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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Simple example for comparison and evaluation of BOP-DMD
%
% Here we fit data generated from 3
% spatial modes, each with time dynamics
% which are exponential in time.
%
% This example is used to compare between this reference code
% and the python version of this code.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% ToDo:
% Loop of sigma
% Scatter plots of eigenvalues
% Alignment using match_vectors

clear all, close all, clc

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generate synthetic data

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% set up modes in space

x0 = 0;
x1 = 1;
nx = 200;

% space

xspace = linspace(x0,x1,nx);

% modes

f1 = sin(xspace);
f2 = cos(xspace);
f3 = tanh(xspace);

% set up time dynamics

t0 = 0;
t1 = 1;
nt = 100;

ts = linspace(t0,t1,nt);

% eigenvalues

e_1 = 1;

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e_2 = -2;
e_3 = 1i;

true_eigenvalues = [e_1;e_2;e_3];

% create clean dynamics

xclean = f1'*exp(e_1*ts) + f2'*exp(e_2*ts) + f3'*exp(e_3*ts);

% corrupt results
% sam= 60;
% Atest2 = zeros(nx,length(ts));
% Arand1 = rand(nx,length(ts));
% Arand2 = rand(nx,length(ts));
% r1k = randperm(length(ts)*n,sam);
%
% for j = 1:sam
%     Atest2(r1k(j))=1;
% end
% Anoise = Atest2.*(i*Arand1);
% Xnoise = xclean+Anoise;

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Run the BOP-DMD for varying levels of noise (sigma)

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% Turn off the rank deficient warning as this triggers constantly
% for the noisiest data.
warning('off', 'MATLAB:rankDeficientMatrix')

% Set an array of varying levels of noise.
sigma_array = [0.1, 0.01, 0.001];

% Determine the number of points along the time dimension.
n = length(ts);

% Set the number of samples to use for each ensemble member.
p = 50;

% Determine the number of realizations of the noisy data.
num_Noisecycles = 25;

% Determine the number of ensemble members for each BOP-DMD solution.
num_cycles = 100;

% Initialize the matrices of eigenvalue solutions...

% ... for optDMD
lambda_vec_optDMD = zeros(3, num_Noisecycles, length(sigma_array));

% ... for BOP-DMD
lambda_vec_ensembleDMD = zeros(3, num_Noisecycles * num_cycles, length(sigma_array));

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% Initialize the random seed
rand(1);

for sigma_ind = 1:length(sigma_array)
    fprintf('Sigma=%f\n', sigma_array(sigma_ind))
    for k = 1: num_Noisecycles
        fprintf('%i \n', k)

        % Create data for this realization of noisy data.
        xdata = xclean + sigma_array(sigma_ind) * randn(size(xclean));

        % Run the optdmd.
        [w_opt,e_opt,b_opt] = optdmd(xdata,ts,3,1,varpro_opts('ifprint',0));

        % Compare to actual eigenvalues
        indices = match_vectors(e_opt, true_eigenvalues);
        lambda_vec_optDMD(:, k, sigma_ind) = e_opt;

        for j = 1:num_cycles
            % Generate the ensemble members.
            unsorted_ind = randperm(n,p);
            % Sort to be in ascending order. NOTE that we now have a
            % variable delta t between members.
            ind = sort(unsorted_ind);

            % Create dataset for this cycle using the ensemble indices.
            xdata_cycle = xdata(:,ind);
            % Provide the index times.
            ts_ind = ts(ind);

            % Run the BOP-DMD with the optDMD modes as the initial conditions.
            [w_cycle,e_cycle,b_cycle] = optdmd(xdata_cycle,ts_ind,3,1,varpro_opts(

            % Compare to actual eigenvalues.
            indices = match_vectors(e_cycle, true_eigenvalues);
            lambda_vec_ensembleDMD(:, j * k, sigma_ind) = e_cycle(indices);

        end
    end
end
end

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Sigma=0.100000
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Warning: Rank deficient, rank = 2, tol = 2.465653e+00.
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Warning: Rank deficient, rank = 2, tol = 2.186103e+00.
Warning: Rank deficient, rank = 2, tol = 2.020117e+00.
Warning: Rank deficient, rank = 2, tol = 1.978243e+00.
Warning: Rank deficient, rank = 2, tol = 2.002827e+00.
Warning: Rank deficient, rank = 2, tol = 1.979157e+00.

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Warning: Rank deficient, rank = 2, tol = 7.016761e+00.
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Warning: Rank deficient, rank = 2, tol = 7.016726e+00.
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Warning: Rank deficient, rank = 2, tol = 7.014523e+00.
Warning: Rank deficient, rank = 2, tol = 7.007816e+00.
Warning: Rank deficient, rank = 2, tol = 6.981095e+00.
Warning: Rank deficient, rank = 2, tol = 6.875891e+00.
Warning: Rank deficient, rank = 2, tol = 6.480499e+00.
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Warning: Rank deficient, rank = 2, tol = 2.826250e+00.
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Warning: Rank deficient, rank = 2, tol = 2.309039e+00.
Warning: Rank deficient, rank = 2, tol = 1.932322e+00.
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Warning: Rank deficient, rank = 2, tol = 1.950776e+00.
Warning: Rank deficient, rank = 2, tol = 1.860858e+00.
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Warning: Rank deficient, rank = 1, tol = 2.106043e+02.
Warning: Rank deficient, rank = 1, tol = 2.092944e+02.
Warning: Rank deficient, rank = 1, tol = 2.041647e+02.
Warning: Rank deficient, rank = 1, tol = 1.852858e+02.
Warning: Rank deficient, rank = 1, tol = 1.298457e+02.
Warning: Rank deficient, rank = 1, tol = 4.562049e+01.
Warning: Rank deficient, rank = 2, tol = 7.644233e+00.
Warning: Rank deficient, rank = 2, tol = 4.156114e+00.
Warning: Rank deficient, rank = 2, tol = 2.408317e+00.
Warning: Rank deficient, rank = 2, tol = 3.422740e+00.
Warning: Rank deficient, rank = 2, tol = 2.453648e+00.
Warning: Rank deficient, rank = 2, tol = 2.242085e+00.
Warning: Rank deficient, rank = 2, tol = 2.208044e+00.
Warning: Rank deficient, rank = 2, tol = 2.238004e+00.
Warning: Rank deficient, rank = 2, tol = 2.209125e+00.

[illegible]

[illegible]

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Sigma=0.010000
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Sigma=0.001000
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for sigma_ind = 1:length(sigma_array)
    sigma = sigma_array(sigma_ind);
    figure()
    hold on
    for j = [1, 2, 3]
        scatter( ...
            real(lambda_vec_ensembleDMD(j, :, sigma_ind)), ...
            imag(lambda_vec_ensembleDMD(j, :, sigma_ind)) ...
        )
    end
end

```



```

scatter(real(true_eigenvalues),imag(true_eigenvalues),'ko', 'filled')
xlim([-3, 3])
ylim([-3, 3])
title(['BOP-DMD MATLAB Reference, Sigma=', num2str(sigma)])
xlabel('Real Component')
ylabel('Imaginary Component')
lgd = legend('1', '2', '3', 'True');
title(lgd, 'Eigenvalues')
fname = ['../py_optDMD/examples/matlab_reference.BOP-DMD.sigma=', num2str(sigma),
saveas(gcf, fname);

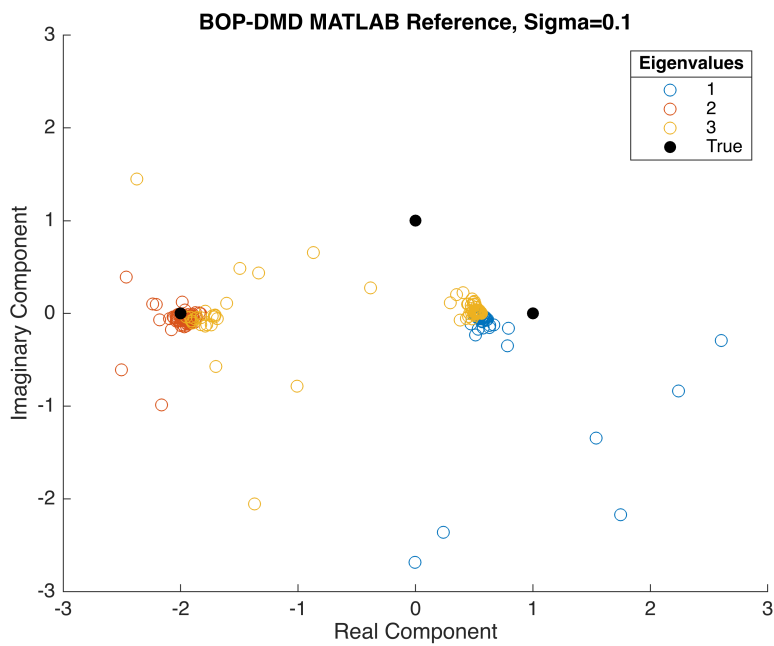
```

end

```

sigma = 0.1000
fname =
'../py_optDMD/examples/matlab_reference.BOP-DMD.sigma=0.1.png'

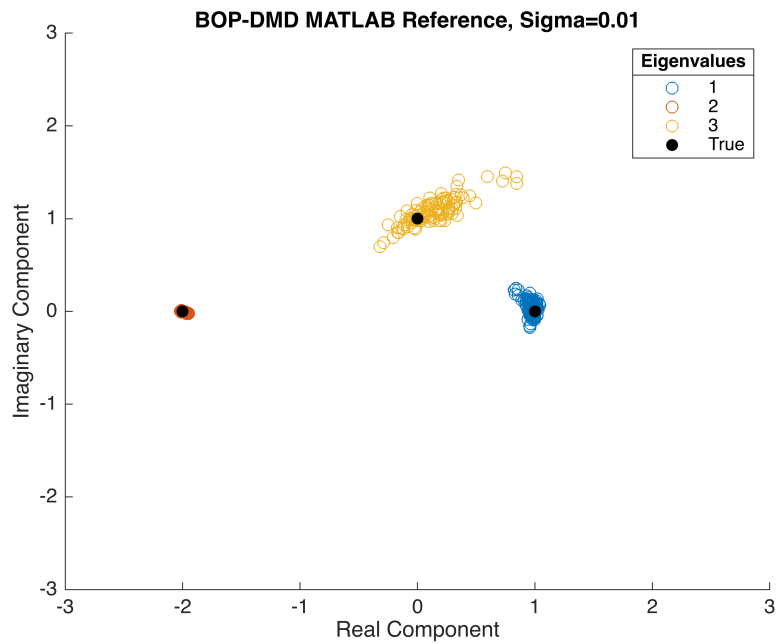
```



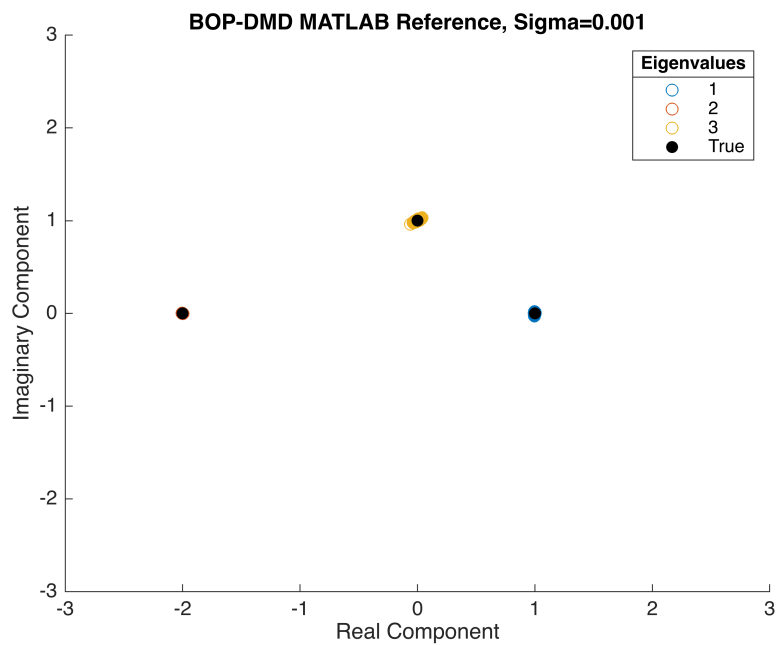
```

sigma = 0.0100
fname =
'../py_optDMD/examples/matlab_reference.BOP-DMD.sigma=0.01.png'

```



```
sigma = 1.0000e-03
fname =
'../py_optDMD/examples/matlab_reference.BOP-DMD.sigma=0.001.png'
```



```
ans =
'../py_optDMD/examples/matlab_reference.optDMD.sigma= .png'
```

```
for sigma_ind = 1:length(sigma_array)
    figure()
    hold on
    for j = [1, 2, 3]
        scatter( ...
            real(lambda_vec_optDMD(j, :, sigma_ind)), ...
            imag(lambda_vec_optDMD(j, :, sigma_ind)) ...
```

```

    )
end
scatter(real(true_eigenvalues),imag(true_eigenvalues),'ko', 'filled')
xlim([-3, 3])
ylim([-3, 3])
title(['optDMD MATLAB Reference, Sigma=', num2str(sigma_array(sigma_ind))])
xlabel('Real Component')
ylabel('Imaginary Component')
lgd = legend('1', '2', '3', 'True');
title(lgd, 'Eigenvalues')

fname = ['./py_optDMD/examples/matlab_reference.optDMD.sigma=', num2str(sigma), ''];
saveas(gcf, fname);

```

end

