det: .10	
			Noun: .30

back:

	book	that	flight

- Noun $\rightarrow$ flight [.30]

Probabilistic CKY Algorithm

- Unary rules

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- Unary rules
  - In *table*, if $A \rightarrow B$ and B is in cell $[i, i+1]$, fill in cell $[i, i+1]$ with A: $P(A \rightarrow B) \times \mathit{table}[i, i+1, B]$

Probabilistic CKY Algorithm

- Unary rules
  - In *table*, if $A \rightarrow B$ and B is in cell $[i, i+1]$, fill in cell $[i, i+1]$ with A: $P(A \rightarrow B) \times table[i, i+1, B]$
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Probabilistic CKY Algorithm

table:

	book	that	flight
	Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$		
		Det: .10	
			Noun: .30

back:

	book	that	flight
	Nominal: Noun		

- Nominal $\rightarrow$ Noun [.75]

Probabilistic CKY Algorithm

table:

	book	that	flight
	Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$ NP: $.15 \times .075 = .01125$		
		Det: .10	
			Noun: .30

back:

	book	that	flight
	Nominal: Noun NP: Nominal		

- NP $\rightarrow$ Nominal [.15]

Probabilistic CKY Algorithm

table:

	book	that	flight
	Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$ NP: $.15 \times .075 = .01125$ VP: $.35 \times .30 = .105$		
		Det: .10	
			Noun: .30

back:

	book	that	flight
	Nominal: Noun NP: Nominal VP: Verb		

- VP $\rightarrow$ Verb [.35]

Probabilistic CKY Algorithm

table:

book	that	flight
Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$ NP: $.15 \times .075 = .01125$ VP: $.35 \times .30 = .105$ S: $.05 \times .105 = .00525$		
	Det: .10	
		Noun: .30

back:

book	that	flight
Nominal: Noun NP: Nominal VP: Verb S: VP		

- S $\rightarrow$ VP [.05]

Probabilistic CKY Algorithm

	book	that	flight
table:	Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$ NP: $.15 \times .075 = .01125$ VP: $.35 \times .30 = .105$ S: $.05 \times .105 = .00525$		
		Det: .10	
			Noun: .30 Nominal: $.75 \times .30 = .225$

	book	that	flight
back:	Nominal: Noun NP: Nominal VP: Verb S: VP		
			Nominal: Noun

- Nominal $\rightarrow$ Noun [.75]

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table:

	book	that	flight
	Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$ NP: $.15 \times .075 = .01125$ VP: $.35 \times .30 = .105$ S: $.05 \times .105 = .00525$		
		Det: .10	
			Noun: .30 Nominal: $.75 \times .30 = .225$ NP: $.15 \times .225 = .03375$

back:

	book	that	flight
	Nominal: Noun NP: Nominal VP: Verb S: VP		
			Nominal: Noun NP: Nominal

- NP $\rightarrow$ Nominal [.15]

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table:	Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$ NP: $.15 \times .075 = .01125$ VP: $.35 \times .30 = .105$ S: $.05 \times .105 = .00525$		
		Det: .10	NP: $.20 \times .10 \times .225 = .0045$
			Noun: .30 Nominal: $.75 \times .30 = .225$ NP: $.15 \times .225 = .03375$

back:

	book	that	flight
back:	Nominal: Noun NP: Nominal VP: Verb S: VP		
			NP: 2 Det Nominal
			Nominal: Noun NP: Nominal

- NP $\rightarrow$ Det Nominal [.20]

Probabilistic CKY Algorithm

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book	that	flight
Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$ NP: $.15 \times .075 = .01125$ VP: $.35 \times .30 = .105$ S: $.05 \times .105 = .00525$		VP: $.20 \times .30 \times .0045 = .00027$
	Det: .10	NP: $.20 \times .10 \times .225 = .0045$
		Noun: .30 Nominal: $.75 \times .30 = .225$ NP: $.15 \times .225 = .03375$

back:

book	that	flight
Nominal: Noun NP: Nominal VP: Verb S: VP		VP: 1 Verb NP
		NP: 2 Det Nominal
		Nominal: Noun NP: Nominal

- VP $\rightarrow$ Verb NP [.20]

Probabilistic CKY Algorithm

table:

	book	that	flight
	Noun: .10 Verb: .30 Nominal: $.75 \times .10 = .075$ NP: $.15 \times .075 = .01125$ VP: $.35 \times .30 = .105$ S: $.05 \times .105 = .00525$		VP: $.20 \times .30 \times .0045 = .00027$ S: $.05 \times .00027 = .0000135$
		Det: .10	NP: $.20 \times .10 \times .225 = .0045$
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back:

	book	that	flight
	Nominal: Noun NP: Nominal VP: Verb S: VP		VP: 1 Verb NP S: VP
			NP: 2 Det Nominal
			Nominal: Noun NP: Nominal

- $S \rightarrow VP$ [.05]

Probabilistic CKY Algorithm

	book	that	flight
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			Nominal: Noun NP: Nominal

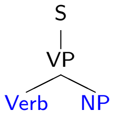
Probabilistic CKY Algorithm

	book	that	flight
back:	Nominal: Noun NP: Nominal VP: Verb S: VP		VP: 1 Verb NP S: VP
			NP: 2 Det Nominal
			Nominal: Noun NP: Nominal

S
|
VP

Probabilistic CKY Algorithm

	book	that	flight
back:	Nominal: Noun NP: Nominal VP: Verb S: VP		VP: 1 Verb NP S: VP
			NP: 2 Det Nominal
			Nominal: Noun NP: Nominal



Probabilistic CKY Algorithm

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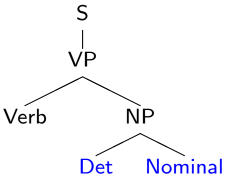

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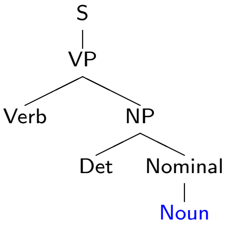

Probabilistic CKY Algorithm

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