The Art of Modeling

An Introduction to Linear Mixed Effects Models with R

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9.5.2025

Agenda

- Motivation: Who is Elmar?
- A bit of linear algebra basics
- A bit of theory
- The Art of Modeling
    - Model building
    - Model checking
    - Model interpretation
    - Trouble shooting
- Outlook: Generalized LMMs
- Hands on: Analyze your own data
- Wrap-up and debrief

Who is Elmar?

Who is Elmar?

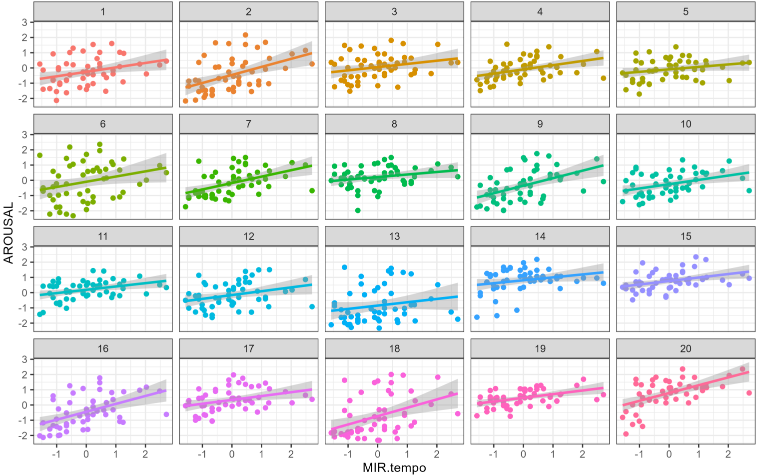
- In 2015, Elke Lange approached me with an idea for a cool study.
- Can perception of emotion expression in music be predicted by audio and other features?
- Setup: 20 audio engineers rated 60 musical excerpts ($\approx 60\,\mathrm{s}$) on 22 variables (emotions, audio quality, modal analogues).
- Extracted 86 audio features with the MIRToolbox.

Who is Elmar?

- We reduced the six emotion variables to two variables `AROUSAL` and `VALENCE` using factor analysis.
- We found that `tempo` correlates moderately with `AROUSAL`, $r = .31$.
- But is this true for all participants?
- Let's see!

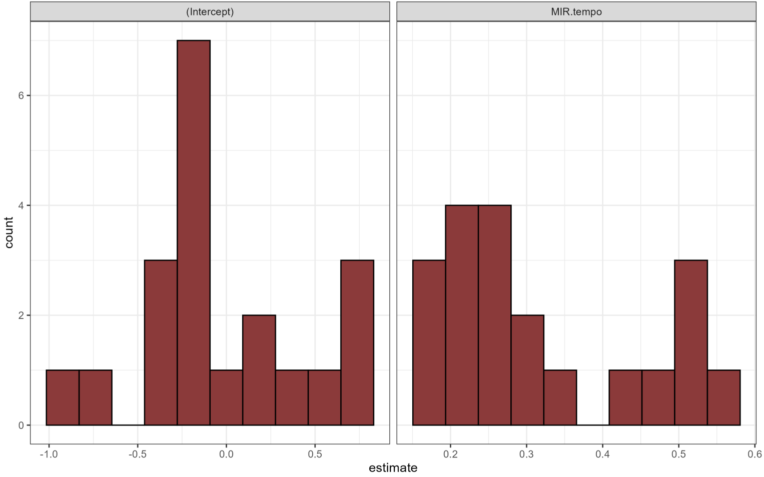
Who is Elmar?

The correlations by participants range from $r = .19$ to $r = .54$ (mean $\bar{r} = .37$).



Who is Elmar?

My first random effects!



Why Linear Mixed Effect Models?

- As the example shows, on many occasions, a constant effect of something on an individual seems not reasonable.
- Fixed effects are average effects.
- Aren't individual differences often more interesting?
- Specifics of experiments, such as stimuli, are often simply a nuisance, from which one wants to abstract to achieve a better generalization.
- LMMs allow to model all this.

Why Linear Mixed Effect Models?

- The first reason to use LMMs was to deal with repeated measurement.
- When using a common (fully) crossed design, repeated measurement ANOVA is not the best option.
- Averaging loses power and detail.
- LMMs allow modeling arbitrary complex models, when the independence assumption is violated due to repetitions or hierarchical nesting.
- Further benefits: Deals smoothly with NA, provide more detailed information.
- LMM is a very general model, unifying, t-tests, ANOVAS, rmANOVAS, ANCOVAS, and multiple linear regressions (and even χ^2 test and non-parametric tests based on ranks).

Why Linear Mixed Models?

Hands On

- Team up with a partner, if you like.
- Git for project: `https://github.com/klausfrieler/lmm_workshop`.
- Owncloud: `https://owncloud.gwdg.de/index.php/s/c2MoDjsRwZGvFLL`
- Start `R`, load the accompanying RStudio project, install missing packages.
- Source the file `setup.R`.
- Run `setup_workspace()`.
- As a warm-up, create a scatter plot like that before with `MIR.pulse_clarity` and `VALENCE`. The data frame is `mer` in the workspace. Bonus question: how many of the individual correlations are significant compared to expectation?

A bit of basic: Linear Algebra

A Bit of Basics: Linear Algebra

A (column) vector of dimension N is a set of N numbers:

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}$$

Also written as boldface $\mathbf{x}$, or with components x_i.
A row vector is the transpose $\mathbf{x}^T = (x_1, x_2, \ldots, x_N)$.
Vectors of same dimension can be added component-wise and mutliplied by numbers:

$$\mathbf{z} = \alpha \mathbf{x} + \beta \mathbf{y},$$

with components

$$z_i = \alpha x_i + \beta y_i$$

A Bit of Basics: Linear Algebra

The scalar product is defined as

$$\vec{x} \cdot \vec{y} = \mathbf{x}^T \mathbf{y} = \sum_{i=1}^{N} x_i y_i$$

The length of a vector is

$$\|x\| = \sqrt{\vec{x}^2} = \sqrt{\sum_i x_i^2}$$

and the (Euclidian) distance between two vectors

$$d(x, y) = \|\mathbf{x} - \mathbf{y}\| = \sqrt{\sum_i (x_i - y_i)^2}$$

A Bit of Basics: Linear Algebra

The (sample) standard deviation of a data vector is proportional to the distance of the vector to the vector of mean values.

$$\hat{\sigma}(\mathbf{x}) = \frac{1}{\sqrt{N-1}} d(\mathbf{x}, \bar{\mathbf{x}}) = \frac{1}{\sqrt{N-1}} \|\mathbf{x} - \bar{\mathbf{x}}\|$$

For two data vectors of dimension N, $\mathbf{x}$, $\mathbf{y}$, the Pearson correlation r_{xy} is defined by the scalar product of the z-transformed vectors as:

$$r_{xy} = \frac{1}{N-1} \mathbf{z}(x) \cdot \mathbf{z}(y) = \frac{\mathbf{x} - \bar{x}}{\|\mathbf{x} - \bar{\mathbf{x}}\|} \cdot \frac{\mathbf{y} - \bar{y}}{\|\mathbf{y} - \bar{\mathbf{y}}\|}$$

A Bit of Basics: Linear Algebra

A **matrix X** of dimension $N \times M$ is a rectangular scheme of numbers, with N rows and M columns with elements x_{ij}.

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1M} \\ \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{NM} \end{pmatrix}$$

A matrix can also be considered a N-dimensional row-vector of M-dimensional column vectors or vice versa.
Matrices of same dimensions can be added and multiplied with a number just like vectors.
The transposed $\mathbf{X}^T$ of a matrix $\mathbf{X}$ switches row and columns, i. e., $x_{ji}^T = x_{ij}$.
A N-dimensional vector $\mathbf{x}$ can be regarded as a $N \times 1$ matrix. $\mathbf{x}^T$ is then a $1 \times N$ vector.

A Bit of Basics: Linear Algebra

Matrices $\mathbf{A}$ and $\mathbf{B}$ can multiplied $\mathbf{C} = \mathbf{AB}$, if the number of columns of $\mathbf{A}$ is the same as the number of rows in $\mathbf{B}$. The elements of the result are defined as

$$c_{ij} = \sum_{k=1}^{M} a_{ik} b_{kj}$$

The elements of $\mathbf{C}$ are the scalar products of the ith row vector of $\mathbf{A}$ with the jth column vector of $\mathbf{B}$

$$c_{ij} = \mathbf{a}_i^T \mathbf{b}_j = \vec{a}_i \cdot \vec{b}_j$$

We have the following computation rules:

$$\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}, \qquad \mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$$

Attention: matrix multiplication is, in general, not commutative : $\mathbf{AB} \neq \mathbf{BA}$.

A Bit of Basics: Linear Algebra

The quadratic ($N \times N$) matrices are special as they can have an inverse matrix, i.e., if a matrix $\mathbf{A}$ there exists a matrix $\mathbf{A}^{-1}$ such that

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}_N,$$

with the identity matrix I_N which has 1's on the diagonal and 0's everywhere else.
The inverse exists, if A has full rank, i. e., no column or row vector is a linear combination of the other vectors or none of them is zero. This is equivalent to $\det \mathbf{A} \neq 0$.
Example of a non-invertible matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

A Bit of Basics: Linear Algebra

Exercise

Let A be the following matrix:

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 0 & 0 \end{pmatrix}$$

and $\mathbf{y}^T = (1, -1, -1)$.
Solve

$$(\mathbf{A}^T\mathbf{A})\mathbf{x} = \mathbf{y}$$

for $\mathbf{x}$.
Use matrix operations in R, to create a matrix: `matrix()`, `t()` for transposition, %*% for matrix multiplication, and `solve()` for matrix inversion.

A bit of theory: Linear Regression

A Bit of Theory: Linear Regression

A linear regression model for one variable and N observations can be written as

$$y_i = \gamma + \beta x_i + \epsilon_i$$

with

- y_i the outcome for participant i, $1 \leq i \leq N$.
- γ the overall intercept
- β the fixed effect coefficient,
- x_i the value of the predictor variable x.
- $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$ the normal distributed residual error with mean 0 and variance σ^2.

A Bit of Theory: Linear Regression

This can be extended to p predictors, each having their own beta coefficient.

$$y_i = \sum_k^p \beta_k x_{ik} + \epsilon$$

and be compactly written with vectors and matrices:

$$\mathbf{y} = \mathbf{X}\beta + \epsilon$$

where $\mathbf{X}$ is the model-matrix, containing the measurements for each participant in rows, and variables are contained in columns.
The global intercept is represented here by adding a 0th-column of ones to $\mathbf{X}$ and a 0th-component to β, i. e., $\gamma = \beta_0$.
Categorial predictors are encoded with numerical dummy variables, e. g., one-hot encoding ('contrast'). K categories add $K - 1$ dummy variables to $\mathbf{X}$. This covers ANOVA and ANCOVA.

A Bit of Theory: Linear Regression

The β-coefficients can be estimated using various methods, e. g., ordinary least squares or maximum likelihood estimation.
In ordinary least squares, one seeks to find estimate $\hat{\mathbf{y}}$ that minimize the error to the observed y (distance in y-direction),

$$\sum_{i=1}^{N} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{N} \left(\sum_{k=1}^{p} x_{ik}\beta_k - y_i \right)^2 = f(\beta_0, \beta_2, \ldots, \beta_p) = \min$$

This is a function of the unknown variables β_i, and a mininum can be found by setting the partial derivatives with respect to all variables to 0.

A Bit of Theory: Linear Regression

Calculating the partial derivative with respect to β_k yields

$$
\begin{aligned}
\frac{\partial}{\partial \beta_k} \sum_{i=1}^{N} \left(\sum_{k=1}^{p} x_{ik}\beta_k - y_i \right)^2 &= 0 \\
\sum_{i=1}^{N} 2 \left(\sum_{k=1}^{p} x_{ik}\beta_k - y_i \right) x_{ij} &= 0 \\
\sum_{i=1}^{N} \sum_{k=1}^{p} x_{ij}x_{ik}\beta_k - \sum_{i=1}^{N} y_i x_{ij} &= 0
\end{aligned}
$$

A Bit of Theory: Linear Regression

Using matrix notation this implies

$$\mathbf{X}^T\mathbf{X}\beta = \mathbf{X}^T\mathbf{y},$$

and, finally,

$$\beta = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

This solution only exists, if the model matrix $\mathbf{X}$ is invertible, which means that it should have full rank. (That's also related to multicollinearity).
The maximum likelihood method has the same result, as it uses the logarithm of a normal distribution, which has a similar quadratic term.

A Bit of Theory: Linear Regression

Exercise

Use the data set `scenario2` and calculate the beta coefficients using the formula above and compare these to those from the linear model

```
coef(lm(y ~x + z, data = scenario2))
```

A Bit of Theory: Hierarchical Linear Regression

Now we incorporate the idea of varying beta coefficient. We use hierarchical models, introducing a linear equation for the betas. Consider

$$y_{ij} = \gamma + x_{ij}\beta_j + \epsilon_{ij}$$

where we introduced a clustering index $1 \leq j \leq q$ for q clusters, indicating participants or stimuli etc. y_{ij} is the ith value in cluster j.
The random effects assumption is that the intercepts and slopes are a normal distribution over the clusters, with mean values γ and β of the fixed effects. Hence, we can write for each cluster

$$\begin{aligned} \gamma_j &= \gamma + u_j^{\gamma} \\ \beta_j &= \beta + u_j^{\beta} \end{aligned}$$

A Bit of Theory: Hierarchical Linear Regression

Inserting this into the original equation yields

$$\begin{aligned} y_{ij} &= \gamma + \gamma_k + x_{ij}(\beta + \beta_j) + \epsilon_{ij} \\ &= (\gamma + x_{ij}\beta) + (\gamma_j + x_{ij}\beta_j) + \epsilon_{ij} \end{aligned}$$

Again using compact matrix notation, this yields to most commonly used formulation for a random effects model

$$\mathbf{y} = \mathbf{X}\beta + \mathbf{Z}u + \epsilon$$

A Bit of Theory: Hierarchical Linear Regression

- In contrast to simple linear regression, no closed solution is available.
- Iterative method based maximum likelihood estimation (ML) or restricted maximum likelihood estimation (REML, default in `lmer`) needed.
- Algorithm might not find always a solution (see Trouble Shooting).
- The fixed coefficients are the same as when using simple linear regression (if the clustering is independent from the predictors).
- It is (mostly) about the standard errors! (= Type I errors).
- In a way, there are only random effects, whereby the means of the random effects are the fixed effects.
- Model only identifiable under certain conditions, measurements need to allow estimation of the RE.
- Sometimes "boundary fits" are produced, which often indicates misspecification (see Trouble Shooting).

Simulation is the Path to Wisdom

As a preparation for the following, let's create some artificial data.

Exercise

- Scenario: N participants rate a set of M musical stimuli in two conditions/Variants, ‘audio-only’ (AO) and 'audio-visual’ (AV) with respect to liking on a 5-point Likert scale (fully crossed design).
- Task: Write a function with parameters `n_raters`, `n_stimuli`, `error` (residual error) and `beta` (effect of condition) that generates liking ratings. (Approximate the 5-point Likert scale with a normal distribution with mean 3.)
- Tips: `expand_grid` or `crossing` create all combinations, `rnorm` produces normal-distributed random numbers for a given mean and standard deviation.
- Bonus Task: Set a parametrizable portion of values to `NA`. Add participant or stimulus-dependent covariates that correlate with `liking` to a certain (parametrizable) extent r.

The Art of Modeling

Roadmap

1. Data exploration
2. Model building
    2.1 Identifying variable types, random effects, fixed effects, covariates.
    2.2 Model optimization by way of model comparison
    2.3 Fixing converge and boundary fit issues
3. Model checking: are assumptions met?
4. Model interpretation: significance testing, marginal effects

Data Exploration

- Know you data!
- First step should be always data exploration.
- Particularly with complex data.
- Histograms and scatterplots of all variables (e. g., `psych::pairs.panels()`, `GGally::ggpairs()`.)
- Data sanity: Missing values, outliers, 'pathological' distributions, unbalanced cluster/cell sizes etc.
- Repeat after me: Know you data!

Model Building: `lmer`

- We will use `lmer` from the `lme4` package.
- In fact, we use the wrapper package `lmerTest`, which adds p-values and provides other useful functions.
- There are other options like `nlme` (old) or `glmmTB` (if you need special stuff such as beta regression).
- `lmer` uses (extended) formula syntax or R.
- Sample call `lmerTest::lmer(liking ~condition + (1 + condition|rater), data = simulate_lmm())`

Model Building: `lmer` Formulas

A `lmer()` formula has the form

```
DV ~ FE + RE
```

The dependent variable is left of the tilde, then fixed and random effects.
Fixed effects contain variables, interactions, and intercepts.

`1 + IV1 + IV2 + ...`	Intercept + independent variable(s)
`0 + IV + ...`, `-1 + IV + ...`	Remove Intercept
IV1 : IV2	Interaction of two variables
IV1 * IV2	Same as `IV1 + IV2 + IV1 : IV2`

The variables correspond to column names in the `data.frame`.

Model Building: `lmer` Formulas

A `lmer()` formula has the form Random effects have the form

    (FE | CV) or (FE || CV)

`FE` is a fixed-effect type formula, defined over a cluster variable `CV`.
A double bar `||` forces the random effects to be uncorrelated instead of being correlated (default, single bar `|`).
Random intercept only is written as `(1|CV)`.
The cluster variables can have nesting using the interaction operator `:` (colon).
The combination `(1|school) + (1|school:class)` can be abbreviated to `(1|school/class)`.

Model Building

- Generally, all units with repeated measurement need to be included as random effects.
- Variables of Interest (VOI) should be identified, i. e., fixed effects as well as additional covariates.
- Fixed effects should be included as random slopes for the random effects if they apply (i. e., if the vary across the cluster).
- Test necessity of random effects with the function `ranova()` from the `lmerTest` package.
- Model comparison is based on likelihood ratio tests.
- For nested models, the difference of deviances ($= -2 logLik$) is approximately χ^2-distributed.
- AIC (and BIC) lower for better fitting models (no sig-test).

Model Building

Exercise

- Simulate data with 10 raters or 10 items (strong/weak random effect with/without slope) using `simulate_lmm()`
- Test if inclusion of random effects (over `rater` and `item`) improves model fit significantly
- Tip: Establish that the simplest random effects is better using `lmerTest::ranova()`, then compare different models with `anova()`
- Bonus: repeated for different effect sizes and rater and item numbers, what is the difference?

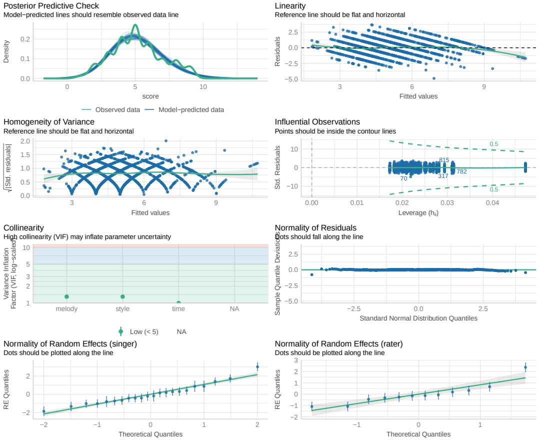
Model Checking

Check the common assumptions using `performance::check_model()`

- Normality of residuals
- Linearity
- Homoscedasticity
- Influential observations
- Variance Inflation
- Normality of random effects

Model Checking

Model check with `performance::check_model()` for the CoVox data set with intercepts-only random effects of `singer` and `rater`.



Model Interpretation

lmerTest::lmer() Output

```
Linear mixed model fit by REML.
t-tests use Satterthwaite's method Formula:
liking ~ condition + (1 + condition | rater) + (1 + condition | item)

REML criterion at convergence: 379.9

Scaled residuals:
     Min       1Q   Median       3Q      Max
-1.99790 -0.56716 -0.02668  0.58813  2.95504

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 rater    (Intercept)  0.1038   0.3222
          conditionAV  0.2291   0.4787   0.57
 item     (Intercept)  0.0919   0.3032
          conditionAV  0.3157   0.5619   0.01
 Residual              0.2651   0.5149
Number of obs: 200, groups:  rater, 10; item, 10

Fixed effects:
             Estimate Std. Error      df t value Pr(>|t|)
(Intercept)    2.5529     0.1491 14.3158   17.13 6.16e-11 ***
conditionAV    0.7701     0.2445 14.9130    3.15  0.00665 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model Interpretation

lmerTest::lmer() Output General Info

```
Linear mixed model fit by REML.
t-tests use Satterthwaite's method Formula:
liking ~ condition + (1 + condition | rater) + (1 + condition | item)

REML criterion at convergence: 379.9

Scaled residuals:
     Min       1Q   Median       3Q      Max
-1.99790 -0.56716 -0.02668  0.58813  2.95504
```

Model interpretation

lmerTest::lmer() Output Random Effects

```
Random effects:
 Groups   Name         Variance Std.Dev. Corr
 rater    (Intercept)  0.1038   0.3222
          conditionAV  0.2291   0.4787   0.57
 item     (Intercept)  0.0919   0.3032
          conditionAV  0.3157   0.5619   0.01
 Residual              0.2651   0.5149
Number of obs: 200, groups:  rater, 10; item, 10
```

True values are .5 for all intercepts and slopes standard deviations and for intercept- slope correlations. Residual error is $\epsilon = .5$.

Model Interpretation

lmerTest::lmer() Fixed Effects

```
Fixed effects:
            Estimate Std. Error      df t value Pr(>|t|)
(Intercept)   2.5529     0.1491 14.3158   17.13 6.16e-11 ***
condition2    0.7701     0.2445 14.9130    3.15  0.00665 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Correlation of Fixed Effects:
            (Intr)
conditionAV 0.177
```

True values are Intercept $= 3.0$, $\beta_{\text{cond}} = 1.0$. $\Rightarrow$ Very small sample size!

Model Interpretation

Test for overall significance using `anova()`. Compute model fit as marginal R_m^2, variance explained by fixed effects, and conditional R_c^2, variance explained by full model. Usually $R_c^2 \gg R_m^2$.

`lmerTest::lmer()` Further metrics

```
anova(mod_r10i10_sws)

Type III Analysis of Variance Table with Satterthwaite's method
          Sum Sq Mean Sq NumDF  DenDF F value   Pr(>F)
condition   2.63    2.63     1 14.913    9.92 0.006651 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

performance::r2(mod_r10i10_sws)

 # R2 for Mixed Models

  Conditional R2: 0.728
     Marginal R2: 0.153
```

Sometimes `MuMIn::r.squaredGLMM()` is needed. Try also `sjPlot::tab_model()`.

Model Interpretation: A Note on Significances

- Even though the main reason to use LMMs is getting the standard errors 'right', there is a bit of a problem with p-values for coefficients.
- The `lme4` deliberately does not provide p-value estimates for the fixed effect coefficients. Problem is to get the degrees of freedom right.
- Alternative methods to establish significance are model comparison, parametric bootstrap, profiled confidence intervals, MCMC sampling, sandwich estimators (package `sandwich`), or simply looking if $t > 2$, if sample size is large enough.
- The `lmerTest` package adds p-values for betas using Satterthwaite's degrees of freedom method or the Kenward-Rogers method, both are not beyond discussion.

Model Interpretation: Extracting Stuff

- `coef()`: Retrieves all coefficients as list of data frames per random effect. Includes fixed effects. Data frames of same size of data, except excluded cases.
- `fixef()`: Retrieves fixed effect coefficients as vector.
- `lme4::ranef()`: Retrieves random effects as data frame.
- `AIC()`, `BIC()`, `logLik()`, `deviance(REML = FALSE)` for global model fit parameters.
- `vcov()`/`VarCorr()` for fixed/random effects variance-covariance matrices.
- `predict()`, `residuals()` Get predictions/residuals.
- `confint()`, get conifidence intervals for all random and fied effects coefficients.
- `broom.mixed::tidy()`, `broom.mixed::glance()` retrieves coefficients / model fit as `tibble`, useful for aggregation.

Model Interpretation

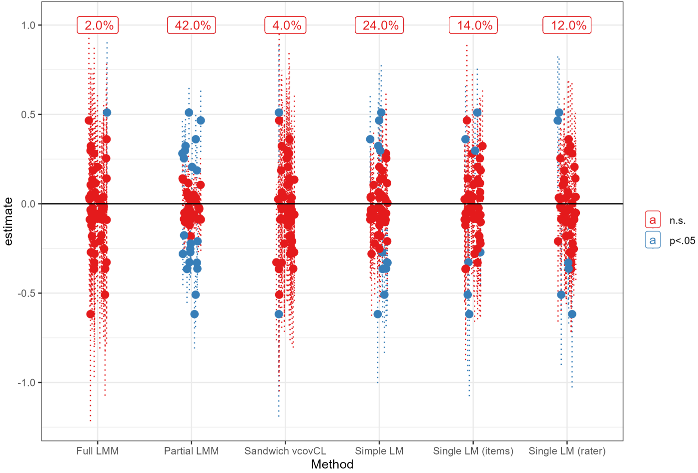
Work on one or more of these problems.

Exercise

- Simulate N data sets with M raters and K items using `simulate_lmm()`, aggregate the fixed effects coefficients and calculate `mean` and `sd`, plot histograms.
- Simulate a data set of you liking with `simulate_lmm()`. Fit a simple model `fit <- lmer(liking ~condition + (1|rater))` Compare the standard deviations of individual random effect estimates with those given by the summary (`summary(fit)$varcor` or `VarCorr(fit)`)? What is going on?
- Simulate a data set with zero fixed effect `simulate_lmm(fixef = 0)`. Fit three models with `condition` as fixed effects and either intercept + slope, intercept-only, simple linear regression. Compare the results (using, e. g., `sjPlot::pltot_models()`.

Model Interpretation: Type I and Type II Errors

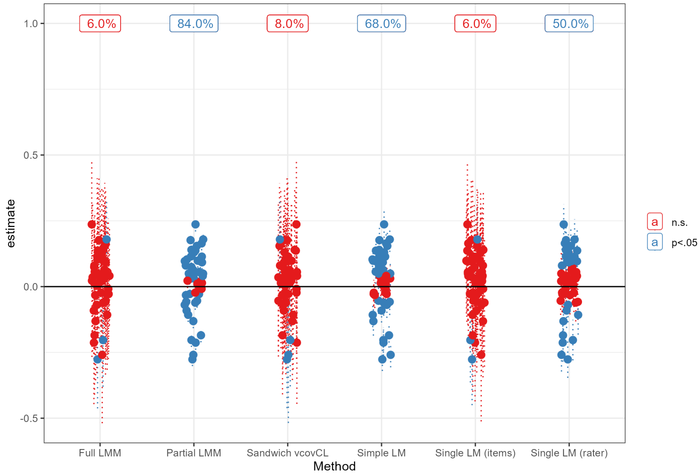
Estimates and Type I errors for null effect, small sample size.



$N_{\text{sim}} = 50,\ N_{\text{raters}} = 10,\ N_{\text{items}} = 10,\ \beta_{\text{cond}} = 0.0$

Model Interpretation: Type I and Type II Errors

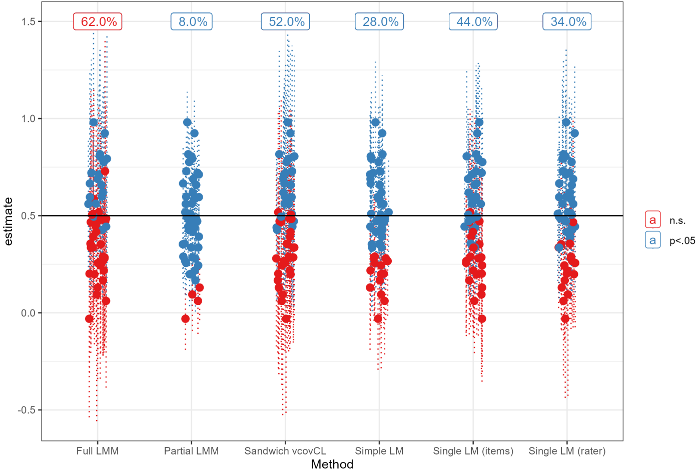
Estimates and Type I errors for null effect, large sample size.



$N_{\text{sim}} = 50,\ N_{\text{raters}} = 200,\ N_{\text{items}} = 20,\ \beta_{\text{cond}} = 0.0$

Model Interpretation: Type I and Type II Errors

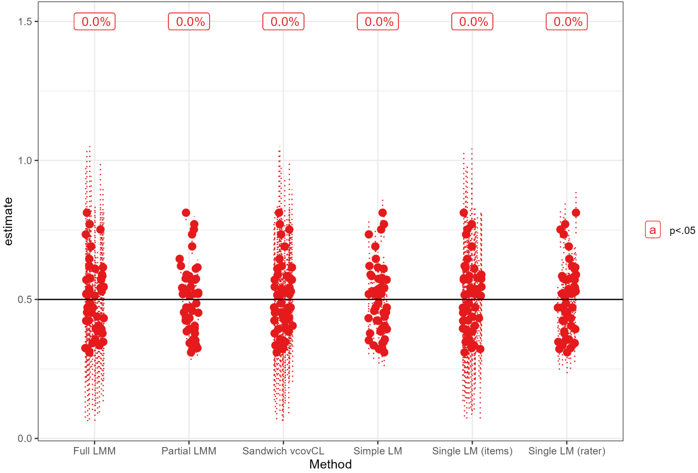
Estimates and Type II errors for weak effect, small sample size.



$N_{\text{sim}} = 50,\ N_{\text{raters}} = 10,\ N_{\text{items}} = 10,\ \beta_{\text{cond}} = 0.5$

Model Interpretation: Type I and Type II Errors

Estimates and Type II errors for weak effect, large sample size.



$N_{\text{sim}} = 50,\ N_{\text{raters}} = 200,\ N_{\text{items}} = 20,\ \beta_{\text{cond}} = 0.5$

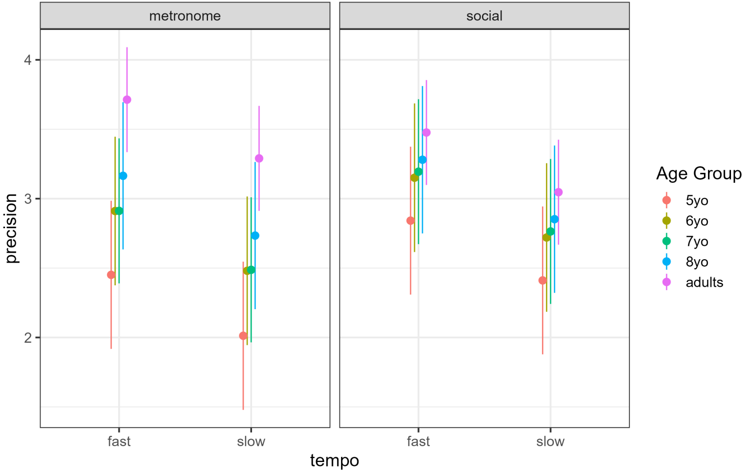
Model Interpretation: Marginal Effects

- For post-hoc testing and visualization you could use two packages: `emmeans` (older) and `marginaleffects` (modern and fancy).
- These allow to aggregate model predictions across conditions and mean covariates etc. taking care of standard errors etc.

Example

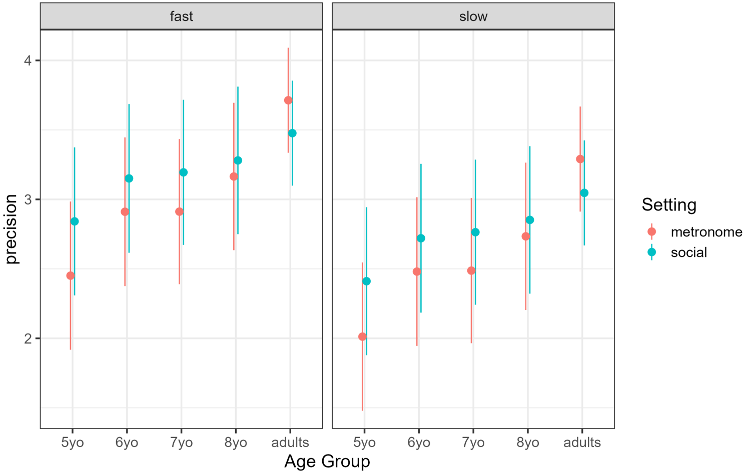
We use the `kid_beats` data set and fit the following model `precision ~age_group * setting + tempo + beat_prod + beat_perc + (1 | experimenter) + (1 | p_id)`
Participants from five different `age_groups` tapped along to a metronome or a human (`setting`) in two different `tempo` conditions (fast: 400 ms, slow: 600 ms)) and the `precision` of the tapping was measured (logarithm of standard deviation of IOIs). For each participant, there were also two beat-related covariates `beat_prod` and `beat_perc`.

Model Interpretation: Marginal Effects




```
marginaleffects::plot_predictions(kid_beats_model, by = c("tempo", "age_group", "setting"))
```

Model Interpretation: Marginal Effects




```
marginaleffects::plot_predictions(kid_beats_model, by = c("age_group", "setting", "tempo"))
```

Trouble Shooting

- Frequently, `lmer` does not converge or shows scary warning messages.
- Singular fit is actually a feature not a bug. It mostly means that some random effects are estimated to 0 or could not estimated at all, which means the model is overspecified. Solution: Inspect the random effects results, possibly simplify the model or decide to ignore.
- Model does not converge. This also mostly means that the model is overspecified or the data do not support the identification of the model or are otherwise peculiar. Solution: Try another optimizer first, or increase tolerance and iteration steps. If that does not work: simplify model.
- An often helpful optimizer is "bobyqa". Usage: `lmer(..., control = lmerControl(optimizer = "bobyqa"))`
- Warning: for large datasets and complicated random effects (random slopes!), the estimation can take a VERY long time. Setting `control = lmerControl(calc.deriv = FALSE)` can speed up things.

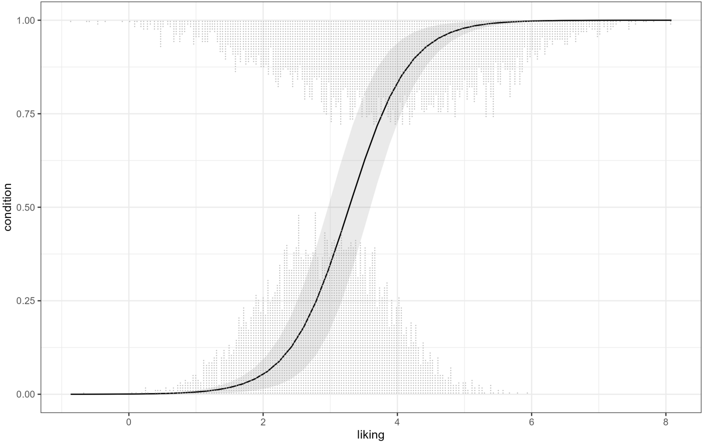
Outlook and Wrap-Up

Outlook: Generalized LMMs

- Generalized Linear Models can be applied to binary (and other) data types, using the `lme4::glmer` function. Syntax is very similar but it needs specification of a link function.
- The link function transforms the outcome to something continuous that can be modeled using LMMs.
- Involves extra steps of transformation, prediction and interpretation might be a bit different, as well as model parameter
- Mixed effects logistic regression uses `family = binomial("logit")`, i.e. $y_i \to \log\left(\frac{p_i}{1-p_i}\right)$.
- Mixed-effects ordinal regression with package `ordinal`, multinomial mixed regression with `mclogit::mblogit`.
- Bayesian mixed models (e. g. the `brms` package), also possible provides different numerical estimation method
- Generalized Additive Mixed Models (GAMMs) and much more ...

Example: Logistic Regression

Predicting `condition` from `liking` with simulated data and strong random effects with slope (200 Rater, 20 Stimuli).



Hands On: Your own models

Try it yourself!

- Take some of your own data and run some models. Show us the results!
- Or explore the `mer`, `covox`, `kid_beats` or `scenario2` data sets in the repository.

Wrap-up

- LMM are state-of-start statistic models, substituting most other methods.
- Need to be used in case of repeated measurements, particularly, the common fully crossed designs.
- Mainly affects estimation of standard error, better inference.
- Comes with some complexities and subtleties, but this is basically true for all statistical models ...
- Happy modeling!

Debrief

- How did you like the workshop?
- Did you learn something?
- Are you eager to use linear mixed effect models now?
- What could be improved?
- Further comments and questions?

The End