ECI 249 Assignment #1

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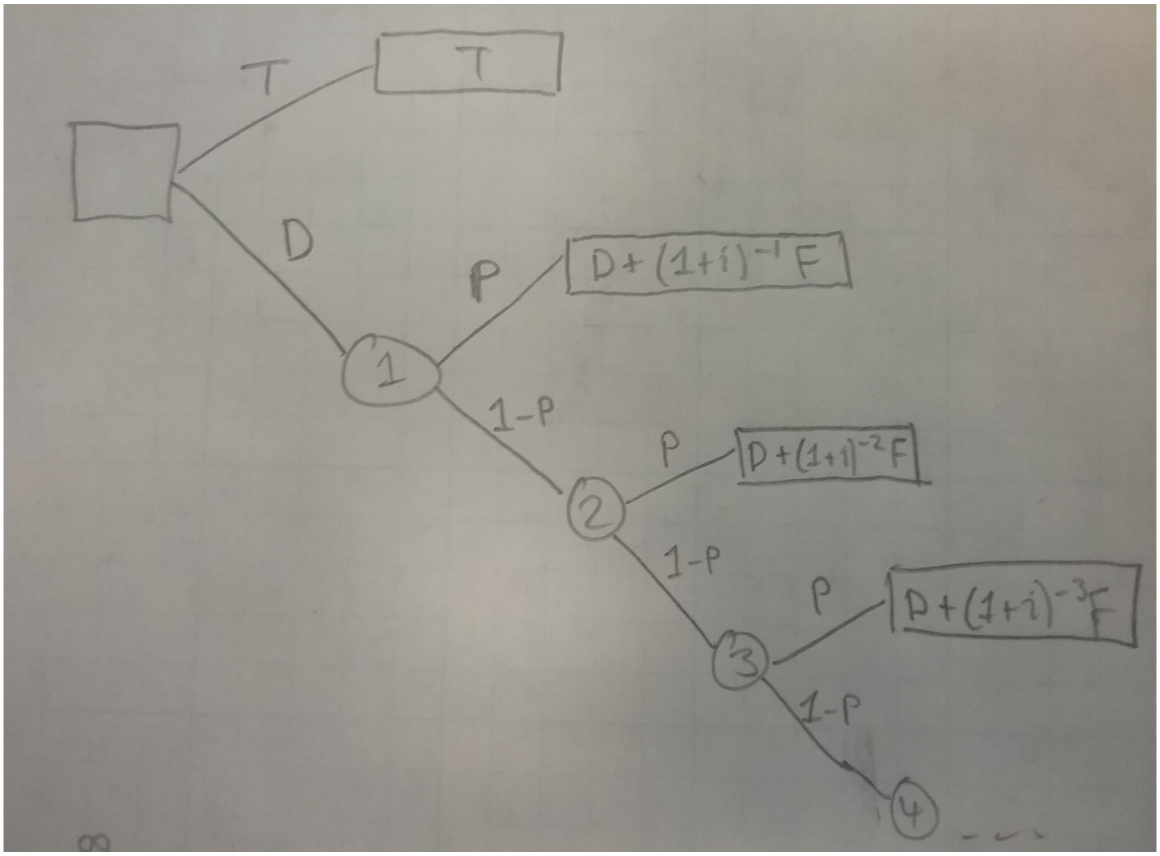
1. Hazardous Waste Disposal

a. Decision Tree and Present Value Cost



$$\boxed{C_T = T}$$

$$C_{LF} = D + P(1+i)^{-1}F + P(1-P)(1+i)^{-2}F + P(1-P)^2(1+i)^{-3}F + ...$$

$$C_{LF} = D + \sum_{k=1}^{\infty} P(1-P)^{k-1}(1+i)^{-k}F = D + PF\sum_{k=1}^{\infty}(1-P)^{k-1}(1+i)^{-k}$$

$$C_{LF} = D + \frac{PF}{1-P}\sum_{k=1}^{\infty}\left(\frac{1-P}{1+i}\right)^k$$

The last term is a geometric series:

$$\sum_{k=1}^{\infty}\left(\frac{1-P}{1+i}\right)^k = \frac{1-P}{1+i}\frac{1}{1-\frac{1-P}{1+i}}$$

Simplifying yields

$$\boxed{C_{LF} = D + \frac{F}{1 + \frac{i}{P}}}$$

b. Determine when each alternative is preferred.

Then the ratio of costs becomes:

$$R = \frac{C_{LF}}{C_T} = \frac{D}{T} + \left(\frac{F}{T}\right)\left(\frac{1}{1 + \frac{i}{P}}\right)$$

When $R < 1$, landfill is preferred; when $R > 1$, treatment is preferred; and when $R = 1$, there is no preference for either alternative.

A natural choice of dimensionless parameters becomes:

$$\pi_1 = \frac{F}{T}, \quad \pi_2 = \frac{1}{1 + \frac{i}{P}}, \quad \pi_3 = \frac{D}{T}$$

The provided range of values for D, F, T, i, and P limit the range of possible values for π_1, π_2, and π_3:

$$0 \leq \pi_1 < \infty, \quad 0 \leq \pi_2 \leq \frac{1}{1.01}, \quad 0 \leq \pi_3 < \infty$$

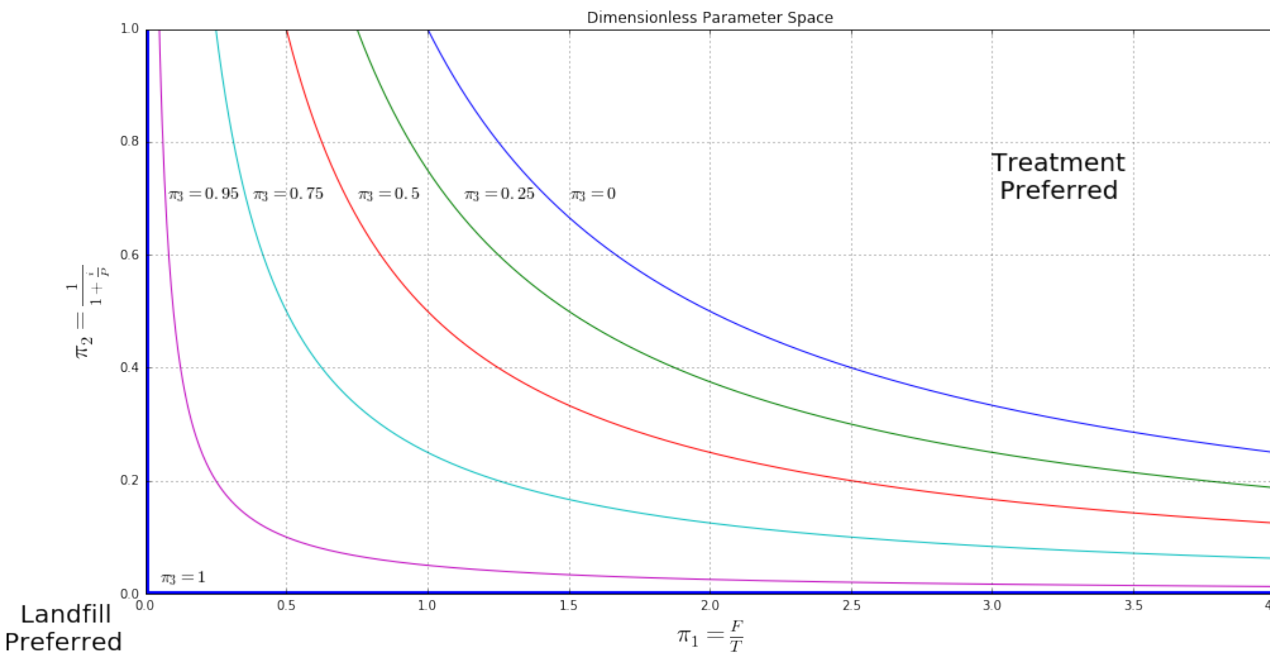
Then R can be written as:

$$R = \pi_3 + \pi_1 \pi_2$$

We can readily visualize the best decisions over the parameter space:

```python
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
%matplotlib inline

x = np.linspace(0.0001, 4, 1000)

def contour(x, pi_3):
    # returns pi_2 for a given pi_3 and pi_1 (x != 0)
    return (1 - pi_3)*1.0/x

fig, ax = plt.subplots(figsize = (16, 8))
ax.set(title='Dimensionless Parameter Space')
ax.set_xlabel(r'$\pi_1 = \frac{F}{T}$', fontsize=20)
ax.set_ylabel(r'$\pi_2 = \frac{1}{1+\frac{i}{P}}$', fontsize=20)
ax.set_xlim(0, 4)
ax.set_ylim(0, 1)
ax.grid()

for pi_3 in [0, 0.25, 0.5, 0.75, 0.95]:
    ax.plot(x, contour(x, pi_3=pi_3), label=r'$\pi_3 = $' + r'$%s$' % str(pi_3))
    ax.text(1.5 - pi_3*1.5, 0.7, r'$\pi_3 = $' + r'$%s$' % str(pi_3), fontsize=14)
```

```
ax.axvline(0, linewidth=5)
ax.axhline(0, linewidth=5)
ax.text(0.05, 0.02, r'$\pi_3 = 1$', fontsize=14)

ax.text(3, 0.7, 'Treatment\n Preferred', fontsize=20)
ax.text(-0.5, -0.1, '  Landfill\nPreferred', fontsize=20)

plt.show()
```



Contour lines are plotted for various values of π_3 on which $R = 1$. For each value of π_3, landfilling is preferred for points below the curve, and treatment is preferred for points above the curve. From here we see that treatment is always preferred for points above the $\pi_3 = 0$ contour, which corresponds to the curve $\pi_2 = \frac{1}{\pi_1}$. When $\pi_3 = 1$, the corresponding contour lines are $\pi_1 = 0$ and $\pi_2 = 0$, along which neither alternative is preferred, but treatment is preferred for the rest of parameter space. Lastly, when $\pi_3 > 1$ (i.e. $\frac{D}{T} > 1$), treatment is preferred across all parameter space.

Value of an Imperfect Test

Given:

	SS	NSS
$P()$	0.05	0.95
$P(+\|\_)$	0.90	0.20
$P(-\|\_)$	0.10	0.80

Applying Bayes Theorem to get the conditional probabilities $P(SS|+)$, $P(SS|-)$, $P(NSS|+)$, $P(NSS|-)$, and the law of total probability to get $P(+)$ and $P(-)$:

	$+$	$-$
$P()$	0.235	0.765
$P(SS\|\_)$	0.1915	0.006536
$P(NSS\|\_)$	0.8085	0.993464

Then we can calculate the overall probability of death, given the test result (or lack thereof) and treatment (or lack thereof).

It is given that the treatment is 99% effective with a 3% mortality rate. Untreated, someone with sleeping sickness has a 20% chance of death.

Overall chance of death:

Test	Treated	Untreated
No Test	0.030097	0.01
$+$	0.03037	0.0383
$-$	0.03001	0.0013072

e.g. $P(\text{D} \mid \text{no test, treat}) = 0.03 + 0.97 \cdot 0.05 \cdot 0.01 \cdot 0.20 = 0.030097$

We can conclude that the test is worthwhile, since the chance of death is lower for treatment given a positive test result, and the chance of death is lower when untreated given a negative test result or no test.