Intro to Radiotherapy Computer Dose Algorithms

MATLAB Student Course

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Exercise I: Variables and Functions

Write a MATLAB function to calculate and return the linear stopping power S (MeV/cm) of a proton with any kinetic energy E (MeV) stopping in a graphite absorber. Use the Bethe-Bloch equation given below.

If you have time, include a logic test so the function will return `-1` if the relativistic particle velocity β is less than 0.1 (indicating the calculation is not valid). Compare your calculations for proton $E = 10$, 100, 500, and 1000 MeV with the NIST PSTAR program results (see web address below).

Bethe-Bloch Equation (Leo 1994, without shell and density effect corrections)

$$S = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e c^2 \gamma^2 \beta^2 W_{\max}}{I^2}\right) - 2\beta^2 \right],$$

where

$$\begin{aligned}
N_a &= \text{Avogadro's number} = 6.022 \times 10^{23}\ \text{mol}^{-1}, \\
r_e &= \text{classical electron radius} = 2.817 \times 10^{-13}\ \text{cm}, \\
m_e &= \text{rest mass of electron} = 0.511\ \text{MeV/c}^2, \\
c &= \text{speed of light}, \\
\rho &= \text{density of absorber (graphite)} = 1.7\ \text{g/cm}^3, \\
Z &= \text{atomic number of absorber (graphite)} = 6, \\
A &= \text{atomic mass of absorber (graphite)} = 12.01\ \text{u}, \\
z &= \text{charge of incident particle (proton)} = 1, \\
\beta &= \text{velocity of incident particle } (\nu/c) = \sqrt{1-(m_o c^2/(E+m_o c^2))^2}, \\
m_o &= \text{rest mass of incident particle (proton)} = 938.272\ \text{MeV/c}^2, \\
\gamma &= (E+m_o c^2)/(m_o c^2), \\
W_{\max} &= \text{maximum energy transfer in a single collision (see following)}, \\
I &= \text{mean excitation potential of absorber (carbon)} = 78.0\ \text{eV},
\end{aligned}$$

and

$$W_{\max} = \frac{2m_e c^2 \eta^2}{1 + 2s\sqrt{(1+\eta^2)} + s^2},$$

and

$$\begin{aligned} \eta &= \beta\gamma, \\ s &= m_e/M, \\ M &= \text{incident particle mass (proton)} = 938.272 \text{ MeV/c}^2. \end{aligned}$$

References

- Leo, W. R., 1994. Techniques for Nuclear and Particle Physics Experiments, 2nd ed., Springer-Verlag. Page 24.
- `http://physics.nist.gov/PhysRefData/Star/Text/PSTAR.html`

Exercise II: Conditional Statements and Visualization

1. Plot the linear stopping power $S(E)$ of a proton stopping in graphite as a function of energy from E = 0 to 1000 MeV with 1 MeV increments. Use a `for` loop and store E and $S(E)$ in 1D arrays. *Note: If you have implemented the logic test in Exercise 1, you should see on your plot where the proton velocity is below the accurate range of the Bethe-Bloch equation. If you have not implemented this, just plot E from 10 to 1000 MeV.*

2. Plot linear stopping power as a function of depth in a graphite slab for a 500 MeV proton. You should see a Bragg peak distribution. Use a `while` loop to iterate over 0.01 cm depth resolution (ΔZ) until the proton energy drops below 10 MeV. Subtract energy loss per step from the proton in each iteration ($\Delta E = S(E)\Delta Z$).

Exercise III: Monte Carlo Radiation Transport Simulations

1. Write a Monte Carlo function to return a randomly-sampled distance t_i (cm) to first interaction for a 100 keV photon incident (normal) on a uniform slab of aluminum. Use the MATLAB pseudo-random number generator `rand` to produce a number ρ_i between 0 and 1. The cumulative probability distribution

   $$F(t) = 1 - \exp(-\mu t)$$

   gives the probability (ranging from 0 to 1) that a photon has interacted by distance t in the aluminum slab. The randomly-sampled distance to interaction t_i can be calculated using a randomly generated number ρ_i (from 0 to 1) and inverting the cumulative probability distribution

$$t_i = F^{-1}(\rho_i) = -\ln(1 - \rho_i)/\mu,$$

where

$$\mu \;=\; \text{probability interaction per unit thickness (100 keV, Al)} = 0.462 \text{ cm}^{-1}.$$

2. Write a Monte Carlo code to randomly sample the Compton scatter angle Θ_i of a photon scattering in aluminum using the Klein-Nishina equation for the differential scattering cross section $\partial\sigma/\partial\Omega$ (Knoll 2000). Implement this as a MATLAB function that accepts photon kinetic energy E (MeV) and atomic number Z of the scattering material and returns both the random scatter angle Θ_i (radians) and the corresponding energy lost ΔE_i (MeV) during the interaction. Recall the Compton scatter equations

$$f(\Theta) = \frac{\partial\sigma}{\partial\Omega} = Zr_o^2\left(\frac{1}{1+\alpha(1-\cos\Theta)}\right)^2\left(\frac{1+\cos^2\Theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\Theta)^2}{(1+\cos^2\Theta)[1+\alpha(1-\cos\Theta)]}\right)$$

where

$$\begin{aligned} Z &= \text{atomic number}, \\ r_o &= \text{classical electron radius} = 2.817 \times 10^{-13} \text{ cm}, \\ \alpha &= E/(m_o c^2), \end{aligned}$$

and

$$E' = \frac{E}{1 + (E/m_o c^2)(1-\cos\Theta)}.$$

Instead of the cumulative probability function inversion method used in part 1, use the Rejection Method (Schultis and Faw 2000) which does not require integration or inversion. It will be necessary to first determine the maximum value M of the Klein-Nishina cross section $\partial\sigma/\partial\Omega$ on the interval from 0 to π. Once you have determined M for the given photon energy and scattering material (Z), use the following rejection method.

(a) Generate a random number ρ_i between 0 and 1.
(b) Let $\Theta_i = a + \rho_i(b-a)$, with $a = 0$ and $b = \pi$.
(c) Generate a second random number ρ_j.
(d) If $\rho_j M \le f(\Theta_i)$, accept Θ_i, otherwise reject Θ_i and return to step 1.

3. Using your functions, for 1000 histories of 100 keV photons in aluminum, tally (1) distance to first interaction (in cm), and (2) the angle of Compton scatter (in radians). Ignore competing

interactions such as photoelectric absorption in this implementation. Create histograms to display your data. If you have time, generate a polar plot of the scatter angles.

References

- Schultis, J. K. and Faw, R. E., 2000. Radiation Shielding, American Nuclear Society. Page 408-30.
- Metcalfe, P., Kron, T., and Hoban, P., 2007. The Physics of Radiotherapy X-rays and Electrons, Medical Physics Publishing. Page 619-50.

Recommended Monte Carlo Codes

- MCNPX, `http://mcnpx.lanl.gov`
- EGSnrc, `http://irs.inms.nrc.ca/software/egsnrc`
- Geant4, `http://www.geant4.org/geant4`