ØV5 — Transformanalyse

Oppgave 1 — Oppgave 4.21 fra Ambardar: Poler og nullpunkter

a. Vi får

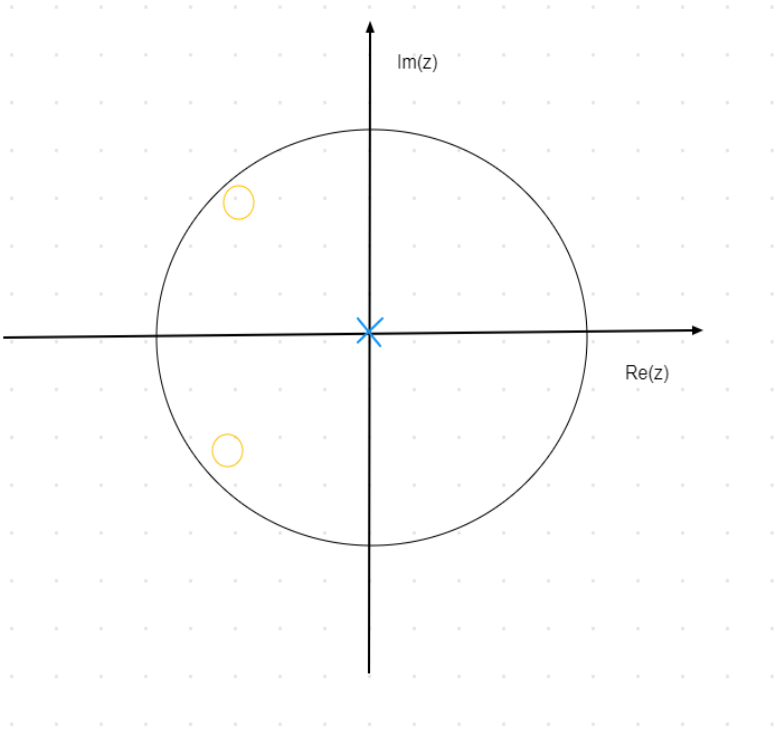
$$X(z) = \sum_{k=\infty}^{\infty} x[k]z^{-k} = \sum_{k=\infty}^{z} x[k]z^{-k} = z + z^{-1} + z^{-2} = \frac{z^2}{z^2} + \frac{z}{z^2} + \frac{1}{z^2} = \frac{z^2 + z + 1}{z^2}$$

Poler:

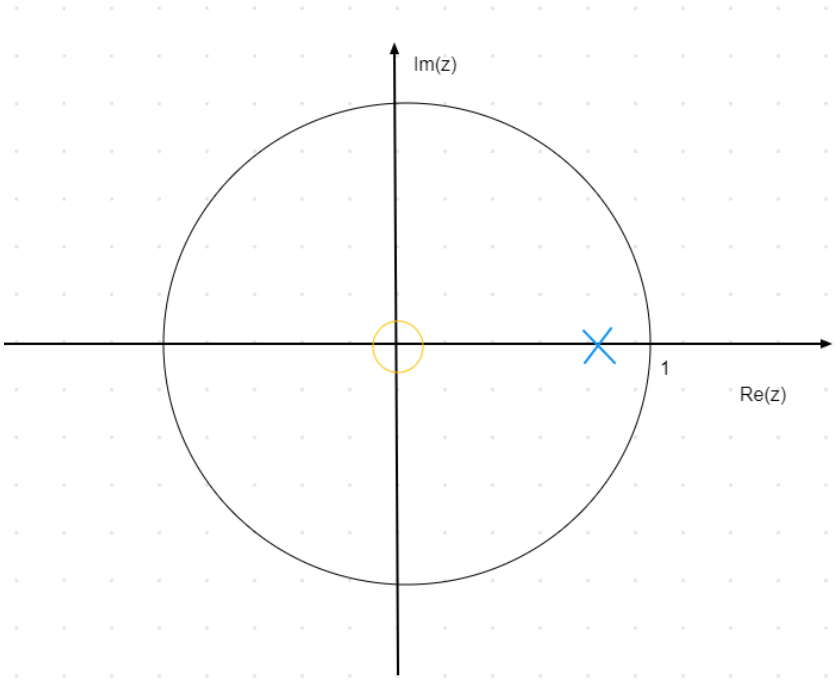
$$z^2 = 0 \rightarrow z = 0$$

Zero:

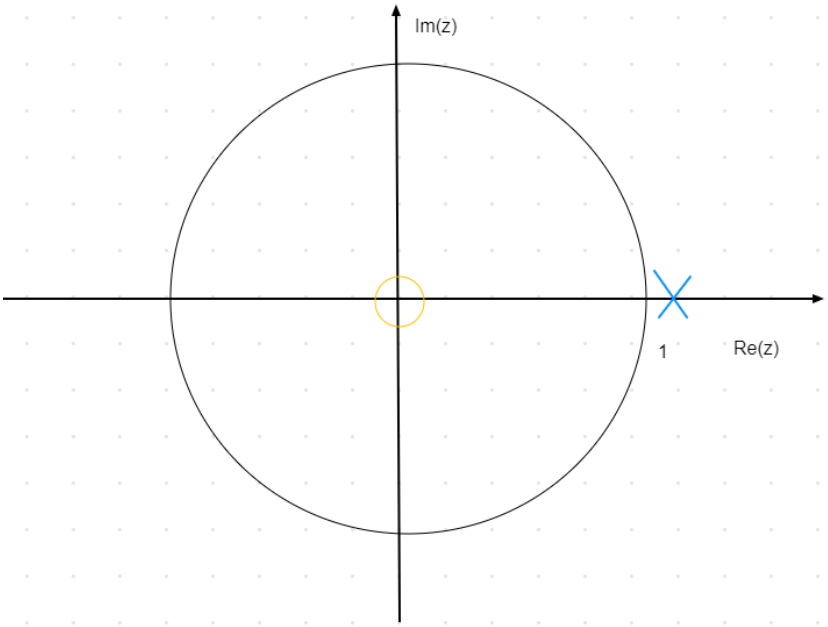
$$z^2 + z + 1 = 0$$
$$z = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$



b. Vi får



c. Vi får



d. Vi får

$$X(z) = \sum_{k=\infty}^{\infty} x[k]z^{-k} = \sum_{k=\infty}^{z} x[k]z^{-k}$$

$$= 0z + 1z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} + 5z^{-5} + 6z^{-6} + 7z^{-7} + 8z^{-8} =$$

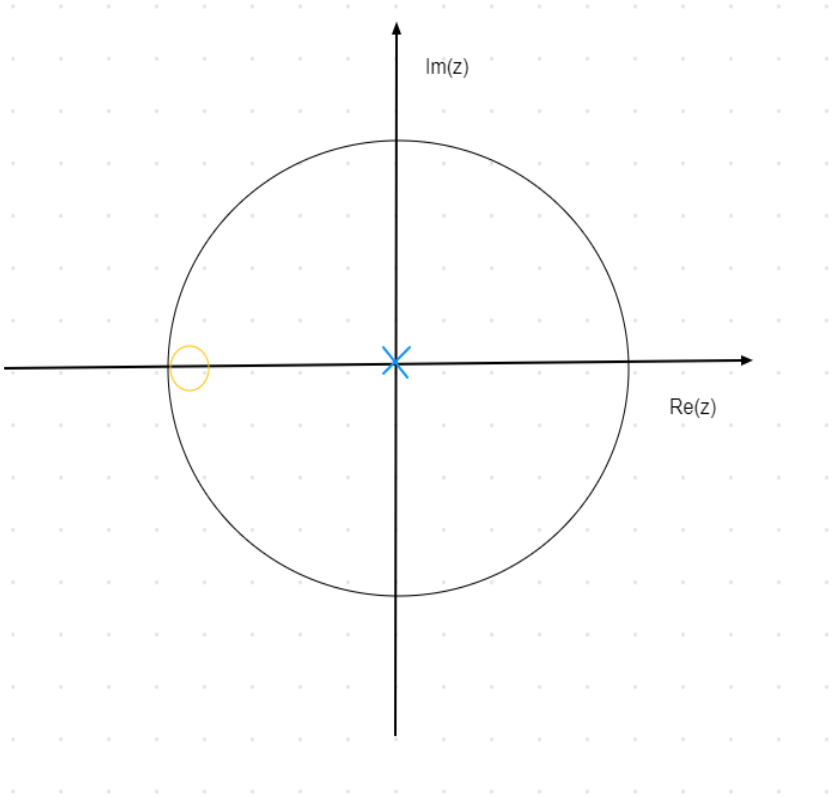
$$= \frac{z^7 + 2z^6 + 3z^5 + 4z^4 + 5z^3 + 6z^2 + 7z + 8}{z^8}$$

Poler:

$$z^8 = 0 \rightarrow z = 0$$

Zero:

$$z^7 + 2z^6 + 3z^5 + 4z^4 + 5z^3 + 6z^2 + 7z + 8 = 0$$
$$z = -1.4$$



e.

Oppgave 2 — Diverse oppgaver

a. Vi får

$$f = \frac{1}{T} = \frac{6}{20 \cdot 60} = \frac{1}{200} Hz$$

Oppgave 3

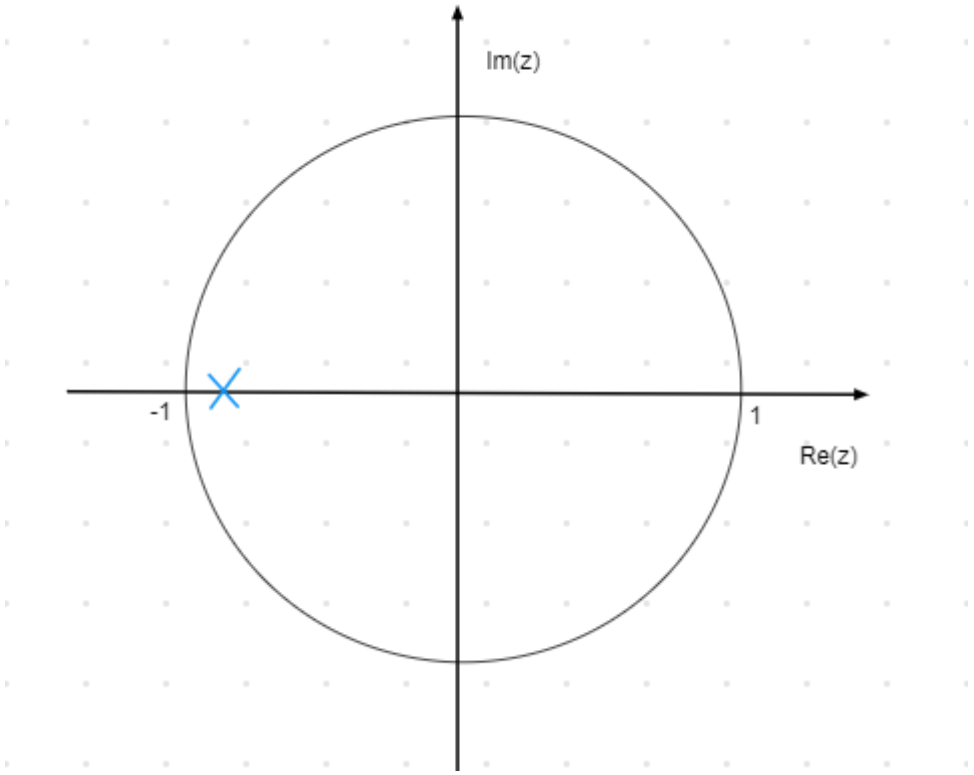
a. Vi finner $H(z)$ fra differanselikning og får

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + 0.9z^{-5}}$$

Poler:

$$1 + 0.9z^{-5} = 0$$

$$z = -\sqrt[5]{\frac{9}{10}}$$



b. Utgangssignalet er gitt som

$$y[n] = h[n] * x[n] = \sum_{k=\infty}^{\infty} h[k]x\{n-k] = \sum_{k=\infty}^{\infty} h[k]h\{n-k]$$

Dersom $n \geq 0$

$$y[n] = \sum_{k=0} h[n-k] =$$

c. Vi får

$$|H(z = e^{j\omega})| = \left|\frac{1}{1 + 0.9e^{-5j\omega}}\right|$$

$$|H(\omega = 0)| = \left|\frac{1}{1 + 0.9}\right| = 0.53$$

$$|H(z = -1)| = \left|\frac{1}{1 + 0.9(-1)^{-5}}\right| = 10$$