

> restart

> with(*LinearAlgebra*)

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA\_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]

> A := Matrix([ [ 2.191489, 0.0869152, 0.8279362, 0.5334933, 0.375243, 0.3672222, 0.0968153, 0.7886894, 0.9305822, 0.1335048 ], [ 0.1809336, 2.3599438, 0.7960174, 0.4092807, 0.9900186, 0.277269, 0.0251958, 0.3093025, 0.2317656, 0.8699428 ], [ 0.1507688, 0.7677984, 2.454561, 0.215663, 0.9470161, 0.9121592, 0.0880068, 0.3078117, 0.6675039, 0.6036517 ], [ 0.5180142, 0.5264675, 0.1915474, 2.0045807, 0.5770334, 0.9771822, 0.8813081, 0.5181077, 0.1464847, 0.8551703 ], [ 0.9058419, 0.8494655, 0.0921422, 0.2569904, 1.9722537, 0.1795926, 0.2596057, 0.7514084, 0.261258, 0.4449708 ], [ 0.8927505, 0.595963, 0.758821, 0.2638501, 0.8952259, 2.7710784, 0.5853018, 0.4921802, 0.6974679, 0.2383001 ], [ 0.0295571, 0.486005, 0.8665615, 0.1049571, 0.1980051, 0.5190647, 2.5916859, 0.3676201, 0.0814937, 0.49547 ], [ 0.6774982, 0.4005898, 0.8906801, 0.3909964, 0.0262317, 0.0240633, 0.7611217, 2.259136, 0.410222, 0.7606841 ], [ 0.1520384, 0.3292257, 0.0131486, 0.1227146, 0.5051157, 0.4677612, 0.2729393, 0.6320958, 2.0344747, 0.1798599 ], [ 0.2179066, 0.2826244, 0.9151035, 0.5044386, 0.5772322, 0.1669885,

(1)

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0.9266215 , 0.1846557, 0.5225408, 2.1408565 ]]);
A := [[ 2.191489, 0.0869152, 0.8279362, 0.5334933, 0.375243, 0.3672222, 0.0968153,
0.7886894, 0.9305822, 0.1335048],
[ 0.1809336, 2.3599438, 0.7960174, 0.4092807, 0.9900186, 0.277269, 0.0251958,
0.3093025, 0.2317656, 0.8699428],
[ 0.1507688, 0.7677984, 2.454561, 0.215663, 0.9470161, 0.9121592, 0.0880068,
0.3078117, 0.6675039, 0.6036517],
[ 0.5180142, 0.5264675, 0.1915474, 2.0045807, 0.5770334, 0.9771822, 0.8813081,
0.5181077, 0.1464847, 0.8551703],
[ 0.9058419, 0.8494655, 0.0921422, 0.2569904, 1.9722537, 0.1795926, 0.2596057,
0.7514084, 0.261258, 0.4449708],
[ 0.8927505, 0.595963, 0.758821, 0.2638501, 0.8952259, 2.7710784, 0.5853018,
0.4921802, 0.6974679, 0.2383001],
[ 0.0295571, 0.486005, 0.8665615, 0.1049571, 0.1980051, 0.5190647, 2.5916859,
0.3676201, 0.0814937, 0.49547],
[ 0.6774982, 0.4005898, 0.8906801, 0.3909964, 0.0262317, 0.0240633, 0.7611217,
2.259136, 0.410222, 0.7606841],
[ 0.1520384, 0.3292257, 0.0131486, 0.1227146, 0.5051157, 0.4677612, 0.2729393,
0.6320958, 2.0344747, 0.1798599],
[ 0.2179066, 0.2826244, 0.9151035, 0.5044386, 0.5772322, 0.1669885, 0.9266215,
0.1846557, 0.5225408, 2.1408565 ]]]

```

```

> u := Vector([ 3.4973496, 3.1826006, 3.424393, 2.926963, 3.5773956, 3.4078488,
1.945066, 4.0296256 , 1.9324165, 3.3537391 ]]);

```

$$u := \begin{bmatrix} 3.4973496 \\ 3.1826006 \\ 3.424393 \\ 2.926963 \\ 3.5773956 \\ 3.4078488 \\ 1.945066 \\ 4.0296256 \\ 1.9324165 \\ 3.3537391 \end{bmatrix}$$

```

> z := LinearSolve(A, u);

```

$$z := \begin{bmatrix} 0.710756563848429 \\ 0.334975288686626 \\ 0.494348371785814 \\ 0.175722340677959 \\ 0.696471489654827 \\ 0.236796381594992 \\ 0.114172690295269 \\ 0.923919140214322 \\ 0.227701558080217 \\ 0.806319388891365 \end{bmatrix} \quad (4)$$

```
> AI := Matrix([ [ 2.191489, 0.0869152, 0.8279362, 0.5334933, 0.375243, 0.3672222,
0.0968153, 0.7886894, 0.9305822, 0.1335048],
[ 0.1809336, 2.3599438, 0.7960174, 0.4092807, 0.9900186, 0.277269,
0.0251958, 0.3093025, 0.2317656, 0.8699428],
[ 0.1507688, 0.7677984, 2.454561, 0.215663, 0.9470161, 0.9121592,
0.0880068, 0.3078117, 0.6675039, 0.6036517],
[ 0.5180142, 0.5264675, 0.1915474, 2.0045807, 0.5770334, 0.9771822,
0.8813081, 0.5181077, 0.1464847, 0.8551703],
[ 0.9058419, 0.8494655, 0.0921422, 0.2569904, 1.9722537, 0.1795926,
0.2596057, 0.7514084, 0.261258, 0.4449708],
[ 0.8927505, 0.595963, 0.758821, 0.2638501, 0.8952259, 2.7710784,
0.5853018, 0.4921802, 0.6974679, 0.2383001],
[ 0.0295571, 0.486005, 0.8665615, 0.1049571, 0.1980051, 0.5190647,
2.5916859, 0.3676201, 0.0814937, 0.49547 ],
[ 0.6774982, 0.4005898, 0.8906801, 0.3909964, 0.0262317, 0.0240633,
0.7611217, 2.259136, 0.410222, 0.7606841]]);
```

```
AI := [[2.191489, 0.0869152, 0.8279362, 0.5334933, 0.375243, 0.3672222, 0.0968153,
0.7886894, 0.9305822, 0.1335048],
[0.1809336, 2.3599438, 0.7960174, 0.4092807, 0.9900186, 0.277269, 0.0251958,
0.3093025, 0.2317656, 0.8699428],
[0.1507688, 0.7677984, 2.454561, 0.215663, 0.9470161, 0.9121592, 0.0880068,
0.3078117, 0.6675039, 0.6036517],
[0.5180142, 0.5264675, 0.1915474, 2.0045807, 0.5770334, 0.9771822, 0.8813081,
0.5181077, 0.1464847, 0.8551703],
[0.9058419, 0.8494655, 0.0921422, 0.2569904, 1.9722537, 0.1795926, 0.2596057,
0.7514084, 0.261258, 0.4449708],
[0.8927505, 0.595963, 0.758821, 0.2638501, 0.8952259, 2.7710784, 0.5853018,
0.4921802, 0.6974679, 0.2383001],
[0.0295571, 0.486005, 0.8665615, 0.1049571, 0.1980051, 0.5190647, 2.5916859,
0.3676201, 0.0814937, 0.49547],
[0.6774982, 0.4005898, 0.8906801, 0.3909964, 0.0262317, 0.0240633, 0.7611217,
```

2.259136, 0.410222, 0.7606841 ]]

>  $uI := \text{Vector}([ 3.4973496, 3.1826006, 3.424393, 2.926963, 3.5773956, 3.4078488, 1.945066, 4.0296256 ])$ ;

$$uI := \begin{bmatrix} 3.4973496 \\ 3.1826006 \\ 3.424393 \\ 2.926963 \\ 3.5773956 \\ 3.4078488 \\ 1.945066 \\ 4.0296256 \end{bmatrix}$$

(6)

>  $zI := \text{LinearSolve}(AI, uI)$ ;

$$zI := \begin{bmatrix} 0.664248366650712 - 0.334546225626174 \_t0_2 + 0.152154215454668 \_t0_1 \\ 0.527597188735546 - 0.00536301419146807 \_t0_2 - 0.237375828980969 \_t0_1 \\ 0.642877640411587 - 0.228070174438584 \_t0_2 - 0.119800337044227 \_t0_1 \\ 0.415882330053850 + 0.0539591568784418 \_t0_2 - 0.313085083836201 \_t0_1 \\ 0.736874323352780 + 0.0363364823918758 \_t0_2 - 0.0603690150880089 \_t0_1 \\ 0.220017313569640 - 0.107877231396878 \_t0_2 + 0.0512735800048730 \_t0_1 \\ 0.155484526884770 + 0.0694563442994125 \_t0_2 - 0.0708492877538001 \_t0_1 \\ 1.10222456893481 - 0.0223985861061430 \_t0_2 - 0.214809712071374 \_t0_1 \\ \_t0_2 \\ \_t0_1 \end{bmatrix}$$

(7)

>

>  $func = \text{sqrt}(zI[1]^2 + zI[2]^2 + zI[3]^2 + zI[4]^2 + zI[5]^2 + zI[6]^2 + zI[7]^2 + zI[8]^2 + zI[9]^2 + zI[10]^2)$

$func$

(8)

$$\begin{aligned} &= \left( (0.664248366650712 - 0.334546225626174 \_t0_2 + 0.152154215454668 \_t0_1)^2 \right. \\ &+ (0.527597188735546 - 0.00536301419146807 \_t0_2 - 0.237375828980969 \_t0_1)^2 \\ &+ (0.642877640411587 - 0.228070174438584 \_t0_2 - 0.119800337044227 \_t0_1)^2 \\ &+ (0.415882330053850 + 0.0539591568784418 \_t0_2 - 0.313085083836201 \_t0_1)^2 \\ &+ (0.736874323352780 + 0.0363364823918758 \_t0_2 - 0.0603690150880089 \_t0_1)^2 \\ &\left. + (0.220017313569640 - 0.107877231396878 \_t0_2 + 0.0512735800048730 \_t0_1)^2 \right) \end{aligned}$$

$$\begin{aligned}
& + (0.155484526884770 + 0.0694563442994125 \_t0_2 - 0.0708492877538001 \_t0_1)^2 \\
& + (1.10222456893481 - 0.0223985861061430 \_t0_2 - 0.214809712071374 \_t0_1)^2 + \_t0_2^2 \\
& + \_t0_1^2)^{1/2}
\end{aligned}$$

> with( Optimization)  
[ ImportMPS, Interactive, LPSolve, LSSolve, Maximize, Minimize, NLPsolve, QPSolve] (9)

> Minimize( sqrt( zI[1]^2 + zI[2]^2 + zI[3]^2 + zI[4]^2 + zI[5]^2 + zI[6]^2 + zI[7]^2 + zI[8]^2  
+ zI[9]^2 + zI[10]^2 ) )  
[ 1.67467531722136687, [ \_t0\_1 = 0.422201736579000, \_t0\_2 = 0.320579456514438 ] ] (10)

> sol := subs( { \_t0\_1 = 0.422201736579000, \_t0\_2 = 0.320579456514438 }, zI )

sol :=

0.621239493461318
0.425657429338214
0.519183117500868
0.300995461139938
0.723035150122100
0.207081883879460
0.147838111664225
1.00435100889945
0.320579456514438
0.422201736579000

(11)