Intro

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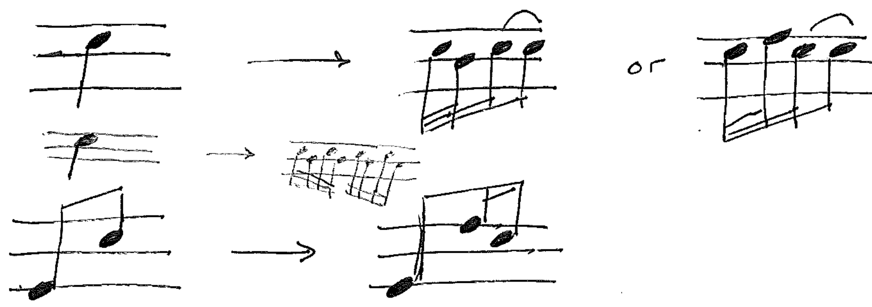


Mordent
Anomalous Dimensions and RG EQ

TRill
Subtraction Schemes + β-Functions

Appoggiatura
~~Subtraction Schemes~~
Formal Matching

Musical Embellishments Are introduced by the performer when it is decided that the music as originally written by the Composer does not sufficiently Express the emotive capacity of the moment the performer desires to create.

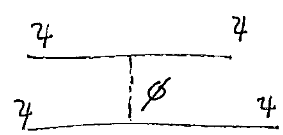
In analogy, this talk will be about the embellishments that come with the Quantum part of Effective Field theories: Anomalous dimensions, Loops, Renormalization, Running Couplings Etc.

- Just like Music, EFT's do not all look the same, but they do have fundamental characteristics / behaviors

The original Composition is the tree level matching that we have done thus far. In this sceneno we were allowed to think of our Effective theories as truncated taylor Expansions

$$\mathcal{L}_{UV} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \tfrac{1}{2}\left[(\partial\phi)^2 - M^2\phi^2\right] + g\phi\bar{\psi}\psi$$

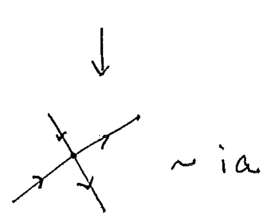
$$[g] = 0$$



+ u, not s

$$\sim \frac{(ig)^2 i}{p^2 - M^2} = i g^2 \frac{1}{M^2}\left(\frac{p^2}{M^2}\right)^n$$

$$\mathcal{L}_{EFF} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + a(\bar{\psi}\psi)^2$$



$\sim ia$

$$\boxed{a = \frac{g^2}{M^2}}$$

And to complete the analogy, the performers are a combination of us and nature... nature making sure the embellishments remain in the style of the original composition and contain physical meaning.

Just as any performer learns to embellish in a style appropriate to the time period, we will here begin to learn to do so in the proper style of nature.

First, Before we go too far, we establish through all of this we would like to keep the Appelquist- Carrazone decoupling theorem

The Statement is: Heavy fields decouple at Low Energies

1.) Power dependence on M in matching conditions for higher Dimension operators (suppressed by $\frac{1}{M}$)

2.) Logarithmic Dependence from running of coupling constants through RG.

One embellishment is the possibility to modify the dimensions of operators and fields that we naively counted last week.

Consider a ϕ^4 theory

$$S = \int d^4x \ \frac{1}{2}\left((\partial\phi)^2 - \frac{\lambda}{4!}\phi^4\right)$$

$[\lambda] = 0$
No dimensions
Scale Invariant
Classically Conformal

When renormalizing, necessary to introduce an artificial mass scale. Many ways to do it, but it must be done. In order to keep bare parameters and observables independent of μ (if $\beta \neq 0$)

$$\lambda(\mu) \quad \text{but } \lambda \text{ is dimensionless} \rightarrow \lambda\left(\frac{\mu}{\Lambda}\right)$$

forced to find a dimensionful quantity. Λ : the "Landau pole" the scale where $\lambda \to \infty$ and theory non-perturbative

Started : theory of Dimensionless parameter

$\rightarrow$ Forced : theory with dimensionful parameter

The renormalized ϕ^4 Lagrangian could be

$$\frac{1}{2} Z_\varphi \partial^\mu \varphi \partial_\mu \varphi - \frac{\lambda}{4!} Z_\lambda \varphi^4$$

In $\overline{MS}$ choose Z's to cancel $1/\varepsilon$ only s.t.

$$Z_\varphi = 1 + \sum_{n=1}^{\infty} \frac{C_n(\alpha)}{\varepsilon^n}$$

shifted to ensure

$$\langle 0 | \varphi(x) | 0 \rangle = 0 \quad \text{and} \quad \langle k | \varphi(x) | 0 \rangle = e^{-ikx}$$

if there was a mass term, it would have a Z_m and m would be tuned to the physical mass of the particle

Separating this into the original + corrections is what Determines the Counter term Lagrangian.

So as I mentioned before, bare parameters and observables by necessity of rationality must be independent of μ, this artificial scale introduced at renormalization: for the propagator,

$$0 = \frac{d}{d\ln\mu} \ln \Delta_0(k^2) \qquad \Delta_0(k^2) = Z_\varphi \Delta(k^2)$$

$$\Delta_0(k^2) = i \int d^d x \, e^{-ikx} \langle 0 | T \varphi_0(x) \varphi_0(0) | 0 \rangle$$

$$= \frac{d\ln Z_\varphi}{d\ln\mu} + \frac{d\ln\Delta(k^2)}{d\ln\mu}$$

$$= \underbrace{\frac{d\ln Z_\varphi}{d\ln\mu}}_{2\gamma_\varphi} + \frac{1}{\Delta(k^2)} \left[\frac{\partial}{\partial\ln\mu} + \underbrace{\frac{d\alpha}{d\ln\mu}}_{-\varepsilon\alpha + \beta(\alpha)} \frac{\partial}{\partial\alpha} + \underbrace{\frac{dm}{d\ln\mu}}_{m\gamma_m(\alpha)} \frac{\partial}{\partial m} \right] \Delta(k^2)$$

$\hookrightarrow$ ~~$\frac{d\alpha_0}{d\ln\mu} = 0 \quad \alpha_0(\alpha, Z_\alpha)$~~

$$\text{i.)} \quad \frac{d\lambda_0}{d\ln\mu} = 0 \quad \lambda_0(\lambda, Z_\lambda)$$

* Anomalous dimensions are very much β-functions for the mass and Z_φ, field renormalization

$$0 = \left[\frac{\partial}{\partial\ln\mu} + \beta(\alpha)\frac{\partial}{\partial\alpha} + \gamma_m(\alpha) m \frac{\partial}{\partial m} + 2\gamma_\varphi(\alpha) \right] \Delta(k^2) \qquad \varepsilon \to 0$$

Why we call them anomalous dimensions:
take massless with $\beta(\alpha^* \neq 0) = 0$

$$\left(\frac{\partial}{\partial\ln\mu} + 2\gamma_\varphi(\alpha) \right) \Delta(k^2) = 0$$

$$\Delta(k^2) = \frac{C_{\alpha^*}}{k^2} \left(\frac{\mu^2}{k^2} \right)^{-\gamma_\varphi(\alpha^*)}$$

the Anomalous dimension Changes the Expected Scaling.

Let us discuss for a moment.

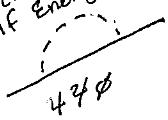
1.) We have seen that in order to calculate anomalous dimensions and β functions

$$\gamma_\varphi = \frac{1}{2}\frac{d\ln Z_\varphi}{d\mu} \qquad \beta(\alpha) = \frac{d\alpha}{d\ln\mu} \qquad \begin{array}{l}\alpha_0(Z_g, Z_\varphi, \alpha)\\ \lambda_0(Z_\lambda, Z_\varphi, \lambda)\end{array}$$

It is sufficient, in an $\overline{MS}$ subtraction scheme to calculate only the divergent, $1/\varepsilon$ pieces of counter terms

ct. in φ^3, $\bar\psi\psi\varphi$

Fermion Self Energy



$\bar\psi\psi\phi$

finite pieces do not have to Enter the RG calculation if Z-factors are more easily attained

Finite parts are the "easy" parts in that some people (myself not yet included) can just write them down. Similar to last week with Expectation of the form, Dimensional Analysis/Power Counting and awareness of the scales available in the problem.

2.) Has the ability to change a marginal operator $\nearrow$ relevant $\searrow$ irrelevant   Ex?

Gravitational EFT where low Energy Quantum Effects are known

2.5.) When ops have diff./mixed γ_φ, can be impossible to calculate leading QCD corrections in full theory.

3.) If the theory is weakly coupled, γ_φ will be small. EFT not only convenience but necessity.

(assumption put into naive dimensional Analysis)

Converse:

If theory is strongly coupled, γ_φ is large

~~While many situations use an EFT out of convenience, ~~ when terms~~

Consider Pions : • In A strongly coupled theory like QCD $[\bar\psi\psi] \sim 3$

when the fundamental d.o.f. are quarks $\bar\psi i\partial\!\!\!/ \psi$

• At low Energies when pions become fundamental d.o.f., behave as scalar $[\varphi] = 1$

or Goldstone boson fields like angles $[\varphi] = 0$

$\gamma_\varphi = -2$: not perturbative.

This is why, if we want to Express a low-energy interaction of pions perturbatively, we do not Use QCD. that would require a nonperturbative calculation like Lattice or some Dual theory.

Moral : choose the d.o.f. wisely.

Weakly-coupled theory of QCD for low energy pions : χPT where coefficients may be matched to QCD but does not suffer from large Chiral Corrections.

More in 2 weeks by Jesse.

TechniColor Models

And in a few more weeks this embellishment is likely to emerge ~~then~~ if Akshay decides to talk about ~~Fermi Landau theory~~ ~~theory~~ Landau theory of Fermi Liquids where low energy Excitations in a conductor are not strongly Interacting Electrons but rather weakly interacting quasi particles described by a different Effective field theory.

Unless on the lattice, dimensional regularization is by far the most popular choice.
Different Choices for Subtraction / renormalization schemes though.

Contribution of Charged Fermion to the QCD β-function

projection into subspace orthogonal to k^μ

$$\frac{i e^2}{2\pi^2}\left(p_\mu p_\nu - p^2 g_{\mu\nu}\right)\left[\frac{1}{6\epsilon} - \frac{\gamma}{6} - \int_0^1 dx\, x(1-x)\log\frac{m^2 - p^2 x(1-x)}{4\pi\mu^2}\right]$$

Ch. 19 of Schwartz

$$= i\Pi_{\mu\nu}(p^2) = i\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)\Pi(p^2)$$

1PI contribution to photon 2-pt function

After Subtraction

$$\beta = \frac{e}{2} M \frac{d}{dM}\Pi(p^2)$$

Mass dependent Subtraction:

$$\Pi(p^2) - \Pi(-M^2)$$

$$\beta(e) = \frac{e^3}{4\pi^2}\int_0^1 dx\, x(1-x)\,\frac{2M^2 x(1-x)}{m^2 + M^2 x(1-x)}$$

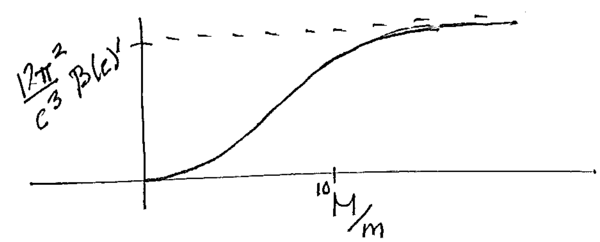
$M \gg m$: $\quad \to \dfrac{e^3}{12\pi^2}$

$M \ll m$: $\quad \to \dfrac{e^3}{60\pi^2}\dfrac{M^2}{m^2}$

$e_R = e_{ff}(m^2)$

as we pass through m, fermion decouples

$$e_{eff}(Q^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{Q^2}{m^2}}$$



Answered the Question: How does a theory itself change as we "integrate out" heavy fields and move to longer distance Scales.

More desirable Subtraction Scheme

$\overline{MS}$ preserves symmetries

Preserves power counting

easy: remove $\frac{1}{\epsilon}$, $\tilde{\mu}^2 = 4\pi\mu^2 e^{-\gamma}$

Does it Answer the same Question?

Mass Independent Subtraction

$$\beta(e) = -\frac{e}{2}\tilde{\mu}\frac{d}{d\tilde{\mu}}\frac{e^2}{2\pi^2}\int_0^1 dx\, x(1-x)\log\frac{m^2 - p^2 x(1-x)}{\tilde{\mu}^2}$$

$$= \frac{e^3}{12\pi^2}$$

regardless of relationship between $\tilde{\mu}$ and m

Looks like

- fermion contributes all the way down?
- does not decouple

Problem

- $\sim \log\frac{m^2}{\tilde{\mu}^2}$ for low momentum p^2 and for $\mu \ll m$, log becomes large and PT breaks.

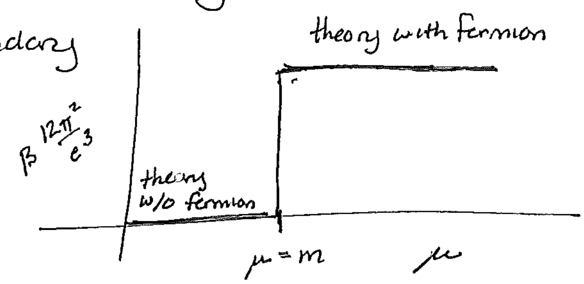
All 2 ~~three~~ of these are indications that we have used the wrong β function

Using Dim Reg + $\overline{MS}$ changes the question to:

- How do we need to modify A theory, using a mass-independent renormalization scheme, in order to get the physics right at low NRG

Decoupling is put in by hand in matching.

Changing theory @ the boundary

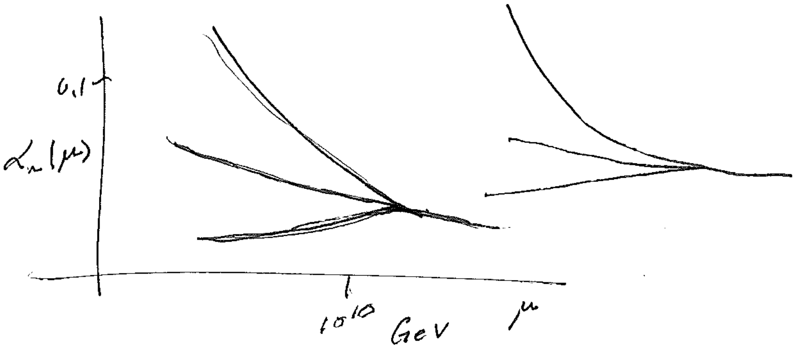


β-functions: discontinuous
scheme-dependent!

Back Pocket:

The idea of β functions being step functions is not foreign to us.

Think of *the* gauge group of the world, SU(5). When couplings are run down, we see a smooth, splitting



Used a momentum-space subtraction scheme. Mass-dependent.

If $\overline{MS}$ or mass dependent subtraction scheme was used, it would not look like this.

$\Downarrow$

Put decoupling in by hand and switch theories at physical thresholds.

think:

$$\beta_{QCD} = -\frac{\alpha}{16\pi^2}\left(11 - \frac{2n_f}{3}\right)$$

Step functions in β at physical particle masses.

Diagramatic Discussion of how these matching conditions are to be implemented.

$$\mathcal{L}_H(\chi,\phi) + \mathcal{L}(\phi) \longrightarrow \mathcal{L}(\phi) + \underbrace{\delta\mathcal{L}(\phi)}$$

- New Light-particle Interaction terms that arise in the Effective theory.
- In the past: becomes taylor Expansion of $\mathcal{L}_H$ in P/M

It is convient to use an $\hbar$ expansion to order and group terms.

$$\text{trees} \rightarrow \mathcal{O}(\hbar) \rightarrow \mathcal{O}(\hbar^2) \rightarrow \cdots$$

Euler's Formula

$$L - 1 = P - V$$

original theory: $e^{-iS/\hbar}$

Propagators : $\hbar$

Vertices : $\hbar^{-1}$

graph $\sim \hbar^{P-V} = \hbar^{L-1}$

this graph in the full theory $\rightarrow$ Vertex in effective theory $S_{eff}/\hbar$

matching becomes $\mathcal{O}(\hbar^L)$

Can think of a loop expansion $\approx$ $\hbar$ Expansion

as $P(E,V)$

$\downarrow$ Euler

$V(L,E)$

for given E, V grows w/ # of loops $\rightarrow$ if perturbative Expansion is possible, Loop/$\hbar$ expansion is too

$$\mathcal{L}_H(\chi,\phi) + \mathcal{L}(\phi) \longrightarrow \mathcal{L}(\phi) + \delta\mathcal{L}^0(\phi) + \delta\mathcal{L}^1(\phi) + \cdots$$

The Matching condition

Equivalence of Light Particle Effective Action

1LPI

$$S^{1LPI}_{\mathcal{L}_H+\mathcal{L}} = S_{\mathcal{L}+\delta\mathcal{L}}$$

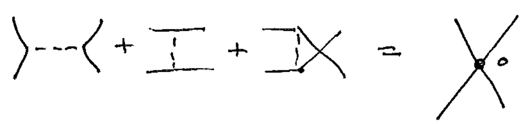
(Actions may also be Expanded in ħ expansion)

At tree level $\mathcal{O}(\hbar^0)$

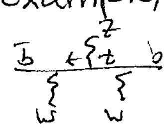
$$S^0_{\mathcal{L}_H+\mathcal{L}} = S^0_{\mathcal{L}+\delta\mathcal{L}}$$

$$\cancel{S^0_{\mathcal{L}}} + \int \begin{Bmatrix}\text{Heavy}\\ \text{Virtual}\end{Bmatrix}^0 = \cancel{S^0_{\mathcal{L}}} + \int \delta\mathcal{L}^0$$

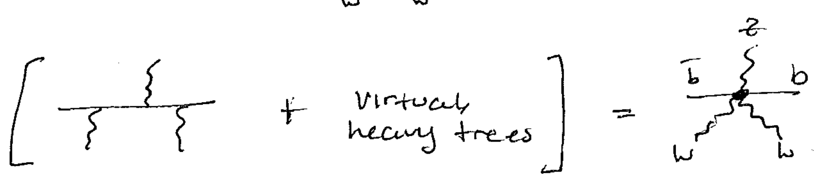
With a scalar theory like the one we started with: with a HLL interaction



In another example, consider the integration of the top quark. from the diagram



. This treelevel matching states



leads to a 5 point vertex in the Effective theory.

- momentum constrained
- $\sim \frac{1}{m_t^2}$

} consistent with decoupling. Expected

Done Some Damage to high Energy physics. Lets match to higher order in ħ

At 1 Loop, $\mathcal{O}(\hbar)$:

$$S^1_{\mathcal{L}_H+\mathcal{L}} = S^1_{\mathcal{L}} + \delta\mathcal{L}^0 + \delta\mathcal{L}^1$$

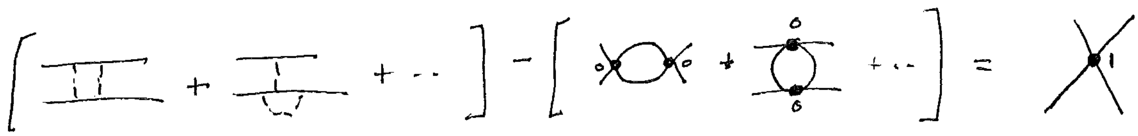
$$S^1_{\mathcal{L}_H+\mathcal{L}} = S^1_{\mathcal{L}+\delta\mathcal{L}^0} + \int \delta\mathcal{L}^1$$

$$S^1_{\mathcal{L}_H+\mathcal{L}} - S^1_{\mathcal{L}+\delta\mathcal{L}^0} = \int \delta\mathcal{L}^1$$

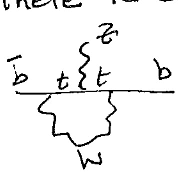
In words: the $\mathcal{O}(\hbar)$ correction term that appears in the effective lagrangian is the difference between loop diagrams (1LPI) that can be constructed out of the full theory and loop diagrams that can be constructed out of the effective theory with the tree-level matching correction

Note: Even if end goal is $\mathcal{O}(\hbar)$ must do matching step-by-step.

for the theory of 2 scalar fields:



There is another diagram to think about. The vertex correction



- Though to some, I have not found intuitive w/ my available knowledge and experience.
- Lived through the 80^s, LEPS
- Hint: represents a process used at the time to constrain m_t
  - so not a small dependence

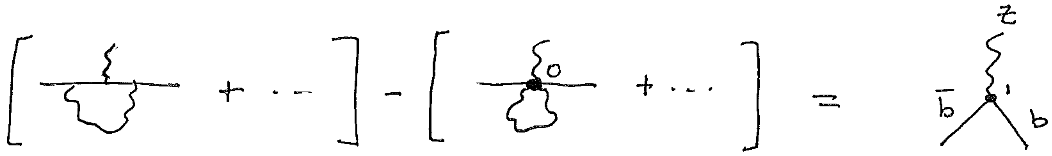
$\sim m_t^2$: Example of mixed heavy-light contributions $-$ no decoupling.

Definitely no taylor expansion

Comes from large momenta in the loop.

Plan: We have already delt (in $\overline{MS}$) with theories that do not formally decouple. The plan is the same. $\overline{MS}$ to remove divergences and reinsert dependence order-by-order in matching.

Loop / $O(\hbar)$ matching looks like:



Notice we have not solved the Equation

$$e^{-i\frac{S_{EFT}}{\hbar}} = \int [d\chi]\, e^{\frac{-iS(\chi,\phi)}{\hbar}}$$

to complete the matching of the Effective theory.

Local limit before light-loop Integration.

Loop momenta being unconstrained $\frac{q^2}{M^2} \to 0$ $\frac{m^2}{M^2} \to 0$

While it may be passible for a simple toy model like the interaction of scalars, it quickly becomes impractical. ($Z\bar{b}b$ Example)

The equation is more of an expression of an idea than it is a useful mathematical tool in this case.

Next week: Bryce Start us off in Examples - finally, Hooray. Put these Embellishments Together.

He will be calculating β functions and Matching at 1 Loop.

He will also not neglect what I have in this discussion: momentum Expansion

Final Word: Make Sure you vote in the Next 3 weeks.