1 Kalman Filter

1.1 How to update Kalman Filter

Given a vector x which follow a gaussian probability distribution, the marginals and the conditions are gaussian distributions.

1.1.1 Linear Model

The Kalman filter assumes a linear transition and observation model with zero mean gaussian noise
For the prediction state update: $x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$

- The first term will take the previous state times a matrix
- The second term will take in the control commands
- The last term will encode covariance or uncertainty into the prediction step

For the measurement: $z_t = C_t x_t + \delta_t$

- Predicted observation based on the state
- C_t will transform from the state space to the observation space
- δ_t is the noise

Matricies explained:

- A_t is an (n, n) matrix which describes how the state evolves from t-1 to t without any controls or noise. More of the environmental factors like blowing wind or current velocity of the robot.
- B_t is an (n, l) matrix which describes how the control changes from state t-1 to t. l is the dimensionality of the control vector and this matrix can transform the odometry to the state measurement
- C_t is an (k, n) matrix which describes how to map the state x_t to an observation z_t
- ϵ_t Represents the process noise at time t, refers to covariance R_t
- δ_t Represents the measurement noise, refers to covariance Q_t

1.2 Deriving the Representation, μ and Σ

1.3 Kalman Filter Algorithm

The predicted mean is just exploiting the linear model. The covariance is just an additive noise from the covariance matrix R_t

The correction step is essentially a weighted sum of the belief and the measurement. The weighting factor is called the **Kalman Gain**. The kalman gain will be high when the covariance is greater in the motion model or small in the measurement model. For the mean correction, kalman gain turns out to be the weight of the residual for the observation/measurement which is really just how much should I trust the measurement I just made when correcting the mean. The covariance correction also uses the kalman gain

Algorithm Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

$\bar{\mu}_t = A_t\ \mu_{t-1} + B_t\ u_t$
$\bar{\Sigma}_t = A_t\ \Sigma_{t-1}\ A_t^T + R_t$

$K_t = \bar{\Sigma}_t\ C_t^T (C_t\ \bar{\Sigma}_t\ C_t^T + Q_t)^{-1}$
$\mu_t = \bar{\mu}_t + K_t(z_t - C_t\ \bar{\mu}_t)$
$\Sigma_t = (I - K_t\ C_t)\ \bar{\Sigma}_t$
return μ_t, Σ_t