ESE 650, Spring 2019 Assignment 1

Due on Friday February 1st, 2019,11:59PM

This assignment is split into two parts. Each part is worth 50 points. The first part consists of a quick probability review and the second part covers localization.

1 Probability Review (50 Points)

Type your answers in LaTeX in the colored boxes and submit the generated pdf on Canvas. Each question is worth 10 points. **Show all your work for full credit**.

Problem 1 *Suppose X and Y are independent random variables each distributed uniformly on the unit interval. What is the distribution of the random variable (X+Y)?*

Problem 2 *Suppose $\mathcal{U}$ and $\mathcal{V}$ are independent random variables, each distributed uniformly in the unit interval.*

1. *Determine the distributions of the random variables $R = \sqrt{-2\log(1-\mathcal{U})}$ and $\Theta = 2\pi\mathcal{V}$*

2. *Determine the joint distribution of $X = Rcos\Theta$ and $Y = Rsin\Theta$*

Problem 3 *A plane with seating for n passengers is fully booked with each of n passengers having a reserved seat. The passengers file in and the first one in sits at an incorrect seat selected at random from the $n-1$ seats assigned to the other passengers. As each of the subsequent passengers file in they sit in their allotted seat if it is available and sit in a randomly selected empty seat if their seat is taken. What is the probability f_n that the last passenger finds his allotted seat free?*

Problem 4 *Suppose X and Y are independent random variables with distribution functions $F(x)$ and $G(y)$, respectively.*

1. *Determine the distribution of max(X, Y).*

2. *Determine the distribution of min(X, Y).*

Problem 5 *A certain hereditary disease can be passed from a mother to her children. Given that the mother has the disease, her children independently will have it with probability 0.5. Given that she doesn't have the disease her children won't have it either. A certain mother who has a $\frac{1}{3}$ probability of having the diseases has 2 children.*

1. *Find the probability that neither child has the disease.*

2. *Is whether the elder child has the disease independent of whether the younger child has the disease?*

3. *You are told that neither child has the disease. Given this information find the probability that the mother has the disease.*

2 Bayes Filter (50 Points)

In this problem we are given a robot operating in a 2D gridworld. Every cell in the gridworld is characterized by a color (0 or 1). The robot is equipped with a noisy odometer and a noisy color sensor. Given a stream of actions and corresponding observations, implement a Bayes filter to keep track of the robot's current position.

The sensor model is given as :

$$\begin{cases} \text{z = true observation} & \text{with probability 0.9} \\ \text{z = false obsrvation} & \text{with probability 0.1} \end{cases}$$

The action model is given as :

$$\begin{cases} x_{t+1} = x_t + u_t & \text{with probability 0.9} \\ x_{t+1} = x_t & \text{with probability 0.1} \end{cases}$$

When the robot is at the edge of the gridworld and is tasked with executing an action that would take it outside the boundaries of the gridworld, the robot remains in the same state with $p = 1$. Start with a uniform prior on all states. For example if you have a world with 4 states (s_1,s_2,s_3,s_4) then $P(x = s_1) = P(x = s_2) = P(x = s_3) = P(x = s_4) = 0.25$

In the given zip file, you are given two Python scripts *example_test.py* and *histogram_filter.py*, as well as a starter file containing some data: *starter.npz*. The *starter.npz* file contains a binary color-map, a sequence of actions, a sequence of observations, and a sequence of the correct belief states. This is provided for you to debug your code. You should implement your code in the histogram filter.py. Be sure not to change the function signature, or your code will not pass the tests on the submit server. Your code should return the belief_grid which is a grid representation of the belief, as well as a belief_state which is a a 1x2 maximum likelihood estimate of the robot position

We will use ENIAC to submit assignments. To get started refer here :

https://alliance.seas.upenn.edu/ cis520/dynamic/2018/wiki/index.php?n=Resources.HomeworkSubmission

After you have uploaded your submission to the ENIAC according to instructions in the above link, you may submit your histogram filter for grading by running the following command on the ENIAC:

turnin -c ese650 -p project0 histogram_filter.py