Turning Kinematically

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Overview

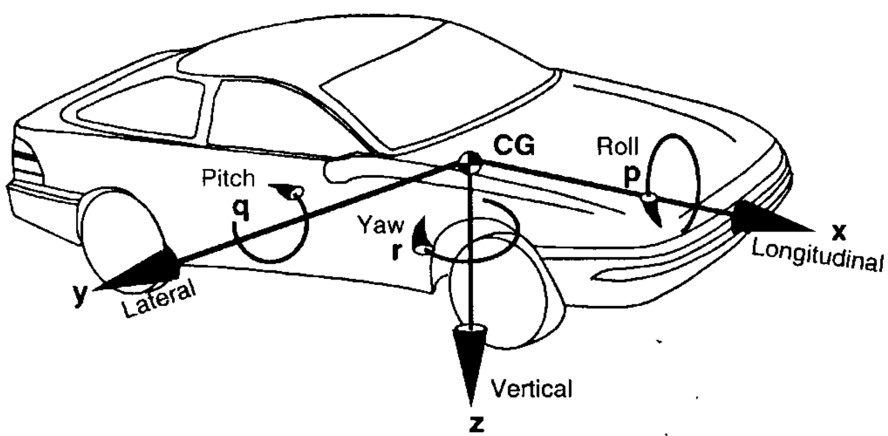
- Recall vehicle coordinate systems and dynamic equations for rigid body
- Global coordinate frame and trajectory calculations
- Yaw on purpose; differential vs. kinematic (or Ackerman) steering
- **Problem 1**: Derivation of the body-fixed kinematic relations for a single-axle, differentially-steered vehicle
- **Problem 2**: Derivation of the inertial-frame kinematic relations for a single-axle, differentially-steered vehicle
- Simulation and animation of the kinematic, differentially-steered vehicle
- **Problem 3**: Adapt code provided for animating simulation of DaNI vehicle driving a 1 meter square
- Kinematic steering – 'tricycle' model and two-axle Ackerman
- Problem 4: Derive kinematic model for two-axle, four-wheel, front kinematically steered vehicle and simulate basic turning maneuver

HW #3 requires submission of Problems 1, 2 and 3

Recall vehicle-fixed coordinate system; SAE standard

Ground vehicle coordinate systems commonly employ a coordinate system standardized by SAE*.

Consider the standard SAE coordinate system and terminology.



*SAE = Society of Automotive Engineers

SAE vehicle axis system

x = forward, on the longitudinal plane of symmetry

y = lateral out the right side of the vehicle

z = downward with respect to the vehicle

p = roll velocity about the x axis

q = pitch velocity about the y axis

r = yaw velocity about the z axis

Recall state space formulation for vehicle ***dynamic*** states in body-fixed axes

Write in terms of velocity states rather than momentum

$$\dot{p}_x = m\dot{v}_x = F_x - m\omega_y v_z + m\omega_z v_y$$

$$\dot{p}_y = m\dot{v}_y = F_y - m\omega_z v_x + m\omega_x v_z$$

$$\dot{p}_z = m\dot{v}_z = F_z - m\omega_x v_y + m\omega_y v_x$$

$$\dot{h}_x = I_x\dot{\omega}_x = T_x - I_z\omega_y\omega_z + I_y\omega_z\omega_y$$

$$\dot{h}_y = I_y\dot{\omega}_y = T_y - I_x\omega_z\omega_x + I_x\omega_x\omega_z$$

$$\dot{h}_z = I_z\dot{\omega}_z = T_z - I_y\omega_x\omega_y + I_x\omega_y\omega_x$$

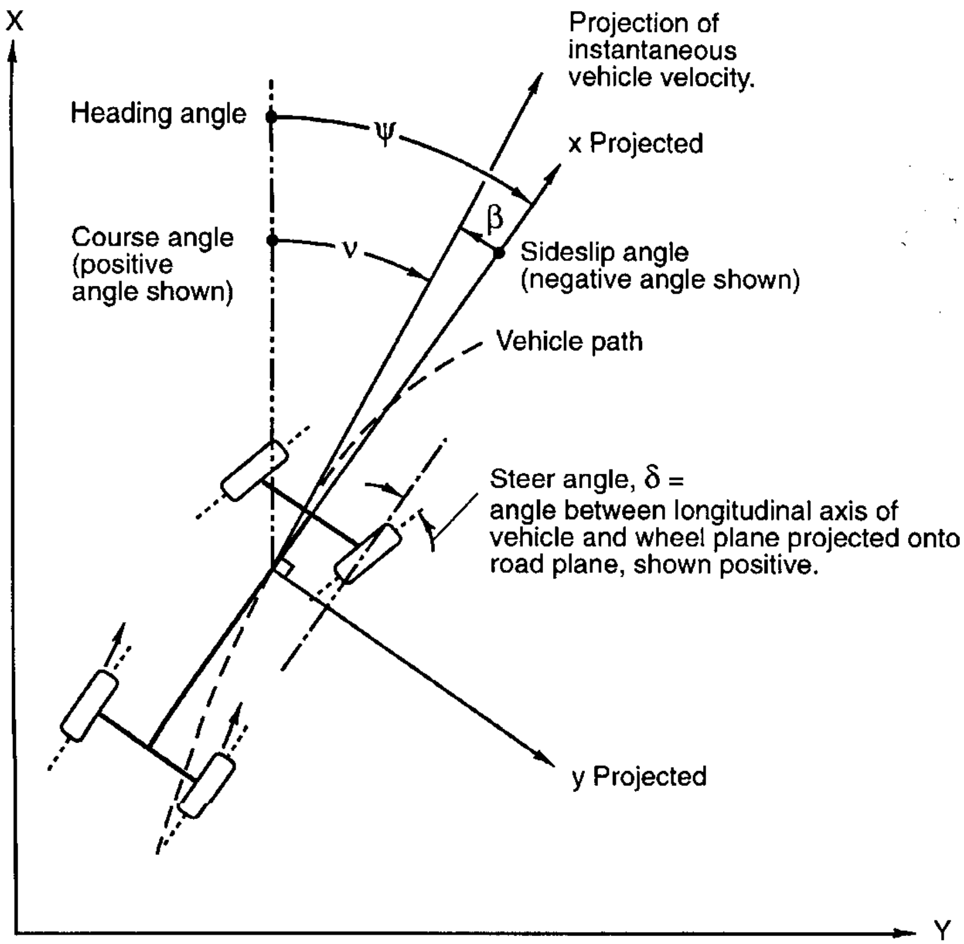
For a single rigid body, we find the velocities by integrating these six equations given known (modeled) externally applied forces/torques (the tough part!).

As we constrain degrees of freedom, naturally we have fewer equations – few degrees of freedom to determine.

Recall we went through this exercise in showing how the performance model for a vehicle was derived. We do the same later for the turning models.

Note these velocities in the body-fixed frame.

Recall the Earth-fixed (or global) coordinate system



X = forward travel

Y = travel to the right

Z = vertical travel (+down)

ψ = heading angle (between x and X in ground plane)

ν = course angle (between vehicle velocity vector and X axis)

β = sideslip angle (between x and vehicle velocity vector)

This is the frame we use to visualize actual motion of our vehicle.

<u>Example</u>: vehicle trajectory in 2D motion

For motion on *XY* plane and given forward and lateral velocity and yaw velocity relative to the body-fixed axes, the trajectory and orientation of the body in Earth-based coordinates can be found from:

$$\dot{X} = v_x \cos(\psi) - v_y \sin(\psi)$$

$$\dot{Y} = v_x \sin(\psi) + v_y \cos(\psi)$$

$$\dot{\psi} = \omega_z$$

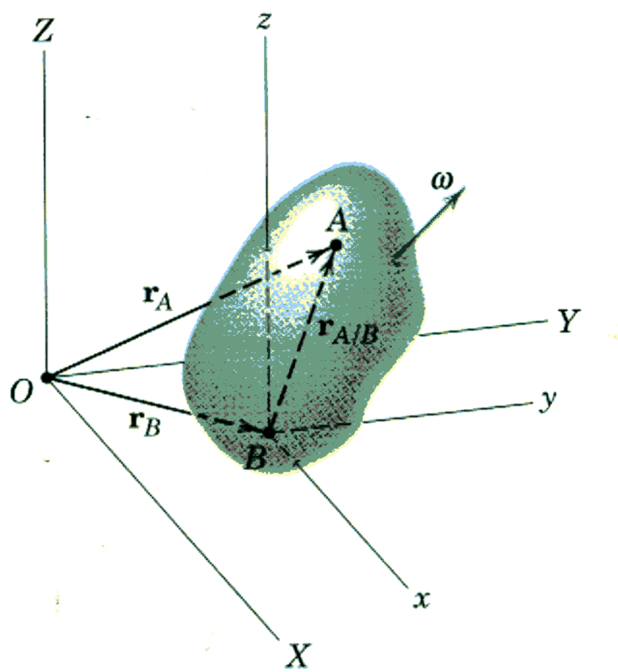
These three differential equations must be solved to determine the <u>trajectory and orientation</u> of a vehicle moving in a plane. The body-fixed velocities must come either from solving the dynamic equations or from kinematic models.

<u>Review</u> concepts for translating and rotating reference frames, helpful in understanding vehicle turning models

It is helpful to have an understanding of the coordinate systems used for rigid body analysis, and the terminology employed for these applications.

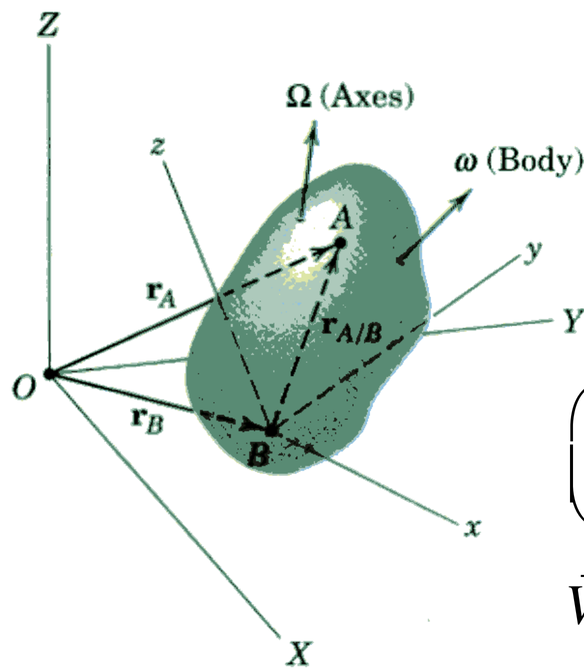
<u>Key result:</u> transformation of a time derivative

For example, see Meriam & Kraige (1997), Ch. 7



<u>Translating reference</u> axes, with body rotating with angular velocity, $\boldsymbol{\omega}$.

<u>Rotating reference</u> axes rotate with velocity $\boldsymbol{\Omega}$, and body rotates with angular velocity, $\boldsymbol{\omega}$.

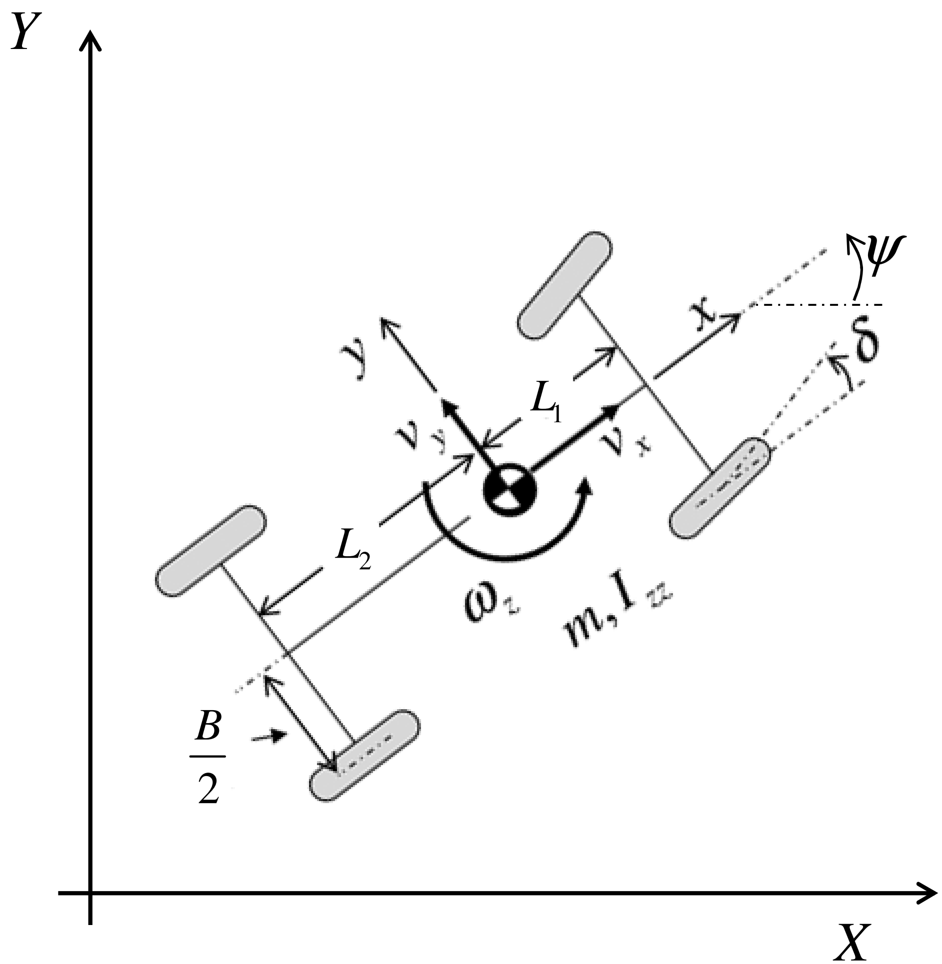


This relationship between vector quantities in xyz and XYZ will prove very useful.

$$\left(\frac{d\vec{V}}{dt}\right)_{XYZ} = \left(\frac{d\vec{V}}{dt}\right)_{xyz} + \vec{\Omega}\times\vec{V}$$

$\vec{V}$ can be any vector quantity.

Example Problems: transferring velocity to wheel locations for a 2D turning vehicle (refer to Karnopp & Margolis problems 1.11 and 1.12)



See following slides

These two examples illustrate useful methods that can be used in studying vehicle motion.

It is worth working through them.

Refer to the handout posted on course log – excerpt from Karnopp and Margolis (2008).

Example: Problem from Karnopp and Margolis (2008) P1.11

1.11 Figure P1.11 shows the top view of a vehicle that has mass m and c.g. moment of inertia about the axis out of the page, I_g. The center of gravity is located a distance a from the front axle and a distance b from the rear axle. The half-width of the vehicle is $w/2$. The front wheels can be steered, indicated by the steer angle δ. A body-fixed coordinate frame is attached to the vehicle at its center of gravity and aligned as shown. The body-fixed velocity components of the center of gravity and the yaw angular velocity are indicated.

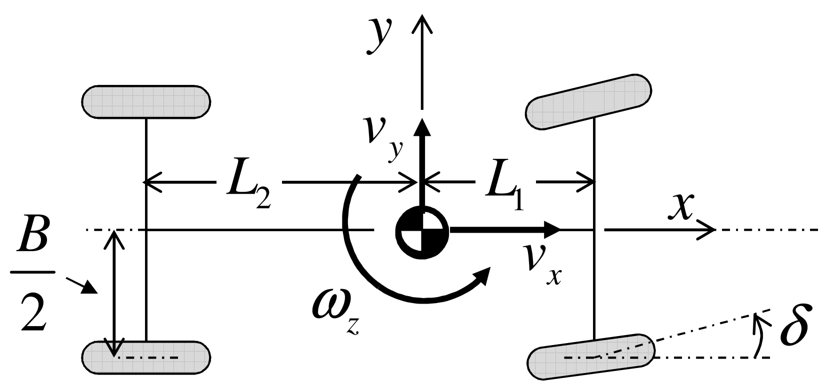
(a) Using arrows and symbols, transfer the c.g. velocity to body-fixed directions at the four wheels.

(b) If each wheel is constrained to have no velocity perpendicular to the plane of the wheel, state the kinematic constraints for each wheel.

(cf. Karnopp & Margolis, eqs. 1.18)

$$\vec{V}_p = \vec{V}_o + \vec{\Omega} \times \vec{R}$$

$$\vec{A}_p = \vec{A}_0 + \dot{\vec{\Omega}} \times \vec{R} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{R} \right)$$



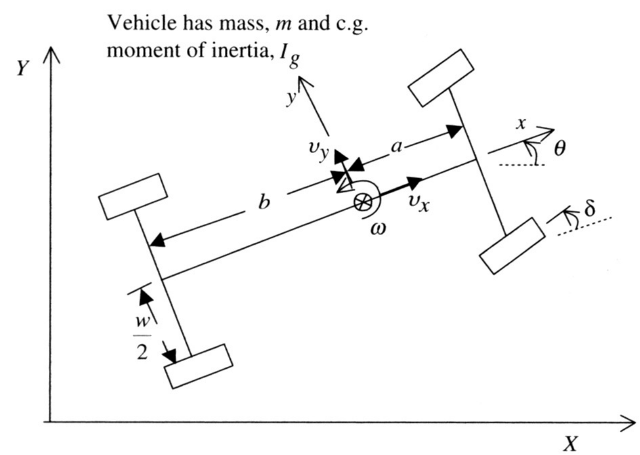


Figure P1.11

Solution to P 1.11 from Karnopp & Margolis (2008)

Problem 1.11
The velocity transfer formula from Eqs. (1.18) was used to transfer the velocity components from the c. g. to the center of each wheel. The cross product terms are realized by using one's right hand and crossing the $\boldsymbol{\omega}$ – vector pointing out of the page into the appropriate components of the $\mathbf{r}$-vector consisting of lengths a and $\frac{w}{2}$ for the front wheels and b and $\frac{w}{2}$ for the rear wheels.

If the sideways velocity of each wheel is constrained to be zero, then

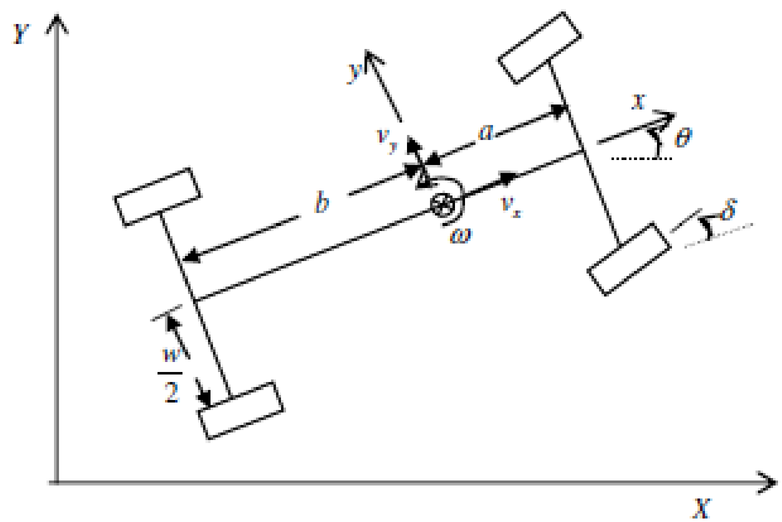
Note how 'sideways' velocity constraint is used.

$(v_y + a\omega)\cos\delta - (v_x + \frac{w}{2}\omega)\sin\delta = 0$ at the front/right

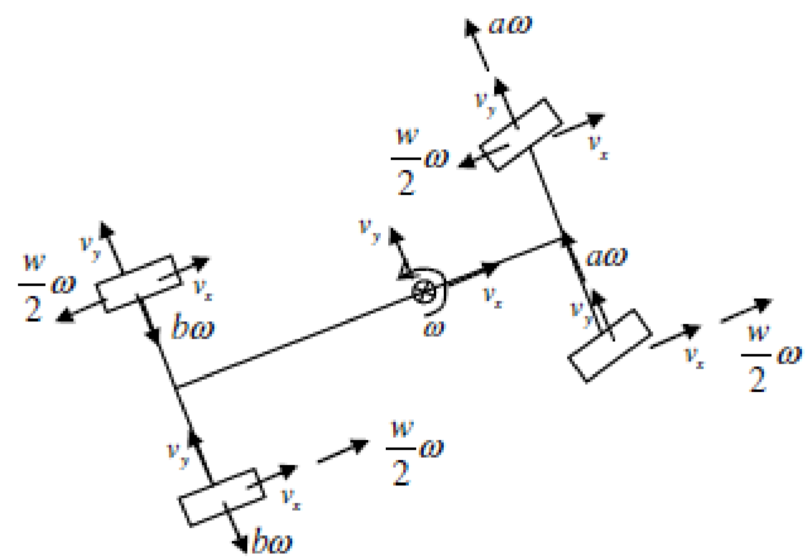
$(v_y + a\omega)\cos\delta - (v_x - \frac{w}{2}\omega)\sin\delta = 0$ at the front/left

$v_y - b\omega = 0$ at the rear/right

$v_y - b\omega = 0$ at the rear/left



Physical system



Velocity diagram

Example: Karnopp and Margolis (2008) P1.12

1.12 This system is identical to that of Problem 1.11, but this time inertial coordinates are used to locate the center of gravity of the vehicle, and the angle θ indicates the angular orientation (Figure P1.12).

(a) Using arrows and symbols, transfer the c.g. velocity to body-fixed directions at the four wheels.

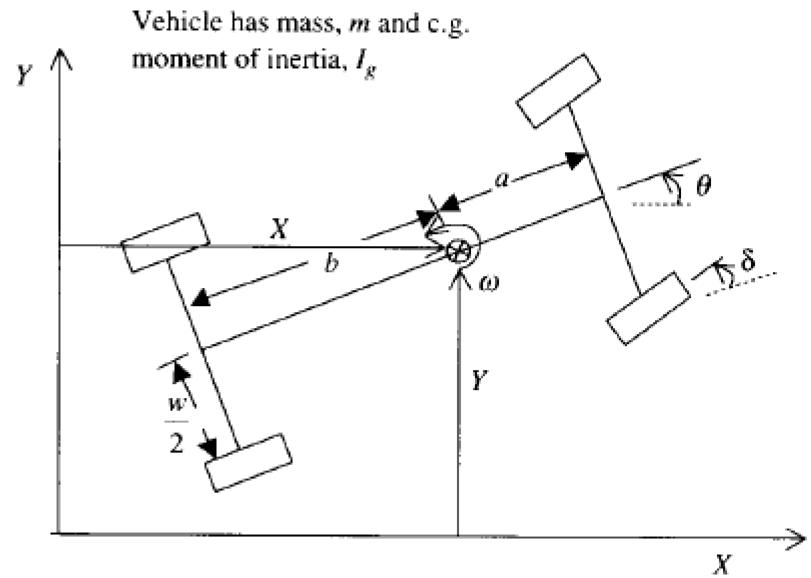


Figure P1.12

(b) If each wheel is constrained to have no velocity perpendicular to the plane of the wheel, state the kinematic constraints for each wheel.

Solution to Problem 1.12 from Karnopp & Margolis (2008)

Problem 1.12

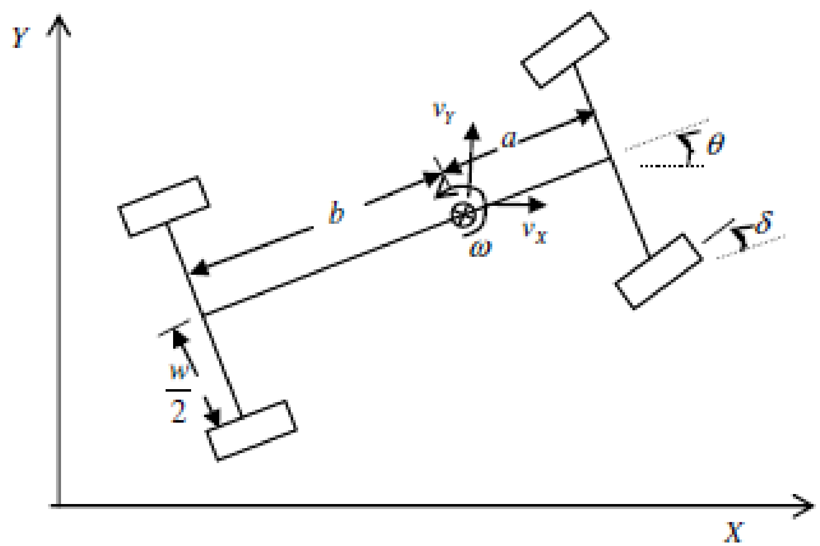
The only difference between this solution and that for Problem 1.11 is the c. g. velocity components are oriented in the inertial horizontal and vertical directions. All the cross product terms are the same and come from appropriate application of Eqs. (1.18). We should recognize that for inertial coordinates we can substitute $v_X = \dot{X}$ and $v_Y = \dot{Y}$. If the sideways velocity at each wheel is constrained to be zero, then the constraint relationships are,

$$v_Y \cos(\theta+\delta) + a\omega\cos\delta - v_X \sin(\theta+\delta) - \frac{w}{2}\omega\sin\delta = 0 \quad \text{at the front/right}$$

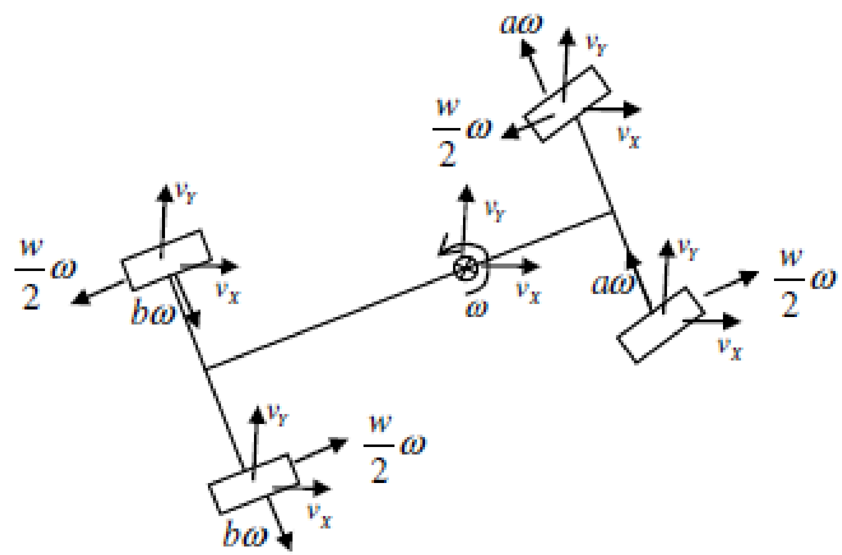
$$v_Y \cos(\theta+\delta) + a\omega\cos\delta - v_X \sin(\theta+\delta) + \frac{w}{2}\omega\sin\delta = 0 \quad \text{at the front/left}$$

$$v_Y \cos\theta - b\omega - v_X \sin\theta = 0 \quad \text{at the rear/right}$$

$$v_Y \cos\theta - b\omega - v_X \sin\theta = 0 \quad \text{at the rear/left}$$



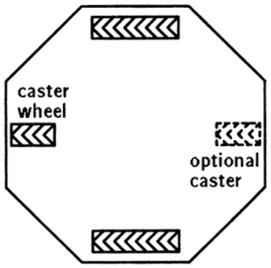
Physical system



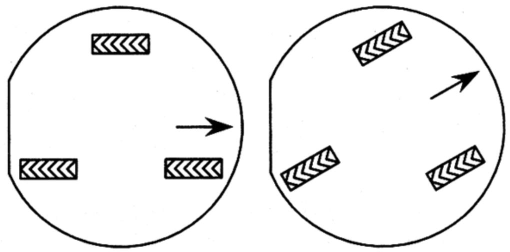
Velocity diagram

<u>Yaw on purpose</u>: common steering mechanisms

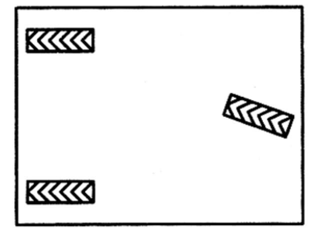
Differential steer

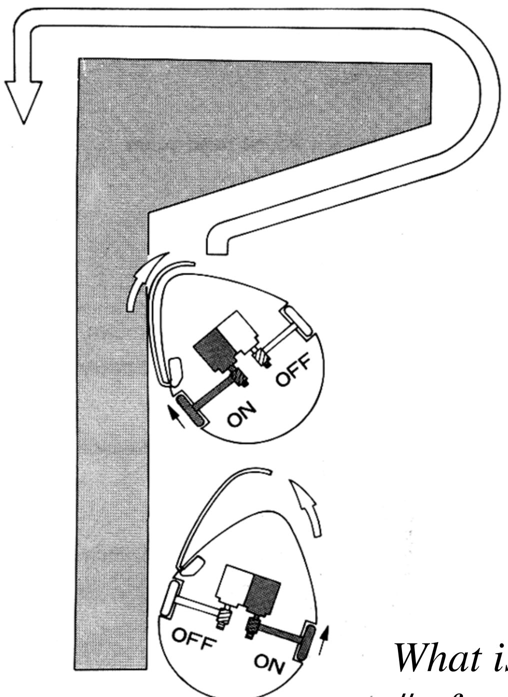


Synchro-drive

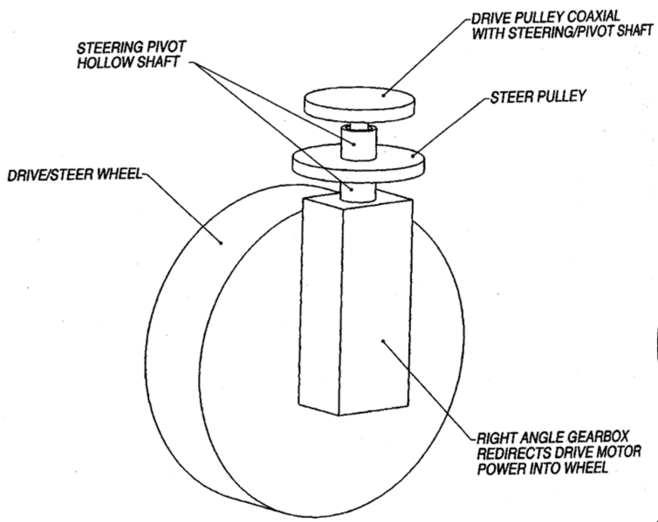


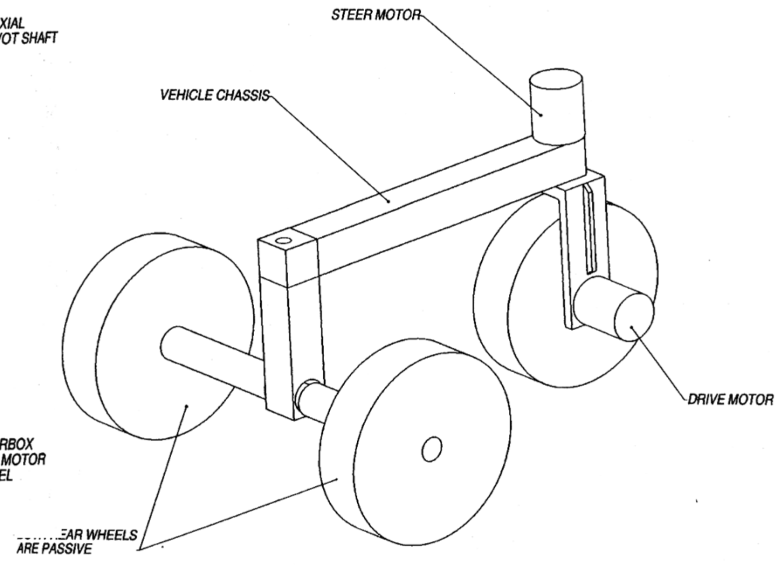
Tricycle





What is minimum
# of actuators?

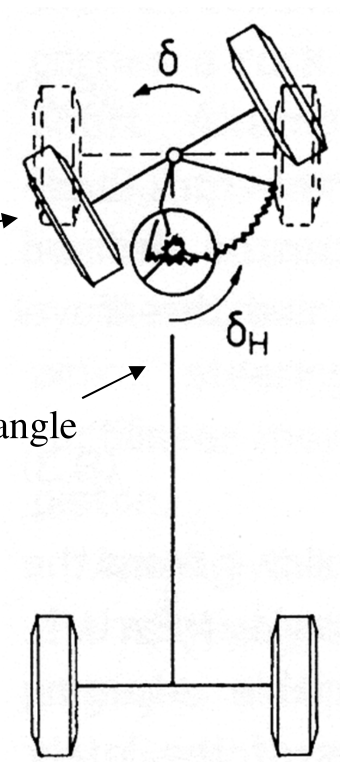




…and some systems also employ **Ackermann-type.**

<u>Yaw on purpose</u>: classical steering mechanisms

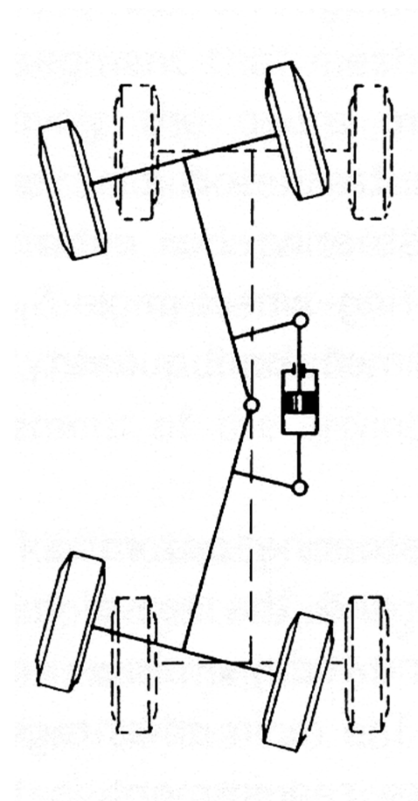
5th wheel steering



δ_H = 'hand wheel' angle

'turntable steering'

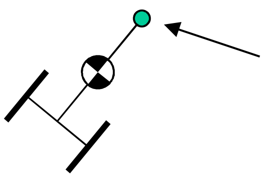
•Likely developed by the Romans, and preceded only by a 2 wheel cart.
•Consumes space
•Poor performance – unstable
•Longitudinal disturbance forces have large moment arms



•Articulated-vehicle steering
•Tractors, heavy industrial vehicles

Why is differential steering commonly used?

- Simple mechanism
- Does not take up a lot of space (e.g., used even for some larger, full-scale vehicles)
- There are disadvantages (tears up the terrain, wear on system, tires, etc.)
- For robotics, very common:



Sliding pivot

Realized as a caster?

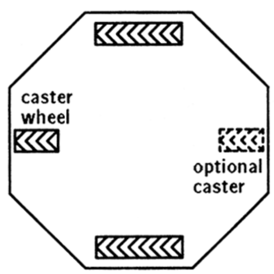
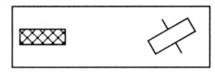
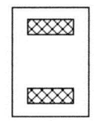
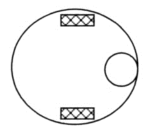
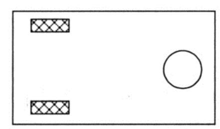
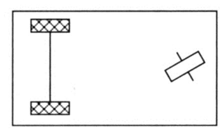
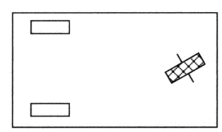
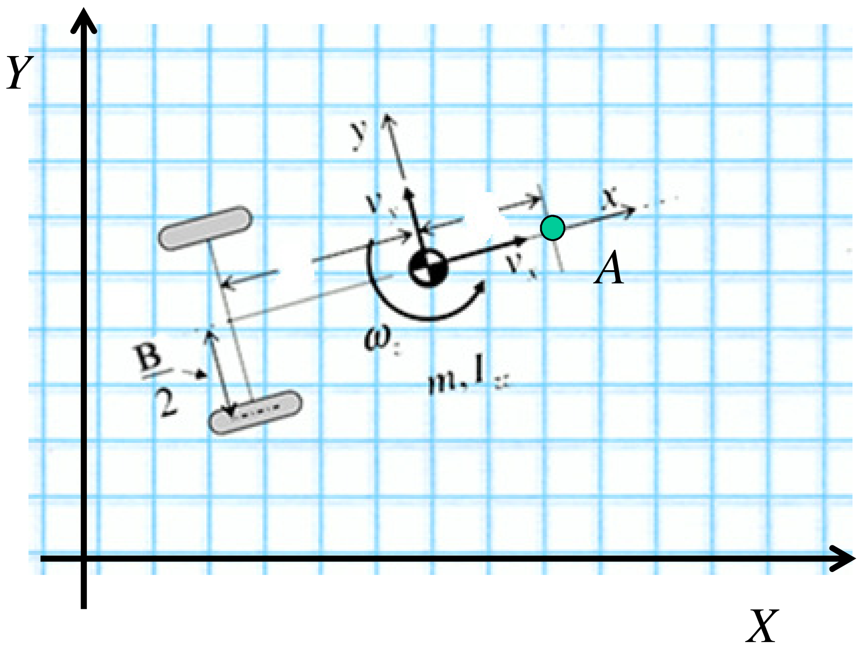


Table 2.1 from Siegwart, et al (2011) provides a nice overview of different wheel configurations used in some robotic vehicles.

Table 2.1
Wheel configurations for rolling vehicles

# of wheels	Arrangement	Description	Typical examples
2		One steering wheel in the front, one traction wheel in the rear	Bicycle, motorcycle
		Two-wheel differential drive with the center of mass (COM) below the axle	Cye personal robot
3		Two-wheel centered differential drive with a third point of contact	Nomad Scout, smartRob EPFL
		Two independently driven wheels in the rear/front, one unpowered omnidirectional wheel in the front/rear	Many indoor robots, including the EPFL robots Pygmalion and Alice
		Two connected traction wheels (differential) in rear, one steered free wheel in front	Piaggio minitrucks
		Two free wheels in rear, one steered traction wheel in front	Neptune (Carnegie Mellon University), Hero-1

Problem 1: A single-axle kinematic vehicle moving and turning in a plane



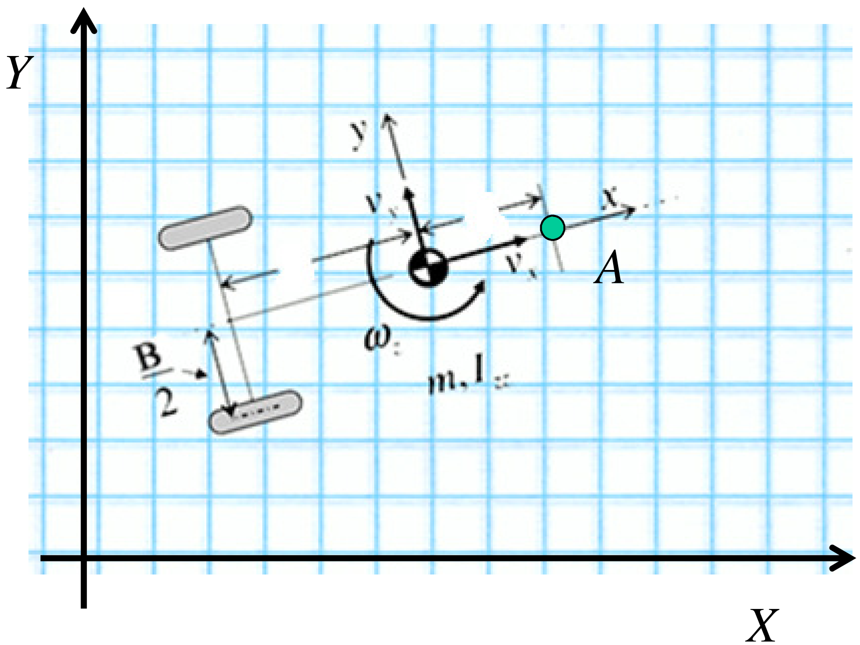
For the simple vehicle model shown to the left, there are negligible forces at contact point *A*, which slides on the ground. This could be a pivot, caster, or some other omni-directional type wheel.

The yaw motion and stability of this vehicle about its CG are of interest, given the angular (differential) speed inputs at the two wheels. Assume the driven wheels roll without slip and cannot slip laterally (along *y*). Designate the right wheel '1' and the left '2'.

Submit a derivation of the three kinematic relations in the body-fixed frame, $\dot{\mathbf{q}} = \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = ?$

The intent of this problem is for you to review relevant principles from basic kinematics as applied to this vehicle motion problem. Fill in all the blanks as if you were trying to explain it to a younger student.

<u>Problem 1:</u> (partial solution)



Assuming the wheels roll without slip, the translational velocities of the vehicle at the axle of each wheel are:

$$\left.\begin{aligned} v_{1x} &= R_w \omega_1 \\ v_{2x} &= R_w \omega_2 \end{aligned}\right\} \begin{array}{l}\text{Velocities at} \\ \text{each wheel}\end{array}$$

<u>You can show</u> that for a rigid body,

$$v_x = \tfrac{1}{2}\left(v_1 + v_2\right) = \tfrac{1}{2} R_w \left(\omega_1 + \omega_2\right)$$

If you assume that the velocity along the rear axle in the *y* direction is constrained to be zero (no lateral slip), <u>you can show</u> that:

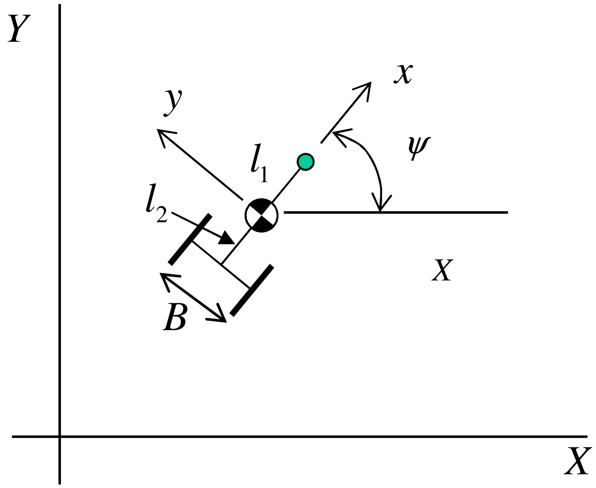
$$v_y = l_2 \omega_z$$

Finally, <u>it can also be shown</u> that for the vehicle body the yaw rate is,

$$\omega_z = \frac{R_w}{B}\left(\omega_1 - \omega_2\right)$$

These are the three kinematic relations relative to the vehicle body-fixed frame.

Problem 2: Kinematic 2D vehicle in inertial frame



Define the vehicle's kinematic state in the inertial frame by, $\mathbf{q}_I = \begin{bmatrix} X \\ Y \\ \psi \end{bmatrix}$

Velocities in the local (body-fixed) reference frame are transformed into the inertial frame by the rotation matrix,

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or, specifically, $\dot{\mathbf{q}} = \mathbf{R}(\psi)\cdot\dot{\mathbf{q}}_I$

Inverting, we arrive at the velocities in the global reference frame,

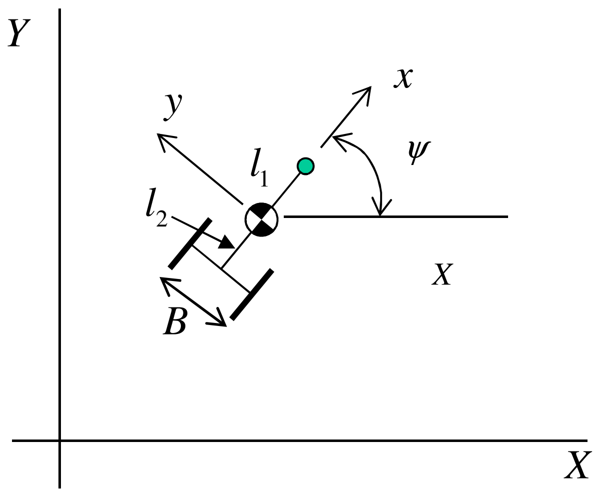
$$\dot{\mathbf{q}}_I = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} U \\ V \\ \Omega \end{bmatrix} = \Psi(\psi)\cdot\dot{\mathbf{q}} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix}$$

So, for our simple (single-axle) vehicle, the velocities in the inertial frame in terms of the wheel velocities are,

$$\dot{\mathbf{q}}_I = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{R_w}{2}(\omega_1+\omega_2)\cos\psi - \frac{l_2 R_w}{B}(\omega_1-\omega_2)\sin\psi \\ \frac{R_w}{2}(\omega_1+\omega_2)\sin\psi + \frac{l_2 R_w}{B}(\omega_1-\omega_2)\cos\psi \\ \frac{R_w}{B}(\omega_1-\omega_2) \end{bmatrix}$$

These three kinematic relations can be solved to find the kinematic state given input velocities.

Problem 2: Kinematic 2D vehicle in inertial frame



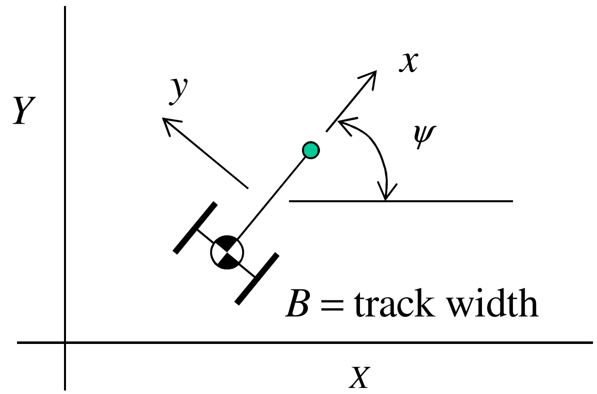
Show any missing steps and convince yourself of the derivation for the final form of the equations:

$$\dot{\mathbf{q}}_I = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{R_w}{2}(\omega_1+\omega_2)\cos\psi - \frac{l_2 R_w}{B}(\omega_1-\omega_2)\sin\psi \\ \frac{R_w}{2}(\omega_1+\omega_2)\sin\psi + \frac{l_2 R_w}{B}(\omega_1-\omega_2)\cos\psi \\ \frac{R_w}{B}(\omega_1-\omega_2) \end{bmatrix}$$

Submit your derivation and discussion.

Example: Differentially-driven single-axle vehicle with CG on axle

For a *kinematic* model of a differentially-driven vehicle, we assume there is **no slip**, and that the wheels have controllable speeds, ω_1 and ω_2. If the CG is on the rear axle,



$$l_1 = L$$
$$l_2 = 0$$

the velocities in the global reference frame are,

$$\dot{\mathbf{q}}_I = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{R_w}{2}(\omega_1 + \omega_2)\cos\psi \\ \frac{R_w}{2}(\omega_1 + \omega_2)\sin\psi \\ \frac{R_w}{B}(\omega_1 - \omega_2) \end{bmatrix}$$

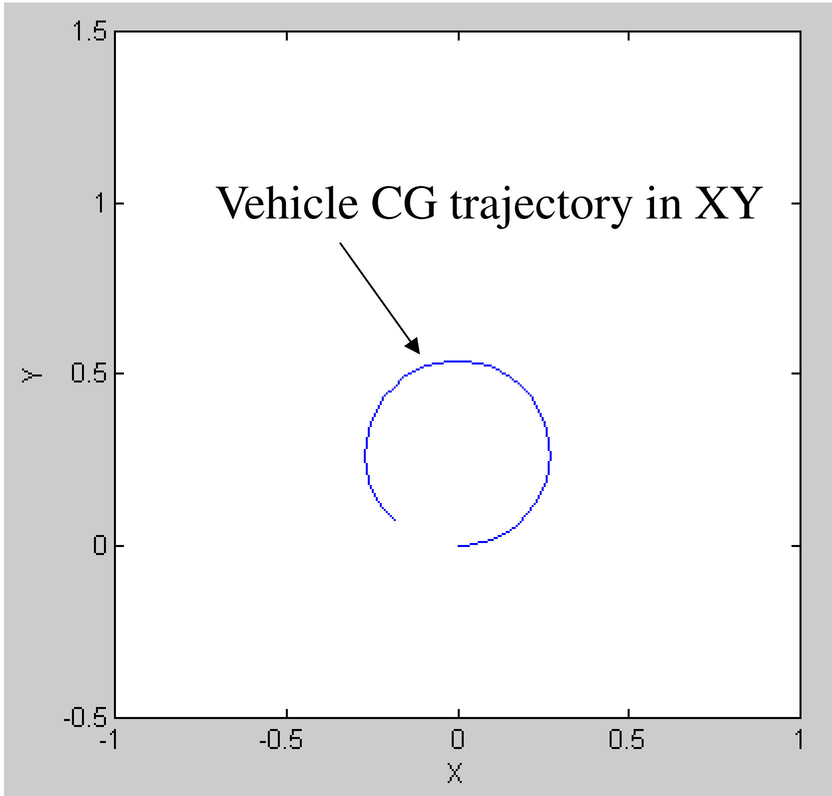
Note: this defaults to the 'mobile robot' model commonly found in robotics literature. Be mindful of when and why this model is applicable to a given situation.

Example: simulation of differentially-steered single-axle vehicle trajectory

A simple code in Matlab to compute and plot out the vehicle trajectory is given below.

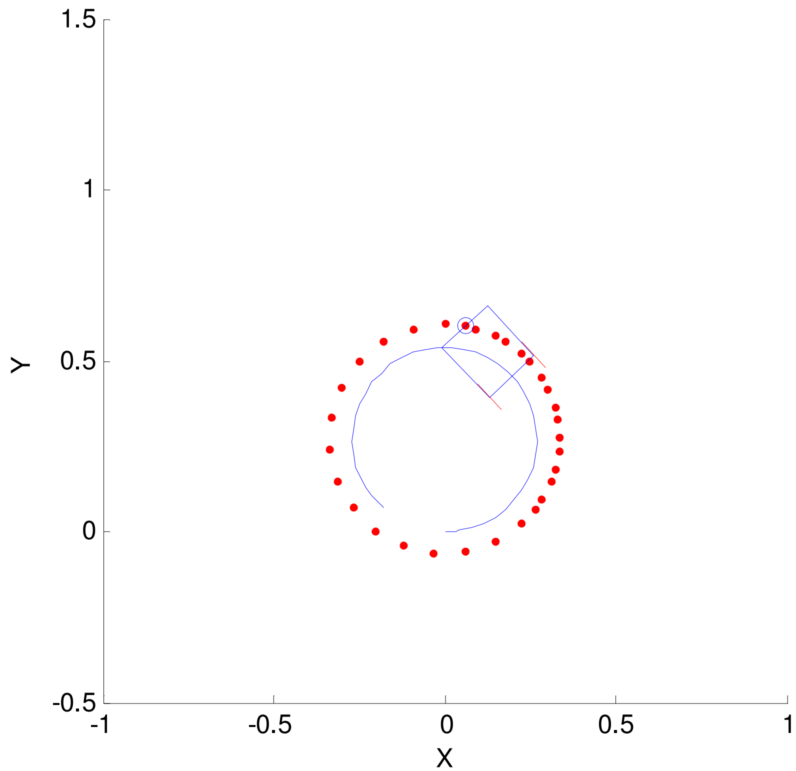
```
% Differentially-steered kinematic vehicle model
% Requires right (#1) and left (#2) wheel velocities, omegaw1 and omegaw2,
% as controlled inputs for single axle, to be passed as global parameters
% Wheel radius, Rw, and axle track width, B, are also required
% Updated 2/20/12 RGL
function Xidot = kin_vehicle_ds(t,Xi)
global Rw B omegaw1 omegaw2

X = Xi(1); Y = Xi(2); psi = Xi(3);
% NOTE: these are global coordinates
% These equations assume CG on single axle
Xdot = 0.5*cos(psi)*R_w*(omegaw1+omegaw2);
Ydot = 0.5*sin(psi)*R_w*(omegaw1+omegaw2);
psidot = R_w*(omegaw1-omegaw2)/B;

Xidot=[Xdot;Ydot;psidot];
```

```
% kin_vehicle_ds_test.m
clear all
global Rw B omegaw1 omegaw2
% Rw = wheel radius, B = track width
% omegaw1 = right wheel speed
Rw = 0.05; B = 0.18;
omegaw1 = 4; omegaw2 = 2;
Xi0=[0,0,0];
[t,Xi] = ode45(@kin_vehicle_ds,[0 10],Xi0);
N = length(t);
figure(1)
plot(Xi(:,1),Xi(:,2)), axis([-1.0 1.0 -0.5 1.5]), axis('square')
xlabel('X'), ylabel('Y')
```



<u>Example</u>: simulation of differentially-steered vehicle with 2D animation

A code in Matlab to plot out the vehicle trajectory including a simple graphing animation of the vehicle body/orientation is provided on the course log. This provides visual feedback on the model results.



The key elements of this code are:

1. Specify and plot initial location and orientation of the vehicle CG.
2. Initiate some 'handle graphics' functions for defining the 'body'.
3. Perform a fixed wheel speed simulation loop to find state, ***q***.
4. The state of the robot is used to define the position and orientation of the vehicle over time.
5. A simple routine is used to animate 2D motion of the vehicle by progressive plotting of the body/wheel positions.

<u>Download</u> the three files and place in the same working directory:
kin_vehicle_ds_2Danim.m, kin_vehicle_ds.m, kin_vehicle_ds_state.m

Problem 3: Study and adapt the example code for the 2D kinematic vehicle simulation and animation to model the DaNI robotic vehicle

Use proper geometric vehicle parameters for the DaNI vehicle and change the graphical rendering so it appears similar to that shown below. Program the wheel velocities (open loop timing) so that the DaNI will drive a 1 meter square. Use realistic wheel velocities.

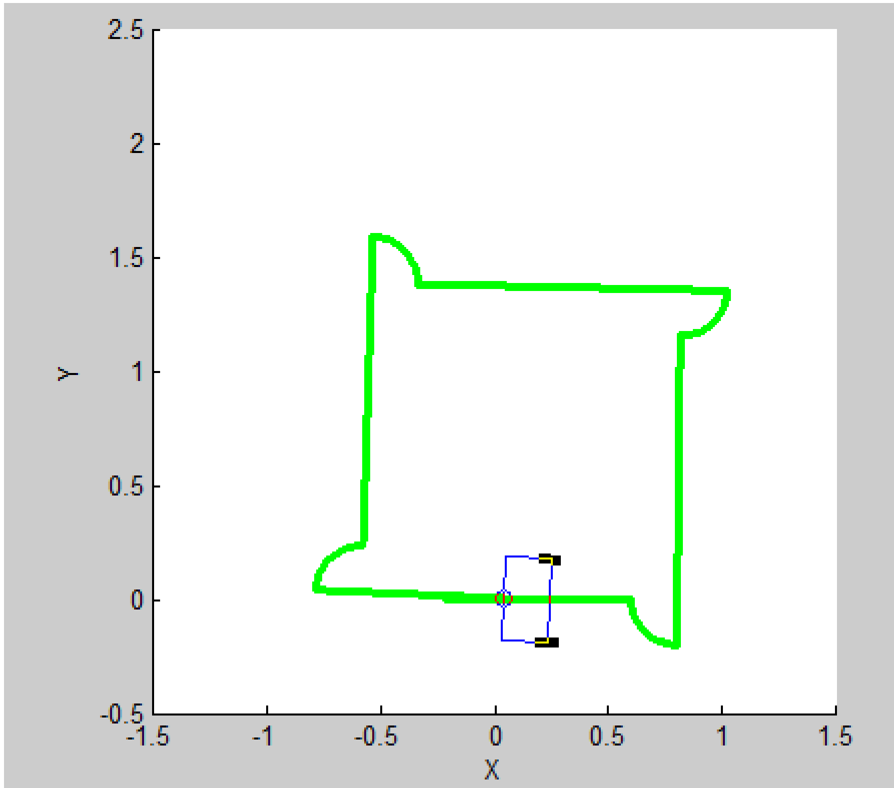
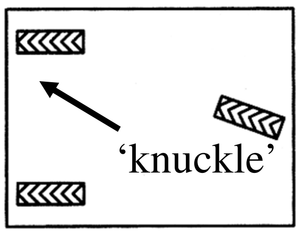
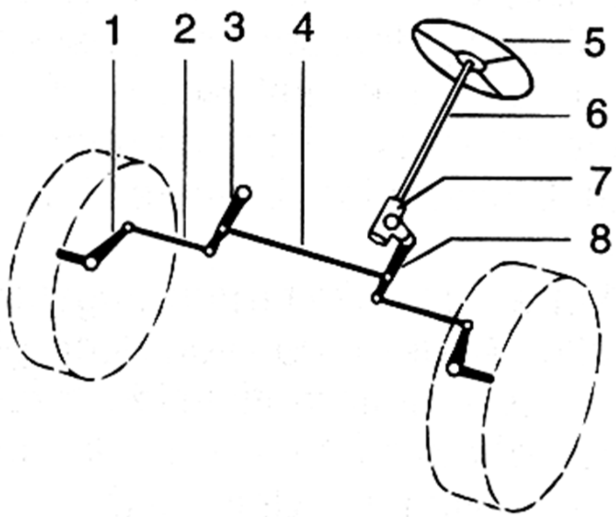


Figure out how to record a movie of your animation. **Submit** your code and the movie (optional) in electronic form. For large file sizes, please provide shared drop/box folder.

<u>Kinematic steering</u> usually refers to steering – i.e., trying to follow a desired heading - by yawing or turning a wheel or tire relative to the vehicle body

Tricycle

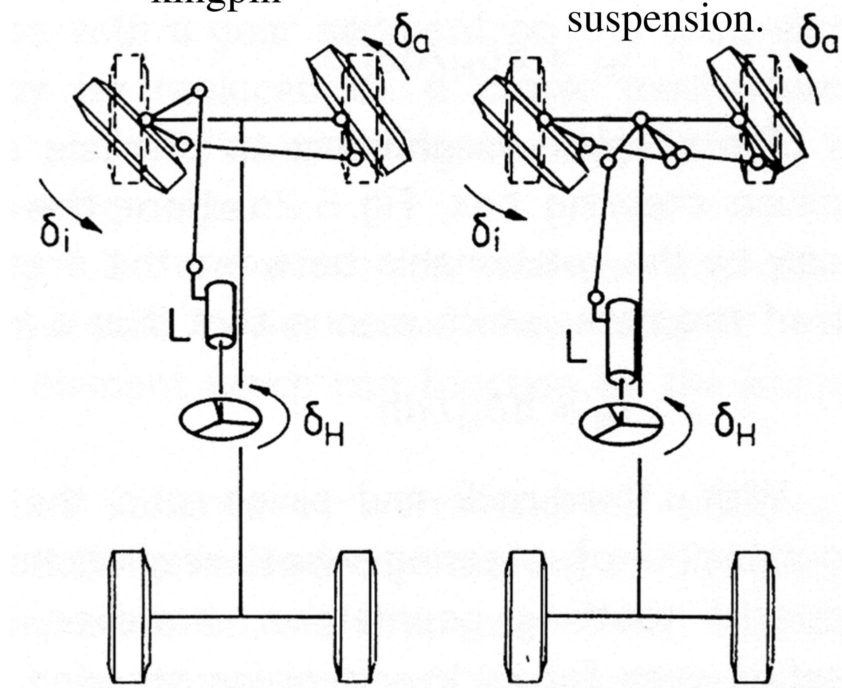




1. Steering arm
2. Drag link
3. Idler arm
4. Tie rod/rack
5. Steering wheel
6. Steering shaft
7. Steering box
8. Pitman arm

Rigid axle with kingpin

Divided track rods for independent suspension.



Consider the single-axle vehicle with front-steered wheel; rolling rear wheels

A wheeled vehicle is said to have *kinematic* steering when a wheel is actually given a steer angle, δ, as shown. A kinematic model for the steered basic vehicle in the inertial frame is given by the equations,

$$\dot{X} = v\cos\psi = R_w\omega\cos\psi$$

$$\dot{Y} = v\sin\psi = R_w\omega\sin\psi$$

$$\dot{\psi} = \frac{v}{L}\tan\delta$$

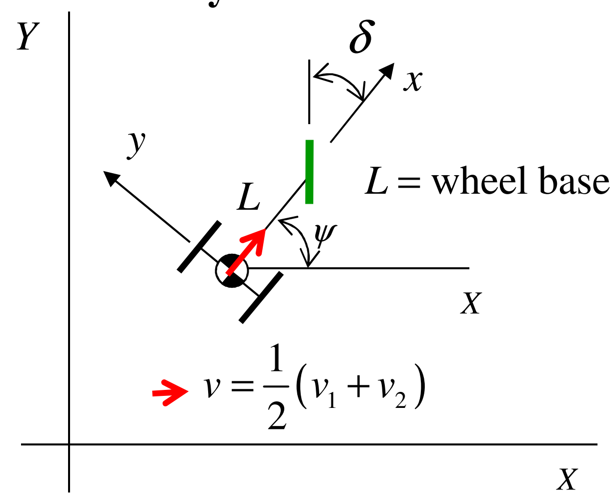
where it is assumed that the wheels do not slip, so we can control the rotational speed and thus velocity at each wheel-ground contact.

So, the input 'control variables' are velocity, $v=R_w\omega$, and steer angle, δ.

In this example, the CG is located on the rear axle.

These kinematic equations can be readily simulated.

'tricycle'

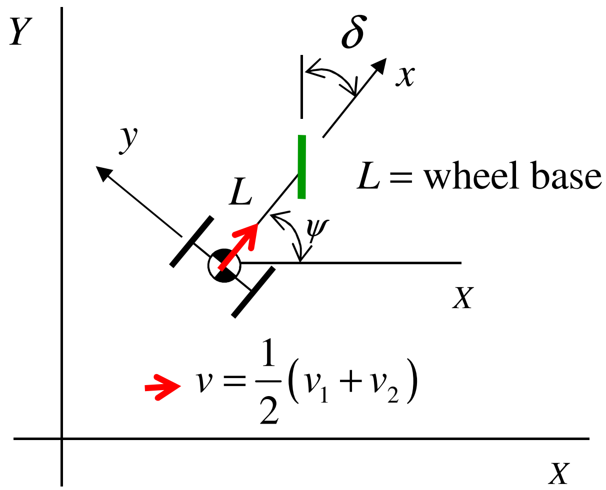


Note:

$$\omega_z = \dot{\psi} = \frac{v_t}{L}$$

$$\tan\delta = \frac{v_t}{v}$$

Example: derivation of equations



Note that the forward velocity at the front wheel is simply, v, but because of kinematic steering the velocity along the path of the wheel must be,

$$v_\delta = \frac{v}{\cos\delta}$$

This means that the lateral velocity at the front steered wheel must be,

$$v_t = v_\delta \sin\delta = v\tan\delta$$

Now we can find the angular velocity about the CG, which is located at the center of the rear axle as,

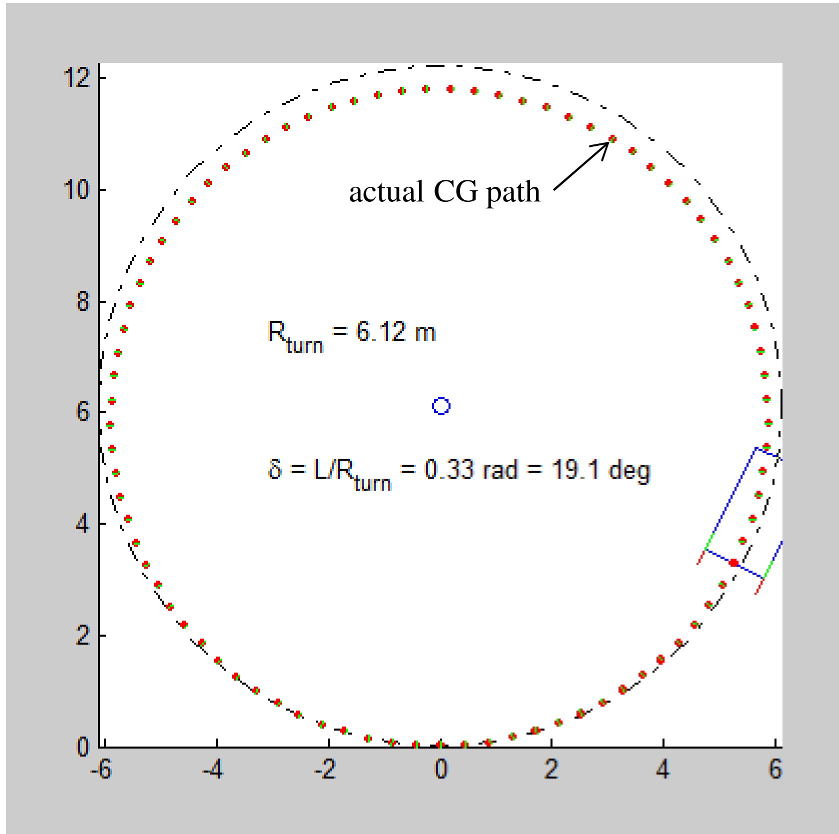
$$\omega_z = \frac{v}{L}\tan\delta$$

Example: simulation and animation of steered tricycle kinematic model

Consider a simple fixed steering angle

```
% --------------------------------------------------
% kin_vehicle_ks1.m
% revised 3/25/15 rgl
% --------------------------------------------------
function qdot = kin_vehicle_ks1(t,q)

global L B Rw vc delta_radc % modified RGL
% L is length between the front wheel axis and rear wheel axis, m
% vc is speed
% delta_radc is the current steering angle

% State variables
x = q(1); y = q(2); psi = q(3);

% Control variables
v = vc; delta = delta_radc;

% vehicle model
xdot = v*cos(psi);
ydot = v*sin(psi);
psidot = v*tan(delta)/L; % from CG to front steer
qdot = [xdot;ydot;psidot];
```

```
% geometric vehicle parameters
L = 2.040; % vehicle wheelbase, m
B = 1.164; % trackwidth, m
Rw = 13/39.37;  % Radius of wheel [m]
% desired turn radius
R_turn = 3*L;
delta_max_rad = L/R_turn;  % steering angle [deg]
```



Example files posted on course log:

kin_vehicle_ks1.m (ODEs)

kin_vehicle_ks1_2Danim.m

vehicle_state_ks1.m

Turning at low speed using kinematic (or Ackermann) steering

- What is **low-speed**?
  - Negligible centrifugal forces
  - Tires need not develop lateral forces to maintain course
- Pure rolling, no lateral sliding (minimum tire scrub).
- For proper geometry in the turn, the steer angles, δ, are given by:

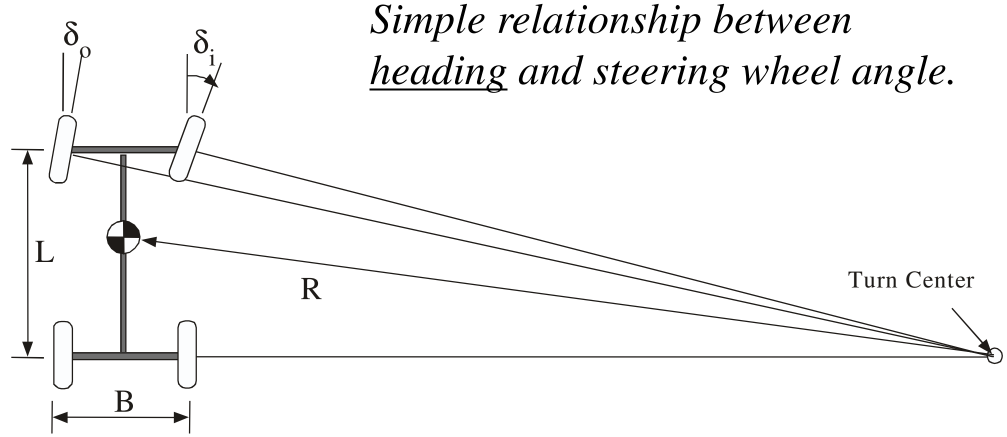
$$\delta_o \cong \frac{L}{R+B/2} < \delta_i \cong \frac{L}{R-B/2}$$

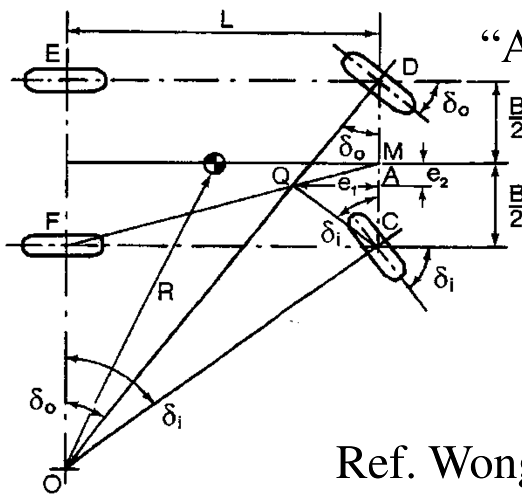
- The average value (small angles) is the **Ackermann angle**,

$$\delta_{\text{Ackermann}} = \frac{L}{R}$$

*Lankensperger was the inventor, Ackermann the patent agent.
http://en.wikipedia.org/wiki/Georg_Lankensperger

Simple relationship between <u>heading</u> and steering wheel angle.





"Ackermann steering geometry"

$$\cot\delta_o - \cot\delta_i = B/L$$

δ_o = steer angle of outside wheel

δ_i = steer angle of inside wheel

B = track

L = wheelbase

Ref. Wong, Ch. 5

"…to allow the front wheels of a carriage to individually follow the natural arc of its turning circle, rather than skidding and slipping when they are forced to each share a common arc with the conventional pivoted axle"

Some different steering geometries and relation between steering angles

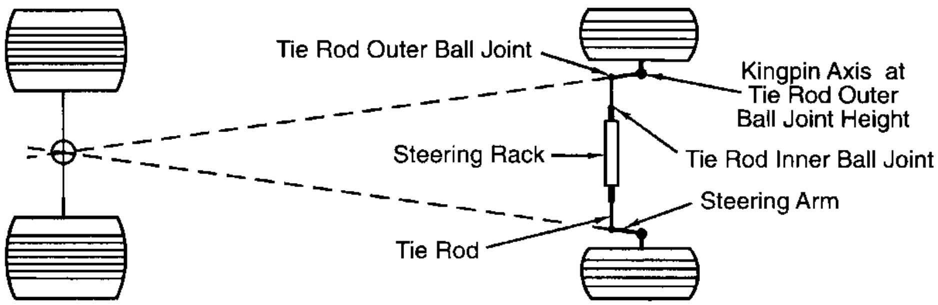
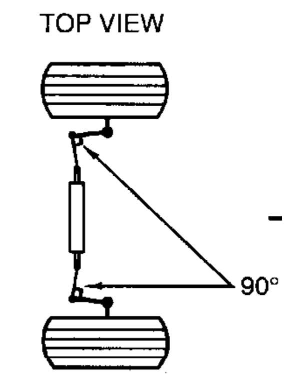
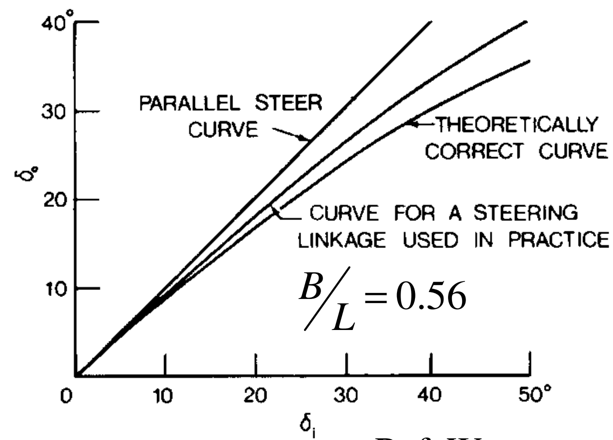


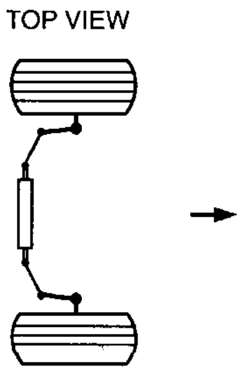
Figure 19.3 Ackermann geometry, with steering rack behind the axle line.



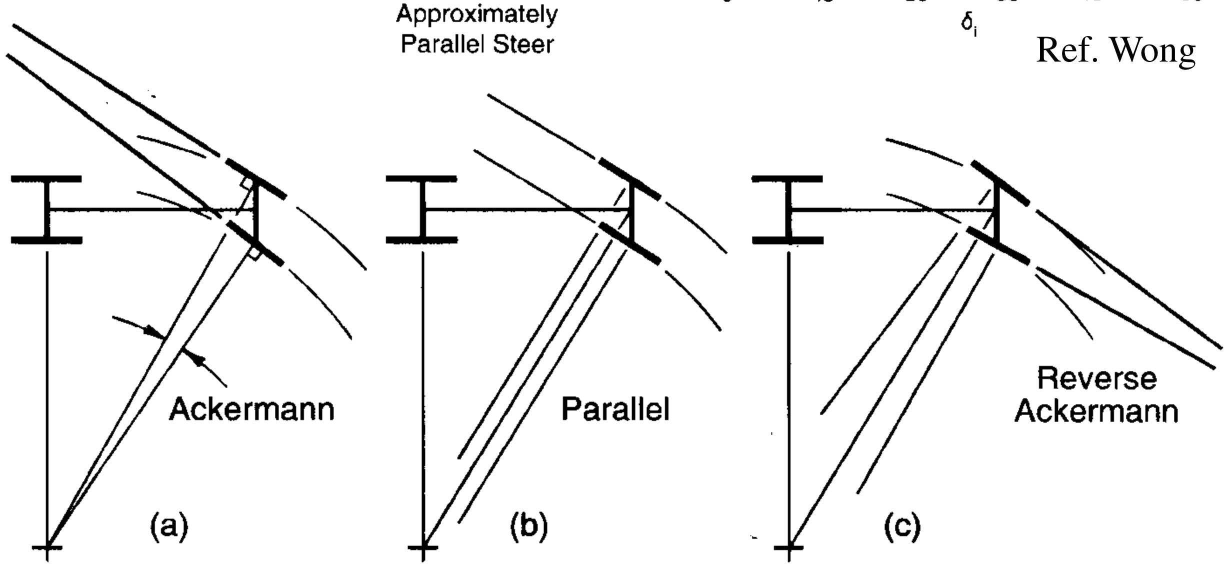
Approximately Parallel Steer



Ref. Wong



"More Than" True Ackermann



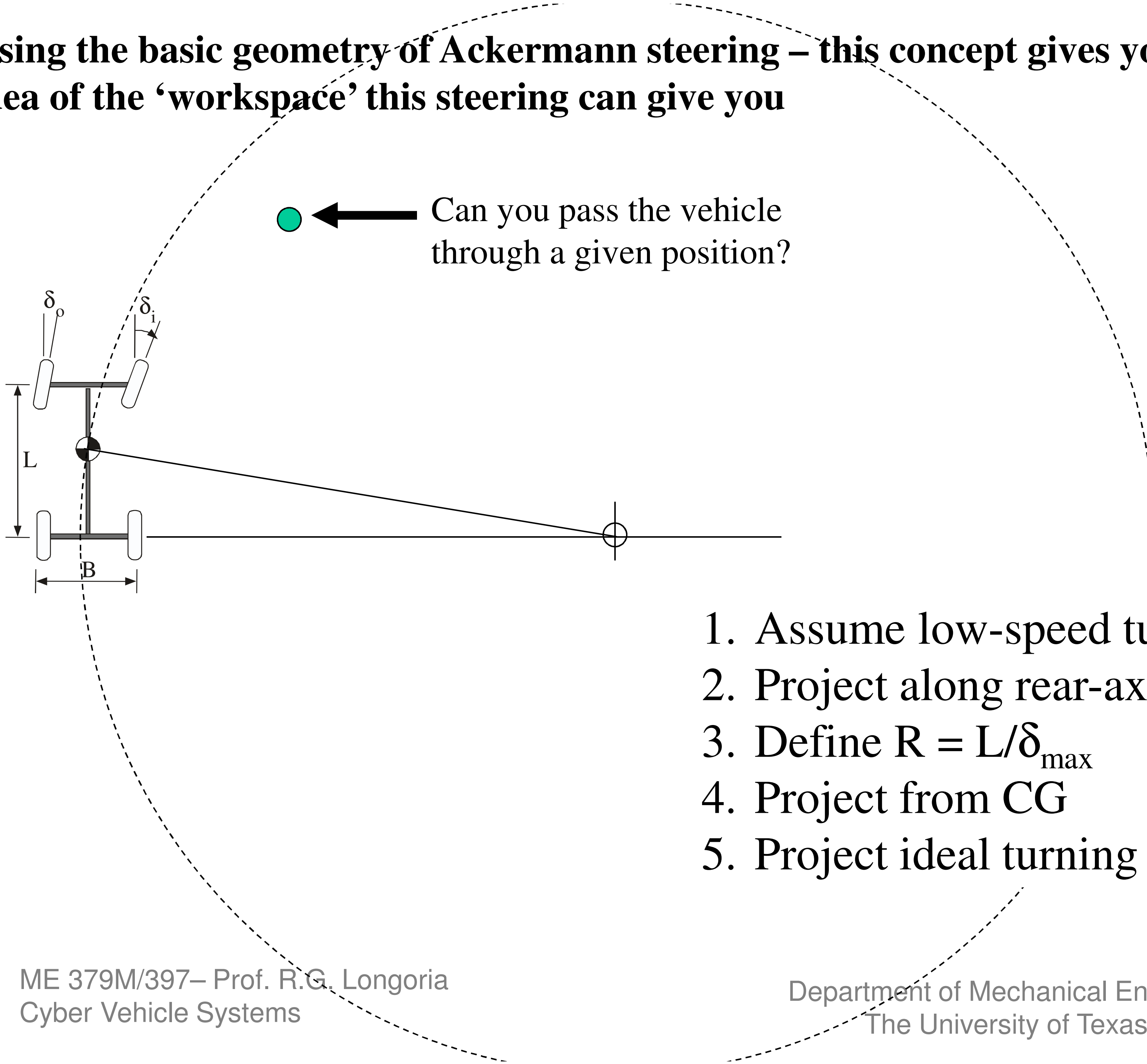
Ref. Milliken & Milliken (1994)

***Figure 19.2** Ackermann steering geometry.*

Additional notes/comments on Ackermann steering

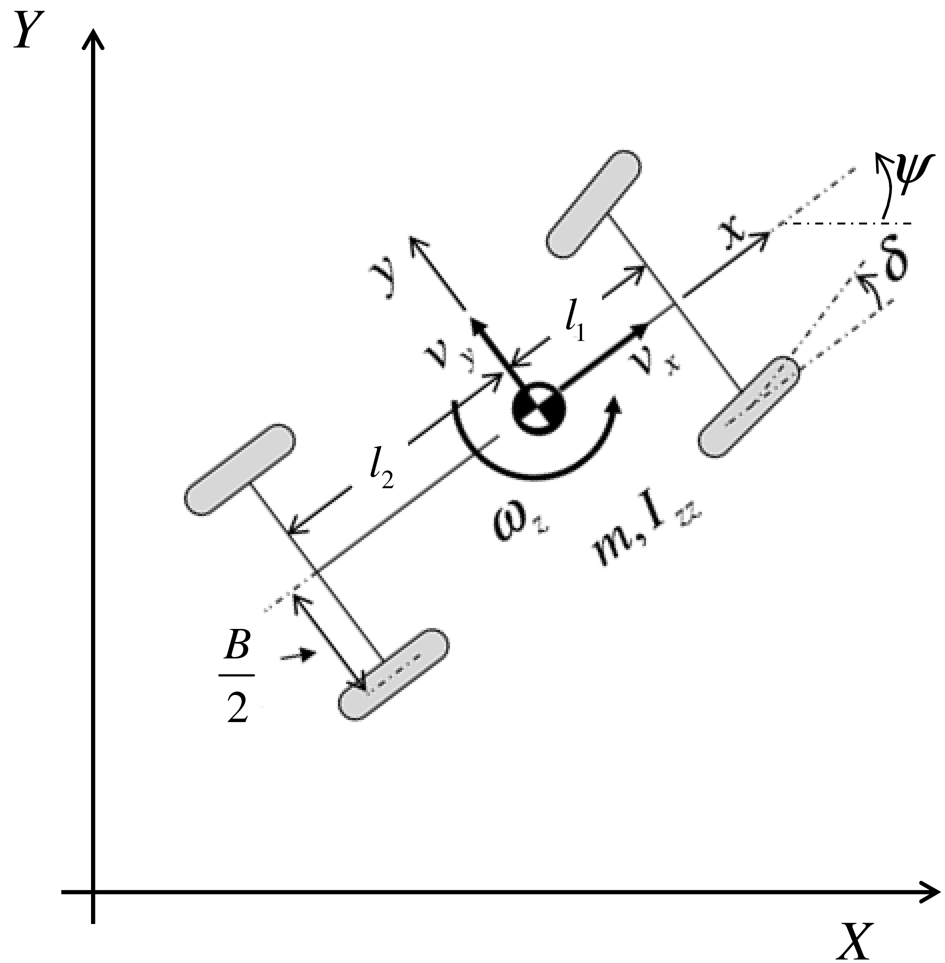
- At **<u>low speed</u>** the wheels primarily roll without **<u>slip angle,</u>** which is the angle between the wheel plane and its actual heading (more on this later).
- If the rear wheels have no slip angle, the center of the turn lies on the projection of the rear axle. Each front-steered wheel has a normal to the wheel plane that passes through the same center of the turn. This is what "Ackermann geometry" dictates.
- Correct Ackermann reduces tire wear and is easy on terrain.
- Ackermann steering geometry leads to steering torques that increase with steer angle. The driver gets feedback about the extent to which wheels are turned. With parallel steer, the trend is different, becoming negative (not desirable in a steering system – positive feedback).
- Off-tracking of the rear wheels, Δ, is related to this geometry. The 'Δ' is $R[1-\cos(L/R)]$, or approximately $L^2/(2R)$.

Using the basic geometry of Ackermann steering – this concept gives you an idea of the 'workspace' this steering can give you



1. Assume low-speed turning
2. Project along rear-axle
3. Define $R = L/\delta_{max}$
4. Project from CG
5. Project ideal turning path

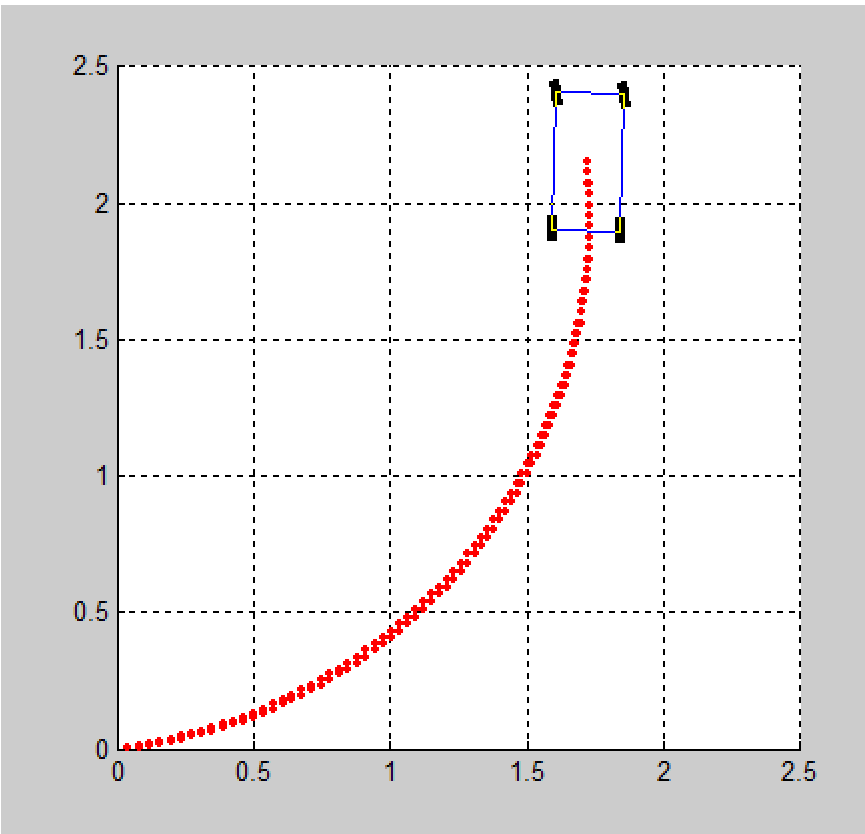
Problem 4: Recall the example worked earlier from the text by Karnopp and Margolis (2008). Derive the kinematic equations and build a simulation with 2D animation for a two-axle vehicle with kinematic steering (with and without Ackermann).

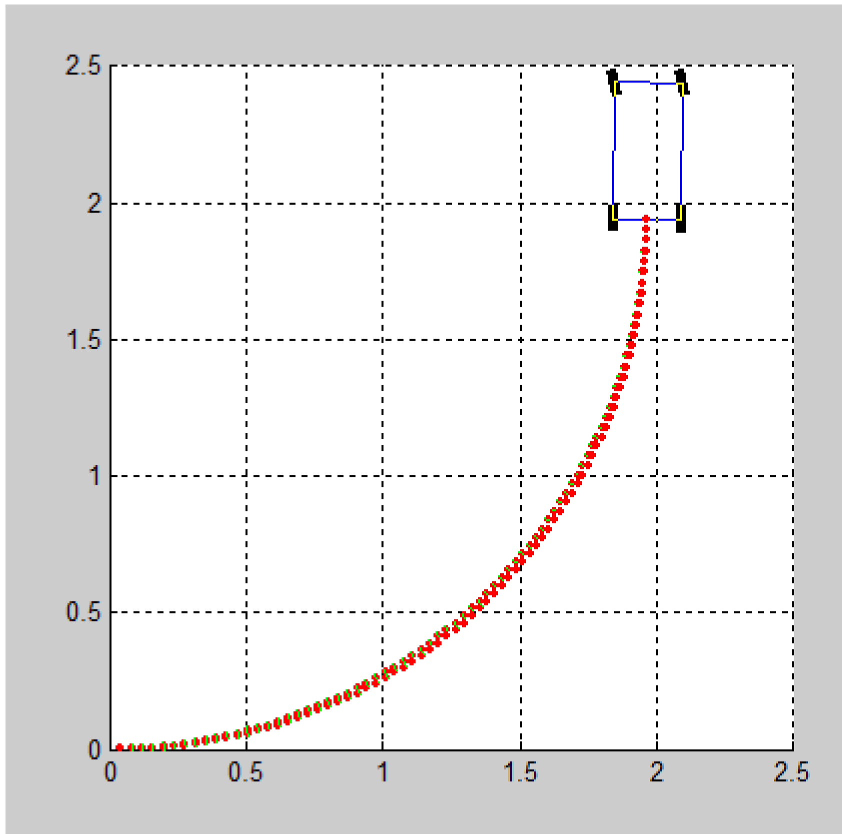


How good is specifying average steer angle by $\delta = L/R$?

Specifying the steering angle using L/R becomes a better approximation to that required to achieve a given turn radius as the CG moves toward rear axle. Below are results using the simple kinematic model for different CG locations (L1 is distance to front axle from CG).

L1 = L/2 **Desired R = 2 m** L1 = L/1.001





<u>Summary</u> of differential and kinematic vehicle turning

- The models introduced here provide a review of fundamental kinematics principles and how they can be applied to vehicle systems.
- The concepts of differential and Ackermann steering are demonstrated through simulations.
- These kinematic models are commonly used in mobile robot applications for path planning, estimation, and control.
- If you know where you want to go, these steering mechanisms can be used to 'estimate' the required control (steer angle) as long as you are at 'low speed' (no slip).
- We'll study in lab how well these can inform our controlled turning maneuvers.
- Lastly, these kinematic models cannot tell us anything about the effect of forces or stability. For that insight, we need dynamic models.

References

1. Den Hartog, J.P., Mechanics, Dover edition. Classic reference on fundamentals of kinematics and dynamics
2. Greenwood, D.T., Principles of Dynamics, Prentice-Hall, 1965. (or any later edition). Good graduate level textbook
3. Hibbeler, Engineering Mechanics: Dynamics, 9th ed., Prentice-Hall. Introductory text often used in course like ME 324.
4. Karnopp, D.C., and D.L. Margolis, Engineering Applications of Dynamics, John Wiley & Sons, New York, 2008. An excellent intermediate engineering dynamics text focused on building mathematical models. Good overview of fundamental material.
5. Meriam, J.L., and L.G. Kraige, Engineering Mechanics: Dynamics (4th ed.), Wiley and Sons, Inc., NY, 1997. Introductory text often used in course like ME 324.
6. Milliken, W.F., and D.L. Milliken, Race Car Vehicle Dynamics, SAE, Warrendale, PA, 1994.