MTH208: Coursework II

- Total marks for the coursework are 15. There are 2 comprehensive questions in the exam. Please write down detailed solution process and submit your completed solution via a submission link provided on the LMO. You may complete the work by using the provided solution sheet in word or tex.
- All the learning materials on the LMO can be referenced during the exam including lecture notes, Lab codes, lecture videos etc. However, you must complete the coursework independently.
- Please name your submission in the form *MTH017Final+ID+ZhangSan.pdf*
- The coursework will be available on 9:00AM May 10th and deadline for submission is 9:00AM May 19th.

Question I: Numerical Quadrature (8 marks)
In this question, we consider different numerical quadratures for approximating the following two integrals

$$I_c = \int_0^1 \frac{\cos x}{\sqrt{x}}\,\mathrm{d}x \textit{ and } \quad I_s = \int_0^1 \frac{\sin x}{\sqrt{x}}\,\mathrm{d}x$$

(1-a) (2 marks) Use the composite trapezoidal rule with n intervals of equal length $h = 1/n$ (n can be hundreds to 1000), "ignoring" the singularity at $x = 0$ (i.e., arbitrarily using zero as the value of the integrand at $x = 0$). Display your observations from the numerical results.
(1-b) (2 marks) Use the composite trapezoidal rule over the interval $[h, 1]$ with $n-1$ intervals of length $h = 1/n$ in combination with a weighted Newton-Cotes rule with weight function $w(x) = x^{-1/2}$ over the interval $[0, h]$. Give a few remarks on the performance by comparing the numerical results obtained in (1-a).
(1-c) (2 marks) Make the change of variables $x = t^2$ and apply the composite trapezoidal rule to the resulting integrals. Make more comparisons on the performance the numerical methods used.
(1-d) (2 marks) You may now consider more advanced methods (e.g., composite Simpson, Gauss-Legendre quadrature etc) on the integrals obtained in (1-c) and show your observations and comparisons.

Question II: IVP (7 marks)
A projectile of mass $m = 0.11$ kg shot vertically upward with initial velocity $v(0) = 80$ m/s is slowed due to the force of gravity, $F_g = -mg$, and due to air resistance, $F_r = -kv|v|$, where $g = 9.8$ m/s^2 and $k = 0.002$ kg/m. The differential equation for the velocity v is given by

$$mv' = -mg - kv|v|.$$

Suppose initially the projectile is at the height $H(0) = 500$ m and finally at some time T the projectile just hits the earth at the height $H(T) = 0$ m.
(2-a) (2 marks) Find the velocity for $t \in [0, 20]$ using Euler's method with the time step sizes $h = 2^{-1}, 2^{-2}, 2^{-3}, \ldots$ until the solutions with h and $h/2$ differ less than 10^{-4} at all the time points of step size h. Plot the velocity as a function of $t \in [0, 20]$.
(2-b) (2 marks) Do you know other time integration methods, e.g. Runge-Kutta methods? Describe the methods and also use them to solve the problem. Compare all the methods you have tried including Euler's method.
(2-c) (2 marks) Based on the numerical solution of the differential equation, use some numerical method e.g. the bisection method to determine when the projectile reaches the maximum height and find the maximum height. How can you ensure that the errors of your computed time and height are less than 10^{-4}?
(2-d) (1 marks) Based on the numerical solution of the differential equation, use some numerical methods to determine when the projectile hits the earth.