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Decentralized overlapping control of a formation of unmanned aerial vehicles☆

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Abstract

Decentralized overlapping feedback laws are designed for a formation of unmanned aerial vehicles. The dynamic model of the formation with an information structure constraint in which each vehicle, except the leader, only detects the vehicle directly in front of it, is treated as an interconnected system with overlapping subsystems. Using the mathematical framework of the inclusion principle, the interconnected system is expanded into a higher dimensional space in which the subsystems appear to be disjoint. Then, at each subsystem, a static state feedback controller is designed to robustly stabilize the perturbed nominal dynamics of the subsystem. The design procedure is based on the application of convex optimization tools involving linear matrix inequalities. As a final step, the decentralized controllers are contracted back to the original interconnected system for implementation.

Keywords: Decentralized control; Robustness; Reliability; Information flows; Large-scale systems

1. Introduction

The ability to form and control long baseline apertures (on the order of kilometers) with Unmanned Aerial Vehicles (UAVs), is envisioned to provide an earth-based low cost alternative to distributed sensing such as Synthetic Aperture Radar (SAR)-interferometry (SAR Interferometry and Surface Change Detection Report, 1994), covering a range of applications, both military (target vertical damage assessment, reconnaissance) and civilian (vegetation growth analysis, rapid assessment of topographical changes as a result of natural events such as flooding or earthquakes). In addition, enhanced coverage for both communication and absolute positioning (via pseudo sources) for operations in remote areas is an area of active research (DARPA ITO Sponsored Research, 1999). Factors which make a UAV system a low-cost alternative to a single large aircraft system, such as rapid reconfigurability in the event of single point failures and dispensability of vehicles, also represent the main challenge from the control design perspective (Fax & Murray, 2001, 2002; Ögren, Fiorelli, & Leonard, 2002; Richards, Bellingham, Tillerson, & How, 2002; Tabuada, Pappas, & Lima, 2001). Thus, decentralized control, as a concept for controlling dynamic systems, is chosen, since it is well known (as stated in Šiljak (1991); Chapter 9: Reliable Control) that decentralized control schemes are superior in terms of reliability with respect to structural reconfigurations to centralized control schemes (e.g. Pachter, D'Azzo, & Proud, 2001). In this paper, each vehicle is modeled using a lateral kinematic model, and an information structure constraint in which each vehicle except the leading one has state information about the vehicle in front of it, is assumed. It is important to note that the imposed information structure is minimal in the sense that there is not more than one

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communication link between any pair of vehicles and therefore it is not information redundant (for more information about information redundancy see Smith and Hadaegh (2002)). As a result of this assumption, communication overhead between vehicles is also minimal, and the formation model can be treated as an interconnected system of overlapping subsystems (the subsystems share common components). This allows one to consider control structures based on overlapping: as introduced in Šiljak (1978) and later developed in Iftar and Özgüner (1990), Ikeda, Šiljak, and White (1984), Ikeda and Šiljak (1986) and Šiljak (1991), it has been shown that such systems can be expanded into a higher-dimensional space in which overlapping subsystems appear as disjoint. Then, fully decentralized control laws can be designed in this expanded space, and contracted back to the original state space of the formation, for implementation. A new method based on the mathematical framework of the inclusion principle, as introduced in Ikeda et al. (1984), is used to ensure that this expansion/contraction procedure is correctly carried over, that is, that solutions of the original system are included in the solutions of the expanded system. For rigorous treatment of various expansion/contraction procedures the reader is referred to Bakule, Rodellar, and Rossell (2000), Iftar and Özgüner (1990), Ikeda et al. (1984), Ikeda and Šiljak (1986), Šiljak (1991), and Stanković and Šiljak (2001).

In this paper, a novel method to design decentralized overlapping control laws that ensure reliable and robust stability of the planar motion of the formation is presented. Stability of the formation is described in terms of the distances between vehicles, and is defined as local Liapunov stability with respect to desired distances between vehicles in the formation. Reliability is understood as stability with respect to structural perturbations (Šiljak, 1978, 1991), such as reconfiguration of the formation, and robustness is understood as stability with respect to vehicle perturbations (e.g., dynamic model uncertainties and wind gust disturbances) that are assumed to be sector bounded functions. The motivation to use decentralized overlapping control comes from the fact that it has already been successfully applied to control a model of a platoon of vehicles (Iftar & Özgüner, 1998; Stanković, Stanojević, & Šiljak, 2000), and similar ideas have been used to control formation flight of UAVs (Wolfe, Chichka, & Speyer, 1996), where perturbations of the nominal system were not considered. Also, the information structure as chosen in this paper can result in weakly coupled subsystems, and this is well suited to overlapping decentralized control. To illustrate this, consider an interconnected system with three subsystems $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$, as shown in Fig. 1. The interconnections between subsystems are depicted with full and dashed lines such that full lines represent strong and dashed lines represent weak interconnections. For example, the full arrow line from the subsystem $\mathbf{S}_1$ to the subsystem $\mathbf{S}_2$ could describe a strong influence such as direct communication of all state variables, and the dashed arrow line from the subsystem $\mathbf{S}_2$ to the subsystem $\mathbf{S}_1$ could describe a weak influence such as partial knowledge of state variables through sensing (for more details on the strong and weak couplings between subsystems, see (Šiljak, 1991)).

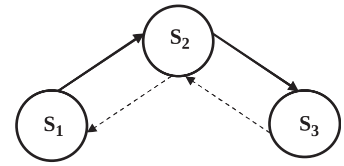

Fig. 1. Interconnected system.

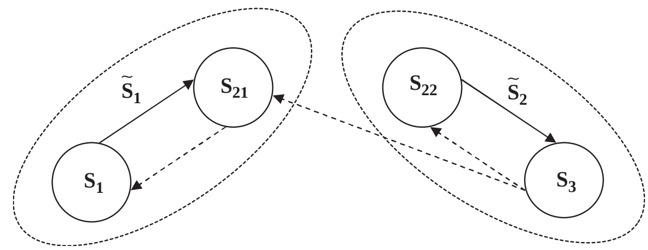

Fig. 2. Expanded interconnected system.

After expanding the interconnected system by mapping subsystem $\mathbf{S}_2$ into subsystems $\mathbf{S}_{21}$ and $\mathbf{S}_{22}$ such that $\mathbf{S}_{21}$ replaces $\mathbf{S}_2$ with respect to $\mathbf{S}_1$ and $\mathbf{S}_{22}$ replaces $\mathbf{S}_2$ with respect to $\mathbf{S}_3$, the obtained expanded system is shown in Fig. 2 (again for rigorous treatment of expansion as transformation of dynamic systems (see Šiljak, 1991 and Section 3 in this paper).

Now, subsystems $\mathbf{S}_1$ and $\mathbf{S}_{21}$ can be combined into one subsystem $\tilde{\mathbf{S}}_1$ in the expanded space, and similarly subsystems $\mathbf{S}_{22}$ and $\mathbf{S}_3$ can be combined into another subsystem $\tilde{\mathbf{S}}_2$. From Fig. 2 it follows that these two subsystems in the expanded space are weakly coupled (that is, there is only weak influence of the subsystem $\tilde{\mathbf{S}}_2$ on the subsystem $\tilde{\mathbf{S}}_1$) and this allows a decentralized control methodology to be applied in the expanded space. Thus, the information structure as shown in Fig. 1 (that is, when the flow of information is dominant in one direction) is well suited to overlapping decentralized control. The information structure constraints imposed on vehicles in a formation are described using unidirectional information flow and thus the subsystems in the expanded space appear as disjoint (that is, the weak connection in Fig. 2 between two subsystems in the expanded space does not even exist).

The main novelty of the results presented in this paper when compared to results presented in Iftar and Özgüner (1998), Stanković et al. (2000), Wolfe et al. (1996), is the capability of designing *robust* decentralized control laws based on the methodology developed in Šiljak and Stipanović (2000). Thus, using convex optimization algorithms formulated in terms of linear matrix inequalities, one is capable of not only stabilizing the nominal system but of robustly stabilizing a perturbed version of the system, where the perturbations are assumed to be Euclidean norm sector bounded functions.

The organization of the paper is as follows. In Section 2, input-state feedback linearization (see, e.g., Sastry, 1999) is used to derive a model of the formation that can be treated as an interconnected system with subsystems that are coupled. The problem of internal stability of the formation, flying at a desired speed, is formulated in terms of stability with respect to the zero equilibrium of the particular interconnected system. The inclusion principle and corresponding expansion and contraction procedures are presented in Section 3. Under the assumption that static feedback control laws obey the information structure constraint, sufficient conditions for the proper contraction of the stabilizing control laws from the expanded to the original space are derived. In Section 4, a procedure for designing robust decentralized control laws for each subsystem in the expanded space is presented. The problem is formulated as a convex optimization problem in terms of linear matrix inequalities (LMIs) (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994; Šiljak & Stipanović, 2000), that can be efficiently solved using powerful convex programming tools. Finally, in Section 5, the design procedure is illustrated by presenting numerical results in the case of a formation of five vehicles.

2. Model description and stability problem formulation

Let us start with the following planar kinematic model for a single vehicle:

$$\dot{X} = \upsilon\cos\psi,$$

$$\dot{Y} = \upsilon\sin\psi,$$

$$\dot{\psi} = \omega, \tag{1}$$

where X and Y denote rectangular coordinates, and ψ is the heading angle in the (X, Y) plane. The speed in the longitudinal direction (in body axes) υ and angular turn rate ω are assumed to be the control inputs. In Ghosh and Tomlin (2000), it is shown that the planar kinematic model corresponds well to the motion of an aircraft, under nonlinear mode-based closed-loop controls. This is because the velocity in the lateral direction in the body axes (or sideslip) is typically regulated at zero and for the purpose of formation flight, the change in altitude is gradual if not zero. Also, υ and ω enter into the aforementioned control scheme as reference inputs.

It can easily be shown that the decoupling matrix of the input-state feedback linearization for the kinematic model (1) is singular. In order to deal with this problem, dynamic extension (Sastry, 1999) is used as follows: speed υ is considered as a new state variable, and acceleration a as a new input variable. Now, the state and input variables are defined as

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ \psi \\ \upsilon \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} a \\ \omega \end{bmatrix}. \tag{2}$$

Using (2) the kinematic model (1) can be rewritten as follows:

$$\dot{\xi} = f(\xi) + g(\xi)\eta \quad \text{with } f(\xi) = \begin{bmatrix} \xi_4\cos(\xi_3) \\ \xi_4\sin(\xi_3) \\ 0 \\ 0 \end{bmatrix},$$

$$g(\xi) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{3}$$

At this point a change of state variables is introduced as

$$z = T(\xi) \quad \text{such that} \quad \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_4\cos(\xi_3) \\ \xi_4\sin(\xi_3) \end{bmatrix} \tag{4}$$

and change of input variables, to define the new input $u \in \mathbb{R}^2$

$$\eta = M(\xi)u \quad \text{with}$$

$$M(\xi) = \begin{bmatrix} \cos(\xi_3) & \sin(\xi_3) \\ -\sin(\xi_3)/\xi_4 & \cos(\xi_3)/\xi_4 \end{bmatrix}. \tag{5}$$

The transformations introduced in (4) and (5) imply the following exact linearization of the nonlinear model (3)

$$\dot{z} = \frac{\partial T}{\partial \xi}\dot{\xi} \Rightarrow$$

$$\dot{z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u = Ez + Fu, \tag{6}$$

which can be rewritten in the compact form as

$$\dot{z} = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & 0_2 \end{bmatrix} z + \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} u \tag{7}$$

with $z \in \mathbb{R}^4$ and $u \in \mathbb{R}^2$ being the state and input to the system, respectively. 0_2 denotes the 2×2 zero matrix and I_2 denotes the 2×2 identity matrix. In order to simplify the notation, from this point and throughout the rest of the paper, these two matrices will be simply denoted as 0 and I.

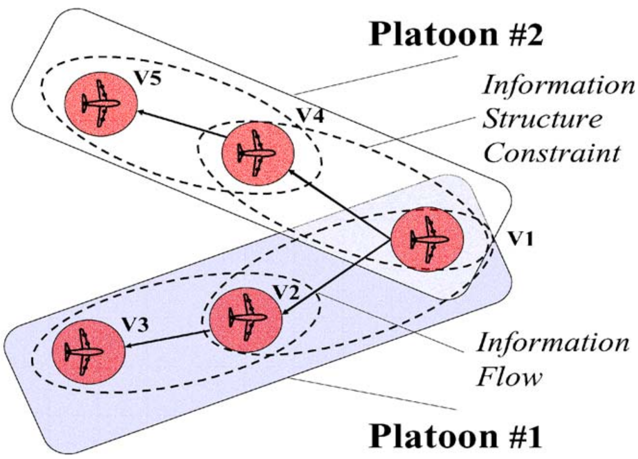

Fig. 3. Leader-follower type formation with five vehicles and two platoons.

At this point the aim is to develop a dynamic model for a leader-follower type of formation of vehicles. Let us introduce the following decomposition for the state variables of the ith vehicle in the formation of q vehicles,

$$z_i = \begin{bmatrix} z_i^{\mathrm{I}} \\ z_i^{\mathrm{II}} \end{bmatrix} \in \mathbb{R}^4 \quad \text{with } z_i^{\mathrm{I}} = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} \in \mathbb{R}^2,$$

$$z_i^{\mathrm{II}} = \begin{bmatrix} z_{i3} \\ z_{i4} \end{bmatrix} \in \mathbb{R}^2 \tag{8}$$

and note that

$$z_i^{\mathrm{I}} = \begin{bmatrix} X_i \\ Y_i \end{bmatrix}, \quad z_i^{\mathrm{II}} = \begin{bmatrix} v_i \cos \psi_i \\ v_i \sin \psi_i \end{bmatrix} \tag{9}$$

with $i \in \{1, 2, \ldots, q\}$. In other words, the idea is to split vector z_i into two subvectors, where the first subvector z_i^{I} includes position coordinates and the second subvector z_i^{II} includes speed coordinates of the ith vehicle. This type of decomposition is chosen due to different treatment of the state variables, that is, the goal is to control the vehicles in a formation by controlling variables that represent distances between vehicles (i.e., not positions of the vehicles), and variables that represent speed coordinates for each independent vehicle. The control input for the ith vehicle as defined in Eq. (5), will be denoted as u_i, where $u_i \in \mathbb{R}^2$.

Motivated by successful results presented in Stanković et al. (2000), where decentralized overlapping control laws were formulated for a platoon of vehicles, a particular structure for the formation, as shown in Fig. 3, is proposed. By imposing the information structure constraint that each vehicle, except the leading one, has state information about the vehicle in front of it, it is natural to decompose the formation into two platoons that share the leading vehicle. In Fig. 3, the number of vehicles in the formation is equal to five and each platoon has three vehicles. Dotted lines encircling pairs of vehicles represent information structure constraints.

It is important to note that under the imposed information structure constraint, analysis of local stability of the formation can be independently formulated in terms of the platoons. The only vehicle that is shared by the platoons is the leader, which does not receive any information from the vehicles behind, and therefore its dynamics are governed independently from the rest of the formation. Thus, control laws for each platoon can be designed independently, and then applied to the formation. Of course, this is valid only in the local sense since large deviations might cause conflicts between vehicles in different platoons.

For simplicity and without loss of generality, let us consider a platoon of r vehicles and introduce the following change of variables:

$$e_1^{\mathrm{II}} = z_1^{\mathrm{II}} - v_{d1} \quad \text{for the leading vehicle}$$

$$\left\{ \begin{array}{c} e_i^{\mathrm{I}} = z_{i-1}^{\mathrm{I}} - z_i^{\mathrm{I}} - d_{i-1} \\ e_i^{\mathrm{II}} = z_i^{\mathrm{II}} - v_{di} \end{array} \right\}, \quad i \in \{2, \ldots, r\}, \tag{10}$$

where $d_{i-1} \in \mathbb{R}^2$ is a constant desired Euclidean distance between the $(i-1)st$ and ith vehicles, $i \in \{2, \ldots, r\}$, and v_{di}, $v_{di} \in \mathbb{R}^2$, represents the desired speed for the ith vehicle, $i \in \{1, 2, \ldots, r\}$. In Fig. 3, for example, platoon 1 would include vehicles 1, 2, and 3, and platoon 2 would include vehicles 1, 4, and 5. Notice that for controlling distances between vehicles, position of the leading vehicle (i.e., z_1^{I}) is not needed. Since the desired Euclidean distances between vehicles are assumed to be constant, the following assumption is necessary:

$$v_{di} = v_d, \quad i \in \{1, 2, \ldots, r\}. \tag{11}$$

In other words, in order to achieve constant desired spacing in the formation, the desired speed for each vehicle must be the same. Then,

$$\dot{e}_1^{\mathrm{II}} = u_1 \text{ for the leading vehicle}$$

$$\left\{ \begin{array}{c} \dot{e}_i^{\mathrm{I}} = e_{i-1}^{\mathrm{II}} - e_i^{\mathrm{II}} \\ \dot{e}_i^{\mathrm{II}} = u_i \end{array} \right\}, \quad i \in \{2, \ldots, r\}. \tag{12}$$

Now, the stability problem can be formulated. Notice that the goal is for the whole platoon (that is, formation) to fly at constant desired speed v_d with desired spacing between vehicles, uniquely determined by desired Euclidean distances between successive vehicles equal to d_i, $i \in \{1, 2, \ldots, r-1\}$, which is accomplished if the system described by Eq. (12) is stable with respect to its zero equilibrium.

Since (due to symmetry) the procedure does not depend on the size of the platoon, for clarity and simplicity of presentation purposes, and without loss of generality, let us assume $r = 3$ (as in Fig. 1). Then, Eq. (12) can be written in the compact form as

$$\underbrace{\begin{bmatrix} \dot{e}_1^{\mathrm{II}} \\ \dot{e}_2^{\mathrm{I}} \\ \dot{e}_2^{\mathrm{II}} \\ \dot{e}_3^{\mathrm{I}} \\ \dot{e}_3^{\mathrm{II}} \end{bmatrix}}_{\dot{e}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} e_1^{\mathrm{II}} \\ e_2^{\mathrm{I}} \\ e_2^{\mathrm{II}} \\ e_3^{\mathrm{I}} \\ e_3^{\mathrm{II}} \end{bmatrix}}_{e} + \underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}}_{u}. \tag{13}$$

Finally, the system described by Eq. (12) (or the special case described by Eq. (13)) can be considered as an interconnected system with subsystems having state variables that are defined as

$$e_1 = e_1^{\mathrm{II}}, \quad e_i = \begin{bmatrix} e_i^{\mathrm{I}} \\ e_i^{\mathrm{II}} \end{bmatrix} \quad \text{for all } i \in \{2, \ldots, r\}. \tag{14}$$

Thus, the dynamic model described by Eq. (13) can be considered as an interconnected system with subsystems that are coupled.

3. Decentralized overlapping control using static state feedback

As mentioned in the Introduction, the goal is to expand the interconnected system represented by Eq. (13) into a space in which the subsystems will be decoupled. Then, in the expanded space, design of static feedback controllers for each decoupled subsystem can be carried over independently, that is, in parallel. For large systems, this procedure will significantly reduce computational complexity when compared to the design of the centralized controller for the whole system. More importantly, this decentralized scheme reflects the physical constraints of the system (individual vehicles running their own control schemes). By stabilizing subsystems independently, the formation will consist of subsystems representing pairs of vehicles that are stable themselves, which provides reliability of the proposed control scheme. Finally, the designed control laws in the expanded space will be contracted back to the original space for implementation. In order to do this, let us recall the definition of the inclusion principle for linear systems (Ikeda et al., 1984). Consider the systems:

$$\mathbf{S}: \dot{x} = Ax + Bu, \quad x(t_0) = x_0,$$

$$x \in \mathbb{R}^n \text{ is the state, } u \in \mathbb{R}^m \text{ is the control input,}$$

$$\tilde{\mathbf{S}}: \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}, \quad \tilde{x}(t_0) = \tilde{x}_0,$$

$$\tilde{x} \in \mathbb{R}^{\tilde{n}} \text{ is the state, } \tilde{u} \in \mathbb{R}^{\tilde{m}} \text{ is the control input} \tag{15}$$

with $\tilde{n} > n$ and $\tilde{m} > m$. Trajectories of system $\mathbf{S}$ are denoted as $x(t; x_0, u)$ and similarly trajectories of system $\tilde{\mathbf{S}}$ are denoted as $\tilde{x}(t; \tilde{x}_0, \tilde{u})$. Without loss of generality, assume that $t_0 = \tilde{t}_0 = 0$, neglect dependence of trajectories on the initial time, and assume that there exists a pair of expansion/contraction matrices (as explained below) for the state

$$V \in \mathbb{R}^{\tilde{n} \times n}, \quad U \in \mathbb{R}^{n \times \tilde{n}}, \quad UV = I \in \mathbb{R}^{n \times n} \tag{16}$$

and, respectively, for the input

$$R \in \mathbb{R}^{\tilde{m} \times m}, \quad Q \in \mathbb{R}^{m \times \tilde{m}}, \quad QR = I \in \mathbb{R}^{m \times m}. \tag{17}$$

Now, recall the following (see, e.g., Šiljak, 1991):

Definition 1 (Inclusion principle). System $\tilde{\mathbf{S}}$ *includes* system $\mathbf{S}$ if for any initial state x_0 and any input $u(t)$, the following is valid: $x(t; x_0, u) = U\tilde{x}(t; Vx_0, Ru)$.

Theorem 1. *System* $\tilde{\mathbf{S}}$ *includes system* $\mathbf{S}$ *if and only if* $A^i = U\tilde{A}^i V$ *and* $A^i B = U\tilde{A}^i \tilde{B} R$ *for* $i \in \{0, 1, 2, \ldots, \tilde{n} - 1\}$.

In other words, the inclusion principle formulates conditions under which the trajectories of the original system $\mathbf{S}$ are included in the set of trajectories of the expanded system $\tilde{\mathbf{S}}$. The following restriction, that is known to be a special case of the inclusion principle, is defined as (Iftar & Özgüner, 1990; Ikeda & Šiljak, 1986):

Definition 2 (Restriction). $\mathbf{S}$ is a *restriction* of $\tilde{\mathbf{S}}$ if one of the following is true:

(a) Given any initial state x_0 and any input $u(t)$, the following is valid: $\tilde{x}(t; Vx_0, Ru) = Vx(t; x_0, u)$ (denoted as restriction type (a)).
(b) Given any initial state x_0 and any input $\tilde{u}(t)$, the following is valid: $\tilde{x}(t; Vx_0, \tilde{u}) = Vx(t; x_0, Q\tilde{u})$ (denoted as restriction type (b)).

Theorem 2. $\mathbf{S}$ *is a restriction of* $\tilde{\mathbf{S}}$ *if one of the following is true*:

(a) $\tilde{A}V = VA$ *and* $\tilde{B}R = VB$ (*restriction type* (a)).
(b) $\tilde{A}V = VA$ *and* $\tilde{B} = VBQ$ (*restriction type* (b)).

It is important to note that conditions (a) and (b) in Definition 2 and Theorem 2 correspond to the two different types of restriction, denoted as restriction type (a) and restriction type (b), respectively. Also, if static feedback control laws for both systems are assumed to be in the following form:

$$u = Kx, \quad K \in \mathbb{R}^{m\times n},$$

$$\tilde{u} = \tilde{K}\tilde{x}, \quad \tilde{K} \in \mathbb{R}^{\tilde{m}\times\tilde{n}}, \tag{18}$$

then even if the open-loop systems satisfy the inclusion principle, it does not necessarily mean that the closed-loop system in the original space

$$\bar{\mathbf{S}} : \dot{x} = (A + BK)x \tag{19}$$

is "included" (understood in the sense of Definition 1 and Theorem 1 for the open-loop systems $\mathbf{S}$ and $\tilde{\mathbf{S}}$ when the control inputs are equal to zero) in the closed-loop system in the expanded space

$$\tilde{\bar{\mathbf{S}}} : \dot{\tilde{x}} = (\tilde{A} + \tilde{B}\tilde{K})\tilde{x}. \tag{20}$$

Conditions for inclusion in the case of the restriction are given as (Iftar & Özgüner, 1990; Ikeda & Šiljak, 1986):

Theorem 3. $\bar{\mathbf{S}}$ *is a restriction of* $\tilde{\bar{\mathbf{S}}}$ *if one of the following is true*:

(a) $\tilde{A}V = VA$, $\tilde{B}R = VB$, *and* $\tilde{K}V = RK$ (*restriction type* (a)).
(b) $\tilde{A}V = VA$, $\tilde{B} = VBQ$, *and* $K = Q\tilde{K}V$ (*restriction type* (b)).

It is interesting to note that given an interconnected system with subsystems that overlap, the expansion procedure can be done by simply repeating overlapping parts such that in the expanded space subsystems appear disjoint. By doing so, based on the information structure, subsystem dynamics in Eq. (13) can be decoupled by introducing the following expansion:

$$\tilde{e}_1 = e_1, \quad \tilde{u}_1 = u_1,$$

$$\tilde{e}_i = \begin{bmatrix} \tilde{e}_i^a \\ \tilde{e}_i^b \\ \tilde{e}_i^c \end{bmatrix} = \begin{bmatrix} e_{i-1}^{\mathrm{II}} \\ e_i^{\mathrm{I}} \\ e_i^{\mathrm{II}} \end{bmatrix},$$

$$\tilde{u}_i = \begin{bmatrix} \tilde{u}_i^a \\ \tilde{u}_i^b \end{bmatrix} = \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix}, \quad i \in \{2,3\}. \tag{21}$$

which can be written in the compact form as

$$\underbrace{\begin{bmatrix} \dot{\tilde{e}}_1 \\ \dot{\tilde{e}}_2 \\ \dot{\tilde{e}}_3 \end{bmatrix}}_{\dot{\tilde{e}}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\tilde{A}_D} \underbrace{\begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \tilde{e}_3 \end{bmatrix}}_{\tilde{e}}$$

$$+ \underbrace{\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}}_{\tilde{B}_D} \underbrace{\begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix}}_{\tilde{u}}. \tag{22}$$

The expansion/contraction matrices for the state are given as

$$V = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$U = \begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}I & \frac{1}{2}I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \tag{23}$$

and similarly for the input

$$R = \begin{bmatrix} I & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad Q = \begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}I & \frac{1}{2}I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}. \tag{24}$$

Using Eqs. (13) and (22)–(24) it is easy to show that $\tilde{A}_D V = VA$ and $\tilde{B}_D R = VB$. Then, from Theorem 2 it follows that this expansion/contraction procedure satisfies the conditions of Definition 2(a) (restriction type (a)).

Now, let us consider the static feedback control laws defined in (18). Since the open-loop systems satisfy Definition 2(a), from Theorem 3 it follows that:

$$\tilde{K}V = RK \tag{25}$$

must be valid in order for the closed-loop systems to satisfy the inclusion principle. Recall that the goal is to design a reliable controller in which each vehicle in the original space (except the leading one) has information about the states of the vehicle in front of it. From Eqs. (10) and (21) it follows that this information structure constraint can be mathematically described in the expanded space as

$$\tilde{K}_D = \begin{bmatrix} \tilde{K}_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_{32} & \tilde{K}_{33} & \tilde{K}_{34} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{55} & \tilde{K}_{56} & \tilde{K}_{57} \end{bmatrix}, \tag{26}$$

where $\tilde{K}_{ij} \in \mathbb{R}^{2\times 2}$ for $i \in \{1,\dots,5\}$ and $j \in \{1,\dots,7\}$. Thus, $\tilde{K}_D \in \mathbb{R}^{10\times 14}$, and to simplify the notation from this point and throughout the rest of the paper (if not noted otherwise), all matrix variables will be of dimension 2×2 and the overall matrix dimension can be determined accordingly. In order to make Eq. (25) solvable for K and following the ideas presented in Ikeda and Šiljak (1986), the matrix $\tilde{K}_D$ is modified as

$$\tilde{K}_{DM} = \begin{bmatrix} \frac{\tilde{K}_{11}+\tilde{K}_{22}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\tilde{K}_{11}+\tilde{K}_{22}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_{32} & \tilde{K}_{33} & \frac{\tilde{K}_{34}+\tilde{K}_{45}}{2} \cdots & 0 & 0 & 0 \\ 0 & \tilde{K}_{32} & \tilde{K}_{33} & 0 & \frac{\tilde{K}_{34}+\tilde{K}_{45}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{55} & \tilde{K}_{56} & \tilde{K}_{57} \end{bmatrix}. \tag{27}$$

Modification of the matrix $\tilde{K}_D$ as given in Eq. (27) guarantees that the closed-loop systems in Eqs. (19) and (20) satisfy the inclusion principle of Definition 1. Solving (25) for K in terms of $\tilde{K}_{DM}$, one obtains

$$K_M = \begin{bmatrix} \frac{\tilde{K}_{11}+\tilde{K}_{22}}{2} & 0 & 0 & 0 & 0 \\ \tilde{K}_{32} & \tilde{K}_{33} & \frac{\tilde{K}_{34}+\tilde{K}_{45}}{2} & 0 & 0 \\ 0 & 0 & \tilde{K}_{55} & \tilde{K}_{56} & \tilde{K}_{57} \end{bmatrix}. \tag{28}$$

Thus, the relation $\tilde{K}_{DM} V = R K_M$ is valid.

Notice that if $\tilde{K}_{11} = \tilde{K}_{22}$, and $\tilde{K}_{34} = \tilde{K}_{45}$, from Eqs. (26) and (27), it follows that the stability of the expanded closed-loop system will be preserved after modification since the structure of $\tilde{K}_D$ is block diagonal and the structure of $\tilde{K}_{DM}$ is lower block triangular, such that both matrices have the same main diagonal blocks. Thus, if a controller in the expanded space is designed in the following form:

$$\tilde{K}_D = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix} \tag{29}$$

and by modifying it according to (27), $\tilde{K}_{DM}$ is computed in the overlapping form as

$$\tilde{K}_{DM} = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix}. \tag{30}$$

Stability of the closed-loop system in the expanded space will be preserved after modification. Then, from Eq. (28) it follows that:

$$K_M = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 \\ \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix} \tag{31}$$

will be a stabilizing feedback in the original space.

At this point let us note that restriction type (b) in Definition 2 (introduced in Iftar and Özgüner (1990) as extension) produces an expanded system that is not stabilizable, and this is the reason why it was not chosen. In general the main advantage of restriction type (b) over restriction type (a) is that if the stabilizing control law is computed in the expanded space it can be directly contracted to the original space as $K = Q\tilde{K}V$ (i.e., there are no modifications). This is not the case with the restriction type (a) where contraction is not necessarily a straightforward procedure (as shown in the work above).

4. Design of robust decentralized static feedback control laws

In this section, a method to compute a feedback gain matrix defined in Eq. (29) that will robustly stabilize the expanded system, implying that its contraction will stabilize the original system, is proposed. Thus, let us start with the perturbed kinematic model

$$\dot{\xi} = f(\xi) + g(\xi)\eta + w, \tag{32}$$

where $w = [w_1, w_2, w_3, w_4]^T \in \mathbb{R}^4$ is a perturbation in the system which represents, for example, wind gust disturbances or uncertainties in the model description. It is important to stress that only sector bounded perturbations will be considered, that is, perturbations that reside in some conical sector emanating from the origin in the state space (for more details see, for example, Šiljak and Stipanović (2000) and references reported therein).

With $z = T(\xi)$ and $\eta = M(\xi)u$ defined in Eqs. (4) and (5), input-state feedback linearization from Section 2 is repeated to obtain

$$\dot{z} = \frac{\partial T}{\partial \xi}\dot{\xi} = \frac{\partial T}{\partial \xi} f(\xi) + \frac{\partial T}{\partial \xi} M(\xi)u + \frac{\partial T}{\partial \xi} w$$
$$= Ez + Fu + \bar{w},$$
$$\bar{w} = \frac{\partial T}{\partial \xi} w. \tag{33}$$

At this point let us note that $w_4 \approx 0$, since it is a perturbation in an artificial equation added to the kinematic model in the process of dynamic extension. It is called an artificial equation since it is added to the kinematic model (1) and therefore it is not connected with the dynamic model of the aircraft which is susceptible to the sector bounded perturbations. From Eqs. (4) and (33) it follows:

$$\bar{w} = [w_1 \quad w_2 \quad -z_4 w_3 \quad z_3 w_3]^T \tag{34}$$

or in the decomposed form (following decomposition of the state variables introduced in Section 2):

$$\bar{w} = \begin{bmatrix} \bar{w}^{\mathrm{I}} \\ \bar{w}^{\mathrm{II}} \end{bmatrix} \in \mathbb{R}^4, \quad \bar{w}^{\mathrm{I}} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \in \mathbb{R}^2,$$
$$\bar{w}^{\mathrm{II}} = \begin{bmatrix} -z_4 \\ z_3 \end{bmatrix} w_3 = J(x^{\mathrm{II}} + v_d) w_3,$$
$$\text{where } J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{35}$$

It is easy to show that after introducing perturbations, the coupled Eqs. (12) become

$$\dot{e}_1 = u_1 + \bar{w}_1^{\mathrm{II}} \quad \text{for the leading vehicle,}$$
$$\left\{ \begin{array}{c} \dot{e}_i^{\mathrm{I}} = e_{i-1}^{\mathrm{II}} - e_i^{\mathrm{II}} + \bar{w}_{i-1}^{\mathrm{I}} - \bar{w}_i^{\mathrm{I}} \\ \dot{e}_i^{\mathrm{II}} = u_i + \bar{w}_i^{\mathrm{II}} \end{array} \right\}, \quad i \in \{2, \ldots, r\}. \tag{36}$$

Let us introduce

$$\hat{w}_1 = \bar{w}_1^{\mathrm{II}} \quad \text{for the leading vehicle,}$$
$$\hat{w}_i = \begin{bmatrix} \hat{w}_i^{\mathrm{I}} \\ \hat{w}_i^{\mathrm{II}} \end{bmatrix} = \begin{bmatrix} \bar{w}_{i-1}^{\mathrm{I}} - \bar{w}_i^{\mathrm{I}} \\ \bar{w}_i^{\mathrm{II}} \end{bmatrix} \quad \text{for } i \in \{2, \ldots, r\}, \tag{37}$$

as perturbations to system (12). From Eqs. (34)–(37) it follows that:

$$\hat{w}_1 = \bar{w}_1^{\mathrm{II}} \quad \text{for the leading vehicle,}$$
$$\hat{w}_i = \begin{bmatrix} \hat{w}_i^{\mathrm{I}} \\ \hat{w}_i^{\mathrm{II}} \end{bmatrix} = \begin{bmatrix} w_{i-1}^{\mathrm{I}} - w_i^{\mathrm{I}} \\ \bar{w}_i^{\mathrm{II}} \end{bmatrix} \quad \text{for } i \in \{2, \ldots, r\}. \tag{38}$$

In the case of three vehicles in the platoon, in the expanded space one obtains

$$\tilde{w}_1 = \hat{w}_1,$$
$$\tilde{w}_i = \begin{bmatrix} \tilde{w}_i^a \\ \tilde{w}_i^b \\ \tilde{w}_i^c \end{bmatrix} = \begin{bmatrix} \hat{w}_{i-1}^{\mathrm{II}} \\ \hat{w}_i^{\mathrm{I}} \\ \hat{w}_i^{\mathrm{II}} \end{bmatrix}, \quad i \in \{2, 3\}. \tag{39}$$

Thus, by introducing perturbation into the kinematic model, Eq. (22) becomes

$$\dot{\tilde{e}} = \tilde{A}_D \tilde{e} + \tilde{B}_D \tilde{u} + \tilde{w}. \tag{40}$$

Now, $\Delta\tilde{K}$ is defined as

$$\Delta\tilde{K} = \tilde{K}_{DM} - \tilde{K}_D, \tag{41}$$

where $\tilde{K}_D$ and $\tilde{K}_{DM}$ are defined in Eqs. (29) and (30), respectively. From this point the reader is referred to $\Delta\tilde{K}$ as a "spillover" and assuming a static feedback $\tilde{u} = \tilde{K}_{DM}\tilde{e}$, Eq. (40) can be rewritten as

$$\dot{\tilde{e}} = (\tilde{A}_D + \tilde{B}_D \tilde{K}_{DM})\tilde{e} + \tilde{w}$$
$$= (\tilde{A}_D + \tilde{B}_D \tilde{K}_D)\tilde{e} + \tilde{B}_D \Delta\tilde{K}\tilde{e} + \tilde{w}. \tag{42}$$

Thus, from Eq. (42) it follows that the system has two types of perturbations. The first is due to the modification of the gain matrix that allows proper contraction, and the second is due to the perturbations of the original kinematic model. It is important to note that the "spillover" term couples subsystem dynamics in the expanded space.

In order to compute stabilizing feedback gains in the expanded space notice that only the ith subsystem from Eq. (40) can be considered (this can be done since all the subsystems are completely decoupled), and is given as

$$\dot{\tilde{e}}_i = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & 0 \end{bmatrix}}_{\tilde{A}} \tilde{e}_i + \underbrace{\begin{bmatrix} I & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix}}_{\tilde{B}} \tilde{u}_i + \tilde{w}_i, \tilde{e}_i = \begin{bmatrix} \tilde{e}_i^a \\ \tilde{e}_i^b \\ \tilde{e}_i^c \end{bmatrix} \in \mathbb{R}^6,$$

$$\tilde{u}_i = \underbrace{\begin{bmatrix} \tilde{K}_1 & 0 & 0 \\ \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix}}_{\tilde{K}} \tilde{e}_i = \begin{bmatrix} \tilde{u}_i^a \\ \tilde{u}_i^b \end{bmatrix} \in \mathbb{R}^4, \tilde{w}_i = \begin{bmatrix} \tilde{w}_i^a \\ \tilde{w}_i^b \\ \tilde{w}_i^c \end{bmatrix} \in \mathbb{R}^6 \tag{43}$$

with $\tilde{w}_i$ residing in the sector, that is,

$$\tilde{w}_i^T \tilde{w}_i \leqslant \alpha^2 \tilde{e}_i^T W^T W \tilde{e}_i, \tag{44}$$

where α is a positive number to be maximized and $W \in \mathbb{R}^{p\times 6}$ (p being an arbitrary positive integer) is a constant matrix (usually set to be identity). Practically, the matrix W is chosen according to a predetermined knowledge about the perturbations, and if no particular knowledge about the perturbations is available it is set to be an identity matrix, meaning that the norm of perturbations is bounded by the scaled norm of the state variables. Due to the fact that the subsystems are identical, the subsystems' parameters $\tilde{A}$, $\tilde{B}$, $\tilde{K}$, α, and W are independent of i.

To stabilize each subsystem, a quadratic Liapunov function $V(\tilde{e}_i) = \tilde{e}_i^T P \tilde{e}_i$, where $P \in \mathbb{R}^{6\times 6}$ is a positive definite matrix (denoted $P \succ 0$), is considered. By computing its derivative with respect to time and given constraints in (44), using the well known S-procedure and the LMI formulation, one obtains (for further details of this standard procedure see Boyd et al. (1994)):

minimize γ

subject to $Y \succ 0$,

$$\begin{bmatrix} \tilde{A}Y + Y\tilde{A}^T + \tilde{B}L + L^T\tilde{B}^T & \tilde{B} & YW^T \\ B^T & -I & 0 \\ WY & 0 & -\gamma I \end{bmatrix} \prec 0, \tag{45}$$

which is an LMI optimization problem (Boyd et al., 1994; Šiljak & Stipanović, 2000) in the scalar variable $\gamma = 1/\alpha^2$, and the matrix variables L and Y (Y is a scaled inverse of P) with an imposed structure as follows:

$$L = \begin{bmatrix} L_1 & 0 & 0 \\ L_2 & L_3 & L_1 \end{bmatrix}, \quad Y = \begin{bmatrix} Y_1 & 0 & 0 \\ 0 & Y_2 & 0 \\ 0 & 0 & Y_1 \end{bmatrix}. \tag{46}$$

A structure for matrices L and Y as given in Eq. (46) guarantees that $\tilde{K} = LY^{-1}$ will have the same structure as in Eq. (43) (Šiljak & Stipanović, 2000).

It is important to notice that the design procedure for the feedback gain of the first subsystem is included in the above analysis and therefore does not have to be treated as a special case. Contraction of the feedback gains is carried over to the original space according to the analysis presented in Section 3 and the results presented in Šiljak and Stipanović (2001).

In terms of complexity, let us assume a leader follower formation as in Fig. 3 consisting of two platoons with m vehicles each (i.e., $2m-1$ vehicles in the formation). In the proposed design one has to solve for only one gain matrix $\tilde{K}$ in Eq. (43) with 12 unknown variables (since each $\tilde{K}_i$, $i \in \{1,2,3\}$, has four variables), compared to the centralized gain matrix with $(4m-2)(8m-6) = 32m^2 - 40m + 12$ entries (since the number of inputs is $2(m-1) = 4m-2$ and the number of state variables is $2 + 4(2(m-1)) = 8m-6$).

Here, it is important to point out that the term due to "spillover" is a coupling term between fully decoupled subsystems as shown in Eq. (22). As discussed in the previous section, since matrix $(\tilde{A}_D + \tilde{B}_D\tilde{K}_D)$ is block diagonal and $\tilde{B}_D\Delta\tilde{K}$ is a lower triangular matrix, stability of $(\tilde{A}_D + \tilde{B}_D\tilde{K}_{DM})$ is equivalent to stability of $(\tilde{A}_D + \tilde{B}_D\tilde{K}_D)$ (i.e., without perturbation $\tilde{w}$). Unfortunately, this is not the case when the robust stability problem is considered, but since the goal is to have a fully decoupled design at the subsystem level in the expanded space, the "spillover" is neglected in the first step of the design procedure (i.e., when one computes the stabilizing controllers while maximizing the sector bounds for the perturbations). Then, by comparing Eqs. (40) and (42) an approximation of the robust performance can be computed using the substitution $\tilde{w} \leftrightarrow \tilde{w} + \tilde{B}_D\Delta\tilde{K}\tilde{e}$ and the following estimation:

$$\|\tilde{w}\|^2 \leqslant \alpha^2\|\tilde{W}\tilde{e}\|^2 - \|\tilde{B}_D\Delta\tilde{K}\tilde{e}\|^2$$
$$\Rightarrow \|\tilde{w} + \tilde{B}_D\Delta\tilde{K}\tilde{e}\|^2 \leqslant \alpha^2\tilde{e}^T\tilde{W}^T\tilde{W}\tilde{e},$$

or

$$\tilde{w}^T\tilde{w} \leqslant \tilde{e}^T(\alpha^2\tilde{W}^T\tilde{W} - \Delta\tilde{K}^T\tilde{B}_D^T\tilde{B}_D\Delta\tilde{K})\tilde{e}$$
$$\Rightarrow [\tilde{w}^T \quad \tilde{e}^T] \begin{bmatrix} I & \tilde{B}_D\Delta\tilde{K} \\ \Delta\tilde{K}^T\tilde{B}_D^T & \Delta\tilde{K}^T\tilde{B}_D^T\tilde{B}_D\Delta\tilde{K} - \alpha^2\tilde{W}^T\tilde{W} \end{bmatrix} \times \begin{bmatrix} \tilde{w} \\ \tilde{e} \end{bmatrix} \leqslant 0, \tag{47}$$

where $\tilde{W} = \mathrm{diag}\{W, W, W\}$. So, the sector's shape changes from $\alpha^2\tilde{W}^T\tilde{W}$ to $\alpha^2\tilde{W}^T\tilde{W} - \Delta\tilde{K}^T\tilde{B}_D^T\tilde{B}_D\Delta\tilde{K}$, that is, the sector volume decreases since $\Delta\tilde{K}^T\tilde{B}_D^T\tilde{B}_D\Delta\tilde{K}$ is a nonnegative definite matrix. Nevertheless, how to incorporate the "spillover" effect into design of controllers is still an open problem. The proposed design is novel in the sense that it guarantees asymptotic stability of the nominal system in the case of restriction type (a) and also maximizes the sector bound on all possible perturbations that the nominal system can withstand without becoming unstable. Restriction type (b) offers more commodities in the design of stabilizing controllers but is more restrictive, and is not applicable in the case considered in this paper. Finally, for a rigorous mathematical analysis of how sector bounds on perturbations are mapped to the original space, the reader is referred to the analysis presented in Šiljak and Stipanović (2001). However, it is important to note that this is a procedure that is both straightforward (along the lines of the inclusion principle) and exact (that is, no approximation involved).

5. Examples

Let us again consider the group of five vehicles flying in the formation as shown in Fig. 3. Assume that the nominal speed v_d (in Eq. (11)) is $[300, 0]$ [ft/s]; desired distances between vehicles (in absolute values) are all equal to $|d_{i-1}| \equiv d = [400, 400]^T$ [ft] for both platoons and all i in Eq. (10), and perturbations are assumed to be sinusoidal functions with magnitudes equal to 10. Also, recall that the only restriction on the perturbation functions is that they be sector bounded. The design procedure as described in Sections 2–4 is applied to compute decentralized overlapping static feedback controllers. Design parameter α which determines the size of the perturbation sector in Eq. (44) was maximized at value 0.95 (this means that the allowed perturbations are of the scale of the state variables), and matrix W

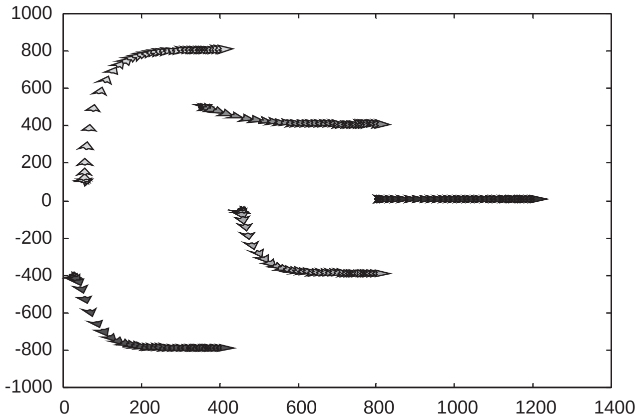

Fig. 4. Snapshots of the formation for one set of initial conditions ($v_d = [300, 0]$ [ft/s]; $d = [400, 400]^T$ [ft]).

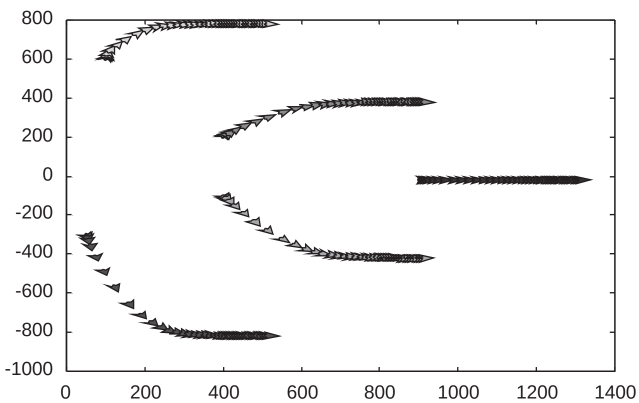

Fig. 5. Snapshots of the formation for a second set of initial conditions ($v_d = [300, 0]$ [ft/s]; $d = [400, 400]^T$ [ft]).

describing the shape of the sector was set to be the identity matrix (thus, from Eq. (44) one obtains the perturbation bound as $\|\tilde{w}_i\| \leqslant 0.95\|\tilde{e}_i\|$ for each subsystem). Simulation results are presented in Figs. 4 and 5 for two different sets of initial conditions using superimposed snapshots of the

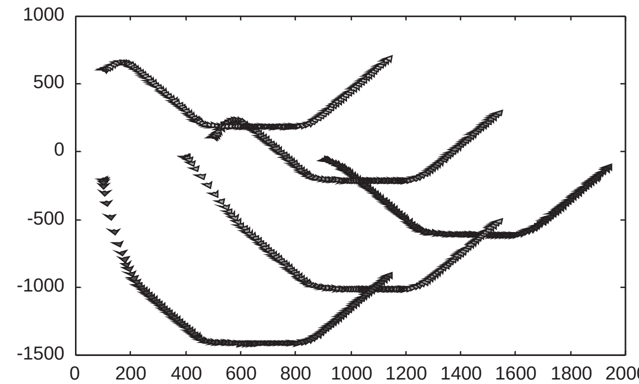

Fig. 7. Snapshots of the formation for the piecewise defined trajectory as shown in the lead vehicle trajectory ($\|v_d\| = 300$ [ft/s]; $d = [400, 400]^T$ [ft]).

formation at representative time instances, which are 40 nonuniform (depending on the nonuniform step size used in simulations) time intervals between 0 and 1.3[s]. Position coordinates are given in feet.

Horizontal distances between vehicles V1 and V2, and V2 and V3 (corresponding to Fig. 3) for the set of initial conditions for the simulation presented in Fig. 4, are given in Figs. 6a and b, respectively. Time is in seconds and distances are in feet.

In Fig. 7, the picture of 120 snapshots of the formation with a desired trajectory that is piecewise linear (as shown by the lead vehicle), is presented (simulation time $\sim$4[s]). The nominal speed v_d takes values $[300\cos(-\pi/3), 300\sin(-\pi/3)]$, $[300, 0]$, and $[300\cos(\pi/3), 300\sin(\pi/3)]$ [ft/s], respectively. The same sinusoidal perturbations as in the previous examples were used.

Finally, to illustrate a case in which the perturbations are outside the bounds determined by the scheme, large sinusoidal perturbations of magnitudes equal to 100 in appropriate units with a phase shift of $\pi/2$ [rad] were applied to the nominal system. These do not satisfy the sector conditions with parameters $\alpha = 0.95$ and $W = I$ (simulation time is

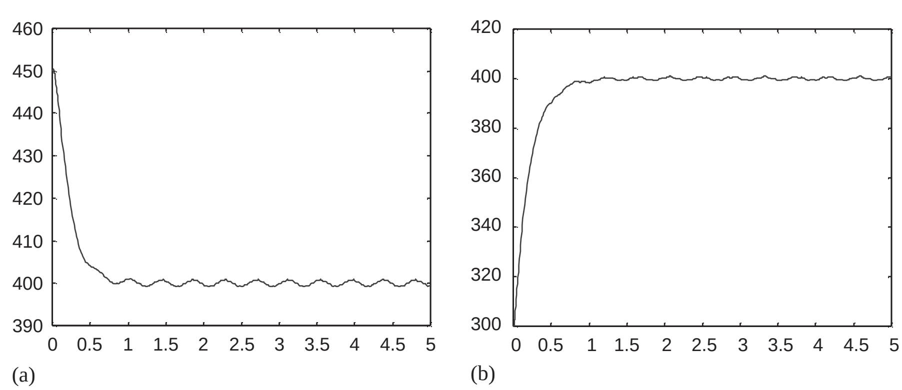

Fig. 6. Horizontal distances between: (a) vehicles V1 and V2; (b) vehicles V2 and V3; ($v_d = [300, 0]$ [ft/s]; $d = [400, 400]^T$ [ft]).

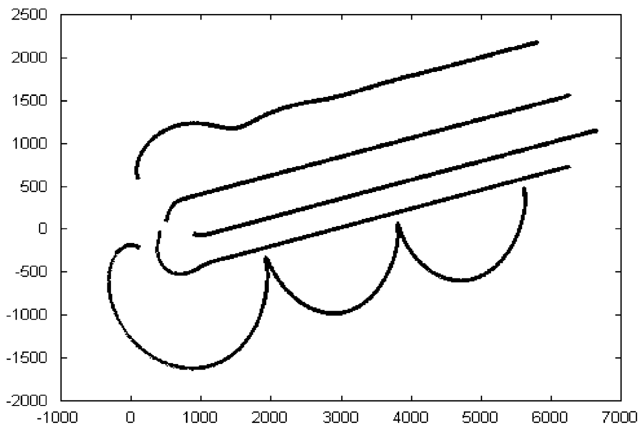

Fig. 8. Snapshots of the formation with perturbations outside of the computed allowed range ($v_d = [300, 0]$ [ft/s]; $d = [400, 400]^T$ [ft]).

20 s). In fact, a phase shift of $\pi/2$ [rad] implies that the perturbations are not equal to zero at the origin of the sector, that is, do not satisfy any sector condition. As expected and as shown in Fig. 8, the formation is not stable.

6. Conclusions

In this paper, an efficient method to design decentralized control laws for a formation of unmanned aerial vehicles with overlapping information flow has been proposed. The formation is modeled as an interconnected system in which the subsystems are defined in such a way that their states are composed of measurements assumed to be available in each vehicle. Static state feedback control laws were designed in the expanded space using an application of convex programming tools, and then contracted back to the original space for implementation. Since the optimization algorithms are formulated in the expanded space where subsystems are disjoint, this method offers significant reduction in computational time due to the possibility of parallel processing. As an example, the procedure was applied to a perturbed leader–follower type of formation of five vehicles, and the obtained results are promising.

In the future research, two possible directions are considered. First is the application of dynamic controllers and the second is the possibility of considering different overlapping structure constraints for a formation of UAVs. In our opinion, the biggest problem to overcome would be the problem of contraction of the controllers from the expanded space under more complicated overlapping information structure constraints.

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