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**Report – Numerical Simulation of Light-Ray Deflection in Schwarzschild Space-Time**

1. **Introduction**

Gravitational lensing - the bending of light by massive bodies - provides one of the most striking confirmations of general relativity. In this project, I simulate photon trajectories around a non-rotating (Schwarzschild) black hole by numerically integrating the null geodesic equations. The primary goal is to reproduce the classic result that light passing at impact parameter is deflected by an angle

in geometrized units . I build an interactive dashboard (using Dash and Plotly) that allows the user to vary the black-hole mass and visualize both individual light-ray trajectories and the dependence of on .

1. **Mathematical Formulation**
   1. **Schwarzschild Metric and Null Geodesics**

In Schwarzschild coordinates , a spherically symmetric, static black hole of mass has line element

.

Restricting motion to the equatorial plane , null geodesics satisfy two first integrals:

, ,

where are the conserved energy and angular momentum per unit mass, and is just a parameter that increases along the light ray’s path. Eliminating and using yields the radial equation (see [theconfused.me, 2017]):

.

Thus we have the system of three first-order ODEs for variables :

We adopt geometrized units throughout, so and all lengths are in the same unit.

* 1. **Impact Parameter and Initial Conditions**

The impact parameter is the asymptotic closest-approach distance for a straight-line light ray. We launch photons from a point with an initial velocity directed along . Converting to polar coordinates,

, ,

and components of the velocity in form are chosen so that the ray is initially moving purely in the direction. One solves

, ,

then sets .

1. **Numerical Method**

I use SciPy’s solve\_ivp routine (RK45 by default) to integrate the ODE system from to ​. The trajectory is sampled uniformly in with step . Trajectories for various values are computed and then transformed back to Cartesian via

, .

1. **Results and Error Analysis**
   1. **Photon Trajectories**

For a representative mass , trajectories with various values from to clearly show increasing deflection for smaller , and capture (plunge into the horizon) when (see Fig.1).

A graph of lines with a black dot

AI-generated content may be incorrect.

Fig.1. The plot of Light Rays Around a Schwarzschild Black Hole

* 1. **Deflection Angle vs. Impact Parameter**

I compare:

* **Theoretical:** .
* **Numerical:** estimated by measuring the asymptotic incoming and outgoing slopes of the trajectory and computing

.

For , numerical and theoretical agree within . As decreases toward the critical value, nonlinear effects introduce larger deviations; at the relative error is a few percent, growing rapidly for .

* 1. **Sources of Error**
* Integration step size : fixed-step sampling of output can poorly resolve rapid curvature near periapsis, leading to slope-estimation error.
* Finite : the photon may not fully exit the curved region within the chosen domain, biasing asymptotic angle.
* Slope estimation: using only two points near the end of the trajectory amplifies noise; a least-squares fit to the outgoing leg would reduce variance.

1. **Conclusions**

I have successfully implemented and visualized light bending around a Schwarzschild black hole, reproducing the classical law in the weak-field regime. The interactive dashboard highlights how deflection grows with mass and falls off with bb, and illustrates the capture threshold at .

**Possible Improvements:**

* Adaptive integration: switch to an adaptive step integrator that refines near periapsis to control local error.
* Extended domain: increase ​ until the angle converges to a desired precision.
* Robust asymptotic fitting: perform linear regression on the far-field segment of to extract the deflection angle more accurately.
* Higher-order methods: implement higher-order integrators tailored to geodesic equations.

Such refinements would reduce numerical error, extend validity closer to the photon-sphere, and provide even more faithful simulations of strong-field lensing.

1. **References**

* Numerical integration of light paths in a Schwarzschild metric, *theconfused.me* (2017), available at <https://theconfused.me/blog/numerical-integration-of-light-paths-in-a-schwarzschild-metric/>