CS114B Lab 7

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## Sequence Labeling

- ► Suppose we observe a list of words *X*. What are the respective parts of speech *Y*?
  - ▶ What is P(Y|X)?

- ► Generative approach: Hidden Markov Models
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Output Independence: the probability of a word at time i depends only on the tag at time i

$$P(X|Y) = \prod_{i=1}^{T} P(x_i|y_i)$$

$$P(Y|X) \propto \prod_{i=1}^{T} P(x_i|y_i) \times \prod_{i=1}^{T} P(y_i|y_{i-1})$$

$$\propto \prod_{i=1}^{T} \left( P(x_i|y_i) \times P(y_i|y_{i-1}) \right)$$

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- ▶ In other words, (log) P(Y|X) decomposes into a product (or sum) of local parts
- ► This allows us to use dynamic programming

- ► Not just for HMMs!
- Discriminative approaches:
  - Conditional random fields
  - Structured perceptrons

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- Discriminative approaches:
  - Conditional random fields
  - Structured perceptrons
- ► As long as the "score" decomposes into a sum of local parts, we can use the Viterbi algorithm

```
function VITERBI(observations of len T.state-graph of len N) returns best-path, path-prob
create a path probability matrix viterbi[N,T]
for each state s from 1 to N do
                                                                ; initialization step
      viterbi[s,1] \leftarrow \pi_s * b_s(o_1)
      backpointer[s,1] \leftarrow 0
for each time step t from 2 to T do ; recursion step
   for each state s from 1 to N do
      viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_{s}(o_{t})
backpointer[s,t] \leftarrow \underset{t}{\operatorname{argmax}} viterbi[s',t-1] * a_{s',s} * b_{s}(o_{t})
\textit{bestpathprob} \leftarrow \max_{s=1}^{N} \ \textit{viterbi}[s,T] \hspace{1cm} ; \text{termination step}
bestpathpointer \leftarrow \underset{}{\operatorname{argmax}} viterbi[s, T] ; termination step
bestpath ← the path starting at state bestpathpointer, that follows backpointer[] to states back in time
return bestpath, bestpathprob
```

Figure 8.10 Viterbi algorithm for finding the optimal sequence of tags. Given an observation sequence and an HMM  $\lambda = (A, B)$ , the algorithm returns the state path through the HMM that assigns maximum likelihood to the observation sequence.

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- ► Also let *N* be the number of states and *V* be the size of the vocabulary

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- ► Three Numpy arrays:
  - ▶ self.initial  $(\pi: \text{shape } (N,))$
  - ▶ self.transition (**A**: shape (N, N))
  - ▶ self.emission ( $\mathbf{B}$ : shape (V, N))
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- We assume you know how to fill them in
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- For each sentence, create two Numpy arrays: (both with shape (N, T))
  - ▶ v (for viterbi)
  - backpointer



- For each state s from 1 to N do  $viterbi[s,1] \leftarrow \pi_s + b_s(o_1)$
- $\blacktriangleright$  Note the use of + instead of  $\times$
- ▶ We'll see how to do this without the for loop in a bit

### Recursion Step

- For each time step t from 2 to T do

  For each state s from 1 to N do  $viterbi[s, t] \leftarrow \max_{s'=1}^{N} viterbi[s', t-1] + a_{s',s} + b_s(o_t)$
- ▶ Note the use of + instead of ×
- ▶ We'll see how to do this without the (inner) for loop in a bit

## Termination Step

- ▶ Best last tag is the argmax of the last column of v
- Follow backpointers in backpointer
  - Nothing fancy; we'll let you figure it out on your own
  - For HW4, you do not have to return the path (log-)probability/score, just the backtrace path

- Perceptrons for sequence labeling
  - Given a sequence  $X = [x_1, \dots, x_T]$ , predict labels  $Y = [y_1, \dots, y_T]$
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- $\hat{Y} = \operatorname*{argmax}_{k \in K^T} Z_k$

- ► Score Z decomposes into a sum of local parts
  - At each time step i, for each possible combination of current tag  $y_i$  and previous tag  $y_{i-1}$ , compute a local score  $z(y_i, y_{i-1})$

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  - At each time step i, for each possible combination of current tag  $y_i$  and previous tag  $y_{i-1}$ , compute a local score  $z(y_i, y_{i-1})$
  - ► Use the Viterbi algorithm to combine the local scores across the sequence, and find the argmax

- ► Suppose that at each time step *i*, we want to predict the current tag *y<sub>i</sub>* using the following features:
  - $\triangleright$  Previous tag  $y_{i-1}$ 
    - ▶ At the beginning of the sentence, let the previous tag y<sub>i-1</sub> be the start symbol <S>
  - Current word x<sub>i</sub>

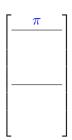
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- ► For simplicity, we will assume that these are the only features, and we will ignore the bias term
- ▶ Let  $\mathbf{f}(X, y_i, y_{i-1}, i)$  be the feature vector at time step i
  - Using f instead of x, because features can include more than just the input

▶ We can arrange our weight matrix **Θ** as follows:

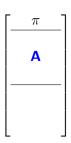


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- ► Initial features
  - ▶  $y_{i-1} = \langle S \rangle, y_i = ...$

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Initial features

$$y_{i-1} = \langle S \rangle, y_i = \dots$$

► Transition features

$$y_{i-1} = \dots, y_i = \dots$$

► Emission features

$$ightharpoonup x_i = \ldots, y_i = \ldots$$

- We want to compute local scores  $z(y_1, \le)$  for each possible  $y_1$ 
  - ▶ These are the elements of  $\mathbf{z}_1 = \mathbf{f}(X, y_1, \langle S \rangle, 1) \cdot \mathbf{\Theta}$

$$\mathbf{z}_1 = \mathbf{f}(X, y_1, \langle \mathbb{S} \rangle, 1) \cdot \mathbf{\Theta}$$
  $= \begin{bmatrix} 1 & & & \\ &$ 

▶ We know that  $y_{i-1} = \langle S \rangle$ 

$$\mathbf{z}_1 = \mathbf{f}(X,y_1, ext{~~}, 1) \cdot \mathbf{\Theta}~~$$
  $= \left[ egin{array}{c|c} 1 & \mathbf{0} & & \\ & \mathbf{B} & \end{array} 
ight] \cdot \left[ egin{array}{c|c} \pi & & \\ & \mathbf{A} & \\ & & \mathbf{B} & \end{array} 
ight]$ 

▶ We know that  $y_{i-1}$  cannot be any other tag

$$\mathbf{z}_1 = \mathbf{f}(X, y_1, \langle \mathrm{S} \rangle, 1) \cdot \mathbf{\Theta}$$
  $= \left[\begin{array}{c|c} 1 & \mathbf{0} & \mathbf{1}\{x_1 = o_1\} \end{array}\right] \cdot \left[\begin{array}{c} \pi & \mathbf{A} \\ \mathbf{B} \end{array}\right]$ 

One-hot vector of the first word

$$\mathbf{z}_1 = \mathbf{f}(X, y_1, \langle \mathbf{S} \rangle, 1) \cdot \mathbf{\Theta}$$

$$= \begin{bmatrix} 1 & \mathbf{0} & \mathbf{1}\{x_1 = o_1\} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{\pi} & \mathbf{A} \\ \mathbf{A} & \mathbf{B} \end{bmatrix}$$

$$= 1 \cdot \pi + \mathbf{0} \cdot \mathbf{A} + \mathbf{1}\{x_1 = o_1\} \cdot \mathbf{B}$$

$$= \pi + \mathbf{B}[o_1]$$

$$\mathbf{z}_{1} = \mathbf{f}(X, y_{1}, \langle \mathbf{S} \rangle, 1) \cdot \mathbf{\Theta}$$

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$$= \pi + \mathbf{B}[o_{1}]$$

▶ These local scores go into the first column of the Viterbi trellis

## Recursion Step

- We want to compute local scores  $z(y_i, y_{i-1})$  for each possible combination of  $y_i$  and  $y_{i-1}$ 
  - We can stack the feature vectors for each possible  $y_{i-1}$ ,  $\mathbf{f}(X, y_i, y_{i-1}, i)$ , on top of each other, in order
    - $\triangleright$  Form a feature matrix  $\mathbf{F}_i$

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    - ightharpoonup Form a feature matrix  $\mathbf{F}_i$
  - ► Compute  $\mathbf{Z}_i = \mathbf{F}_i \cdot \mathbf{\Theta}$

$$\mathbf{Z}_i = \mathbf{F}_i \cdot \mathbf{\Theta}$$
 
$$= \begin{bmatrix} \mathbf{0} & & & \\ & & & \\ & & & \end{bmatrix} \cdot \begin{bmatrix} \frac{\pi}{\mathbf{A}} & & \\ & \mathbf{A} & & \\ & & \mathbf{B} & \end{bmatrix}$$

▶ We know that  $y_{i-1} \neq \langle S \rangle$ 

$$\mathbf{Z}_i = \mathbf{F}_i \cdot \mathbf{\Theta}$$
 
$$= \left[ egin{array}{c|c} \mathbf{0} & \mathbf{I} & & & \\$$

► Identity matrix!

$$\mathbf{Z}_i = \mathbf{F}_i \cdot \mathbf{\Theta}$$
 
$$= \left[ \begin{array}{c|c} \mathbf{0} & \mathbf{I} & \mathbf{1}\{x_i = o_i\} \end{array} \right] \cdot \left[ \begin{array}{c} \pi & \\ \mathbf{A} & \\ B \end{array} \right]$$

Stack of one-hot vectors

$$\mathbf{Z}_{i} = \mathbf{F}_{i} \cdot \mathbf{\Theta}$$

$$= \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{1} \{x_{i} = o_{i}\} \end{bmatrix} \cdot \begin{bmatrix} \frac{\pi}{\mathbf{A}} \\ \mathbf{A} \end{bmatrix}$$

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Use the Viterbi algorithm to combine these local scores with scores from the rest of the sequence



▶ Use the Viterbi algorithm to combine these local scores with scores from the rest of the sequence

$$\mathbf{Z}_i = \mathbf{F}_i \cdot \mathbf{\Theta} = \mathbf{A} + \mathbf{B}[o_i]$$

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$$\mathbf{Z}_i = \mathbf{F}_i \cdot \mathbf{\Theta} = \mathbf{A} + \mathbf{B}[o_i]$$

$$Viterbi[:, i] = max(Viterbi[:, i - 1:i] + \mathbf{A} + \mathbf{B}[o_i], axis=0)$$

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  - In other words, for each time step *i*:
    - ▶ Increment weights for features in  $y_i$ , decrement weights for features in  $\hat{y}_i$

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