1 Extreme value statistics

– Random distribution $p(x)$ in $[0, \infty[$

– Largest variable $x_{\max}$ out of N random variables

$$\int\limits_{x_{\max}}^{\infty} p(x)dx \sim \frac{1}{N} \quad \rightsquigarrow \quad \text{estimate for } x_{\max}$$

More precisely, we get:

$$M_N(x) = N\left[1 - P(x)\right]^{N-1} p(x); \quad P(x) = \int\limits_{x}^{\infty} p(y)dy$$

a) $p(x) = e^{-x}$

$$\int\limits_{x_{\max}}^{\infty} e^{-x}dx = e^{-x_{\mathrm{xmax}}} \approx \frac{1}{N}$$

$$\rightsquigarrow x_{\max} \approx \ln(N)$$

b) $p(x) = \mu x^{-(1+\mu)}\ x > 1; \mu > 0$

$$\int\limits_{x_{\max}}^{\infty} \mu x^{-(1+\mu)}dx = x_{\max}^{-\mu} \sim \frac{1}{N}$$

$$\Rightarrow x_{\max} \sim N^{\frac{1}{\mu}}$$

c) $p(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{else} \end{cases}$

$$\int\limits_{x_{\max}}^{1} dx = (1 - x_{\max}) \sim \frac{1}{N}$$

$$\rightsquigarrow x_{\max} \sim 1 - \frac{1}{N}$$

Perfectly absorbing boundary conditions

Diffusion equation

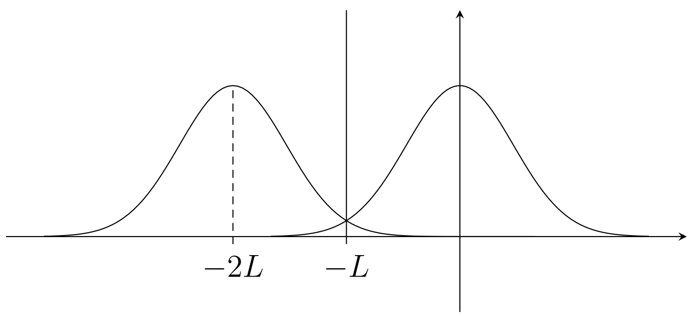
$$\partial_t p(x,t) = D\partial_x^2 p(x,t)$$

We consider the initial condition

$$p(x,0) = M\delta(x)$$

and boundary conditions:

$$p(-L,t) = 0$$



blabla

For the case of two boundary conditions, we have to add an imaginary source at $x = 2L$. After a while, however the imaginary random walk may cross the boundary at $\pm L$. Therefore we need multiple imaginary walks.

In particular, we need negative image at $x = \pm 2L$, positive at $x \pm 4L$ and so forth.

Therefore: $p(x,t) = \frac{M}{\sqrt{4\pi Dt}} \sum_{n=-\infty}^{\infty} \left[-\exp\left(-\frac{(x+(4n-2)L)^2}{4Dt} \right) + \exp\left(-\frac{(x+4nL)^2}{4Dt} \right) \right]$ where $-L < x < L$. Nummer 3: $p(l) = \rho e^{-\rho l}$ $\rho L = l$ has to be taken into account