University of Victoria

Department of Mechanical Engineering

MECH 395

Laboratory # 2

Heat Sink Experiment

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Section B01 – Session A

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Summary

The following report recommends a solution for the overheating of an electronic device. The recommended solution involves securing a finned array to the device in conjunction with inducing forced air flow. Several configurations of finned arrays are available. In the preliminary report, a model was developed in MATLAB to determine the temperature of the electronic device for any given finned array and environmental parameters. The objective of this report is to validate the model and use it to recommend the most effective finned array.

Experiments were used to validate the model. A control station was set up for each finned array that allowed the measurement of device temperature, power dissipation and fan air speed at steady state conditions. In addition, the base and tip temperatures of a sample of fins were measured in order to obtain the experimental average convection coefficient.

Using the measured input values, the model predicted the device temperature within 12.5% at each control station. Because the variance in the performance of the finned arrays was significant; the model provided a feasible relative performance rating for each finned array. Out of the three available finned arrays, the validated model predicted Plate 13 to have the best performance in terms of heat dissipation. Plate 13 was predicted to yield the lowest electronic device temperature at nominal power dissipation.

The recommended solution is to use Plate 13 in conjunction with forced air flow. In order to avoid premature failure of the device, the recommended minimum air speed from the fan at nominal power is 1000 ft/min. The model predicts the device temperature to be 50 ºC for Plate 13 at 1000 ft/min air speed and nominal power, 10 ºC below the maximum allowable temperature for the electronic device.

Table of Contents

1. Background 1
   1.1. Objectives 1
2. Methods 1
   2.1. Model Validation 1
   2.2. Determination of the Experimental Convection Coefficients 1
3. Model 2
   3.1. Assumptions 2
   3.2. Limitations 3
4. Results 3
5. Discussion 5
   5.1. Experimental Uncertainties 5
   5.2. Model Uncertainties 6
6. Recommendations 7
   6.1. Finned Array 7
   6.2. Operating Conditions 8
7. Conclusion 9
8. References 10

Appendix A: Model Methodology 11
   Thermal Circuit 11
   Determination of Convection coefficients 13
   Approximation of the Contact Resistances 14
Appendix B: Available Finned Arrays 15
Appendix C: Air Speed Analysis 15
Appendix D: MATLAB Model Source Code 16

1. Background

The purpose of this report is to propose a solution to the premature failure of an electronic device. The source of the premature failure is overheating; the device is exceeding its maximum allowable temperature of 60°C. The nominal power dissipation of the device is 40W while operating at room temperature. The proposed solution involves inducing forced air flow with a fan and securing a finned array to the electronic device. There are three finned arrays available that vary in the material, configuration and number of fins.

1.1. Objectives

The objectives of this report are

(i) to validate a modeling tool for heat sink design through experimental testing, and
(ii) to recommend a solution for the premature failure of the electronic device.

2. Methods

In the previous stage of this analysis, a model was developed to predict the temperature of the electronic device as a function of its power dissipation, environment, and finned array parameters. The main method involved in development of the model was a thermal circuit. A full description of the methods used to develop the model is available in Appendix A. Refer to section 3 for the input parameters, development assumptions and limitations of the model. The methodology used for validation as well as the experimental determination of the convection coefficients is discussed in the following sections.

2.1.Model Validation

The validity of the model is determined through comparison with experimental results. Experiments are conducted on a control system that contains the electronic chip, fan, and one of the available finned arrays. There is a control system for each finned array. The following measurements are obtained at steady state:

(i) Electronic device temperature
(ii) Power dissipation
(iii) Fan air speed
(iv) Air temperature entering the finned array
(v) Air temperature exiting the finned array
(vi) Base and tip temperatures of a sample of fins

The measured power dissipation, fan air speed and entrance air temperature are used as input for the model. The electronic device temperature computed by the model is compared with the measured electronic device temperature.

2.2. Determination of the Experimental Convection Coefficients

By measuring the base and tip temperatures of the fins at different locations of the array a set of experimental convection coefficient is obtained. First, the temperature ratio of $\theta(L)$ equation (2) and θ_b equation (3) is determined using the measured tip and base temperatures $T(L)$ and T_b, respectively. Equation (1) is than utilized to find the corresponding convection coefficient for the fin at that location.

$$\frac{\theta(x)}{\theta_b} = \frac{\left(\cosh m(L-x) + \left(\frac{h}{mk}\right)\sinh m(L-x)\right)}{\left(\left(\frac{h}{mk}\right)\sinh(mL) + \cosh(mL)\right)} \tag{1}$$

where

$$\theta(x) = T(x) - T_{air} \tag{2}$$
$$\theta_b = T_b - T_{air} \tag{3}$$

The assumptions included for this analysis are as follows:

(1) A single base temperature was taken for the analysis thus assuming base temperature is constant throughout the fin.
(2) Ambient air temperature remains constant throughout and is taken equal to entrance air temperature
(3) Fins and base are assumed to be made from pure materials. See Appendix B for finned array properties

3. Model

The purpose of the model is to predict the temperature of an electronic device with a finned array and forced air flow given the following input parameters.

(i) Power dissipation
(ii) Fan air speed
(iii) Air temperature
(iv) Base plate contact resistance
(v) Finned array dimensions
(vi) Finned array thermal conductivity
(vii) Fin contact resistance

The following section discusses the assumptions made during the development of the model.

3.1. Assumptions

The following list provides the assumption made to construct the model. Refer to Appendix A for justification of assumptions.

Thermal Circuit Assumptions:

(1) Steady state conditions
(2) One dimensional heat transfer
(3) Uniform temperature within electronic device
(4) No heat loss from bottom of the electronic device
(5) Radiation is negligible

Determination of Convection Coefficient Assumptions:

(6) The convection coefficient of forced flow over a single cylinder is equal to the convection coefficient of an individual fin in a finned array.
(7) The convection coefficient over a flat plate is equal to the convection coefficient of a base plate in a finned array
(8) The temperature of the air entering the finned array is equal to the temperature of the air exiting the finned array
(9) Fins have equal tip and base temperatures

Approximation of the Contact Resistances:

(10) The contact resistance between the plate and the electronic device is approximately equal to the contact resistance of a chip/aluminum interface with 0.02-mm epoxy
(11) The contact resistance between the fin and the plate is approximately equal to the contact resistance of a solid/solid interface.

3.2. Limitations

In order to provide a valid result, the model parameters in Table 1 must remain within their respective bounds. The model is programmed to warn the user if one of the bounds is exceeded. In addition, the model is developed for forced air flow and will not yield valid results for natural convection.

Table 1 Parameter Restrictions

Parameter	**Restriction**
Reynolds Number	0.4 < Re < $4\ x\ 10^5$
Prandlt Number	0.6 < Pr < 60
Air Temperature (K)	250 < T < 300

4. Results

The following results pertain to the selected finned arrays Plate 3, Plate 7, and Plate 13. The parameters for the finned arrays are available in Appendix B.

Table 2 Experimental Results

Trail	**Plate**	**Power [W]**	**Air Speed [ft/min]**	**Entrance Air Temperature [ºC]**	**Exit Air Temperature [ºC]**	**Electronic Device Temperature [ºC]**
1	13	18	900	21.7	28.2	34.4
2	3	17	-	-	-	35.7
3	7	12	550	22.4	25.9	40
4	7	12	900	22.8	24.3	35.9

The experimentally obtained power dissipation, air speed, and entrance air temperature is inputted into the model to determine the theoretical electronic device temperature. A comparison of the experimental and theoretical results is shown below in table 3. The model was unable to predict the temperature of the device in trail 2 due to the lack of forced air flow. In the preliminary report, it was determined that there is a risk of electronic device overheating if it is run at nominal power without forced air flow (See Appendix C for justification).

Table 3 Comparison of Experimental and Theoretical Electronic Device Temperatures

Trail	**Experimental Temperature [ºC]**	**Theoretical Temperature [ºC]**	**Error %**
1	34.4	34	1.2
2	35.7	-	-
3	40	43.63	9.1
4	35.9	40.4	12.5

From Table 3 it is concluded that the model represents the fin temperatures quite accurately. The maximum error is calculated from Fin 7 (12.5%). An interpretation of these errors is discussed in section 5. Overall the model predicts device temperatures from the initial parameters quite accurately in comparison to the measured value.

The estimated value for convection coefficient is obtained by the method described in Section 2.2. The average experimental values and the theoretical values for convection coefficient is shown in Table 4. The model largely predicts a theoretical convection coefficient with a significant error in comparison to the experimental value. Trial 1 for Fin 13 was the most satisfactory. Even though an error of 20.2% was obtained for the h_f from model, the experimental analysis did reveal a modal value for h_f to be 130.67 W/ m^2.K. This suggests that the model did not take into account the change in h_f spatially and so only the first few values of h_f were in agreement to the model. Trial 2 was done for Fin 3 with no forced air flow. Experimental calculations revealed an average h_f of 48.02 W/K.m^2 which was within the range for convection coefficients for free flow of air.

Table 4 Comparison of Experimental and Theoretical convection coefficient for fins

Trail	**Experimental** $\overline{h_{fins,e}}$ **W/(Km^2)**	**Theoretical** $\overline{h_{fins,t}}$ **W/(Km^2**	**Error %**
1	165.17	131.67	20.2
2	48.02	10-100	Within range
3	640.6843429	102.87	83
4	651.25	131.67	79.7

In order to find h_f average for the previous section, a number of value of spatial h_f were found and the mean was taken. For all the three fins the mode of the set of hf value generally were close to a small range. However, some h_f values found were also significantly different than the mode of the set. Table 4 shows an example for the local hf value obtained for fin 13. As is seen from the table the mode for the set of hf is within the range (125-135) W/K.m^2, which agrees closely with the model. It was seen that the value of hf deviated (increased) from the model value as row number analyzed was increased. This was observed in all three fins. This was basically due to a different actual ambient temperature, base temperature and fluid mixing as row number observed was increased.

Table 5 Temperature Measurements of Plate 13

Row	Column	Tx ©	h (exp)	θx/θb
1	3	27.4	130.6557014	0.656999725
1	7	27.4	130.6559083	0.65699934
1	9	27.5	123.2574723	0.670999156
3	1	24.7	471.4457061	0.302629457
3	4	26.5	204.7337224	0.538999977
3	7	27.3	137.7545491	0.643999064
3	9	26.7	185.8607691	0.565699755
5	9	26.1	246.1564837	0.486841127
5	7	25.9	269.7817831	0.460499522
5	4	26.1	246.1926802	0.486799013

5. Discussion

The following section discusses the uncertainties in both the experimental and model uncertainties which contribute to the error found in the results section.

5.1. Experimental Uncertainties

The following lists describes the uncertainties in the experimentally obtained values.

(1) Measuring the fin and base temperature with the thermocouple was difficult and often times not steady. The ends of the thermocouples often touched other parts of the fin not intended in the analysis due to force from air.

(2) Measuring the base surface temperature with the thermocouple also resulted in some uncertainties. The radiation from adjacent fins and the lower temperature air-flow over the base resulted in errors for the measured temperature. Only one base temperature reading was taken for the experiment in order to find convection coefficient but it was seen that the base surface temperature varied depending on spatial location.

(3) There were a lot of sources of uncertainties in the experiment. Firstly the exact material properties of the fins were not known and the expected values of temperature changed drastically upon changing the thermal conductivity from pure Aluminum/Brass to Alloys.

(4) Uncertainties in the calculations for average convection coefficients:
While trying to measure average convective coefficient a set of localized convection coefficient values are obtained at different position of the fins keeping symmetry in mind. However, the base temperature and the ambient air temperature for all the calculations were

kept to one single value. The base temperature at the middle, and the ambient air temperature at the entrance were used only instead of finding local values of the temperatures.

5.2. Model Uncertainties

Due to the approximations made to develop the model, there are uncertainties associated with its results. In the experiment, several assumptions were found to be inaccurate; specifically, assumptions (8) and (9) below.

(8) The temperature of the air entering the finned array is approximately equal to the temperature of the air exiting the finned array
(9) Fins have equal tip and base temperatures

During the experiment, the difference between the entrance and exit temperature of the air was found to be at least 1.5 ^{o}C at each control station. This results in a variation in the fin convection coefficients and is unaccounted for by the model. A constant surface temperature and an increased air temperature yield to a decreased convection coefficient. However, in this application, the tips of the fins and base of the plate were found to vary in temperature, making it difficult to predict how the convection coefficient varies spatially. When inputting the entrance air temperature into the model, two results are observed:

1. The average theoretical convection coefficients are underestimated relative to the average experimental convection coefficients.
2. The theoretical electronic device temperature is overestimated relative to the experimental electronic device temperature.

The overestimation of the theoretical electronic device temperature is expected after learning the average theoretical convection coefficients are underestimated. However, accounting for the increased air temperature in the model, while not accounting for the variation in fin temperature, will increase the error in the results. Assumptions (2) and (4) also contribute to the overestimation of the electronic device temperature:

(2) One dimensional heat transfer

(4) No heat loss from bottom of the electronic device

In reality, heat is lost through the sides of the plate and bottom of the electronic device that is not being accounted for by the model.

It is also notable that the error for predicted electronic device temperature and convection coefficients was substantially larger than that for Plate 13. The configuration of fins on Plate 7 is staggered whereas Plate 13's fins are aligned. This indicates that assumption (6) provided a better approximation for the fin configuration of Plate 13 compared to Plate 7.

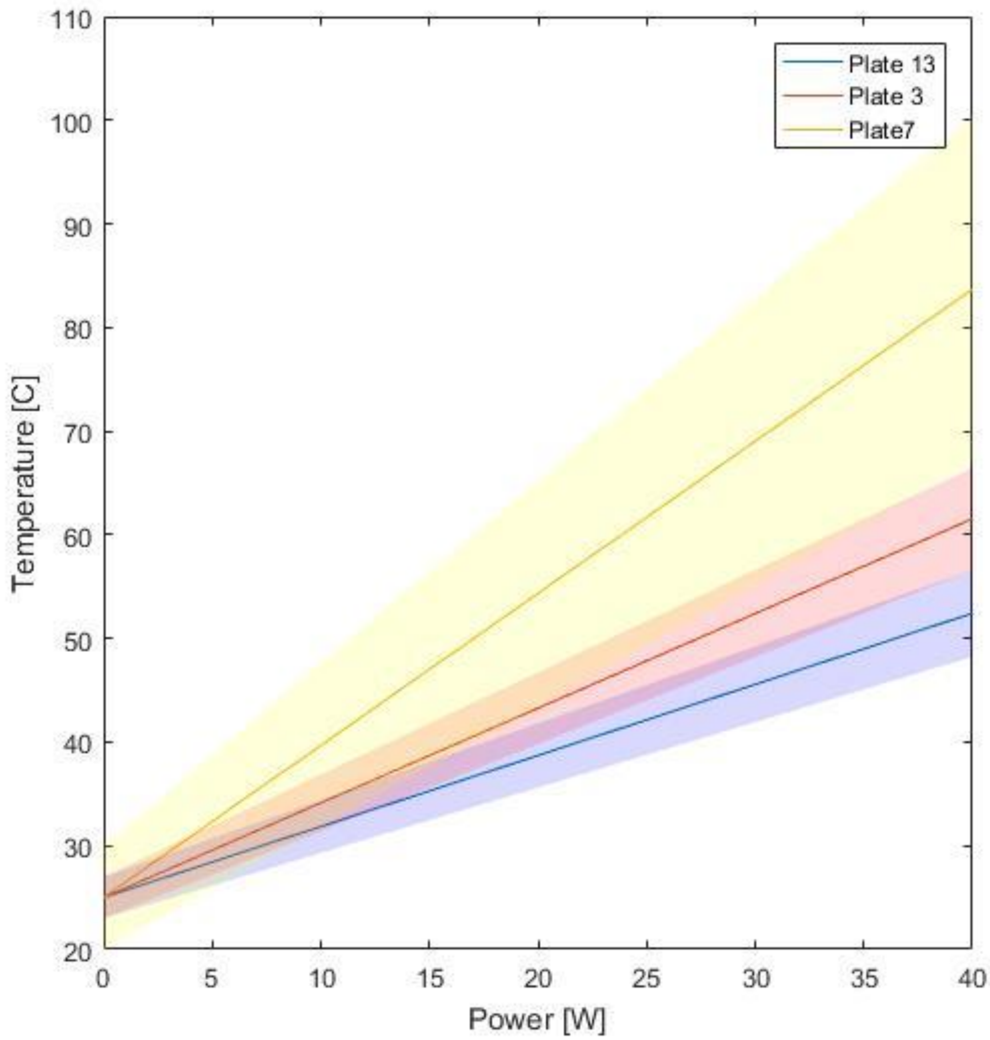
(6) The convection coefficient of forced flow over a single cylinder is equal to the convection coefficient of an individual fin in a finned array.

6. Recommendations

Overall, the model preformed well in predicting the electronic device temperature. Therefore, the model provides can provide a valid conclusion on the finned array best suited o solve the overheating of the electronic device. The following sections discuss the recommended finned array and operating conditions.

6.1. Finned Array

The model is used to compare the predicted temperatures of each finned array under the same operating conditions. Figure 1 shows the predicted temperature of the electronic plate for an air speed of 900 ft/min.



The colour bands depict the allowable error bounds in order for the relative performance rating to be valid; the relative performance rating and allowable error bands are summarized in Table 6 and Table 7, respectively.

Figure 1 Predicted Temperatures for each plate at an air speed of 900 ft/min. The color bands display the allowable error for a valid recommendation.

Table 6 Relative Performance Rating

Plate	Predicted Device Temperature at Nominal Power for 900 ft/min fan speed	Relative Performance Rating
13	52	1
3	63	2
7	84	3

To provide an approximation on the feasibility of the relative power rating. The allowable error is compared to the experimentally obtained error from for a fan air speed of 900 ft/min.

Table 7 Comparison of Experimental and Allowable Error

Plate	Allowable Error at Nominal Power Dissipation %	Experimental Error at Decreased Power Dissipation %
13	8.5	1.05
3	8.5	-
7	21	12.5

The experimental error is within the allowable error; however, the experimental error pertains to a decreased power dissipation. For increased power dissipation the error is expected to increase due to an increased spatial variation of temperature in the finned array. However due to the significant buffer between the allowable error and experimental error, it can be concluded that the relative performance rating provided in Table 6 is feasible. The recommended solution to the overheating of the device is to use Plate 13 in conjunction with forced air flow.

6.2. Operating Conditions

For Plate 13 at nominal power, the minimum forced air speed from the fan is recommended to be 1000 ft/min to ensure the electronic device temperature does not exceed its allowable temperature of 60 ^{o}C. The predicted temperature of the electronic device at 1000 ft/min is approximately 50 ^{o}C. If an improved efficiency system is desired, it is recommended to improve the accuracy of the model to predict a safe temperature before decreasing the minimum fan air speed. The models accuracy can be improved by accounting for the variation in temperature of the air and fins.

7. Conclusion

From the experimental analysis performed, it was found that the model predicts the temperature of the electronic device for a given power dissipation with satisfactory accuracy. The average error from the model for all four trials was found to be less than 10%. This error could be largely accounted to a changing ambient and base temperature within the fin. This error could be minimized by taking more step readings of base and ambient temperatures within the fin and or finding an expression for the change of these temperatures within the fin. For estimation of the convection coefficient, the model in most cases gave hf value with a significant error. The highest error was found to be for the staggered fin (Fin 7) being around 80% for both trials. Due to the two errors being so close to each other (83% and 79%), it was concluded that the model did not take into account the effect of fluid mixing for staggered fin which resulted in a similar percent error for the two trials.

Regardless of the error for the average convection coefficient, the model does a good job in realizing the best fin for the application. The recommended solution is to use Plate 13 in conjunction with forced air flow. In order to avoid premature failure of the device, the recommended minimum air speed from the fan at nominal power is 1000 ft/min. The model predicts the device temperature to be 50 ^{o}C for Plate 13 at 1000 ft/min air speed and nominal power, 10 ^{o}C below the maximum allowable temperature for the electronic device.

8. References

[1] T. L. Bergman, A. S. Lavine, F. P. Incropera and D. P. Dewitt, Fundamentals of Heat and Mass Transfer, John Wiley & Sons, 2011.

Appendix A: Model Methodology

The following sections discuss the methods used to develop model.

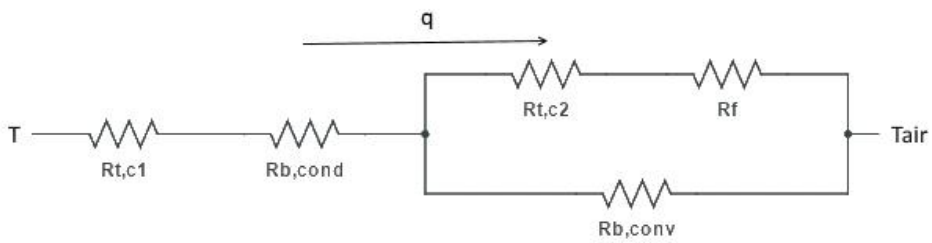
Thermal Circuit

The system is modeled as a thermal circuit; the following assumptions were made.

Assumptions:

(1) Steady state conditions
(2) One dimensional heat transfer
(3) Uniform temperature within electronic device
(4) One face of the electronic device is well insulated
(5) Radiation is negligible

Figure A1 displays the thermal circuit diagram of the simplified system.



The parameters denoted in Figure A1 are summarized below.

Parameter	Description
q	Power dissipation of electric chip
T	Temperature of the electric chip
T_{air}	Temperature of the surrounding air
$R_{t,c1}$	Contact resistance between base plate and electronic device
$R_{t,c2}$	Contact resistance between N fins and base plate
$R_{b,cond}$	Conduction resistance of base plate
$R_{b,conv}$	Convection Resistance of base plate
R_f	Resistance of N fins

The expression for the power dissipation is analogous to Ohm’s law,

$$\boldsymbol{q = \frac{T - Tair}{R_T}} \tag{1}$$

where R_T is the total thermal resistance.

$$\boldsymbol{R_T = R_{t,c1} + R_{b,cond} + \left(\frac{1}{R_{t,c2} + R_f} + \frac{1}{R_{b,conv}}\right)^{-1}} \tag{2}$$

Section 14 provides approximations for the contact resistances. The resistance for the conduction through the base plate is as follows,

$$\boldsymbol{R_{b,cond} = \frac{t}{kA}} \tag{3}$$

where, A and k are the area and thermal conductivity of the base plate, respectively. The expression for the convection resistance,

$$\boldsymbol{R_{b,conv} = \frac{1}{h_b A_b}} \tag{4}$$

where A_b is exposed area of the base plate. The convection coefficient for the base plate, h_b, describes the heat transfer from the exposed base plate to the surrounding air. Lastly, R_f, the resistance of N fins, is as follows,

$$\boldsymbol{R_f = \frac{1}{N n_f h_f A_f}} \tag{5}$$

where n_f and A_f are the fin efficiency and effective area, respectively. The convection coefficient for describing the heat transfer from a single fin to the surrounding air is denoted as h_f. For a fin with an active tip, the fin efficiency is

$$\boldsymbol{n_f = \frac{(hPKA_c)^{0.5}\left(\sinh(mL) + \left(\frac{h}{mk}\right)\cosh(mL)\right)}{(hA_f)\left(\cosh(mL) + \left(\frac{h}{mk}\right)\sinh(mL)\right)}} \tag{6}$$

Where m is defined as follows.

$$\boldsymbol{m = \left(\frac{hP}{kA_c}\right)^{0.5}} \tag{7}$$

The available finned arrays consist of a square base plate with N number of cylindrical fins. Considering the tip of the fins as part of the effective heat transfer area yields the following equivalent expressions for the area parameters.

Area Parameter	Description	Equivalence
A	Total Area of base plate	w^2
A_c	Area of tip of fin	$\pi D^2/4$
A_f	Effective Area of fins	$A - NA_c$
A_b	Exposed area of base plate	$\pi DL + A_c$

Determination of Convection coefficients

In order to approximate the resistances of convection, an analytical solution is required for the convection coefficients h_b and h_f of the base plate and fins, respectively. The following relationship between the average of the Nusselt Number $\overline{Nu}$, Reynolds Number, R_e, and Prandtl number $\mathrm{P_r}$ can be used to determine the average convection coefficient over a surface [1].

$$\overline{\boldsymbol{Nu}} = \frac{\overline{\boldsymbol{h}}\boldsymbol{L}}{\boldsymbol{k_{fluid}}} = \boldsymbol{f}(\boldsymbol{R_e}, \mathbf{P_r}) \tag{8}$$

The relation $f(R_e, \mathrm{P_r})$ is dependent on the type of flow and geometry of the object. The $f(R_e, \mathrm{P_r})$ relations for these applications are available below [1]

Nusselt Number	Constraints	
Laminar Flow over Flat Plate: $\overline{Nu} = \frac{hL}{k} = 0.664 Re^{\frac{1}{2}} Pr^{\frac{1}{3}}$	$\mathrm{Pr} \geq 0.6$	$Re < 5x10^5$
Turbulent Flow over Flat Plate $\overline{Nu} = \frac{hL}{k} = 0.322 Re_x^{\frac{4}{5}} Pr^{\frac{1}{3}}$	$60 < \mathrm{Pr} \leq 0.6$	$5x10^5 \leq Re \leq 10^7$

Nusselt Number for flow over a cylinder [1]:

$$\overline{Nu_{cyl}} = \frac{hD}{k} = CRe_D Pr^{\frac{1}{3}}$$

Where the constants m and C are defined below.

Re_D	C	m
0.4 -4	0.989	0.330
4 -40	0.911	0.385
40-4000	0.683	0.466
4000 - 40000	0.193	0.618
40000 - 400000	0.027	0.805

The Reynolds number for the base plate can be approximated as fluid flow over a flat plate [2]

$$\boldsymbol{R_e = \frac{V_{avg}L}{v}} \tag{9}$$

Where V_{avg} is the average velocity, L is the length of the plate and *v* is the kinetic velocity of the fluid. Similarly, the Reynolds number can be determined for the fins by modeling fluid flow over a cylinder; note that the length, L, is replaced by the cylinder diameter, D in equation (8) and (9).

Approximation of the Contact Resistances

The contact resistances Rtc1'' and Rtc2'' are the resistance between the plate and the electronic device, and the fins and the plate respectively. Rtc1'' is approximated as a silicon chip/aluminum interface with 0.02-mm epoxy [1]. Rtc2'' is approximated according the material and the fit of the fins to the base. The contact resistance, Rtc2'', was approximated to be less for a tight fit between the fins and the base relative to a loose fit.

	Rtc1'' Km^2/W	**Rtc2''** Km^2/W
Plate 3	3.5 E-05	5.0 E-06
Plate 7	3.5 E-0.5	1.4 E -05
Plate 13	3.5 E-0.5	1.0 E-06

Appendix B: Available Finned Arrays

The available finned arrays are Plate 3, Plate 7 and Plate 13. The dimensions the length of the fins, width and height of the base plate are 1 inch, 2 inches and 0.5 inches, respectively for all of the finned arrays. The other parameters are shown below.

	Number of Fins N	**Fin Diameter D, m**	**Thermal Conductivity k, W/mK**
Plate 3	36	0.125	237
Plate 7	25	0.125	110
Plate 13	81	0.125	237

Appendix C: Air Speed Analysis

In order to gauge the feasibility of each finned array. The air speed required to maintain an electronic device temperature of 60°C at the nominal power 40W was predicted by the model. The results are summarized in the Table below.

Finned Array	**Air speed [m/s]**
Plate 3	4.3
Plate 7	17.95
Plate 13	0.78

Plate 13 the finned array predicted by the model to perform the best requires an air speed of 0.78 m/s to maintain an electronic device temperature of 60°C at the nominal power 40W. Therefore, natural convection in conjunction with a finned array is not a feasible design for nominal power dissipation.

Appendix D: MATLAB Model Source Code

```
% The following program plots the predicted device temperature for
% three finned arrays.

function main
qnom = 40;
airSpeedMetersPerSecond = 5;
TairC = 22.5;

q = linspace(0,qnom, 80);
% Predicted chip temperature given q nominal
T1 = deviceTemperature(q,airSpeedMetersPerSecond,TairC,'Plate13.txt');
T2 = deviceTemperature(q,airSpeedMetersPerSecond,TairC,'Plate3.txt');
T3 = deviceTemperature(q,airSpeedMetersPerSecond,TairC,'Plate7.txt');

plot(q,T1,q,T2,q,T3)
drawPatch(q,T1,0.085,[0 0 1]);
drawPatch(q,T2,0.085,[1 0 0]);

drawPatch(q,T3,0.21, [1 1 0]);
T1(end)
T2(end)
T3(end)
xlabel('Power [W]');
ylabel('Temperature [C]');
legend('Plate 13','Plate 3', 'Plate7');
end

function drawPatch(q,T,error, color)
err= error*T;
c = color;
error1Patch = patch([q fliplr(q)],[T+err fliplr(T-err)],c);
error1Patch.EdgeColor = 'none';
alpha(error1Patch,0.15);
end

% The following function plots the predicted chip
% temperature as a function of heat dissapation
% for a constant air speed.

function T = deviceTemperature(q, airSpeedMetersPerSecond, TairC,
plateString)

% Parameter Description                         Units
%
% qmax      Maximum allowable heat transfer     W
% qnom      Nominal heat dissapation            W
% Tmax      Maximum allowable temperature       K
% T         Temperature of electical device     K
% Tair      Temperature of surrounding air      K
% h         Convection coefficient              W/(Km^2)
```

```
% Square Base Plate Parameters:
% k         conduction coefficient              W/mK
% t         Thickness                           m
% w         Width                               m

% Fins
% N         Number of Fins
% D         Diameter                            m
% L         Length                              m

% Resistances Unit: K/W
% Rtc1      Contact resistance between base plate
%           and electronic device
% Rtc2      Contact resistance between N fins and
%           base plate
% Rbcond    Conduction resistance of  base plate
% Rbconv    Convection Resistance of base plate
% Rf        Resistance of N fins
% Ro        Resistance of finned array
% RT        Total resistance of thermal circuit


Tmax = convtemp(60, 'C', 'K');
Tair = convtemp(TairC, 'C', 'K');

% Read Finned Array Parameters from File
% File Format: N    D(in)   L(in)   w(in)   t(in)   k(W/mk) Rtc1''(m^2K/W)
Rtc2''(m^2k/W)
finnedArrayParameters = importdata(plateString);

%Fin Parameters
N = finnedArrayParameters.data(1);
D = convlength(finnedArrayParameters.data(2), 'in', 'm');
L = convlength(finnedArrayParameters.data(3), 'in', 'm');

% Base Plate Parameters
w = convlength(finnedArrayParameters.data(4), 'in', 'm');
t = convlength(finnedArrayParameters.data(5), 'in', 'm');
k = finnedArrayParameters.data(6);

%Areas
A = w^2;
Ac = pi*D^2/4;
Ab = A - N*Ac;

% Contact Resistances
Rtc1Area = finnedArrayParameters.data(7);
Rtc2Area = finnedArrayParameters.data(8);
Rtc1 = Rtc1Area/A;
Rtc2 = Rtc2Area/(N*Ac);

[hf,hb] = convectionCoefficients(airSpeedMetersPerSecond,Tair,w,D)

% Resistances
Rbcond = t/(k*A);
```

```
Rbconv = 1/(hb*Ab);
nf = finEfficiency(hf,k,D,L);
Rf = finsResistance(hf, nf, N, D, L);
Ro = (1/(1/(Rf + Rtc2) + 1/(Rbconv)));
RT = Rtc1 + Rbcond + Ro;

Tkelvin = q*RT + Tair;
T = convtemp(Tkelvin, 'K','C');
end

function nf = finEfficiency(h, k, D, L)

p = pi*D;
Ac = pi*D^2/4;
Af = pi*D*L + Ac;

m = sqrt(h*p/(k*Ac));
nf = (sqrt(h*p*k*Ac)/(h*Af))*(sinh(m*L) + (h/(m*k))*cosh(m*L))/(cosh(m*L) +
(h/(m*k))*sinh(m*L));

end
function Rf = finsResistance(hf, finEfficiency, numberOfFins, finDiameter,
finLength)
N = numberOfFins;
L = finLength;
D = finDiameter;
nf = finEfficiency;

Ac = pi*D^2/4;
Af = pi*D*L + Ac;
Rf = 1/(N*nf*hf*Af);

end


function [hf,hb] = convectionCoefficients(airVelocityMetersPerSecond,
airTempK, plateLength, finDiameter)
%  h convection coefficent [W/Km^2]
%  A Matrix of constants and restrictions for cylinder calculation

A = [0.4 4 0.989 0.330; 4 40 0.911 0.385; 40 4000 0.683 0.466; 4000 40000
0.193 0.618; 40000 400000 0.027 0.805];
k250 = 0.0223;         % Thermal Conductivity of air at 250 fahrenheit, 1 atm
k300 =  0.0263;      % Thermal Conductivity of air at 300 fahrenheit, 1 atm
Pr250 = 0.720;       % Prandtl Number of air at 250 fahrenheit, 1 atm
Pr300 = 0.707;       % Prandtl Number of air at 300 fahrenheit, 1 atm
v250 = 11.44*10^-6 ;   % Kinematic Velocity of air at 250 fahrenheit, 1 atm
v300 = 15.89*10^-6;    % Kinematic Velocity of air at 300 fahrenheit, 1 atm
L = plateLength;
D = finDiameter;


x = (airTempK - 250)/50;
k = (k300-k250)*x + k250;
Pr = (Pr300-Pr250)*x + Pr250;
```

```
v = (v300-v250)*x + v250;

ReCylinder = airVelocityMetersPerSecond*finDiameter/v;
RePlate = airVelocityMetersPerSecond*plateLength/v;
if (RePlate < 5*10^5 && RePlate > 0) % laminar flow
    NuPlate = 0.664*((RePlate)^(1/2))*((Pr)^(1/3));

elseif (RePlate > 5*10^5 && RePlate < 10^7) % Turbulent flow
    NuPlate = 0.037*((RePlate)^(4/5))*((Pr)^(1/3));
else
    RePlate
    warning('Re must be within the bounds: 0 < Re < 10^7')
end

for n = 1: length(A)
    if ( ReCylinder > A(n,1) && ReCylinder <=A(n,2))
        c = A(n,3)
        m = A(n,4)
        break
    end
end
NuCylinder = (c*ReCylinder^m)*Pr^(1/3);

checkRestrictions(ReCylinder,Pr,airTempK);

hb = NuPlate*k/L;
hf = NuCylinder*k/D;

end

function checkRestrictions(ReCylinder,Pr,airTempK)

if (airTempK > 300)
    airTempK
    warning('Air Temperature must be within the bounds: 250 K < airTempK <
300 K')
elseif (airTempK < 250)
    airTempK
    warning('Air Temperature must be within the bounds: 250 K < airTempK <
300 K')
elseif (Pr <= 0.7 || Pr >60)
    Pr
    warning('Prdantl Number must be within the bounds 0.7 < Pr < 60')
elseif(ReCylinder <= 0.4 || ReCylinder > 400000)
    ReCylinder
    warning('Re must be within the bounds: 0.4 < ReCylinder <= 400000')
end

end
```