Three-Dimensional Markov Chain Model for Performance Analysis of the IEEE 802.11 Distributed Coordination Function

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***Abstract*—This paper introduces an accurate analysis using three dimensional Markov chain modeling to compute the IEEE 802.11 DCF performance under heavy traffic conditions and absence of hidden terminals. The proposed model matches the real implementation of the DCF as presented in the standard through considering the impact of retry limits of control and data frames jointly on the performance of DCF mechanism. Moreover, transmission errors are added to the model as constant frame error probabilities. In addition to the throughout efficiency, this analytical analysis calculates the average packet delay, the packet drop probability and the average packet drop time for the DCF access modes, basic and RTS/CTS. We show that our proposed model is a more general model compared to the other models that are presented in the literature, which leads to more accurate performance analysis. Simulation results validate the accuracy of our analytical analysis. Moreover, we prove the generality and validate the correctness of our analysis by showing that other models appeared in literature are special cases from our proposed model. Moreover, the impact of the retry limits and the network size on the performance of IEEE 802.11 DCF is presented.**

***Index Terms*—IEEE 802.11, DCF, Markov model, performance analysis, saturated throughput, BEB.**

I. INTRODUCTION

Medium Access Control (MAC) of IEEE 802.11 defines two medium access methods, the mandatory Distributed Coordination Function (DCF) and the optional Point Coordination Function (PCF). PCF is out of scope of this paper. DCF is based on Carrier Sense Multiple Access with Collision Avoidance (CSMA-CA) scheme, which adopts a slotted Binary Exponential Backoff (BEB) scheme as a medium access control to resolve collisions and to control network congestion. The DCF protocol defines two modes for packet transmission: the mandatory basic access mode and the optional Request-To-Send (RTS)/Clear-To-Receive (CTS) access mode.

In literature, there is a lot of research work in modeling the IEEE 802.11 DCF and studying its performance metrics. The first Markov chain model introduced by Bianchi [3] has become the most common method for calculating the saturated throughput for single hop wireless networks, whereas a 2-dimensional Markov chain model to calculate the saturated throughput is proposed under the assumptions of error-free channels, no hidden terminals, no capture effect, and unlimited packet retransmissions. Tay [4] developed a simple mathematical model to derive the probability of collision and the saturation throughput under idealistic assumptions, whereas packet retransmissions are unlimited and no channel errors. Additionally, number of transmissions per packet is considered as geometric distribution. Wu [5] extended Bianchi model by considering the impact of packet's retransmission limits on throughput analysis. Neither [3] nor [5] studied the packet delay, the packet drop probability, the transmission errors, or the drop time of the transmitted packet. Chatzimisios [6] extended [5] to develop the average packet delay for basic and RTS/CTS access modes, whereas retry limits are considered but transmission errors are ignored. Therefore, the model does not predict the 802.11 frame delay in an accurate way. [6] is extended in [9] to consider the impact of transmission errors in addition to the retry limits on the performance of basic mode, whereas transmission errors are considered as a constant frame error probabilities only for data frames and ignored for control frames. Tickoo [7] introduced an analytical model for evaluating the DCF packet queuing delays under finite load conditions by modeling each node as a discrete time G/G/1 queue. Vukovic [8] simplified Wu model [5] into a one dimensional Markov chain and computed the average delay of three backoff schemes.

The proposed analytical models for IEEE 802.11 DCF did not consider the actual specification as it is described in the standard [2]. The proposed models either evaluate the performance of DCF under the basic mode by considering data retry limit and channel bit errors or evaluate the DCF under the RTS/CTS scheme by considering RTS retry limit and channel bit errors but ignoring the data retry limit.

In this paper, we develop a theoretical framework to evaluate the impacts of control and data retry limits as well as the quality of the received data on the performance of DCF mechanism. We propose a new 3-dimensional Markov model that directly integrates backoff process as well as data and control retry limits into one model. The x-dimension used to model the backoff process, the y-dimension used to model the short retry count (backoff stage) and the z-dimension allows us to accurately model the long retry count (backoff layer). Transmission errors are modeled by assuming a Gaussian wireless error channel with a constant Bit Error Rate (BER), whereas all transmitted frames have the same Frame Error Rate (FER). By exploiting the theoretical framework, we derive

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the the transmission probability, which we then use to derive throughput, packet delay, packet drop probability and packet drop time. We conduct extensive simulations to verify the theoretical analysis. It shows that analytical results match simulation outcomes quite well. Further, the effects of different parameters, such as network size and data/control retry limits are deeply investigated.

The remainder of this paper is organized as follows: Section II reviews the IEEE 802.11 DCF protocol. Section III describes the proposed analytical framework and develops an analytical analysis to derive the DCF throughput, average packet delay, average packet drop time, and packet drop probability. Section IV presents analysis and simulation results. Finally, section V concludes the paper.

II. IEEE 802.11 Distributed Coordination Function

In 802.11 DCF, priority levels for accessing the channel are provided through the use of Interframe Spaces (IFs) such as: Short InterFrame Space (SIFS), DCF InterFrame Space (DIFS) and Extended InterFrame Space (EIFS). A station with new packet to transmit monitors the channel, if the channel is sensed to be idle for an interval larger than DIFS period, the station transmits the packet. Otherwise, if the channel is sensed busy (either immediately or during the DIFS), the station defers its transmission and keeps monitoring the channel until it becomes idle for a DIFS period. Then, the station generates a random backoff period before transmitting the packet. To avoid channel capture problem, the random backoff period is selected between successive transmissions. The Contention Window (CW) value depends on the number of retransmissions. It starts with a minimum value (CW_{min}) and doubles after each unsuccessful transmission up to a maximum value $CW_{max} = 2^m CW_{min}$, (m is a positive-integer number, which limits the value of CW). The backoff time is randomly and uniformly chosen from the range $(0, CW - 1)$ time slots. It is a slotted time, the duration of each time slot (σ) is carefully set equal to the time needed by any station to detect the transmission of other stations within a certain range. The backoff time is decremented once every time slot for which the channel is detected idle, frozen when a transmission is detected on the channel, and resumed when the channel is sensed idle again for a DIFS period. The station transmits when the backoff time reaches zero. Time duration between successive empty time slots is variable and depends on the status of the medium. Two successive empty time slots should be proceeded by an idle DIFS period. DCF sets a threshold for the number of retransmissions, as the number of retransmission exceeds this threshold, the frame is dropped from the MAC queue. More, as CW reaches it is maximum value, it keeps on this value in subsequent retransmission attempts.

In the basic access mode, as the backoff time equals zero, the source node transmits a data frame and waits for a timeout period in order to receive an acknowledgment packet (ACK) from a destination node. The destination node waits for a SIFS period immediately following the successful reception of the data frame and replies with the ACK to indicate that the data packet has been received correctly. While the data frame is being transmitted, other nodes hearing the data frame transmission adjust their Network-Allocation Vector (NAV), which is used for virtual carrier sense at the MAC layer, correctly based on the duration field value in the data frame received. This includes the SIFS and the ACK frame transmission time, which are following the data frame.

In the RTS/CTS access mode, two small control packets, RTS and CTS, are handshaked between a source and a destination nodes prior to the transmission of an actual data frame in order to capture the channel, to prevent other nodes from transmission and to shorten the collision time interval. A node that needs to transmit a packet follows the rules of backoff mechanism. As the backoff counter reaches zero, the source node sends an RTS frame. As the destination receives the RTS frame, it responds with a CTS frame after a SIFS period. The source node is allowed to transmit its data frame if and only if it received the CTS frame correctly. Successful data transmission is acknowledged by the destination node. RTS and CTS used by other stations to update their NAVs using duration fields information. If a collision occurs with two or more RTS frames, less bandwidth is wasted as compared to the situation when larger data frames are collided.

In both access modes, if the ACK frame is received correctly, the transmitting node resets its CW to CW_{min} and reenters the backoff process if it has further frames in its MAC queue. If the source node does not receive the ACK, the data frame is assumed to be lost and the source node doubles its CW and reschedules the frame retransmission according to the backoff rules. The transmitted data packet is dropped from the MAC queue after specific number of retransmission attempts. The standard proposed two thresholds, which are maintained by each station and take an initial value of zero for every new packet: station short retry count (ssrc) and station long retry count (slrc). slrc represents the maximum number of retransmission attempts of a data frame, which is incremented with each unsuccessful data frame transmission. ssrc represents the maximum number of retransmission attempts for the RTS control frame, which is incremented with each unsuccessful data or RTS frame transmission. As either of these two limits is reached first, the frame is discarded from the MAC queue, the CW is reset to CW_{min}, and both retry limits are set to zero.

III. System performance analysis

A. Network Model Assumptions

We assume a network of a finite number of stations (n), all stations run IEEE 802.11 DCF mechanism and use the same channel access mode. Moreover, all stations are in direct communications (no hidden stations) and they operate under heavy traffic conditions. Received frames may have errors due to transmission errors or collisions. Transmission errors appear due to poor channel conditions resulted from different sources such as: multipath fading, path loss, thermal noise and interference. Our model is based on two key assumptions. The first assumption is that collision probability of a transmitted

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packet is constant and independent of the retransmissions history of that packet. The second assumption is that FERs of data frame and control frames are independent, whereas FER depends on the bit error probability (P_b) and the number of bits per frame. Assume the length of DATA, ACK, RTS, and CTS frames are constants and equal to l_{rts}, l_{cts}, l_{data} and l_{ack} bits respectively. Given that the bit errors are uniformly distributed over the whole frame and P_b, then P_e^{rts}, P_e^{cts}, P_e^{data} and P_e^{ack} are:

$$\begin{cases} P_e^{rts} & = 1-(1-P_b)^{l_{rts}} \\ P_e^{cts} & = 1-(1-P_b)^{l_{cts}} \\ P_e^{data} & = 1-(1-P_b)^{l_{data}} \\ P_e^{ack} & = 1-(1-P_b)^{l_{ack}} \end{cases} \tag{1}$$

Collision occurs due to simultaneous transmission by two or more stations, whereas station's transmission encounters collision if at least one of the remaining $n-1$ stations transmit simultaneously. Assuming that each station transmits with probability τ and it collides with a constant and independent probability P_c, then the probability of collision is:

$$P_c \quad = 1-(1-\tau)^{n-1} \tag{2}$$

B. Transmission Probability

Let $b(t)$ be a stochastic process represents the backoff time counter, $s(t)$ be a stochastic process represents the backoff stage, and $l(t)$ a stochastic process represents the backoff layer (number of data frame retransmission attempts) for a given station at time slot t. The 3-dimensional process $\{b(t), s(t), l(t)\}$ can be modeled with a discrete-time Markov chain depicted in Fig. 1. let $b_{i,k}^{(j)} = lim_{t\to\infty} P\{s(t)=i, b(t)=k, l(t)=j\}, j \in (0, d-1)$, $i \in (j, s)$, and $k \in (0, W_i - 1)$ be the stationary distribution of the Markov chain. In the proposed model, there are *d* layers, transition from layer *j* to layer *j+1* is triggered by unsuccessful data frame transmission. Each layer j has *s-j+1* stages, transition from the backoff stage *i* to the next backoff stage *i+1* in the same layer j is triggered by unsuccessful *rts* frame transmission. Each stage *i* has $W_i - 1$ backoff states, transmission from state *k+1* to state *k* represents the backoff counter decrement process.

Transmission probabilities p_r and p_d between stages and layers, respectively, are constant and independent regardless of number of retransmissions. Let p_r represents the probability that a RTS frame collides with another RTS frame or the transmitted RTS or CTS frame is an erroneous frame. Let p_d represents the probability that the transmitted DATA or ACK frame is an erroneous frame. Assuming these events are independent, p_r and p_d can be expressed as:

$$\begin{cases} p_r & = 1-(1-P_c)(1-P_b)^{l_{rts}+l_{cts}} \\ p_d & = 1-(1-P_b)^{l_{data}+l_{ack}} \end{cases} \tag{3}$$

From figure 1, the transitions probabilities are given in (4). Assume the current backoff process is at backoff layer $d-1$, backoff stage $i-1$ and backoff state $k+1$. First and second equations show that moving from current backoff stage to the

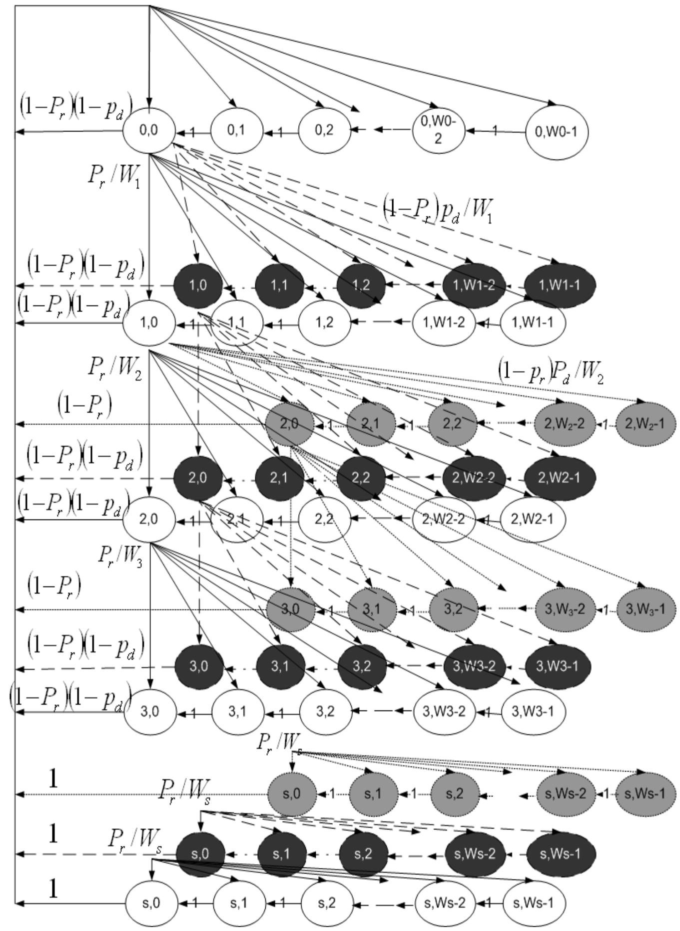

Fig. 1. Markov chain model for the backoff window size

next backoff stage within the current or the next backoff layer is triggered by an erroneous unsuccessful channel reservation or data frame transmission respectively. Third equation accounts for the fact that, at the beginning of each time slot, the backoff time is decremented. Fourth equation indicates that once the backoff stage reaches the stage *s*, the backoff process is reset regardless of data transmission status. Fifth equation accounts for the fact that, at backoff layers $L_0, \ldots, L_{d-2}$, successful data transmission followed by reseting the backoff process. Finally, the last equation holds for layer L_{d-1}, the *slrc* count reaches its maximum limit and as a result the backoff process is reset regardless of the data transmission status. Reseting the backoff process implies $CW = CW_{min}$, $slrc = 0$ and $ssrc = 0$. Those transition probabilities can be expressed in a highly reduced mathematical form in terms of $b_{(0,0)}^{(0)}$ and the two conditional probabilities p_r and pd as shown in (5), where $u(x)$ is a unit step function[1] and $a_{i,j}$, a positive-integer coefficient, is given by:

$$a_{i,j} \quad = \begin{cases} 0 & ; i<0,\ j<0 \\ 1 & ; j=0,\ i=0 \\ a_{j,i-1}+a_{j-1,i-1} & ; Otherwise \end{cases} \tag{6}$$

[1] if $x \le 0$, $u(x) = 0$, otherwise $u(x) = 1$

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$$
\begin{cases}
P\left\{i^{(j)},k|i-1^{(j)},0\right\}=\frac{p_r}{W_i} & j\in(0,d-1)\,, \quad i\in(j+1,s)\,, \quad k\in(0,W_i-1)\\
P\left\{i^{(j)},k|i-1^{(j-1)},0\right\}=\frac{(1-p_r)(p_d)}{W_i} & j\in(1,d-1)\,, \quad i\in(j,s)\,, \quad k\in(0,W_i-1)\\
P\left\{i^{(j)},k|i^{(j)},k+1\right\}=1 & j\in(0,d-1)\,, \quad i\in(j,s)\,, \quad k\in(0,W_i-2)\\
P\left\{0^{(0)},k|i^{(j)},0\right\}=\frac{1}{W_0} & j\in(0,d-1)\,, \quad i=s, \quad k\in(0,W_0-1)\\
P\left\{0^{(0)},k|i^{(j)},0\right\}=\frac{(1-p_r)(1-p_d)}{W_0} & j\in(0,d-2)\,, \quad i\in(j,s-1)\,, \quad k\in(0,W_0-1)\\
P\left\{0^{(0)},k|i^{(j)},0\right\}=\frac{(1-p_r)}{W_0} & j=d-1, \quad i\in(j,s-1)\,, \quad k\in(0,W_0-1)
\end{cases} \tag{4}
$$

$$
b_{i,k}^{(j)} = \frac{(W_i-k)}{W_i} b_{0,0}^{(0)}
\begin{cases}
\begin{array}{l}
\sum_{w=0}^{d-2}\sum_{r=w}^{s-1} p_r^{r-w}.p_d^{w}.\left(1-p_r\right)^{w+1}.\left(1-p_d\right).a_{r,w}+\\
\sum_{w=0}^{d-1} p_r^{s-w}.p_d^{w}.\left(1-p_r\right)^{w}.a_{s,w}+\\
\sum_{r=d-1}^{s-1} p_r^{r-d+1}.p_d^{d-1}.\left(1-p_r\right)^{d}.a_{r,d-1}
\end{array} & i=0,j=0\\
p_r^{i-j}.p_d^{j}.\left(1-p_r\right)^{j}.a_{i,j} & Otherwise
\end{cases} \tag{5}
$$

Based on the fact that, transmission is only allowed when the backoff timer value is zero, the transmission probability τ (the probability that any station transmits in a randomly chosen time slot) is:

$$
\begin{aligned}
\tau &= \sum_{j=0}^{d-1}\sum_{i=0}^{s} b_{i,0}^{(j)}\\
&= b_{0,0}^{(0)}\left[\sum_{j=0}^{d-1}\sum_{i=0}^{s} p_r^{i-j}.p_d^{j}.\left(1-p_r\right)^{j}.a_{i,j}\right]
\end{aligned} \tag{7}
$$

$b_{0,0}^{(0)}$ can be determined by imposing the normalization condition as follows:

$$
\sum_{j=0}^{d-1}\sum_{i=j}^{s}\sum_{k=0}^{W_i-1} b_{i,k}^{(j)} = 1 \tag{8}
$$

Assume the size of data and control frames are constants, then P_f^{rts}, P_f^{cts}, P_f^{data} and P_f^{ack} are constants. Let $K_1 = \left(1-P_b\right)^{l_{rts}+l_{cts}}$ and $K_2=\left(1-P_b\right)^{l_{data}+l_{ack}}$, then p_r can be expressed in terms of τ as:

$$
\begin{cases}
p_r &= 1-\left(1-\tau\right)^{n-1}K_1\\
p_d &= 1-K_2
\end{cases} \tag{9}
$$

Now, p_r can be inverted to express $\tau^*\left(p_r\right)$ as:

$$
\tau^*\left(p_r\right) \quad = 1-K_1\left(1-p_r\right)^{\frac{1}{n-1}} \tag{10}
$$

equations 7 and 10 represent a nonlinear system with two unknowns p_r and τ, which can be solved using numerical techniques.

In the basic access mode, no control frames are used to reserve the channel and a station's frame transmission encounters collision if at least one of the remaining $n-1$ stations transmit simultaneously. All previous derivations are valid for the basic access mode by assuming $p_r = 0$, $p_d = 1-\left(1-\tau\right)^{n-1}K_2$ and $s=d-1$.

Our model is a general one, where the impact of transmission errors on the data and control frames is considered for both access modes. Other proposed models [3], [5], [9] can be derived from our model as shown in table I.

TABLE I
EVALUATION PARAMETERS

rts/cts access mode $p_r = 1-(1-\tau)^{n-1}K_1$, $p_d = 1-K_2$				
s	d	K_1	K_2	Model
∞	1	1	1	[3]
$ssrc$	1	1	1	[5]
basic access mode $p_r = 0$, $p_d = 1-(1-\tau)^{n-1}K_2$				
∞	∞	x	1	[3]
$slrc-1$	slrc	x	1	[5]
$slrc-1$	$slrc$	x	$(1-P_b)^{l_{data}+l_{ack}}$	[9]

C. Throughput

Let S be the normalized throughput defined as the fraction of time the channel is used to successfully transmit useful payload bits $E(P)$. $E\left(P\right)$ is the average packet payload size. In this paper, we assume a constant packet payload size ($E\left(P\right)=P$). To compute S, we need to analyze the expected events that may occur in a randomly chosen time slot and their corresponding probabilities and durations. In each time slot, one of the following three events may occur: an empty time slot (σ) with a probability P_{idle}, a successful data transmission with duration T_s and probability P_s, or unsuccessful data transmission due to one of the following reasons: collision, an error data frame, an error ACK frame, an error RTS frame, or an error CTS frame. Each reason has its own duration (T_k) and probability (P_k). Now, S can be expressed as:

$$
S \quad = \frac{P_s.E[P]}{P_{Idle}.\sigma + P_s.T_s + \sum_k (P_k.T_k)} \tag{11}
$$

In the basic access mode, the sequence of data transmission is "DATA-ACK". Then, P_s represents the probability that only one station transmits on the channel and that transmission is free from DATA and ACK errors. Unsuccessful transmission occurs due to collisions, an error data frame, or an error ACK frame with probabilities and durations (P_1, T_1), (P_2, T_2), and (P_3, T_3) respectively. Let $H = h_{rtr} + h_{phy} + h_{mac}$ be the packet header ($l_{data} = H \; + \; P$), and δ be the propagation delay. Equations 12 and 13 show the probabilities and their corresponding time durations:

$$
\begin{cases}
P_{idle} &= \left(1-\tau\right)^n\\
P_s &= n\tau\left(1-\tau\right)^{n-1}.\left(1-P_e^{data}\right).\left(1-P_e^{ack}\right)\\
P_1 &= 1-\left(1-\tau\right)^n - n\tau\left(1-\tau\right)^{n-1}\\
P_2 &= n\tau\left(1-\tau\right)^{n-1}.P_e^{data}\\
P_3 &= n\tau\left(1-\tau\right)^{n-1}.\left(1-P_e^{data}\right).P_e^{ack}
\end{cases} \tag{12}
$$

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$$\begin{cases} T_{idle} & = \sigma \\ T_s & = l_{data}/R + \delta + SIFS + l_{ack}/R + \delta + DIFS \\ T_1 & = l_{data}/R + \delta + EIFS \\ T_2 & = l_{data}/R + \delta + EIFS \\ T_3 & = l_{data}/R + \delta + SIFS + l_{ack}/R + \delta + EIFS \end{cases} \tag{13}$$

In the RTS/CTS access mode, the sequence of data transmission is "RTS-CTS-DATA-ACK". Then, P_s is the probability that only one station transmits on the channel and that transmission is free from RTS, CTS, DATA and ACK errors. Unsuccessful transmission could occur due to collision, an error RTS frame, an error CTS frame, an error data frame, or an error ACK frame with probabilities and durations (P_1, T_1), (P_2, T_2), (P_3, T_3), (P_4, T_4) and (P_5, T_5) respectively. Equations (14) and (15) show the probabilities and their corresponding time durations respectively:

$$\begin{cases} P_{idle} & = (1-\tau)^n \\ P_s & = n\tau(1-\tau)^{n-1}.\left(1-P_e^{rts}\right).\left(1-P_e^{cts}\right). \\ & \quad \left(1-P_e^{data}\right).\left(1-P_e^{ack}\right) \\ P_1 & = 1-(1-\tau)^n - n\tau(1-\tau)^{n-1} \\ P_2 & = n\tau(1-\tau)^{n-1}.P_e^{rts} \\ P_3 & = n\tau(1-\tau)^{n-1}.\left(1-P_e^{rts}\right).P_e^{cts} \\ P_4 & = n\tau(1-\tau)^{n-1}..\left(1-P_e^{rts}\right).\left(1-P_e^{cts}\right).P_e^{data} \\ P_5 & = n\tau(1-\tau)^{n-1}..\left(1-P_e^{rts}\right).\left(1-P_e^{cts}\right). \\ & \quad \left(1-P_e^{data}\right).P_e^{ack} \end{cases} \tag{14}$$

$$\begin{cases} T_{idle} & = \sigma \\ T_s & = l_{rts}/R + \delta + SIFS + l_{cts}/R + \delta + SIFS \\ & \quad + l_{data}/R + \delta + SIFS + l_{ack}/R + \delta + DIFS \\ T_1 & = l_(rts)/R + \delta + EIFS \\ T_2 & = l_(rts)/R + \delta + EIFS \\ T_3 & = l_(rts)/R + \delta + SIFS + l_(cts)/R + EIFS \\ T_4 & = l_(rts)/R + \delta + SIFS + l_(cts)/R + \delta \\ & \quad + SIFS + l_(data)/R\delta + EIFS \\ T_5 & = l_(rts)/R + \delta + SIFS + l_(cts)/R + \delta \\ & \quad + SIFS + l_(data)/R + \delta + SIFS \\ & \quad + l_(ack)/R + \delta + EIFS \end{cases} \tag{15}$$

D. Packet Drop Probability

Each data frame has several retransmission attempts, *ssrc* and *lsrc* indicate the maximum number of retransmission attempts of RTS and data frames respectively. Data packet is discarded from the MAC queue as either of these two limits is reached first. Therefore, packet is dropped when an RTS or a data packet encounters unsuccessful transmission at any backoff stages *s* or the data packet encounters unsuccessful transmission at any of L_{d-1} backoff stages. The packet drop probability can be expressed as:

$$\begin{aligned} P_{drop} & = \left(p_r + p_d(1-p_r)\right)\sum_{j=0}^{d-1} p_d^j p_r^{s-j}(1-p_r)^j.a_{s,j} \\ & + \sum_{i=d-1}^{s-1} p_d^d p_r^{i-d+1}(1-p_r)^d.a_{i,d-1} \end{aligned} \tag{16}$$

E. Average packet delay

Average packet delay $E\left[T_{delay}\right]$ for successfully transmitted packets is the average period of time between a packet at the head of MAC queue is ready for transmission and the packet's ACK is received. Dropped packets are not considered in the average packet delay calculations. $E\left[T_{delay}\right]$ is:

$$E\left[T_{delay}\right] = E\left[Y\right].E\left[slot\right] \tag{17}$$

$E\left[Y\right]$ is the average number of time slots required for a packet to be transmitted successfully. $E\left[Y\right]$ can be found by multiplying the average number of time slots a packet is delayed until it reaches stage *i* by the probability the packet reaches the stage *i* is tramsitted sussessfuly. The average value of the backoff counter at stage *i* is $(W_i+1)/2$,

$$E\left[Y\right] = (1-p_d)\sum_{j=0}^{d-1} p_d^j (1-p_r)^{j+1}\sum_{i=j}^{s} p_r^{i-j}.a_{i,j}.t_i \tag{18}$$

$$t_i = \sum_{k=0}^{i}\frac{W_k+1}{2} \tag{19}$$

$E\left[slot\right]$ is the average time duration for a station to transfer from a state to another. It can be derived as follows. If there is no medium activity on the channel, the time slot would be the system time slot σ. If the medium is busy, the slot time would be either the time to complete a successful transmission or the time to handle a failed transmission. The average time slot is:

$$E\left[slot\right] = P_{Idle}.\sigma + P_s.T_s + \sum_k (P_k.T_k) \tag{20}$$

F. Average time to drop a packet

As mentioned above, packet may be dropped at any of the *s* backoff stages or at any of L_{d-1} backoff stages. Then, the average time $E\left(T_{drop}\right)$ to drop a packet is:

$$E\left[T_{drop}\right] = E\left[X\right].E\left[slot\right] \tag{21}$$

$E\left[X\right]$ can be found by multiplying the average number of time slots the packet is delayed until it dropped from a stage *i* by the probability that the packet reached the stage *i*.

$$\begin{aligned} E\left[X\right] & = \sum_{j=0}^{d-1} p^{s-j}(1-p)^j.a_{s,j}.t_s \\ & + (1-p)^d\sum_{i=d-1}^{s-1} p^{i-d+1}.a_{i,d-1}.t_i \end{aligned} \tag{22}$$

if $p_r = 0$ or $p_d = 0$, then $p = 1$. Otherwise, $p = p_r$.

IV. Simulation and Verification

We use network simulator (*ns*) [1] to implement our simulation. Free Space propagation model is used to predict the received signal power. Simulation area is chosen such that the received signal strength is always detectable. *ns* does not consider BER but uses thresholds to determine the correctness of a received frame. By assuming constant BER, ns *wireless-phy* module is modified in order to calculate a FER. Then, the FER is used to predict whether the frame is received correctly or erroneously. All placed stations are static and run the same access mode. Unless otherwise specified, table II shows the system parameters, whereas the reported values are for the Direct Spread Sequence Spectrum (DSSS) physical layer used in 802.11b standard.

For the basic mode, fig. 2 shows the normalized throughput versus the number of stations at different *slrc* values. The throughput decreases as number of stations increases because

TABLE II
DSSS SYSTEM PARAMETERS

Parameter	Value
Packet payload, P	8184 bits
P_b	10^{-5}
W_{min}	32 time slots
m	5
Routing header, h_{rtr}	160 bits
MAC header, h_{mac}	272 bits
Physical header, h_{phy}	192 bits
ACK	112 + h_{phy} bits
RTS	160 + h_{phy} bits
CTS	112 + h_{phy} bits
Channel bit rate, R	1Mbps
Propagation time, δ	$1\mu sec.$
Slot σ	$50\mu sec.$
SIFS	$28\mu sec.$
DIFS	2 * Slot + SIFS
EIFS	SIFS + DIFS + ACK/R
Transmission range, R_x	107m
Simulation region	67x67m^2
Simulation time	90sec

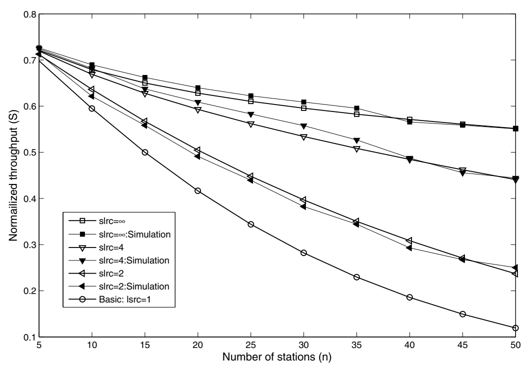

Fig. 2. The normalized saturation throughput for basic access mode

the probability of collision increases. More, at low *slrc* values, stations have low CW sizes, which increases the collision probability. In fact, transmission errors as well as collisions increase the packet drop rate. At large *slrc* values, stations have higher CW sizes. Hence, the probability of collision is lower and the system throughput is higher. Fig 2 clearly shows the throughput enhancement as *slrc* value increases. For ***RTS/CTS*** mode, fig. 3 shows that the throughput does not strongly depend on the number of stations. Additionally, the performance of the network at $ssrc = 5$, 7, and ∞ are comparable. Although the probability of collision increases with increasing number of stations but less bandwidth is wasted as compared to the situation when larger data frames are collided as in the basic access case. Moreover, at high *ssrc* values, the impact of *slrc* is eliminated. The throughput at *(ssrc, slrc)* equal *(7,4)* and *(7,1)* are similar. Figs. 2 and 3 show that the analytical results coincide with the simulation results. in both basic and RTS/CTS cases.

Figs. 4 and 5 show the impact of number of station as well as retry limits on the packet drop probability. Equation (16)

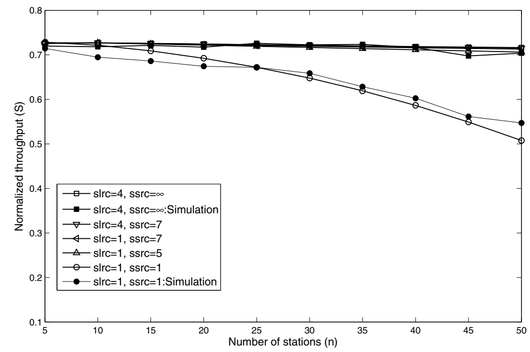

Fig. 3. The normalized saturation throughput for RTS/CTS access mode

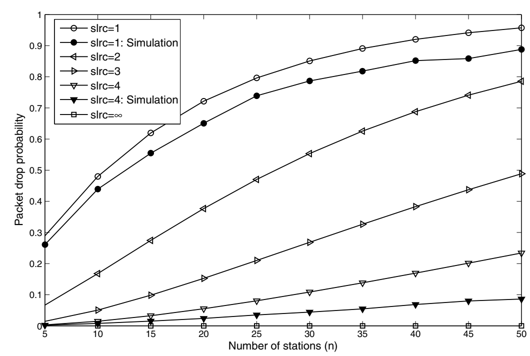

Fig. 4. The packet drop probability for the basic access mode

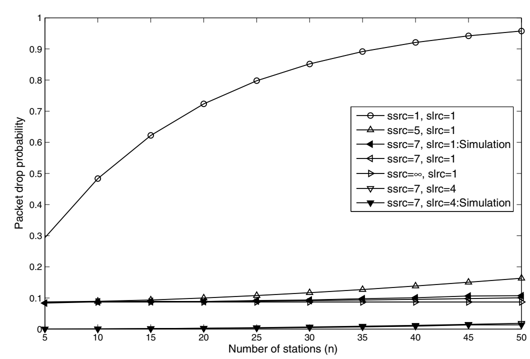

Fig. 5. The packet drop probability for the RTS/CTS access mode

shows that P_{drop} is a function of $ssrc = s$, $slrc = d$, and P_c. As mentioned above, the collision rate is directly proportional to the number of stations, and inversely proportional to the *ssrc* and *slrc* values. As a result, decreasing *ssrc* and *slrc* as well as increasing number of stations will increase P_{drop}. At large number of stations $(n > 40)$ and small value of *ssrc* or *slrc*, P_{drop} approaches *1* as shown in figs. 4 and 5 . Moreover, at $(ssrc, slrc) = (\infty, 1)$, fig. 5 shows that $P_{drop} \approx 0.0845$ due to transmission errors (at $P_b = 10^{-5}$, $FER = 0.0845$).

Results of Figs. 6 and 7 indicate that $E\left[T_{delay}\right]$ increases with increasing number of stations due to the increase rate

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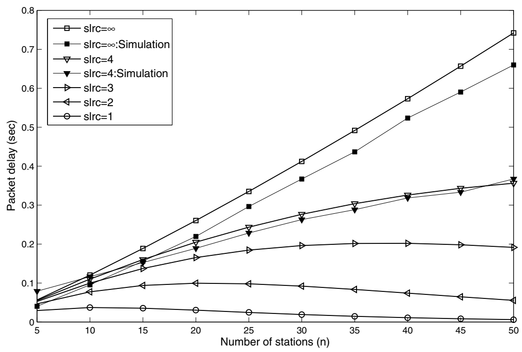

Fig. 6. The average packet delay for basic access mode

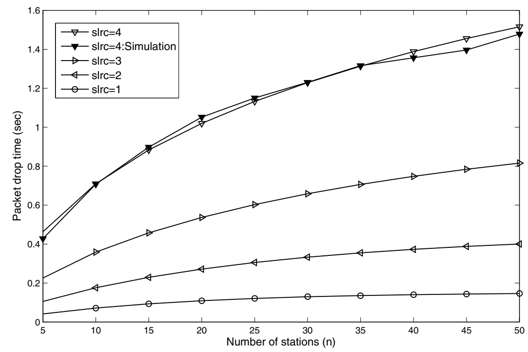

Fig. 8. The average packet drop time for basic access mode

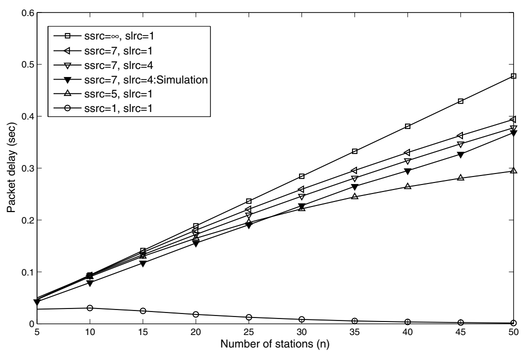

Fig. 7. The average packet delay for RTS/CTS access mode

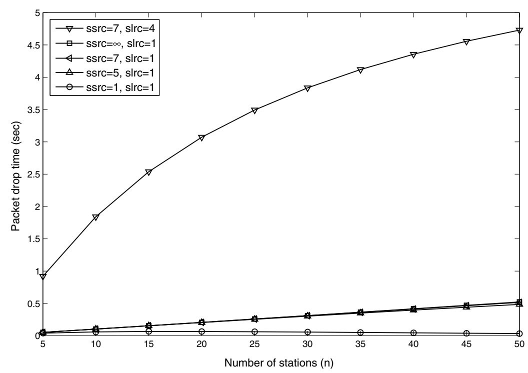

Fig. 9. The average packet drop time for RTS/CTS access mode

of collisions and hence number of retransmitting increases for each sharing station. Thus, high retry limits implies more retransmission attempts, which increases $E\left[T_{delay}\right]$ significantly. For a limited retry limit, the RTS/CTS mode provides lower delay compared to the basic mode due to the short collision times ($E^{RTS/CTS}\left[Slot\right] < E^{basic}\left[Slot\right]$). For the same reasons, the average time to drop a packet is increased as the retry limits increased as shown in figs. 8 and 9. Fig. 9 shows that at $(ssrc, slrc) = (\infty, 1)$, $E\left[T_{drop}\right]$ is lower compared to the case $(ssrc, slrc) = (7, 1)$. This unexpected behavior is attributed to the fact that the packet drop probability at $lsrc = 1$ is larger and therefore $E\left[T_{drop}\right]$ is lower.

V. Conclusion

In this paper, we propose a new 3-dimensional Markov model as a theoretical framework to evaluate the impacts of control and data retry limits as well as the quality of the received data on the performance of DCF mechanism. Transmission errors are modeled by assuming a Gaussian wireless error channel with a constant BER, whereas all transmitted frames have the same FER. Throughput, packet delay, packet drop probability and packet drop time are analyzed and verified by simulations, whereas analytical results match simulation outcomes quite well. As future work, the equality of stations radio transmission will be modeled by considering the impact of channel fading on the BER. Moreover, the proposed model will be extended in order to analyze the performance of multihop ad hoc networks.

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