CS 540: Introduction to Artificial Intelligence
Homework Assignment 3

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Section 1
0 Late Days Used

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Contents

1 Question 1: Probabilities

The following two tables provide the full joint distribution for three boolean variables X, Y and Z.

Table 1: Z

	Y	$\bar{Y}$
X	0.2	0.12
$\bar{X}$	0.04	0.24

Table 2: $\bar{Z}$

	Y	$\bar{Y}$
X	0.1	0.08
$\bar{X}$	0.06	0.16

1.1 a) What is $P\left(\bar{Y}\right)$?

$$\begin{aligned} P(\bar{Y}) &= P\left(\sum X, \bar{Y}, \sum Z\right) \\ &= P(X, \bar{Y}, Z) + P(X, \bar{Y}, \bar{Z}) + P(\bar{X}, \bar{Y}, Z) + P(\bar{X}, \bar{Y}, \bar{Z}) \\ &= 0.12 + 0.08 + 0.24 + 0.16 \end{aligned}$$

$$\boxed{P\left(\bar{Y}\right) = 0.6}$$

1.2 b) What is $P\left(X \middle| \bar{Y}\right)$?

$$\begin{aligned} P(X|\bar{Y}) &= \frac{P(X, \bar{Y})}{P(\bar{Y})} \\ P(X, \bar{Y}) &= P(X, \bar{Y}, \sum Z) \\ &= P(X, \bar{Y}, Z) + P(X, \bar{Y}, \bar{Z}) \\ &= 0.12 + 0.08 = 0.2 \\ P(\bar{Y}) &= P(X, \bar{Y}, Z) + P(X, \bar{Y}, \bar{Z}) + P(\bar{X}, \bar{Y}, Z) + P(\bar{X}, \bar{Y}, \bar{Z}) \\ &= 0.12 + 0.08 + 0.24 + 0.16 = 0.6 \\ P(X|\bar{Y}) &= \frac{0.2}{0.6} \end{aligned}$$

$$\boxed{P\left(X|\bar{Y}\right) = \frac{1}{3}}$$

1.3 c) What is $P\left(\bar{Z}\middle|\bar{X}, Y\right)$?

$$
\begin{aligned}
P\left(\bar{Z}\middle|\bar{X}, Y\right) &= \frac{P\left(\bar{Z}, \bar{X}, Y\right)}{P\left(\bar{X}, Y\right)} \\
P(\bar{Z}, \bar{X}, Y) &= 0.06 \\
P(\bar{X}, Y) &= P(\bar{X}, Y, \sum Z) \\
&= P(\bar{X}, Y, Z) + P(\bar{X}, Y, \bar{Z}) \\
&= 0.04 + 0.06 = 0.1 \\
P\left(\bar{Z}\middle|\bar{X}, Y\right) &= \frac{0.06}{0.1} \\
\Aboxed{P\left(\bar{Z}\middle|\bar{X}, Y\right) &= 0.6}
\end{aligned}
$$

Note on Independence

Two events A, B, are independent if:

$$P(A, B) = P(A) \times P(B) \tag{1}$$
$$P(A|B) = P(A) \tag{2}$$
$$P(B|A) = P(B) \tag{3}$$

1.4 d) Verify whether or not X and Y are independent.

Test the conditions in the section labeled "Independence".

Condition 1

We attempt to verify (1).

$$P(X, Y) \stackrel{?}{=} P(X) \times P(Y)$$

First we simplify the left hand side:

$$
\begin{aligned}
P(X, Y) &= P(X) \times P(Y|X) \\
&= P(X) * \frac{P(Y, X)}{P(X)} \\
&= P(X, \sum Y, \sum Z) * \frac{P(X, Y, \sum Z)}{P(X, \sum Y, \sum Z)} \\
&= (0.2 + 0.12 + 0.1 + 0.08) * \frac{0.2 + 0.1}{0.2 + 0.12 + 0.1 + 0.08} \\
&= 0.2 + 0.1 = 0.3 \\
\Aboxed{P(X, Y) &= 0.3}
\end{aligned}
$$

Next we simplify the right hand side.

$$
\begin{aligned}
P(X) \times P(Y) &= P(X, \sum Y, \sum Z) \times P(Y, \sum X, \sum Z) \\
&= (0.2 + 0.12 + 0.1 + 0.08) \times (0.2 + 0.04 + 0.1 + 0.06) \\
&= 0.5 \times 0.4 \\
\boxed{P(X) \times P(Y) = 0.2}
\end{aligned}
$$

Now we can compare the two:

$$
\begin{gathered}
P(X,Y) \stackrel{?}{=} P(X) \times P(Y) \\
\boxed{0.3 \neq 0.2}
\end{gathered}
$$

Since (1) is not true, we know that X and Y *are not* independent.

1.5 e) Verify whether or not Y and Z are independent.

Condition 1

We attempt to verify (1).

$$P(Y,Z) \stackrel{?}{=} P(Y) \times P(Z)$$

First we simplify the left hand side:

$$
\begin{aligned}
P(Y,Z) &= P(Y) \times P(Z|Y) \\
&= P(Y, \sum X, \sum Z) \times \frac{P(Z, Y, \sum X)}{P(Y, \sum X, \sum Z)} \\
&= P(Y, Z, \sum X) = P(X, Y, Z) + P(\bar{X}, Y, Z) = 0.2 + 0.04 \\
\boxed{P(Y,Z) = 0.24}
\end{aligned}
$$

Now we simplify the right hand side:

$$
\begin{aligned}
P(Y) \times P(Z) &= P(Y, \sum X, \sum Z) \times P(Z, \sum X, \sum Y) \\
&= (0.2 + 0.04 + 0.1 + 0.06) \times (0.2 + 0.04 + 0.12 + 0.24) \\
&= 0.4 \times 0.6 \\
\boxed{P(Y) \times P(Z) = 0.24}
\end{aligned}
$$

Now we compare the two:

$$
\begin{gathered}
P(Y,Z) \stackrel{?}{=} P(Y) \times P(Z) \\
\boxed{0.24 = 0.24}
\end{gathered}
$$

We have satisfied (1), and now must move on to the next condition.

Condition 2

We attempt to verify (2).

$$P(Y|Z) = P(Y)$$

First we simplify the left hand side:

$$\begin{aligned}
P(Y|Z) &= \frac{P(Y,Z)}{P(Z)} \\
&= \frac{P(Y,Z,\sum X)}{P(Z,\sum X,\sum Y)} \\
&= \frac{0.2+0.04}{0.2+0.12+0.04+0.24} = \frac{0.24}{0.6} \\
\boxed{P(Y|Z) = 0.4}
\end{aligned}$$

Next we simplify the right hand side:

$$\begin{aligned}
P(Y) &= P(Y,\sum X,\sum Z) \\
&= 0.2+0.4+0.1+0.06 \\
\boxed{P(Y) = 0.4}
\end{aligned}$$

Now we compare the two sides:

$$P(Y|Z) \stackrel{?}{=} P(Y)$$

$$\boxed{0.4 = 0.4}$$

We have successfully verified (2). One more condition to verify!

Condition 3

We attempt to verify (3).

$$P(Z|Y) = P(Z)$$

First we simplify the left hand side:

$$\begin{aligned}
P(Z|Y) &= \frac{P(Z,Y)}{P(Y)} \\
&= \frac{P(Z,Y,\sum X)}{P(Y,\sum X,\sum Z)} \\
&= \frac{0.2+0.04}{0.2+0.04+0.1+0.06} = \frac{0.24}{0.4} \\
\boxed{P(Z|Y) = 0.6}
\end{aligned}$$

Next we simplify the right hand side:

$$\begin{aligned}
P(Z) &= P(Z,\sum X,\sum Y) \\
&= 0.2+0.12+0.04+0.24 \\
\boxed{P(Z) = 0.6}
\end{aligned}$$

Now we compare the two sides:

$$P(Z|Y) \stackrel{?}{=} P(Z)$$
$$\boxed{0.6 = 0.6}$$

We have satisfied (3).

Conclusion

We have satisfied (1), (2), and (3). Therefore, we can conclude that Y and Z *are* independent.

2 Question 2: Neural Networks

The following problems were answered using the provided neural network on page 2 of "hw3v2.pdf". Note that all bias nodes b have an output of 1.

Definition of linear perceptron output

$$a = \sum_{d=0..D} w_d \times x_d \tag{4}$$

$$\sigma(x) = \frac{1}{1 + e^{(-w'x)}} \tag{5}$$

Output of all nodes with inputs x_i and weights w_i are calculated with (4), and the classification is designated by the sigmoid function (5).

2.1 a) Calculate the output of this network for the input $\{x_1 = 0, x_2 = 1\}$.

h_1

First we calculate the value of $w'x$ for h_1 with (4):

$$w'x_{h_1} = -1 * b + 2 * x_1 + 1 * x_2$$
$$w'x_{h_1} = -1 * 1 + 2 * 0 + 1 * 1$$
$$w'x_{h_1} = -1 + 1$$
$$\boxed{w'x_{h_1} = 0}$$

Now we apply (5) to find h_1:

$$h_1 = \sigma\left(w'x_{h_1}\right)$$
$$h_1 = \frac{1}{1+e^{\left(-w'x_{h_1}\right)}}$$
$$h_1 = \frac{1}{1+e^{(0)}}$$
$$\boxed{h_1 = \frac{1}{2}}$$

h_2

Now we calculate the value of $w'x$ for h_2 with (4):

$$w'x_{h_2} = 0.5 * b - 3 * x_1 - 2 * x_2$$
$$w'x_{h_2} = 0.5 * 1 - 3 * 0 - 2 * 1$$
$$w'x_{h_2} = 0.5 - 2$$
$$\boxed{w'x_{h_2} = -1.5}$$

Next we apply (5) to find h_2:

$$h_2 = \sigma\left(w'x_{h_2}\right)$$
$$h_2 = \frac{1}{1+e^{\left(-w'x_{h_2}\right)}}$$
$$h_2 = \frac{1}{1+e^{(1.5)}}$$
$$h_2 = 0.1824255238$$
$$\boxed{h_2 \approx 0.18}$$

k

Finally, we can calculate $w'x$ for k with (4):

$$w'x_k = -0.5 * b - 1 * h_1 + 0.8 * h_2$$
$$w'x_k = -0.5 * 1 - 1 * 0 + 0.8 * (-1.5)$$
$$w'x_k = -0.5 - 1.2$$
$$\boxed{w'x_k = -1.7}$$

Again, we apply (5) to find k:

$$
\begin{aligned}
k &= \sigma\left(w' x_k\right) \\
k &= \frac{1}{1+e^{(-w' x_k)}} \\
k &= \frac{1}{1+e^{(1.7)}} \\
k &= 0.1544652651 \\
&\boxed{k \approx 0.15}
\end{aligned}
$$

Thus the output of the network will be $y \approx 0.15$.

2.2 b) Now you are going to compute one step of the backpropagation algorithm. The weights of the output node k (red node) are fixed. The input for the training instance is $\{x_1 = 0, x_2 = 1\}$ and the output of this training instance is $y = 1$. Please compute the updated weights for the hidden layer (the two blue nodes) by performing ONE step of gradient descent. Let the step size α equal 0.1.

For output unit k, compute error term δ_k:

$$
\begin{aligned}
&\delta_k \leftarrow \left(o_k - y_k\right) o_k \left(1 - o_k\right) \\
&\delta_k \leftarrow (0.15 - 1.) * 0.15 * (1 - 0.15) \\
&\delta_k \leftarrow -0.85 * 0.15 * 0.85 \\
&\boxed{\delta_k \leftarrow 0.11}
\end{aligned}
$$

Now compute error term for hidden node h_1:

$$
\begin{aligned}
&\delta_{h_1} \leftarrow \left(\sum_{i \in succ(h_1)} w_{ih_1} \delta_i\right) o_{h_1} \left(1 - o_{h_1}\right) \\
&\delta_{h_1} \leftarrow \left(w_{kh_1} \delta_k\right) o_{h_1} \left(1 - o_{h_1}\right) \\
&\delta_{h_1} \leftarrow (-1 * 0.11) * \frac{1}{2} * \left(1 - \frac{1}{2}\right) \\
&\boxed{\delta_{h_1} \leftarrow -0.0275}
\end{aligned}
$$

The error term for node h_2 is similar:

$$
\begin{aligned}
&\delta_{h_2} \leftarrow \left(w_{kh_2} \delta_k\right) o_{h_2} \left(1 - o_{h_2}\right) \\
&\delta_{h_2} \leftarrow (0.8 * 0.11) * 0.18 * (1 - 0.18) \\
&\boxed{\delta_{h_2} \leftarrow 0.131}
\end{aligned}
$$

Now we need to update all non-fixed weights with (6).

$$w_{ji} \leftarrow w_{ji} - \alpha \delta_j x_{ji} \tag{6}$$

- $w_{h_i b}$
- $w_{h_i x_1}$
- $w_{h_i x_2}$

w_{bh_i}

$$\begin{aligned} &w_{h_1 b} \leftarrow w_{h_1 b} - \alpha \delta_{h_1} b \\ &w_{h_1 b} \leftarrow -1 - 0.1 * -0.0275 * 1 \\ &\boxed{w_{h_1 b} \leftarrow -0.99725} \end{aligned}$$

$$\begin{aligned} &w_{h_2 b} \leftarrow w_{h_2 b} - \alpha \delta_{h_2} b \\ &w_{h_2 b} \leftarrow 0.5 - 0.1 * 0.131 * 1 \\ &\boxed{w_{h_2 b} \leftarrow 0.4869} \end{aligned}$$

$w_{h_i x_1}$

$$\begin{aligned} &w_{h_1 x_1} \leftarrow w_{h_1 x_1} - \alpha \delta_{h_1} x_1 \\ &w_{h_1 x_1} \leftarrow 2 - 0.1 * -0.0275 * 0 \\ &\boxed{w_{h_1 x_1} \leftarrow 2} \end{aligned}$$

$$\begin{aligned} &w_{h_2 x_1} \leftarrow w_{h_2 x_1} - \alpha \delta_{h_2} x_1 \\ &w_{h_2 x_1} \leftarrow -3 - 0 \\ &\boxed{w_{h_2 x_1} \leftarrow -3} \end{aligned}$$

$w_{h_i x_2}$

$$\begin{aligned} &w_{h_1 x_2} \leftarrow w_{h_1 x_2} - \alpha \delta_{h_1} x_2 \\ &w_{h_1 x_2} \leftarrow 1 - 0.1 * -0.0275 * 1 \\ &\boxed{w_{h_1 x_2} \leftarrow -0.99725} \end{aligned}$$

$$\begin{aligned} &w_{h_2 x_2} \leftarrow w_{h_2 x_2} - \alpha \delta_{h_2} x_2 \\ &w_{h_2 x_2} \leftarrow -2 - 0.1 * 0.131 * 1 \\ &\boxed{w_{h_2 x_2} \leftarrow -2.0131} \end{aligned}$$

2.2.1 Answers

All weights have been updated. To maintain good organization, all final weight values have been listed below.[1]

- $w_{h_1 b} \leftarrow -0.99725$
- $w_{h_2 b} \leftarrow 0.4869$
- $w_{h_1 x_1} \leftarrow 2$
- $w_{h_2 x_1} \leftarrow -3$
- $w_{h_1 x_2} \leftarrow -0.99725$
- $w_{h_2 x_2} \leftarrow -2.0131$

3 Question 3: Bayes Nets

All questions in this section were addressed using the Bayes Net provided in "hw3v2.pdf" on page 3.
Note the following abbreviations that will be employed:

- Storm $\rightarrow S$
- Camp Fire $\rightarrow CF$
- Lightning $\rightarrow L$
- Thunder $\rightarrow T$
- Forest Fire $\rightarrow FF$

[1] Note that w_{ji} is the weight from unit i to unit j.

3.1 a) What is the probability of a forest fire?

The probability of a forest fire is the sum of probabilities of a forest fire joint with all other possible sets of events.

$$
\begin{aligned}
\Pr(FF) &= \Pr\left(FF, \sum L, \sum CF\right) \\
\Pr(FF) &= \Pr(FF, L, CF) + \Pr\left(FF, L, \bar{CF}\right) + \Pr\left(FF, \bar{L}, CF\right) + \Pr\left(FF, \bar{L}, \bar{CF}\right) \\
\Pr(FF) &= [\Pr(FF|L, CF)\Pr(L)\Pr(CF)] + \left[\Pr(FF|L, \bar{CF})\Pr(L)\Pr(\bar{CF})\right] \\
&\quad + \left[\Pr(FF|\bar{L}, CF)\Pr(\bar{L})\Pr(CF)\right] + \left[\Pr(FF|\bar{L}, \bar{CF})\Pr(\bar{L})\Pr(\bar{CF})\right] \\
\Pr(L) &= \Pr\left(L, \sum S\right) = \Pr(L|S)\Pr(S) + \Pr(L|\bar{S})\Pr(\bar{S}) \\
\Pr(L) &= 0.5 * 0.1 + 0.05 * 0.9 \\
\Pr(L) &= 0.05 + 0.045 = 0.095 \\
\Pr(\bar{L}) &= \Pr\left(\bar{L}, \sum S\right) = \Pr(\bar{L}|S)\Pr(S) + \Pr(\bar{L}|\bar{S})\Pr(\bar{S}) \\
\Pr(\bar{L}) &= 0.5 * 0.1 + 0.95 * 0.9 \\
\Pr(\bar{L}) &= 0.05 + 0.855 = 0.905 \\
\Pr(FF) &= [0.5 * 0.095 * 0.75] + [0.4 * 0.095 * 0.25] \\
&\quad + [0.1 * 0.905 * 0.75] + [0.01 * 0.905 * 0.25] \\
\Pr(FF) &= [0.035625] + [0.0095] \\
&\quad + [0.067875] + [0.0022625] \\
\Pr(FF) &= 0.1152625 \\
&\boxed{\Pr(FF) \approx 0.115}
\end{aligned}
$$

3.2 b) What is the probability of a forest fire given thunder?

$$\Pr(FF|T) = \frac{\Pr(FF,T)}{\Pr(T)}$$

$$\Pr(FF,T) = \Pr\left(FF,T,\sum CF,\sum L,\sum S\right)$$

$$\begin{aligned}
\Pr(FF,T) = {} & \Pr(FF,T,CF,L,S) + \Pr(FF,T,CF,L,\bar{S}) \\
& + \Pr(FF,T,CF,\bar{L},S) + \Pr(FF,T,CF,\bar{L},\bar{S}) \\
& + \Pr(FF,T,\bar{CF},L,S) + \Pr(FF,T,\bar{CF},L,\bar{S}) \\
& + \Pr(FF,T,\bar{CF},\bar{L},S) + \Pr(FF,T,\bar{CF},\bar{L},\bar{S})
\end{aligned}$$

$$\begin{aligned}
\Pr(FF,T,CF,L,S) &= \Pr(FF|L,C)\Pr(T|L)\Pr(CF)\Pr(L|S)\Pr(S) \\
&= 0.5 * 0.95 * 0.75 * 0.5 * 0.1 = 0.0178125 \\
\Pr(FF,T,CF,L,\bar{S}) &= \Pr(FF|L,C)\Pr(T|L)\Pr(CF)\Pr(L|\bar{S})\Pr(\bar{S}) \\
&= 0.5 * 0.95 * 0.75 * 0.05 * 0.9 = 0.01603125 \\
\Pr(FF,T,CF,\bar{L},S) &= \Pr(FF|\bar{L},CF)\Pr(T|\bar{L})\Pr(CF)\Pr(\bar{L}|S)\Pr(S) \\
&= 0.1 * 0.2 * 0.75 * 0.5 * 0.1 = 0.00075 \\
\Pr(FF,T,CF,\bar{L},\bar{S}) &= \Pr(FF|\bar{L},CF)\Pr(T|\bar{L})\Pr(CF)\Pr(\bar{L}|\bar{S})\Pr(\bar{S}) \\
&= 0.1 * 0.2 * 0.75 * 0.95 * 0.9 = 0.012825 \\
\Pr(FF,T,\bar{CF},L,S) &= \Pr(FF|L,\bar{CF})\Pr(T|L)\Pr(\bar{CF})\Pr(L|S)\Pr(S) \\
&= 0.4 * 0.95 * 0.25 * 0.5 * 0.1 = 0.00475 \\
\Pr(FF,T,\bar{CF},L,\bar{S}) &= \Pr(FF|L,\bar{CF})\Pr(T|L)\Pr(\bar{CF})\Pr(L|\bar{S})\Pr(\bar{S}) \\
&= 0.4 * 0.95 * 0.25 * 0.05 * 0.9 = 0.004275 \\
\Pr(FF,T,\bar{CF},\bar{L},S) &= \Pr(FF|\bar{L},\bar{CF})\Pr(T|\bar{L})\Pr(\bar{CF})\Pr(\bar{L}|S)\Pr(S) \\
&= 0.01 * 0.2 * 0.25 * 0.5 * 0.1 = 0.000025 \\
\Pr(FF,T,\bar{CF},\bar{L},\bar{S}) &= \Pr(FF|\bar{L},\bar{CF})\Pr(T|\bar{L})\Pr(\bar{CF})\Pr(\bar{L}|\bar{S})\Pr(\bar{S}) \\
&= 0.01 * 0.2 * 0.25 * 0.95 * 0.9 = 0.0004275
\end{aligned}$$

$$\begin{aligned}
\Pr(FF,T) &= 0.0178125 + 0.01603125 + 0.00075 + 0.012825 \\
&\quad + 0.00475 + 0.004275 + 0.000025 + 0.0004275 \\
\Pr(FF,T) &= 0.05689625 \\
\Pr(T) &= \Pr(T,\sum L) = \Pr(T,L) + \Pr(T,\bar{L}) \\
\Pr(T) &= \Pr(T|L)\Pr(L) + \Pr(T|\bar{L})\Pr(\bar{L}) \\
\Pr(T) &= 0.95 * 0.095 + 0.2 * 0.905 = 0.27125 \\
\Pr(FF|T) &= \frac{0.05689625}{0.27125} \\
\Pr(FF|T) &= 0.20975576036866359447004608294931
\end{aligned}$$

$$\boxed{\Pr(FF|T) \approx 0.21}$$

3.3 c) What is the probability that there is a storm given that there is a forest fire?

$$\begin{aligned}
\Pr(S|FF) &= \frac{\Pr(S,FF)}{\Pr(FF)} \\
\Pr(S,FF) &= \Pr\left(S,FF,\sum L,\sum CF\right) \\
&= \Pr(S,FF,L,CF) + \Pr(S,FF,L,\bar{CF}) + \Pr(S,FF,\bar{L},CF) + \Pr(S,FF,\bar{L},\bar{CF}) \\
\Pr(S,FF,L,CF) &= \Pr(S)\Pr(FF|L,CF)\Pr(L|S)\Pr(CF) \\
&= 0.1 * 0.5 * 0.5 * 0.75 = 0.01875 \\
\Pr(S,FF,L,\bar{CF}) &= \Pr(S)\Pr(FF|L,\bar{CF})\Pr(L|S)\Pr(\bar{CF}) \\
&= 0.1 * 0.4 * 0.5 * 0.25 = 0.005 \\
\Pr(S,FF,\bar{L},CF) &= \Pr(S)\Pr(FF|\bar{L},CF)\Pr(\bar{L}|S)\Pr(CF) \\
&= 0.1 * 0.1 * 0.5 * 0.75 = 0.00375 \\
\Pr(S,FF,\bar{L},\bar{CF}) &= \Pr(S)\Pr(FF|\bar{L},\bar{CF})\Pr(\bar{L}|S)\Pr(\bar{CF}) \\
&= 0.1 * 0.01 * 0.5 * 0.25 = 0.000125 \\
\Pr(S,FF) &= 0.01875 + 0.005 + 0.00375 + 0.000125 = 0.027625 \\
\Pr(FF) &= 0.1152625 \\
\Pr(S|FF) &= 0.23967031775295521093156924411669 \\
&\boxed{\Pr(S|FF) \approx 0.24}
\end{aligned}$$

3.4 d) What is the probability of thunder given that there is no storm?

$$\begin{aligned}
\Pr(T|\bar{S}) &= \frac{\Pr(T,\bar{S})}{\Pr(\bar{S})} \\
\Pr(T,\bar{S}) &= \Pr(T,\bar{S},\sum L) \\
&= \Pr(T,\bar{S},L) + \Pr(T,\bar{S},\bar{L}) \\
&= \Pr(T|L)\Pr(\bar{S})\Pr(L|\bar{S}) + \Pr(T|\bar{L})\Pr(\bar{S})\Pr(\bar{L}|\bar{S}) \\
&= 0.95 * 0.9 * 0.05 + 0.2 * 0.9 * 0.95 = 0.04275 + 0.171 \\
&= 0.21375 \\
\Pr(\bar{S}) &= 0.9 \\
\Pr(T|\bar{S}) &= \frac{0.21375}{0.9} \\
&\boxed{\Pr\left(T\middle|\bar{S}\right) = 0.2375}
\end{aligned}$$

3.5 e) What is the probability of a camp fire and a forest fire?

$$\begin{aligned}
\Pr(CF, FF) &= \Pr(CF, FF, \sum L) \\
&= \Pr(CF, FF, L) + \Pr(CF, FF, \bar{L}) \\
&= \Pr(CF)\Pr(FF|CF, L)\Pr(L) + \Pr(CF)\Pr(FF|CF, \bar{L})\Pr(\bar{L}) \\
&= 0.75 * 0.5 * 0.095 + 0.75 * 0.1 * 0.905 = 0.035625 + 0.067875
\end{aligned}$$

$$\boxed{\Pr(CF, FF) = 0.1035}$$

Question 4: Spam Classification with a Naïve Bayes Classifier

This work has been completed separately and handed in electronically.