Homework №3

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1 Part 1

1.1 Graph coloring problem, the DSatur (Degree of Saturation) algorithm.

see code

1.2 Testing

see code

1.3 Estimation of the work required in θ-notation

- V: Number of vertices in the graph
- E: Number of edges in the graph
- d: Average degree of a vertex $\left(d = \frac{2E}{V}\right)$

1. Initialization:
   - Compute the degree of each vertex: $O(V + E)$
   - Initialize saturation degrees: $O(V)$
   - Total: $O(V + E)$
2. Main Loop (Iterates V times, once for each vertex), for each uncolored vertex: per iteration $O(V)$
3. Assign a Color: per iteration $O(d)$
4. Update Saturation Degrees: $O(V + d)$ for one vertex

Overall, in Main Loop:

$$O(V) \cdot O(V + d) = O(V^2 + Vd) \tag{1}$$

Total Complexity
Adding the initialization step:

$$O(V + E) + O(V^2 + Vd) = O(V^2 + Vd + E) \tag{2}$$

using $d = \left(\frac{2E}{V}\right)$ we can substitute Vd as $O(E)$, so

$$O(V^2 + E) \tag{3}$$

In θ-notation

$$\theta(V^2 + E) \tag{4}$$

This complexity makes DSatur efficient for sparse graphs ($E = O(V)$), but less efficient for dense graphs($E = O(V^2)$). For a complete graph ($E = \frac{V(V-1)}{2}$) the complexity simplifies to $\theta(V^2)$

2 Part 2

2.1 Combine all the learned methods

see code

2.2 Provide reasoning, algorithm designing steps, and resulting complexity estimation

Approximation Guarantee:

1. Greedy Coloring: base alorithm
2. Kempe Chain Refinement: reduces the number of colors by identifying unnecessary color usage in substructures of the graph
3. Planar Graphs: if the graph is planar, guarantee at most 4 colors via Four Color Theorem

Complexity Analysis

- Initialization: $O(V + E)$
- Color Assignment: $O(V^2)$
- Refinement: $O(V^2)$
- Total: $O(V^2 + E)$

This is efficient for sparse graphs and provides an approximate solution with minimal deviation from the optimal coloring.