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Introduction to Radiotherapy Computer Dose Algorithms

Intro to Monte Carlo Method

Objective

- Calculate an accurate dose to a patient in a radiation field

- Difficulties
  - Patient is heterogeneous material w/ irregular geometry
  - Radiation fields often contain mixed particle types w/ wide range of energies & often produce 2° particles and/or activate matter

Ideal dose calculation

- Accounts for heterogeneous patient composition
- Considers all radiation particles and all possible physical interactions

→ This field of study is Radiation Transport Theory

Possible approaches

1) Make gross approximations to simplify calculations

- e.g., patient is a sphere of water and we only consider energy loss of 1° radiation type

- Advantages: Fast, easy to implement, dose calculation possible in seconds

Deterministic Approach

2) Assemble a very large differential equation to describe all possible physical interactions for the particle spectrum and patient anatomy

- Linearized Boltzmann Transport Equation

- Typically, so many variables are needed to accurately describe the radiation field (and patient anatomy) that the differential equations are almost impossible to solve

- Uncharged particles (photons & neutrons) work fairly well, charged particles harder to solve

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Possible approaches (continued)

3) **Stochastic Approach**

Monte Carlo Method

- Assemble a very long list of all possible physical interactions for all particle types and all geometries & materials (of the patient)
- Don't write 1 large equation to describe everything
- Instead, focus on having very accurate equations for individual interactions and probabilities of interaction
- Stochastically simulate the (random) behavior of a single particle as it traverses matter
  - track length b/w interactions
  - probabilities of different interaction types
  - energy loss & direction changes
  - Use self-contained (highly accurate) equations to model each step

(e.g. energy loss in a voxel)

- Repeat simulations for large numbers of particles and look at <u>average behavior</u>

Monte Carlo Overview

- Define initial conditions
  - particle types (p, e^-, n, γ . . .)
  - energy distributions
  - direction
- Define geometry and materials radiation will interact with (e.g., patient)
  - Any coordinate system may be used, pick simplest
- Define physics interaction types
  - Analytical models
  - Measured cross-section libraries
- Define tallies
  - Points or volumes at which to record properties of the radiation field (e.g. energy deposition, 2° neutron fluence, . . .)
  - Tallies may be single volumes, or meshes, concentric rings, etc. . .
  - Use symmetry to your advantage (increased volume ⇒ increased counts ⇒ better statistics)
- Simulate histories of incident particles
  - track 2° particles also
  - repeat until statistics are good

Typical flowchart to calculate 1 history

1) Select an incident particle type, kinetic energy, and direction (randomly sample this from distribution if you have a mixed particle field)

2) Determine geometry & materials the particle will interact with (ρ, Z, A, mixtures ...)

3) Calculate distance to 1st interaction

- Generate a pseudo random number
- Randomly sample the probability distribution that a particle reaches depth w/out interaction

  (e.g., $f(x) = e^{-\mu x}$)

  $\mu \equiv$ total linear attenuation coefficient

4) Determine type of 1st interaction

- If competing interactions, randomly sample the probability of each interaction type

5) Calculate effect of interaction

- ΔE, change in direction (again, randomly sample these angular distributions & energy loss)
- If any 2° particles are generated, store particle type, kinetic energy, & direction & calculate histories for these (at a later time)

6) If interactions occur within the bounds of a tally, record the interaction data

Typical flowchart to calculate 1 history (continued)

7) Repeat process (distance to next interaction + interaction type) until kinetic energy of initial particle is entirely lost (or falls below a threshold at which the remaining range or dosimetric effect is negligible)

8) Repeat process for all 2° particles generated along the way

$\boxed{\text{END Calculate 1 history}}$

Uncertainties

- Events such as energy deposition in a tally voxels are often random + follow Poisson statistics

$$\therefore \quad \sigma \propto \frac{1}{\sqrt{N}}$$

- Typically must quadruple the number of histories (and hence computation time) to reduce uncertainty by a factor of 2

Random Numbers

- ideally have random access to an infinite set of numbers b/w $\underline{\underline{0}}$ + $\underline{\underline{1}}$

- Practically, a very large repository of random integers will work
  - for integers from 1 to N, divide by N

- Also, sequentially generated pseudorandom numbers are most practical approach (+ works well if you have a good pseudorandom # generator)

- MATLAB and other software packages typically have pseudorandom number generators
  - these are most likely <u>inadequate</u> for Monte Carlo dose calculations

- If you are serious about writing your own Monte Carlo code, read up on mathematical methods to produce pseudorandom numbers

(Today, we will just use MATLAB's <u>rand</u> function) for example

Stochastic Sampling Method: Invert Cumulative Distribution Function

Background

- Consider a Probability Density Function (PDF), $f(x)$ where $f(x)$ is limited to a range from a to b

- The probability that x lies b/w a & b is written

$$P\{a < x < b\} = \int_a^b f(x)\,dx$$

- Related to the PDF, a cumulative probability distribution (CPD) → $F(x)$ is written

$$F(x) = \int_a^x f(x')\,dx'$$

and the probability that a randomly selected value of x is less than x_1 is given by $F(x_1)$

method

1) Normalize $f(x)$ so that $F(a) = \emptyset$ & $F(b) = 1$
(equivalently $P\{a < x < b\} = 1$)

2) Select a number ρ_i randomly b/w 0 & 1

3) Invert CPD, $F(x_i) = \rho_i \Rightarrow \boxed{x_i = F^{-1}(\rho_i)}$
to solve a randomly sampled x_i

Stochastic Sampling Method: Invert CPD - Example

e.g., To compute distance to 1st interaction

- The PDF that a particle reaches a depth $\underline{\underline{t}}$ in a material without interaction is

$$f(t) = \mu e^{-\mu t}$$

- The corresponding CPD gives the probability that a random interaction ~~depth~~ will occur at a depth less than $\underline{\underline{t}}$

$$F(t) = 1 - e^{-\mu t} \left(= \int_0^t \mu e^{-\mu t'} dt' \right)$$

- Here the limits of interaction depth would be 0 & $+\infty$

$$\underbrace{F(t=0) = \underline{\underline{0}} \quad \& \quad F(t \to \infty) = \underline{\underline{1}}}$$

Good, this means we can use a random number ρ_i b/w 0 & 1

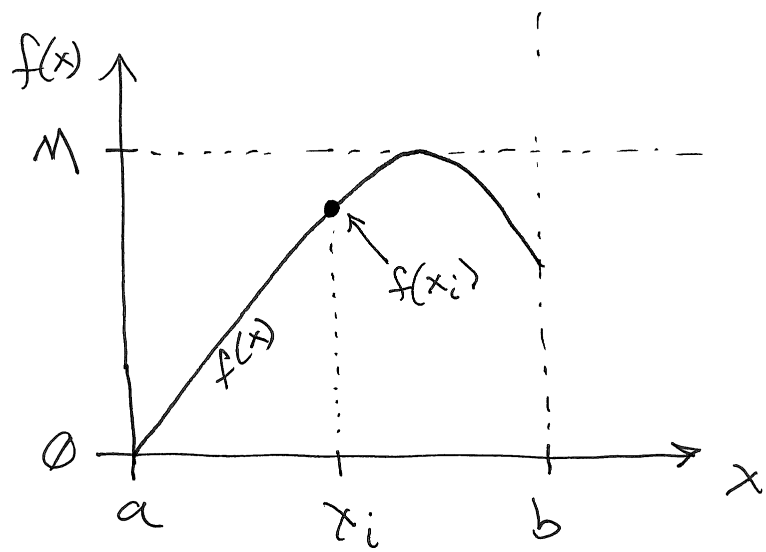
- Invert the CPD to find the depth of a random interaction for a given random # ρ_i (t_i)

$$\boxed{t_i = F^{-1}(\rho_i) = -\ln(1-\rho_i)/\mu}$$

Stochastic Sampling Method: Rejection Method

- Use when PDF $f(x)$ cannot be integrated analytically <u>or</u> if $F^{-1}(\rho_i)$ is difficult to determine analytically

e.g.,



Rejection Method

1) Generate a random number ρ_i b/w 0 + 1

2) Let $x_i = a + \rho_i(b-a)$ [not done yet, haven't used PDF yet]

   a = lower bound of x to sample
   b = upper bound of x

3) Select another random number ρ_j

4) If $\rho_j M \le f(x_i)$, accept x_i, otherwise reject x_i and return to step 1 (repeat until acceptable x_i is found)