ØV2 — Frekvensdomene-representasjon av signaler

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Oppgave 1

a) Vi bruker linearitet og får

$$F(u[n] - u[n-5]) = F(u[n]) - F(u[n-5])$$

Vi bruker homogenitet og får

$$F(u[n]) - F(u[n-5]) = \frac{jd}{d\omega}\, F(u[n]) - \frac{jd}{d\omega} F(u[n-5])$$

Videre får vi

$$\frac{jd}{d\omega}\, F(u[n]) - \frac{jd}{d\omega}(e^{-j\omega 5} F(u[n]))$$

Tabellen gir $F(u[n]) \rightarrow$

$$\frac{1}{1 - e^{-j\omega}}$$

Resultatet blir:

$$\begin{aligned} F((u[n] - u[n-5]) &= \frac{1}{1 - e^{-j\omega}} - e^{-j\omega 5} \frac{1}{1 - e^{-j\omega}} \\ &= e^{-j\omega 4} + e^{-j\omega 3} + e^{-j\omega 2} + e^{-j\omega 1} + 1 \end{aligned}$$

b) Vi bruker linearitet, og får:

$$F(\alpha^n\big((u[n] - u[n-8])\big) = F(\alpha^n u[n]) - F(\alpha^n u[n-8])$$

Videre får vi:

$$F(\alpha^n u[n]) - F(\alpha^n u[n-8]) = jF\left(\frac{1}{j}\alpha^n u[n]\right) - F(\frac{1}{j}\alpha^n u[n-8])$$

$$jF\left(\frac{1}{j}\alpha^n u[n]\right) - F\left(\frac{1}{j}\alpha^n u[n-8]\right) = \frac{jd}{d\omega} F(\alpha^n u[n]) - \frac{jd}{d\omega} F(\alpha^n u[n-8])$$

$$\frac{jd}{d\omega} F(\alpha^n u[n]) - \frac{jd}{d\omega}\left(e^{-j\omega 8}\alpha^n F(u[n])\right)$$

Tabellen gir at $\alpha^n \rightarrow$

$$\frac{1}{1 - \alpha e^{-j\omega}}$$

Og $F(u[n]) \rightarrow$

$$\frac{1}{1-e^{-j\omega}}$$

Derfor:

$$F(\alpha^n\big(n(u[n]-u[n-8])\big) = \frac{1}{1-ae^{-j\omega}}\big(e^{-j\omega 7}+e^{-j\omega 6}+e^{-j\omega 5}+e^{-j\omega 4}+e^{-j\omega 3}+e^{-j\omega 2}+e^{-j\omega 1}+1\big) = \frac{\big(e^{-j\omega 7}+e^{-j\omega 6}+e^{-j\omega 5}+e^{-j\omega 4}+e^{-j\omega 3}+e^{-j\omega 2}+e^{-j\omega 1}+1\big)}{1-ae^{-j\omega}}$$

c) Vi bruker linearitet og får:

$$F\left(n\left(\frac{1}{2}\right)^{|n|}\right) = F\left(n\left(\frac{1}{2}\right)^{n}u(n)\right) + F\left(n\left(\frac{1}{2}\right)^{-n}u(-n-1)\right)$$

Videre får vi at:

$$j\,F\left(\frac{n}{j}\left(\frac{1}{2}\right)^{n}u(n)\right) + 0.5F\left(\frac{n}{j}\left(\frac{1}{2}\right)^{-n-1}u(-n-1)\right)$$

$$j\,F\left(\frac{n}{j}\left(\frac{1}{2}\right)^{n}u(n)\right) + 0.5F_{-\omega}\left(\frac{n}{j}\left(\frac{1}{2}\right)^{n-1}u(n-1)\right)$$

$$\frac{jd}{d\omega}F\left(\left(\frac{1}{2}\right)^{n}u(n)\right) + \frac{j}{d\omega}0.5F_{-\omega}e^{j\omega}\left(\left(\frac{1}{2}\right)^{n}u(n)\right)$$

Tabellen gir at $\left(\frac{1}{2}\right)^{n} \rightarrow$

$$\frac{1}{1-\frac{1}{2}e^{-j\omega}} = \frac{2}{2-e^{-j\omega}}$$

Derfor:

$$F\left(n\left(\frac{1}{2}\right)^{|n|}\right) = \frac{2}{2-e^{-j\omega}} + \frac{1}{2}e^{j\omega}\,\frac{2}{2-e^{-j\omega}} = \frac{2+e^{j\omega}}{2-e^{-j\omega}}$$

d) Eulers identitet gir

$$F\big(a^{|n|}u[n]sin\omega_0 n\big) = F\left(a^{|n|}u[n]\frac{1}{2j}(e^{j\omega_0 n}-e^{-j\omega_0 n})\right)$$

Vi bruker linearitet og får

$$F\left(a^{|n|}u[n]\frac{1}{2j}(e^{j\omega_0 n}-e^{-j\omega_0 n})\right) = \left(\frac{1}{2j}F\big(a^n u[n]e^{j\omega_0 n}\big) + \frac{1}{2j}F\big(a^n u[-n-1]e^{j\omega_0 n}\big)\right) - \left(\frac{1}{2j}F\big(a^n u[n]e^{-j\omega_0 n}\big) + \frac{1}{2j}F\big(a^n u[-n-1]e^{j\omega_0 n}\big)\right)$$

Dermed får vi

$$\left(\frac{1}{2j}F\left(a^n u[n]e^{j\omega_0 n}\right)+\frac{1}{2j}F\left(a^n u[-n-1]e^{j\omega_0 n}\right)\right)$$
$$-\left(\frac{1}{2j}F\left(a^n u[n]e^{-j\omega_0 n}\right)+\frac{1}{2j}F\left(a^n u[-n-1]e^{j\omega_0 n}\right)\right)$$
$$=\left(\frac{1}{2j}F_{\omega-\omega_0}(a^n u[n])+\frac{1}{2j}F_{\omega_0}(a^n u[-n-1])\right)$$
$$-\left(\frac{1}{2j}F_{\omega-\omega_0}(a^n u[n])+\frac{1}{2j}F_{\omega_0}(a^n u[-n-1])\right)$$
$$=\left(\frac{1}{2j}\frac{j}{d\omega}F_{\omega-\omega_0}(a^n u[n])+\frac{1}{2j}\frac{j}{d\omega}F_{\omega_0}e^{j\omega}(a^n u[n])\right)$$
$$-\left(\frac{1}{2j}\frac{j}{d\omega}F_{\omega-\omega_0}(a^n u[n])+\frac{1}{2j}\frac{j}{d\omega}F_{\omega_0}e^{j\omega}(a^n u[n])\right)$$

Tabellen gir at $\alpha^n \rightarrow$

$$\frac{1}{1-\alpha e^{-j\omega}}$$

Derfor

$$F\left(a^{|n|}u[n]sin\omega_0 n\right)=\left(\frac{\frac{1}{2j}}{1-\alpha e^{-j\omega}}+\frac{\frac{1}{2j}e^{j\omega}}{1-\alpha e^{-j\omega}}\right)-\left(\frac{\frac{1}{2j}}{1-\alpha e^{-j\omega}}+\frac{\frac{1}{2j}e^{j\omega}}{1-\alpha e^{-j\omega}}\right)$$
$$=\left(\frac{j}{2-2\alpha e^{-j\omega}}+\frac{je^{j\omega}}{2-2\alpha e^{-j\omega}}\right)-\left(\frac{j}{2-2\alpha e^{-j\omega}}+\frac{je^{j\omega}}{2-2\alpha e^{-j\omega}}\right)$$
$$=\left(\frac{j+je^{j\omega}}{2-2\alpha e^{-j\omega}}\right)-\left(\frac{j+je^{j\omega}}{2-2\alpha e^{-j\omega}}\right)=\ 0?$$