Reinforcement Learning

DSE 220

Outline

1- Reinforcement Learning Intro

2- Policy Gradients

3- Markov Decision Process

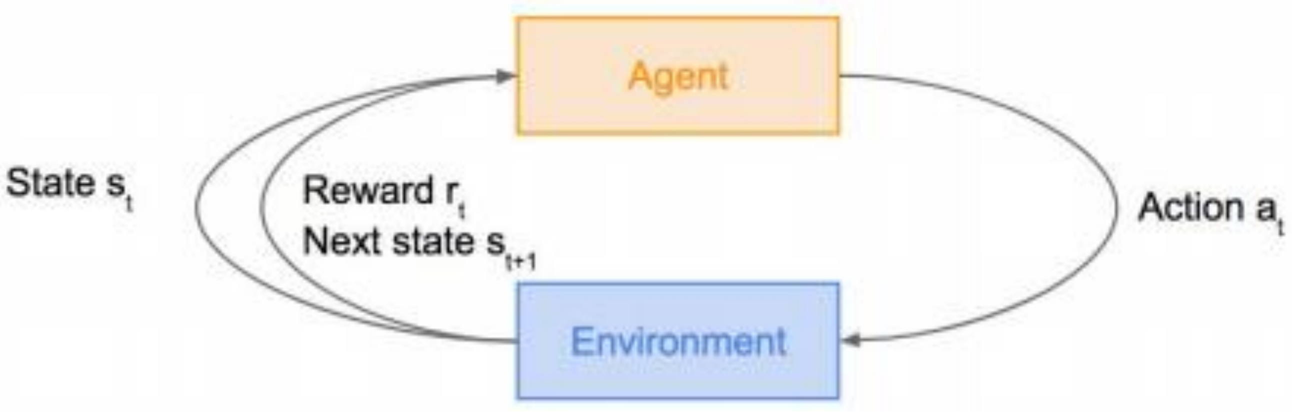
4- Q-learning

5- Comparison

What is Reinforcement Learning?

Reinforcement Learning (RL) is an area of machine learning where an *agent* ought to take some *actions* in an *environment* so as to maximize some notion of cumulative *reward*.

Goal: Learn how to take actions in order to maximize reward



Why do we need RL?

- Learn to make good sequences of decisions
- Goal is to find an optimal way to make decisions
  - Yielding best outcomes or at least very good outcomes
- Explicit notion of utility of decisions
  - Example: finding minimum distance route between two cities given network of roads

Why do we need RL?

- In supervised learning each decision is independent:
  - There is a label related to each decision
- In reinforcement learning we have a sequential decision-making process:
  - Not every decision has a label, sequences have labels (reward or no reward)

Why do we need RL?

- Consider the board game "Go":
  - We do not have the label for the optimal decision at each step
  - Collecting such data is infeasible/expensive
  - Instead of SL, we can use RL to imitate and beat human decisions
  - Instead of learning from the labels, we can learn from the reward that is observed at the end.

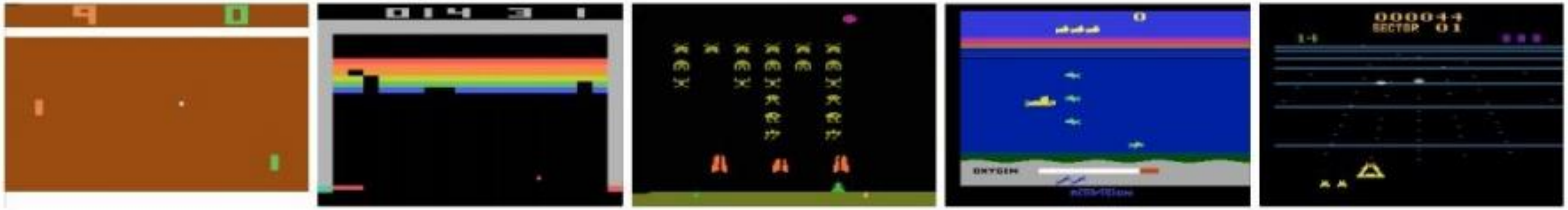
RL vs Supervised vs Unsupervised

- Supervised - prediction tasks
- Unsupervised – understand data
- RL – decision making

RL Examples

- Game playing AI. Teaching an agent to play games like Pong, Chess, Go, etc.
- Article recommendations. Teaching an agent to make better article suggestions to a user.
- Robotics. Teaching an agent to navigate a course.

RL Examples: Atari Games



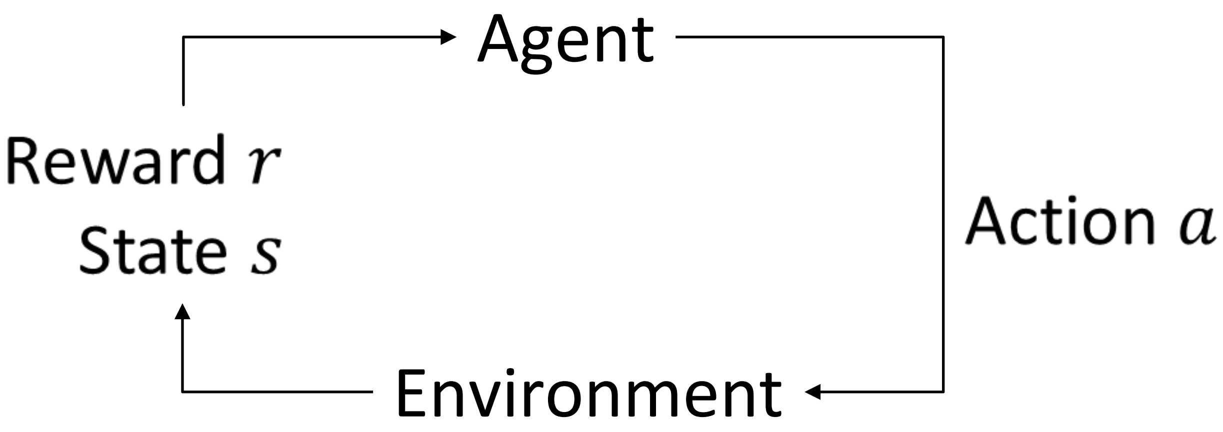
Objective: Complete the game with the highest score

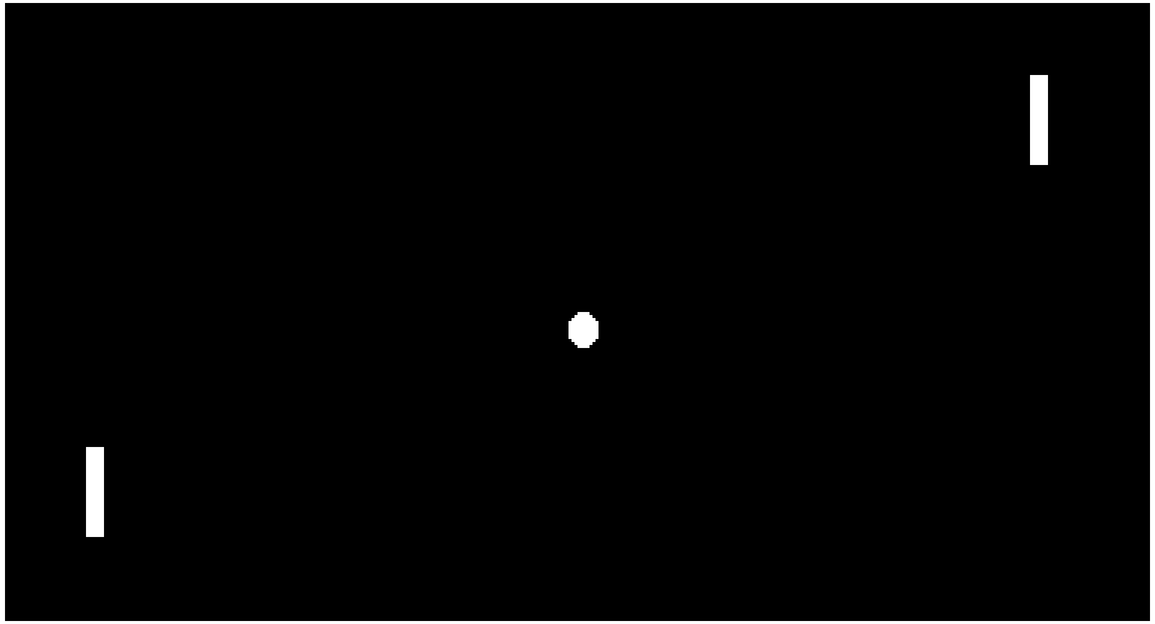
State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

RL System





RL System

Consider an AI playing the ATARI game **Pong.**

Environment- The setup of the game

Agent - One of the players

Action - Move up / Move down

State - Current positions of the paddles and the ball

Reward - +1 for winning

-1 for losing

0 otherwise

Objective

The objective of an agent in an RL setup is to achieve maximum (cumulative) reward.

In the Pong example, the objective is to keep winning to achieve maximum reward.

Objective

The objective of an agent in an RL setup is to achieve maximum (cumulative) reward.

In the Pong example, the objective is to keep winning to achieve maximum reward.

$$E_{x\sim p(x|\theta)}[f(x)]$$

Objective

The objective of an agent in an RL setup is to achieve maximum (cumulative) reward.

In the Pong example, the objective is to keep winning to achieve maximum reward.

$$E_{x \sim p(x|\theta)}[f(x)]$$

$$x - Action$$

$$f(x) - Reward\ for\ action\ x$$

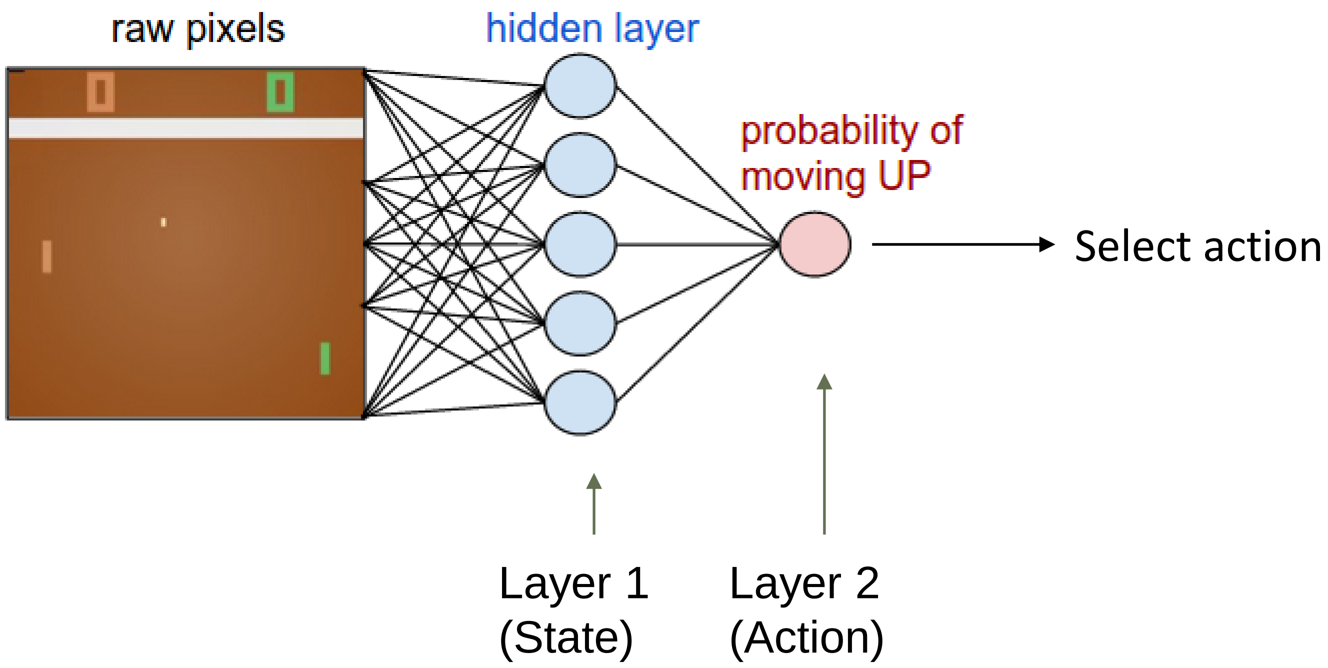
$$p(x|\theta) - Policy\ (Decision\ maker)$$

Model

A fully connected 2 layer Neural Network.

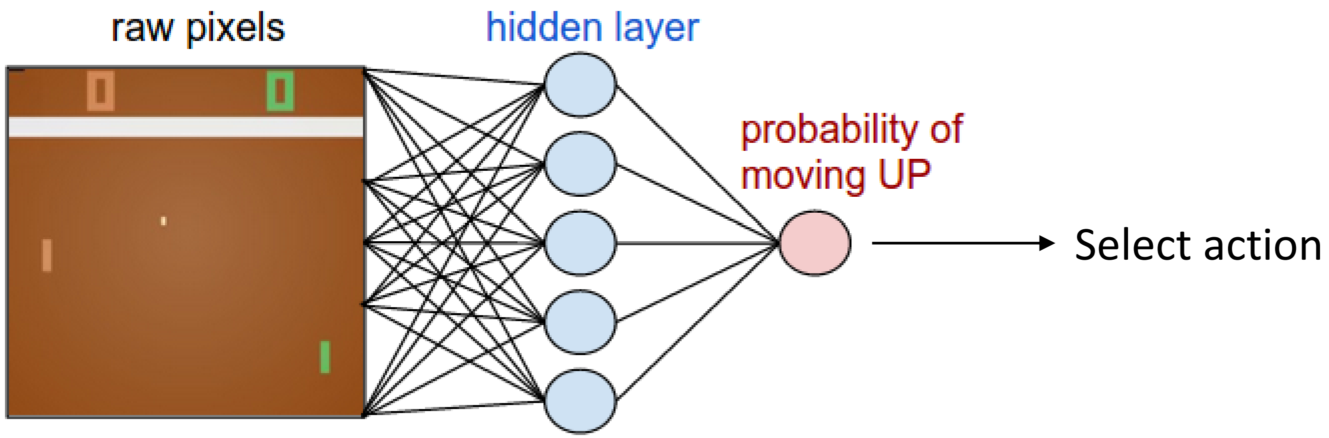
The first layer will take as input a vector of H x W x 3 pixels and capture the state of the game.

The second layer will capture the action needed to perform.



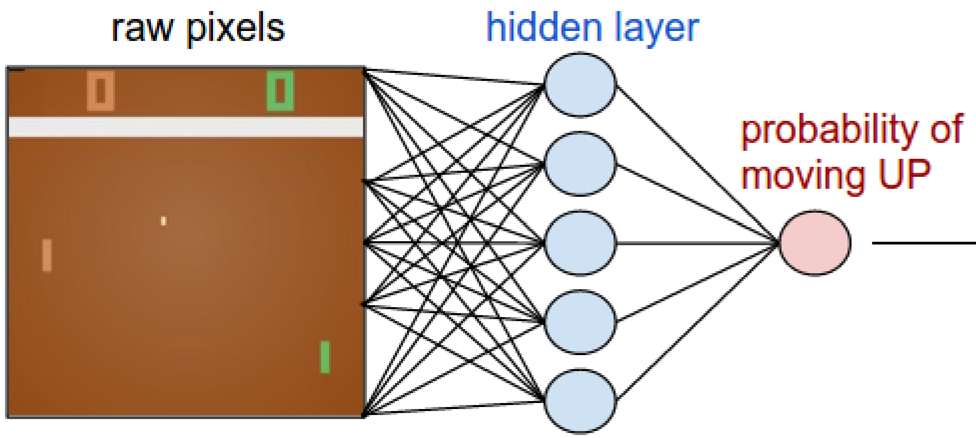
How to train?

Can we train in a Supervised learning fashion?



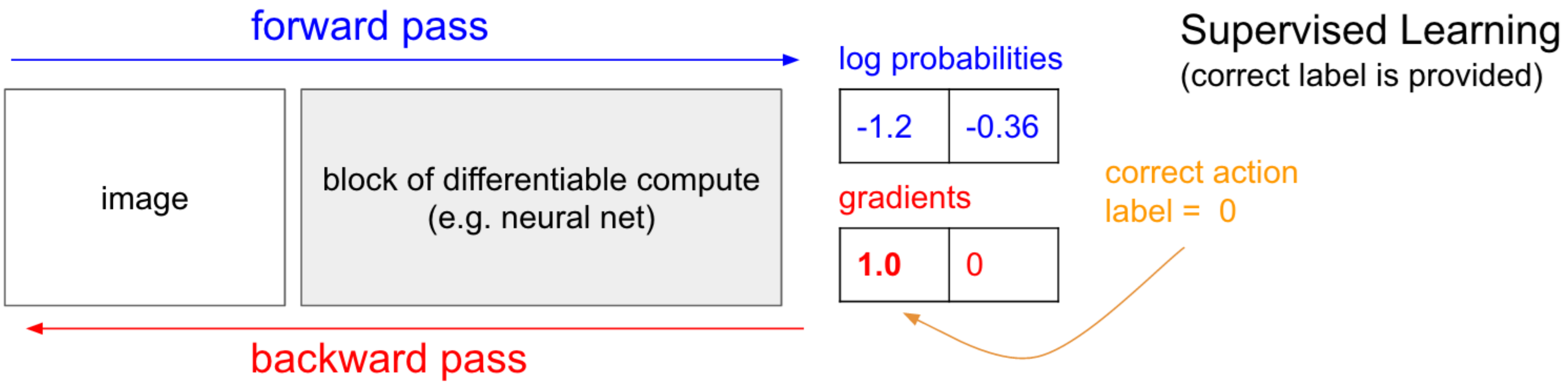
How to train?

Can we train in a Supervised learning fashion?



Select action ← Correct action??

How to train? (cont.)



In supervised learning we need a *correct label* to perform backward pass to train the system.

Solution

What do we do if we do not have the correct label in the Supervised Learning setting?

Solution

What do we do if we do not have the correct label as in the Supervised Learning setting?

Use Policy Gradients

Policy Gradient

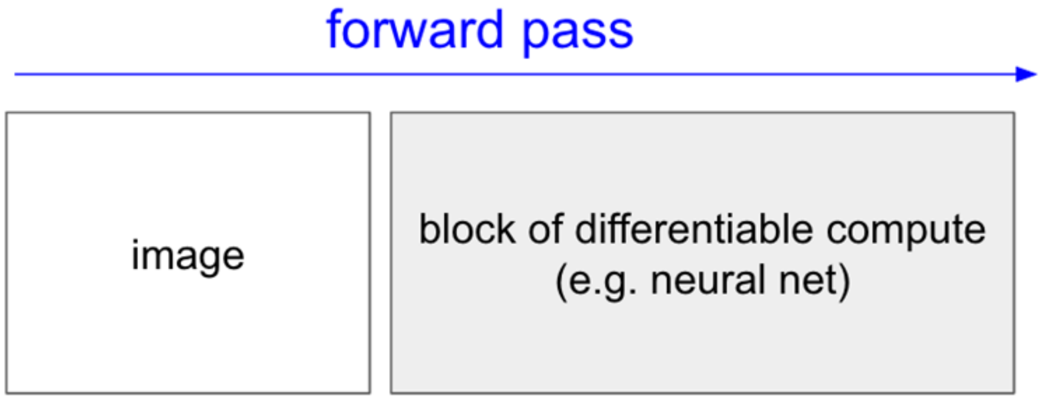
Can we learn a policy directly, e.g. finding the best policy from a collection of policies?

- The policy gradient methods target at modeling and optimizing the policy directly.

Policy Gradient - Step 1

Run a forward pass on the network with the present state.

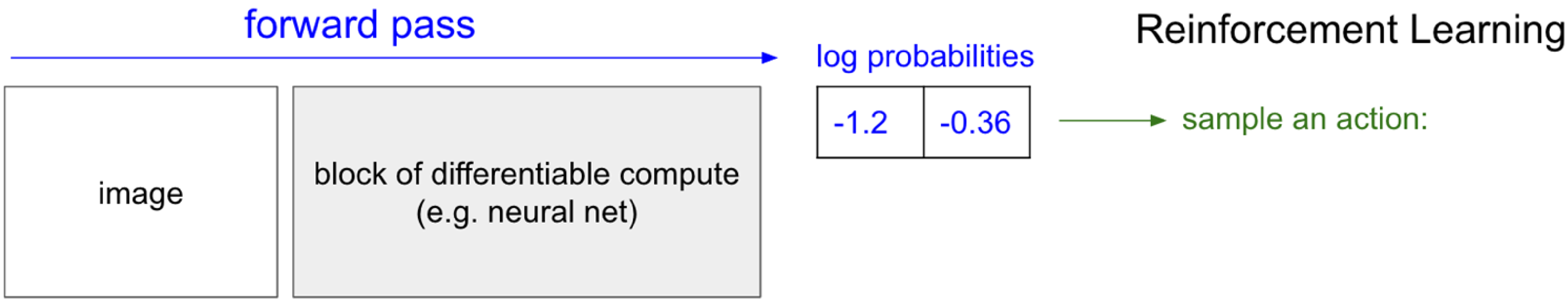
Let’s say, our policy network calculated probability of going UP as 30% (logprob -1.2) and DOWN as 70% (logprob -0.36).



Policy Gradient - Step 2

Sample an action from the probability distribution obtained from Step 1. Execute the action.

We will now sample an action from this distribution; E.g. suppose we sample DOWN, and we will execute it in the game.

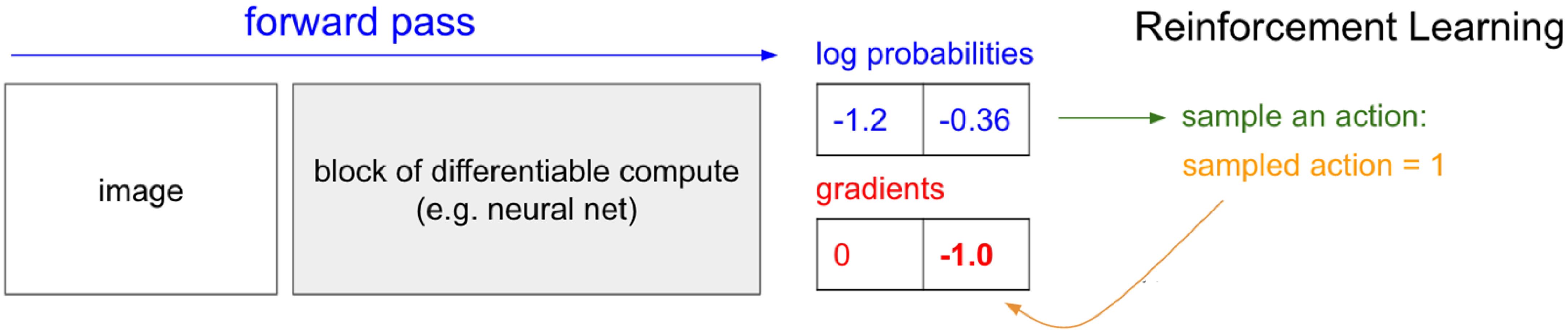


Policy Gradient - Step 3

Calculate the gradient assuming the sampled action is the correct action.

The sampled action might not be correct. But that is ok. We can wait for a bit and see!

Note: We are not updating the network yet. We are only calculating the gradients.

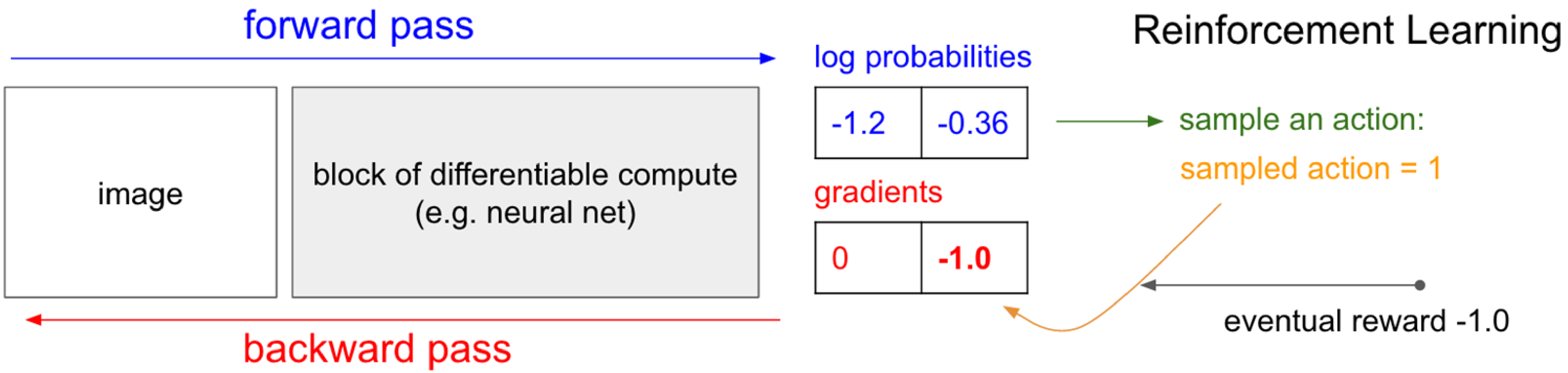


Policy Gradient - Step 4

Repeat Steps 1 - 3 till the game ends.

Update the network using the calculated gradients when the **“eventual reward”** received.

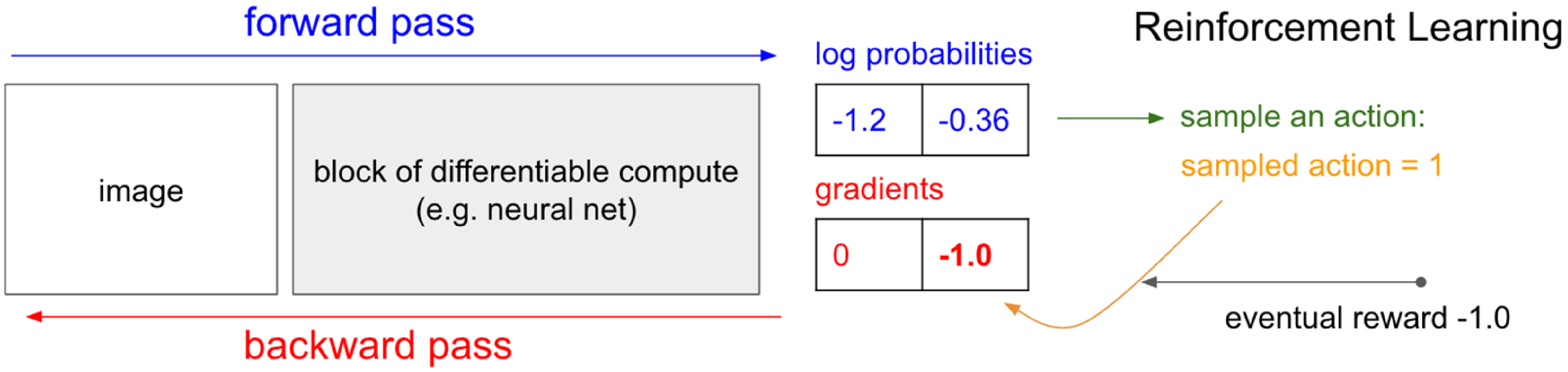
In Pong the eventual reward will be +1 (winning) or -1 (losing).



Policy Gradient - Intuition

Let’s say going DOWN ended up in losing the game (-1 reward). So if we combine that with the gradient and do backprop we will find a gradient that ***discourages*** the network to take the DOWN action for that input in the future.

Policy Gradient



Deriving Policy Gradients

Policy Gradients are a special case of a more general *score function gradient estimator*.

The general case is that we are trying to maximize an expression of the form:

$$E_{x \sim p(x \mid \theta)} [f(x)]$$

$$\mathrm{E}[X] = \sum_{i=1}^{k} x_i \, p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k .$$

$$\begin{aligned}
\nabla_\theta E_x[f(x)] &= \nabla_\theta \sum_x p(x) f(x) && \text{definition of expectation} \\
&= \sum_x \nabla_\theta p(x) f(x) && \text{swap sum and gradient} \\
&= \sum_x p(x) \frac{\nabla_\theta p(x)}{p(x)} f(x) && \text{both multiply and divide by } p(x) \\
&= \sum_x p(x) \nabla_\theta \log p(x) f(x) && \text{use the fact that } \nabla_\theta \log(z) = \frac{1}{z} \nabla_\theta z \\
&= E_x[f(x) \nabla_\theta \log p(x)] && \text{definition of expectation}
\end{aligned}$$

Policy Gradient (cont.)

Supervised learning objective :

$$\sum_i \log(p(y_i|x_i))$$

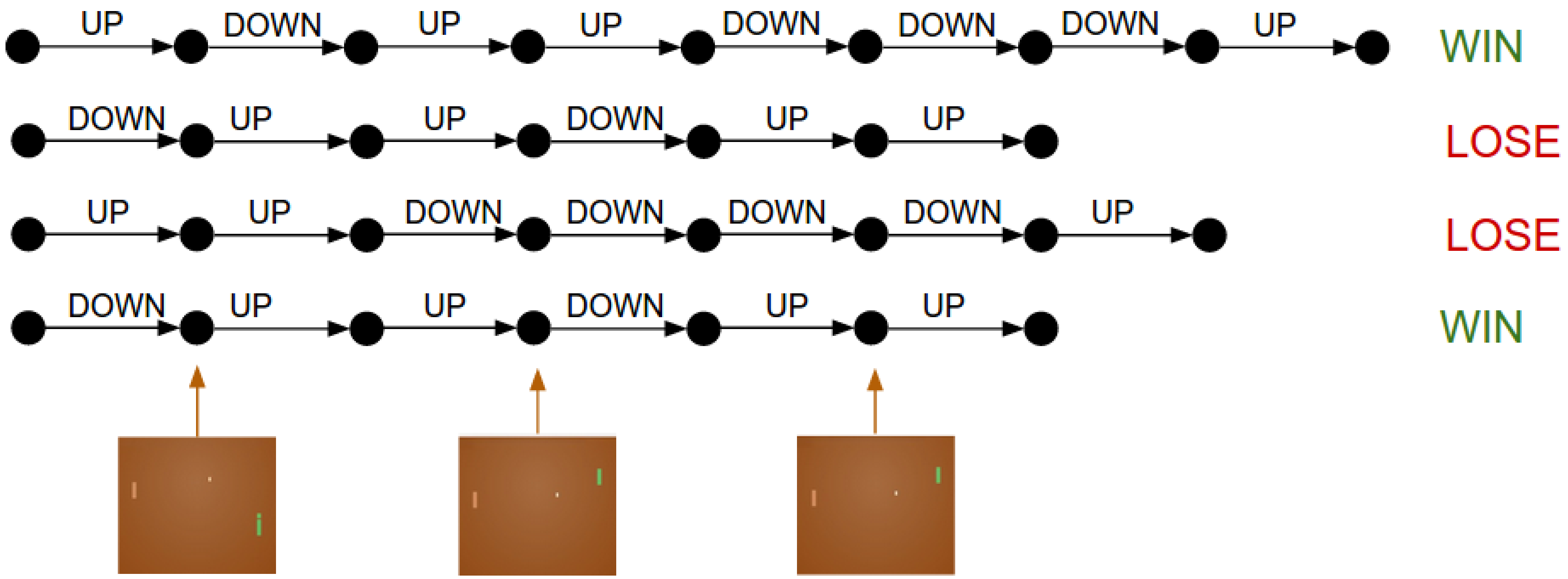
Policy Gradient learning objective :

$$\sum_i A_i\, log(p(y_i|x_i))$$

- A_i is advantage. In pong +1 if we win the episode and -1 otherwise
- y_i is the action we sampled from the distribution output of network

- This will ensure that we maximize the log probability of actions that led to good outcome and minimize the log probability of those that didn't.

Policy Gradient (cont.)



- Each black circle is a game state
- Each arrow is an action that sampled in the related state
- Ended up with 2 WINs and 2 LOSEs.
- Encourage the actions in WINs and discourage the actions in LOSEs.

More general Advantage functions

In a more general RL setting we would receive some reward r_t at every time step

One common choice is to use a discounted reward, so the “eventual reward” would become:

$$R_t = \sum_{k=0}^{\infty} \gamma^k \, r_{t+k}$$

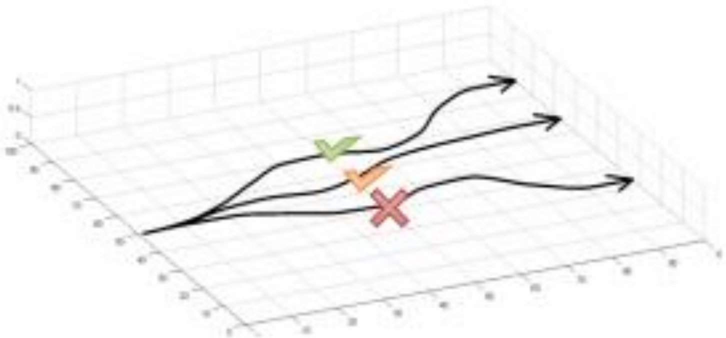
γ is a number between 0 and 1 called *discount factor*

The total reward is the weighted sum of all rewards afterwards, but later rewards are exponentially less important.

Results

Updating the Policy

$$\theta^\star = \underbrace{\arg\max_\theta E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right]}_{J(\theta)}$$



$$J(\theta) = E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right] \approx \frac{1}{N}\sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

sum over samples from π_θ

Updating the Policy

$$\text{recall: } J(\theta) = E_{\tau \sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right] \approx \frac{1}{N}\sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

$$\nabla_\theta J(\theta) = E_{\tau \sim \pi_\theta(\tau)}\left[\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)\right)\left(\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)\right)\right]$$

Updating the Policy

$$\text{recall: } J(\theta) = E_{\tau \sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right] \approx \frac{1}{N}\sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

$$\nabla_\theta J(\theta) = E_{\tau \sim \pi_\theta(\tau)}\left[\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)\right)\left(\sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)\right)\right]$$

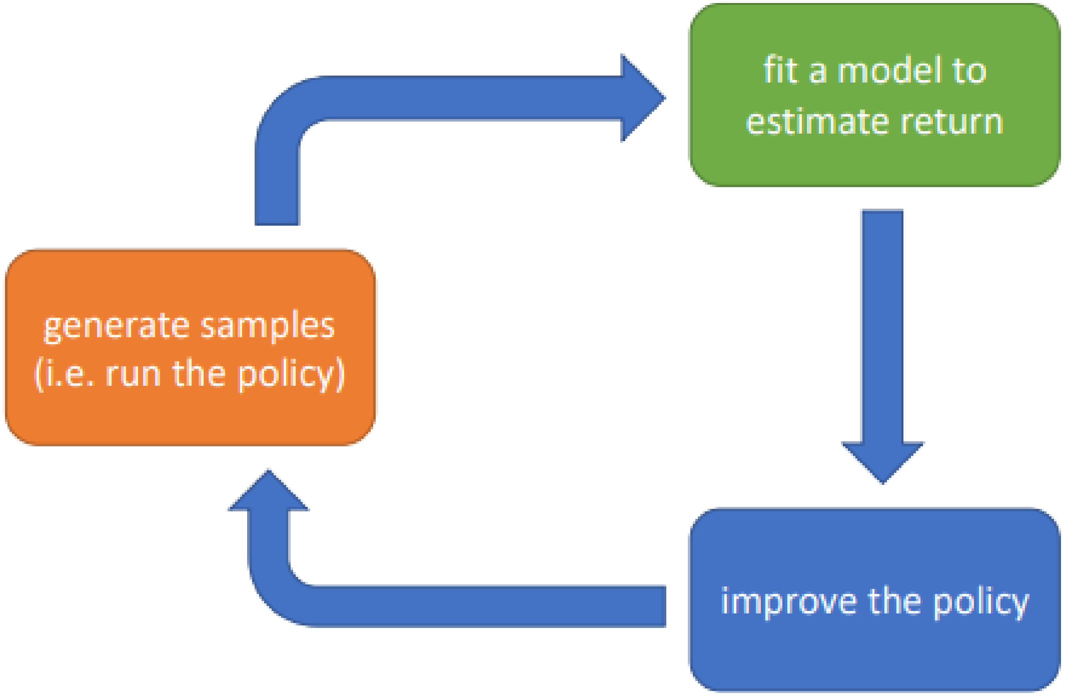
$$\nabla_\theta J(\theta) \approx \underline{\frac{1}{N}\sum_{i=1}^{N}}\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})\right)\underline{\left(\sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})\right)}$$

$$\underline{\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)}$$

Updating the Policy

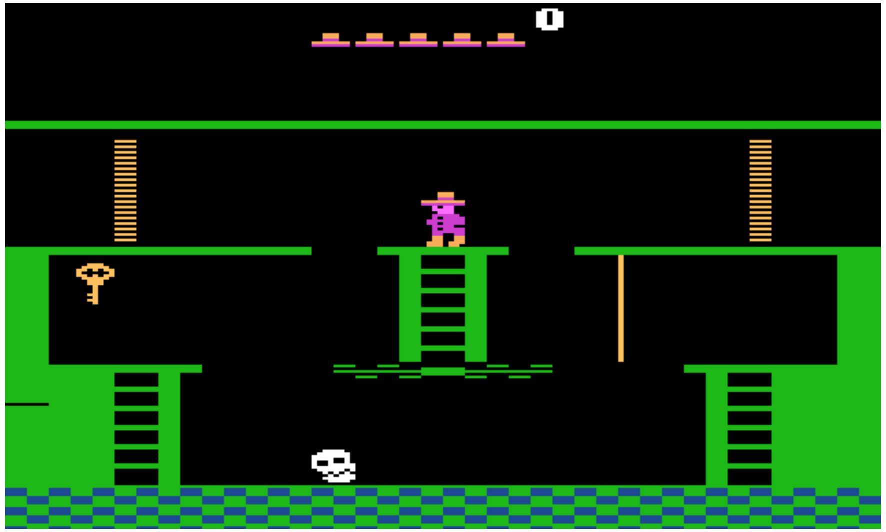
REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



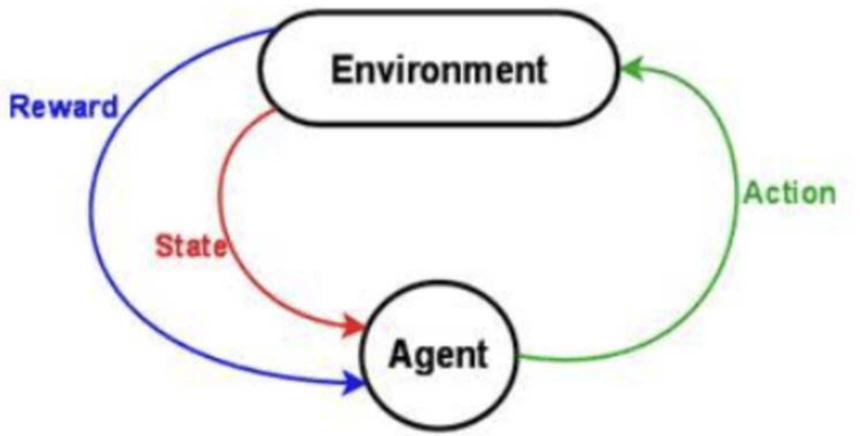
Disadvantages of PG

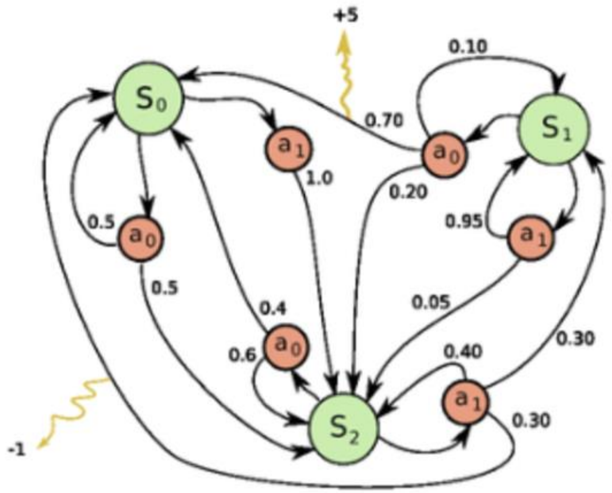
- In Policy Gradients, the models have to experience the positive awards to shift the policy towards high rewarding moves.
- As the action space increases the policy samples billions of random actions which will result in negative rewards mostly.
- It becomes harder for the policy to “stumble upon” the positive rewarding situation.
- It becomes increasingly difficult for the model to learn a good policy which can maximize the cumulative reward.



Montezuma's Revenge

RL as a Markov Decision Process





RL as a Markov Decision Process

- Mathematical formulation of the RL problem
- **Markov property**: Current state completely characterises the state of the world

Defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$: set of possible states
$\mathcal{A}$: set of possible actions
$\mathcal{R}$: distribution of reward given (state, action) pair
$\mathbb{P}$: transition probability i.e. distribution over next state given (state, action) pair
γ : discount factor

RL as a Markov Decision Process

- At time step t=0, environment samples initial state $s_0 \sim p(s_0)$
- Then, for t=0 until done:
  - Agent selects action a_t
  - Environment samples reward $r_t \sim R(. | s_t, a_t)$
  - Environment samples next state $s_{t+1} \sim P(. | s_t, a_t)$
  - Agent receives reward r_t and next state s_{t+1}

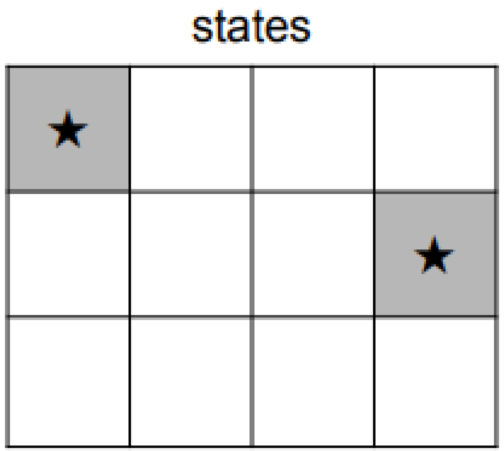
- A policy **π** is a function from S to A that specifies what action to take in each state
- **Objective**: find policy **π*** that maximizes cumulative discounted reward: $\sum_{t \geq 0} \gamma^t r_t$

Markov Decision Process Example

actions = {

1. right
2. left
3. up
4. down

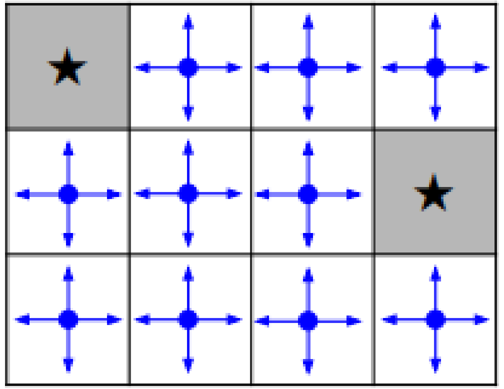
}



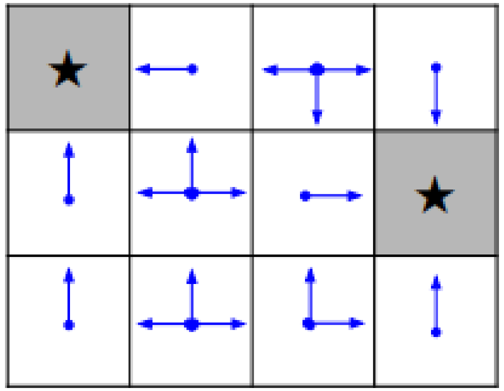
Set a negative "reward" for each transition (e.g. $r = -1$)

Objective: reach one of terminal states (greyed out) in least number of actions

Markov Decision Process Example

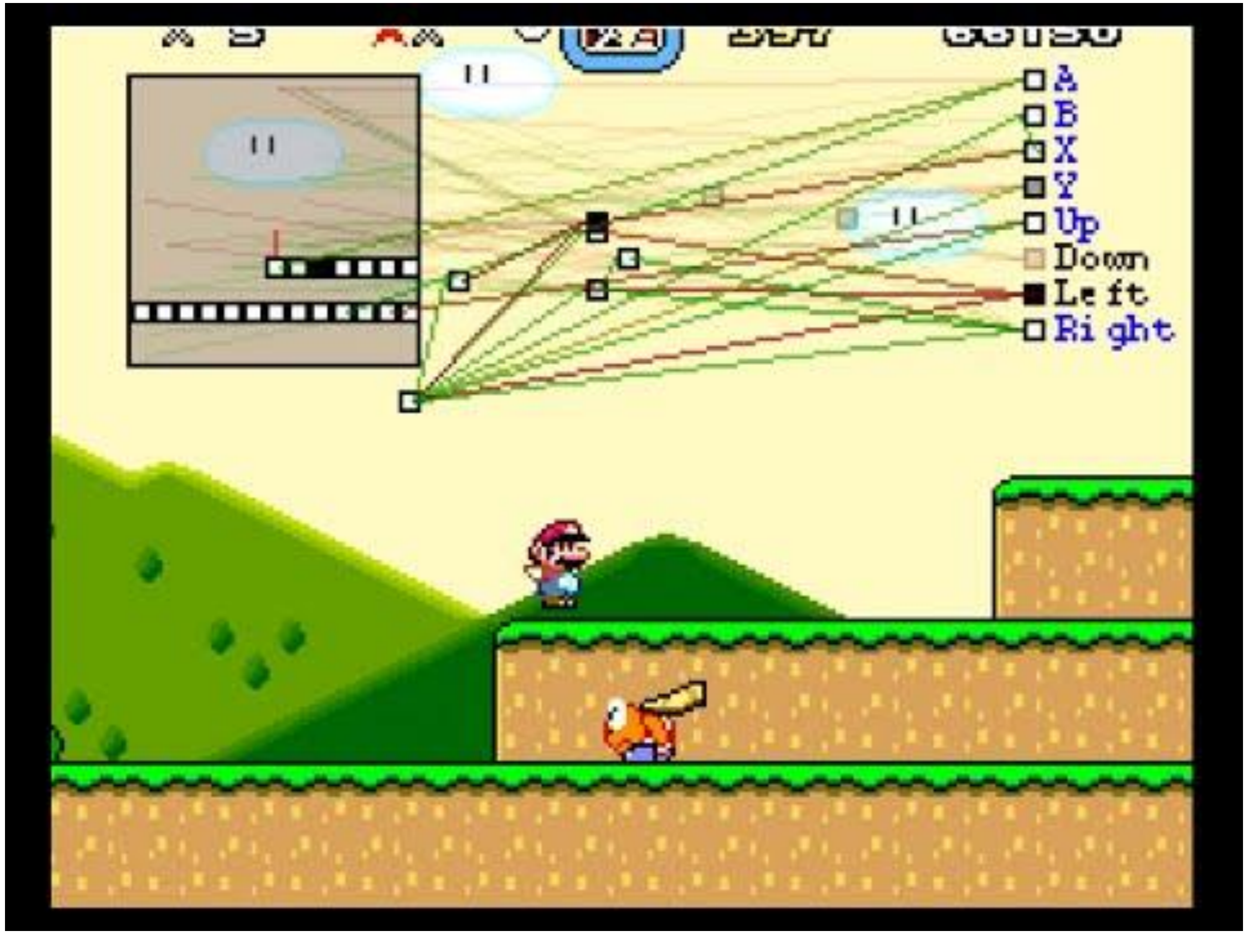


Random Policy

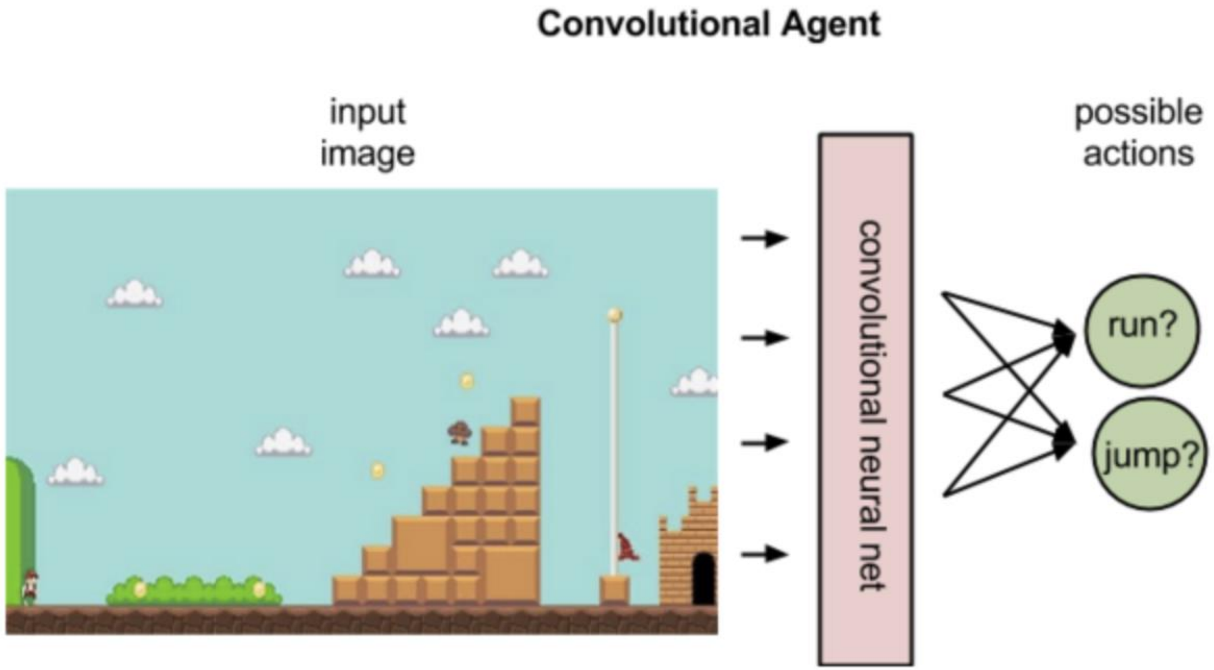


Optimal Policy

Eg. RL for MarI/O



Goal: Learn a Policy



$$a = \pi(s)$$

Rewards

- Total reward in a markov chain:

$$R = r_1 + r_2 + r_3 + \ldots + r_n$$

- Total Future Reward from time-step t:

$$R_t = r_t + r_{t+1} + r_{t+2} + \ldots + r_n$$

- Discounted Future Reward:

$$R_t = r_t + \gamma(r_{t+1} + \gamma(r_{t+2} + \ldots)) = r_t + \gamma R_{t+1}$$

Q-Learning:

In Q-learning we define a function **Q(s, a)** representing the maximum discounted future reward when we perform action a in state s, and continue optimally from that point on.

In other words: *The best possible score at the end of the game after performing action a in state s*

Policy

- Our goal is to find the optimum policy: that maximizes the reward
- If we have our Q function, deriving optimal policy is easy:

$$\pi(s) = argmax_a \, Q(s,a)$$

But how do we get the Q function?

- Recall:

$$R_t = r_t + \gamma(r_{t+1} + \gamma(r_{t+2} + \ldots)) = r_t + \gamma R_{t+1}$$

$$Q(s,a) = r + \gamma max_{a'} Q(s', a')$$

Learning- update rule

$$Q(s_t, a_t) \leftarrow (1-\alpha) \cdot \underbrace{Q(s_t, a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \cdot \overbrace{\left(\underbrace{r_t}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \cdot \underbrace{\max_a Q(s_{t+1}, a)}_{\text{estimate of optimal future value}} \right)}^{\text{learned value}}$$

Q-learning Algorithm

initialize *Q[num_states, num_actions]* arbitrarily

observe initial state *s*

repeat:

select and carry out an action *a*

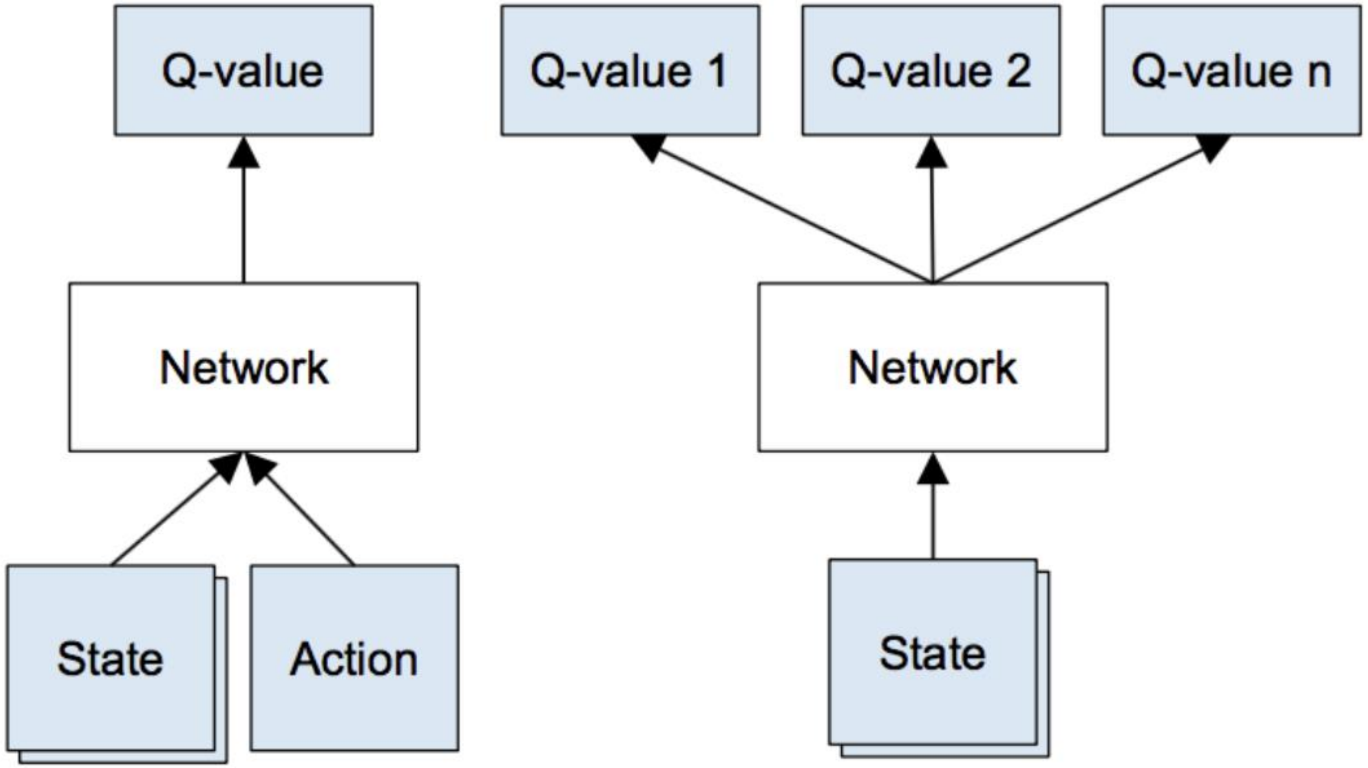
observe reward *r* and new state *s'*

$Q[s, a] = Q[s, a] + \alpha(r + \gamma \max_{a'} Q[s, a'] - Q[s, a])$

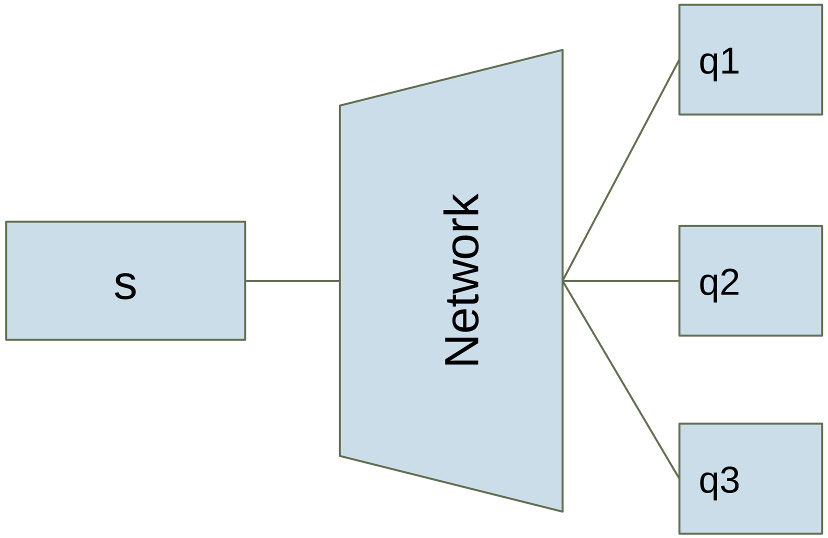
$s = s'$

until terminated

How to implement the Q-value function?

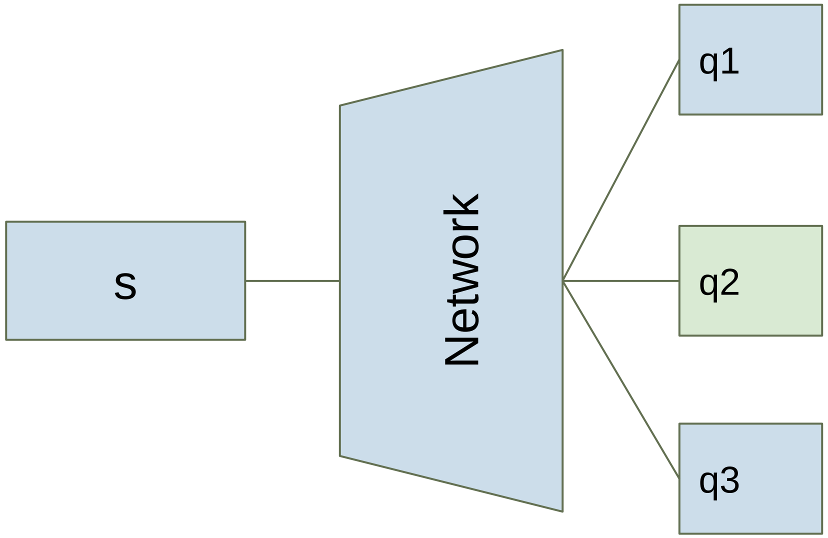


Deep Q-learning

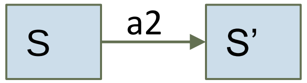


- Do a feedforward pass for the current state s to get predicted Q-values for all actions.

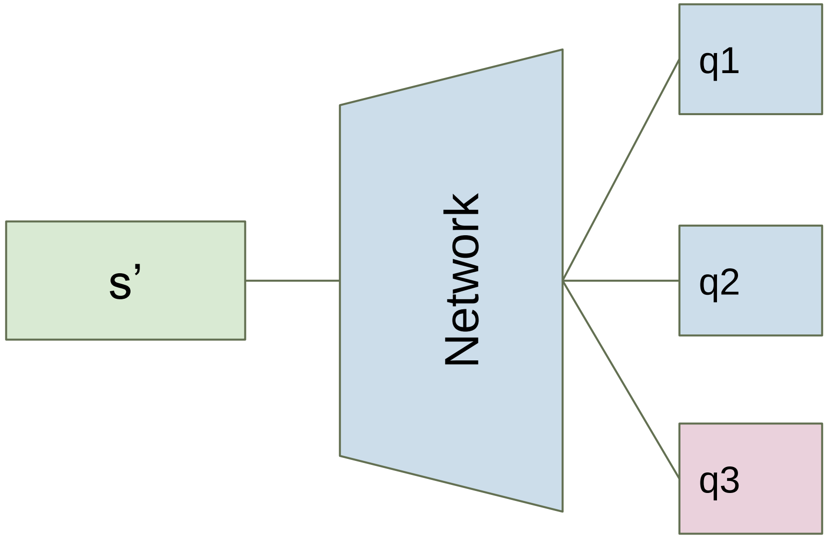
Deep Q-learning



- Do a feedforward pass for the current state s to get predicted Q-values for all actions.
- Sample an action, note the immediate reward r, and move to state s'.

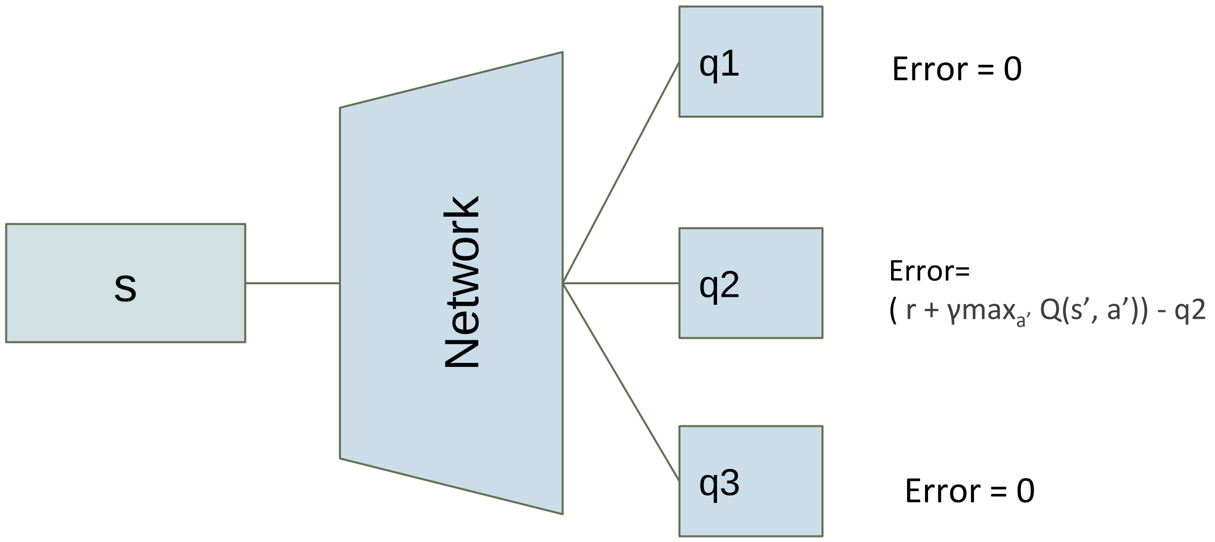


Deep Q-learning



- Do a feedforward pass for the next state s' and calculate maximum overall network outputs $\max_{a'} Q(s', a')$.

Deep Q-learning



- Set Q-value target for action to:

  $r + \gamma \max_{a'} Q(s', a')$ (max in step 2).

- For all other actions:

  Set the Q-value target to the value returned from step 1, making the error 0 for those outputs.

- Update the weights using backpropagation.

Summary: Deep Q-learning

$$L = \frac{1}{2}[\underbrace{r + max_{a'}Q(s', a')}_{\text{target}} - \underbrace{Q(s, a)}_{\text{prediction}}]^2$$

- Do a feedforward pass for the current state s to get predicted Q-values for all actions.
- Do a feedforward pass for the next state s' and calculate maximum overall network outputs $max_{a'}$ Q(s', a').
- Set Q-value target for action to r + $\gamma max_{a'}$ Q(s', a') (use the max calculated in step 2). For all other actions, set the Q-value target to the same as originally returned from step 1, making the error 0 for those outputs.
- Update the weights using backpropagation.

Conclusion

- Deep Q-learning became popular after the paper "Playing Atari with Deep Reinforcement Learning" from Deepmind.
- Deep Q learning in practice requires several tricks/hacks to work properly - experience replay, target network, error clipping, reward clipping.
- Many improvements to deep Q-learning have been proposed since its first introduction – Double Q-learning, Prioritized Experience Replay, Dueling Network Architecture and extension to continuous action space to name a few.

How does Q-learning stand against PG?

- Like most RL algorithms, both Q-learning and PG require careful hyper-parameter tuning to work well.
  - Reward definition, discount factor, learning rate, policy network, exploration-exploitation etc.

- PG is preferred because it is end-to-end: there's an explicit policy and a principled approach that directly optimizes the expected reward.

- The authors of the original DQN paper have shown Policy Gradients work better than Q Learning when tuned well.

State-of-the-art RL Algorithms

- Policy gradient methods
  - Vanilla policy gradient (including A3C)
  - Natural policy gradient and trust region methods (including TRPO)
- Q-function methods
  - Deep Q-Network and relatives (DQN, Deep Recurrent Q-Learning)
  - SARSA: also found to perform well
- Comparison: Q-function methods are more sample efficient when they work but don’t work as generally as policy gradient methods
  - Policy gradient methods easier to debug and understand

Useful Resources

- CSE 234: Reinforcement Learning, Emma Brunskill, Stanford
- CSE 231n: CNN for Visual Recognition, Fei-Fei Li & Justin Johnson & Serena Yeung, Stanford
- Blog Post: Write an AI to win at Pong from scratch with Reinforcement Learning
- Blog Post: Reinforcement Learning - A mathematical introduction to Policy Gradient
- Python RL Library: OpenAI

References

1. http://karpathy.github.io/2016/05/31/rl/
2. http://web.stanford.edu/class/cs234/schedule.html
3. http://joschu.net/docs/2016-bayareadlschool.pdf
4. https://hackernoon.com/reinforcement-learning-and-supervised-learning-a-brief-comparison-1b6d68c45ffa
5. http://rail.eecs.berkeley.edu/deeprlcourse-fa17/f17docs/lecture_4_policy_gradient.pdf