Laser Engineering

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Contents

I. Laser Amplification Process in Pulsed Laser Amplifiers

I.1. basic level schemes

To operate a laser we will always need a so called inversion, meaning, that there are more ions in the excited state than in the ground state. To better understand the basic processes, we can use the following basic models:

2 level scheme
- no laser operation under optical pumping as pump and laser operate at the same wavelength!
- as soon as inversion reaches equality under pumping, no pump radiation is absorbed any more
- no gain!

3 level scheme
- pump excites into a separate level from where the atoms quickly relax into a meta stable level.
- as the ground state is still occupied, a certain amount of atoms got to be excited to achieve inversion
- in pump pulsed mode this can be a strong loss process!
- often used to describe laser media with re-absorption (Yb^{3+})

4 level scheme
- laser pumps into a separate level from where the atoms quickly relax into a meta stable level.
- from this level the laser process goes to a non occupied ground level from which it quickly relaxes back into the ground state.
- inversion is achieved with the first excited atom!
- used to describe laser materials with high quantum defect.

These basic models can be thought of as some idealized laser medium. Real laser materials typically involve a much higher number of energy levels. In the following we will derive a model that takes this into account. Therefore, we use a system of 2 manifolds with a continuous level distribution in each representing a set of energy levels.

For the interaction with a radiation field we have to take care, that only the available transitions are included with the correct weight, by introducing the Boltzmann occupation factors for the upper (1) and lower manifolds (2). Further for practical reasons we use the photon density Φ to describe the radiation field. Hence the rate equation for the interaction with an incident photon density reads:

$$\frac{d\Phi}{dz} = -\sigma\Phi \cdot (f_1 N_1 - f_2 N_2) \tag{I.1}$$

$$\frac{d\Phi}{dt}\frac{dt}{dz} = -\frac{dN_1}{dt}\frac{1}{c} = \sigma\Phi \cdot (f_1 N_1 - f_2 N_2) \tag{I.2}$$

$$\frac{dN_1}{dt} = -\sigma\Phi c \cdot (f_1 N_1 - f_2 N_2) \tag{I.3}$$

Here f denotes the Boltzmann occupation factors as used in the Mc-relation:

$$f_i = \frac{e^{-\frac{E_i}{kT}}}{Z_x} \tag{I.4}$$

If we now consider the pump and laser process with their appropriate cross sections for the emission and absorption, and the spontaneous emission the rate equation for the complete laser cycle is:

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = -\frac{\mathrm{d}N_1}{\mathrm{d}t} = \underbrace{\sigma_\mathrm{a}(\lambda_\mathrm{p})c\Phi_\mathrm{p}(f_{1\mathrm{p}}N_1 - f_{2\mathrm{p}}N_2)}_{1} \underbrace{-\frac{N_2}{\tau_\mathrm{f}}}_{2} \underbrace{-\sigma_\mathrm{e}(\lambda_\mathrm{l})c\Phi_\mathrm{l}(f_{2\mathrm{l}}N_2 - f_{1\mathrm{l}}N_1)}_{3} \tag{I.5}$$

Here "p" denotes the parameters involved in the pumping process and "l" the corresponding ones in the laser process. The under set numbers 1 to 3 denote the parts of the equation representing the pump process, the spontaneous emission and the amplification process respectively.

I.2. Pumping

During the pump process the extracting laser field will be neglected. hence we obtain:

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = -\frac{\mathrm{d}N_1}{\mathrm{d}t} = \sigma_\mathrm{a}(\lambda_\mathrm{p})c\Phi_\mathrm{p}(f_{1\mathrm{p}}N_1 - f_{2\mathrm{p}}N_2) - \frac{N_2}{\tau_\mathrm{f}} \tag{I.6}$$

As typical for most diode pumped lasers we will now consider an end pumped setup, meaning, that the pump beam is co-propagating with the laser beam through the laser medium. For a realistic representation we got to consider as well the number densities as the photon density dependent on space. We will do so using the propagation direction z in 1D.

To obtain a more convenient representation of our formula we further introduce some substitutions:

- inversion

$$\beta(t,z) = \frac{N_2(t,z)}{N_{dop}} \tag{I.7}$$

- pump rate

$$R(t,z) = \sigma_a c\Phi_P(t,z) \tag{I.8}$$

- equilibrium inversion (the name will become clearer looking at the final formula)

$$\beta_{eq} = \frac{f_1}{f_1 + f_2} \tag{I.9}$$

$$\beta_{eq} = \frac{\frac{1}{Z_1} \cdot e^{-\frac{E_i}{kT}}}{\frac{1}{Z_1} \cdot e^{-\frac{E_i}{kT}} + \frac{1}{Z_2} \cdot e^{-\frac{E_j}{kT}}} \tag{I.10}$$

$$\beta_{eq} = \frac{e^{-\frac{E_i}{kT}}}{e^{-\frac{E_i}{kT}} + \frac{Z_1}{Z_2} e^{-\frac{E_j}{kT}}} \tag{I.11}$$

$$\beta_{eq} = \frac{e^{-\frac{E_i}{kT}}}{e^{-\frac{E_i}{kT}} + \frac{Z_1}{Z_2} e^{-\frac{E_i}{kT}} e^{\frac{E_{ZL} - h\nu}{kT}}} \tag{I.12}$$

$$\beta_{eq} = \frac{1}{1 + \frac{Z_1}{Z_2} e^{\frac{E_{ZL} - h\nu}{kT}}} \tag{I.13}$$

$$\beta_{eq} = \frac{\sigma_a}{\sigma_a + \sigma_a \frac{Z_1}{Z_2} e^{\frac{E_{ZL} - h\nu}{kT}}} \tag{I.14}$$

$$\beta_{eq} = \frac{\sigma_a}{\sigma_a + \sigma_e} \tag{I.15}$$

after substitution we obtain:

$$\frac{\partial \beta(t,z)}{\partial t} = R(t,z) f_{1\mathrm{p}} - \beta(t,z) \cdot \left(\frac{R(t,z) f_{1\mathrm{p}}}{\beta_{\mathrm{eq}}(z)} + \frac{1}{\tau_{\mathrm{f}}} \right) \tag{I.16}$$

As a second equation we would need to describe the evolution of the pump rate during the propagation through the material. For this we utilize equation I.1 and carry out the according substitutions to obtain:

$$\frac{\partial R(t,z)}{\partial z} = -N_{\mathrm{dop}} \sigma_{\mathrm{a}}(z) \cdot \left(1 - \frac{\beta(t,z)}{\beta_{\mathrm{eq}}(z)} \right) R(t,z) \tag{I.17}$$

The time dependent rate equation now is a differential equation of the shape $y'(x) + \mathrm{f}(x) \cdot y(x) = g(x)$, with the known solution $y(x) = e^{-F(x)} \int g(x) \cdot e^{F(x)} dx$ (cf. e.g. [Bro13]). We identify:

- $$y \to \beta \tag{I.18}$$

- $$y' \to \frac{\partial \beta}{\partial t} \tag{I.19}$$

- $$f(x) \to \frac{R(t,z) f_{1\mathrm{p}}}{\beta_{\mathrm{eq}}(z)} + \frac{1}{\tau_{\mathrm{f}}} \tag{I.20}$$

  and hence

  $$F(x) \to \int \frac{R(t,z) f_{1\mathrm{p}}}{\beta_{\mathrm{eq}}(z)} + \frac{1}{\tau_{\mathrm{f}}} dt = \int \frac{R(t,z) f_{1\mathrm{p}}}{\beta_{\mathrm{eq}}(z)} dt + \frac{t}{\tau_{\mathrm{f}}} \tag{I.21}$$

- $$g(x) \to R(t,z) f_{1\mathrm{p}} \tag{I.22}$$

Hence we get:

$$\begin{aligned}\beta(t,z) &= \exp\left[-\int_0^t \frac{R(t',z)f_{1p}}{\beta_{eq}}\mathrm{d}t' - \frac{t}{\tau_f}\right]\cdot \\ &\cdot\left\{\int_0^t R(t',z)f_{1p}\cdot\exp\left[\int_0^{t'}\frac{R(t",z)f_{1p}}{\beta_{eq}}\mathrm{d}t" - \frac{t'}{\tau_f}\right]\mathrm{d}t' + \beta_0(z)\right\}\end{aligned} \tag{I.23}$$

Further by integrating equation I.17 one obtains:

$$R(t,z) = \underbrace{R_0(t)\cdot e^{-N_{dop}\sigma_a z}}_{R_a}\cdot\underbrace{\exp\left(N_{dop}\sigma_a\cdot\int_0^z\frac{\beta(t,z')}{\beta_{\mathrm{eq}}(z')}\mathrm{d}z'\right)}_{R_{\mathrm{b}}} \tag{I.24}$$

The first R_a term corresponds to the Lambert-Beer law, while the R_b term acts as a correction factor for non zero inversion. We now derived a solution of the differential equation system in the integral form (which is advantageous as such expressions can be well handled by numerical integration). Though still one cannot directly calculate the result as both equations are still linked with each other, it is now possible to obtain a solution based on an iterative approach.

As a start we assume two cases:

zero inversion at all times

$$R_b^1 = R_b(\beta^0_{low}(t,z)=0) = 1 \tag{I.25}$$

maximum inversion at all times

$$R_b^1 = R_b(\beta^0_{high}(t,z)=\beta_{eq}) = \frac{1}{R_a} \tag{I.26}$$

Using now an iterative algorithm calculating

$$R_b^{(m+1)}(t,z) = R_b(\beta^m_{low/high}(t,z)) \tag{I.27}$$

and

$$\beta^{(m+1)}_{low/high}(t,z) = \beta(R_a R_b^{(m+1)}(t,z)) \tag{I.28}$$

one obtains successive new estimations of minimum and maximum inversions which approach a common value representing the solution (The claim, that solutions originating from the maximum and minimum assumptions always stay maximum and minimum assumptions that are always closer to the real solution can of course be proofed. I desist from this here as it is not complicated and saves some time).

One can now numerically calculate the successive estimations for both cases together with a condition to abort the calculation as soon as the difference between both solutions is reasonably small. This then is the solution for the pump process. Solutions for successive iterations for a sample case are shown in figure I.1.

Though this might seem a rather complex calculation it is pretty effective and accurate. You can have a try here with the supplied small program using this algorithm.

note: f_{1p} can usually be approximated with 1 as the pump process typically starts at the lowest level of the lowest manifold.

Manipulating the input parameters for the pump process one can see several effects, that are present in an actual laser material (please feel free to use the given simulation to test the statements below):

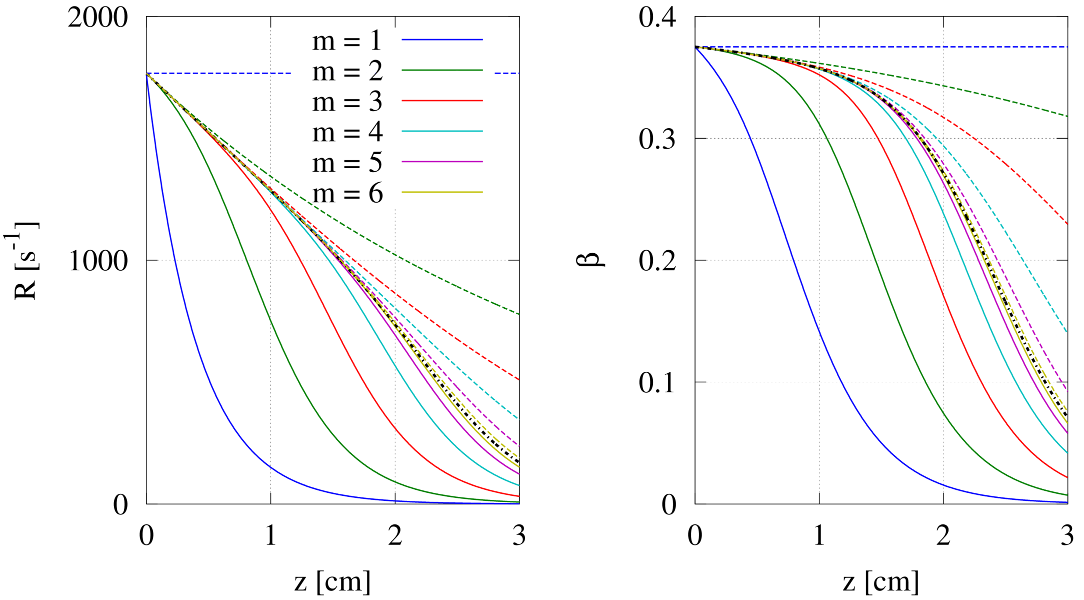

Figure I.1.: Sample iteration steps for the calculation of the inversion in a laser medium (here Yb:CaF_2 pumped at 980nm). The dashed lines show the upper limit assumptions and the solid lines the lower limit assumption. the black line corresponds to the final result. m denotes the iteration number. Taken from [Kör14].

- For a pump duration much longer than the fluorescence lifetime the introduced changes to the materials inversion are rather small. Hence it does not make sense to pump much longer than the lifetime. In practice most lasers work with a pump duration in the range of the fluorescence lifetime. In extreme cases a maximum of twice this value is used.

- For low pump densities the pump density and, hence, the inversion follow an exponential decay as known from Lambert Beers law.

- For high pump intensities (for a rule of thumb what is high and what is low see next section) the inversion is somehow "pushed through the material along the propagation axis (the inversion in at the entrance face of the crystal stays rather constant, before it converts back to the exponential decay for later times)

- The maximum inversion, that we can achieve for high pump intensities approaches β_{eq} and therefore is strongly dependent on the wavelength. The higher cross sections are, the less pump intensity is needed to approach this limit.

I.2.1. Pump saturation intensity

The pump saturation is the pump intensity which is needed to achieve $\frac{\beta_{eq}}{2}$ for infinite pump duration (on surface!). It can be used as an order of magnitude when saturation effects are needed to be taken into account.

We start with the base equation and include that for infinite pump duration there is no more change in the inversion:

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = R_{sat} f_{1\mathrm{p}} - \beta \cdot \left(\frac{R f_{1\mathrm{p}}}{\beta_{\mathrm{eq}}} + \frac{1}{\tau_f} \right) = 0 \tag{I.29}$$

$$\rightarrow 0 = R_{sat} f_{1\mathrm{p}} \cdot \left(1 - \frac{\beta}{\beta_{eq}} \right) + \frac{\beta}{\tau_f} \tag{I.30}$$

$$\rightarrow R_{sat} = \frac{\beta}{f_{1\mathrm{p}} \tau_f \cdot \left(1 - \frac{\beta}{\beta_{eq}} \right)} \tag{I.31}$$

We replace $\beta = \frac{\beta_{eq}}{2}$:

$$R_{sat} = \frac{\beta_{eq}}{f_{1\mathrm{p}} \tau_f} \tag{I.32}$$

$$R_{sat} = \frac{\sigma_a}{f_{1\mathrm{p}} \tau_f (\sigma_a + \sigma_e)} \tag{I.33}$$

This can also be expressed in terms of intensity:

$$R_{sat} = \sigma_a \Phi_{sat} c \tag{I.34}$$

$$R_{sat} = \sigma_a \frac{I_{sat}}{h\nu} \tag{I.35}$$

$$\rightarrow I_{sat} = \frac{h\nu}{f_{1\mathrm{p}} \tau_f \cdot (\sigma_a + \sigma_e)} \tag{I.36}$$

Here are some example materials:

- Yb:CaF_2 pumped at 940 nm: 52 kw/cm^2
- Yb:CaF_2 pumped at 980 nm: 7.6 kw/cm^2
- Yb:YAG pumped at 940 nm: 22.4 kw/cm^2

I.2.2. storage efficiency

The maximum efficiency can be calculated directly using the achieved inversion. For this, one has to calculate the maximum fluence, that can be extracted on the laser wavelength, given by the amount of excited ions minus the amount of ions needed for transparency of the laser material per area:

$$F_{ex} = N_{dop} h\nu_l \cdot \int_0^l \beta(z) - \beta_{eq}(\nu_l) dz \tag{I.37}$$

Comparing this with the incident pump fluence given by

$$F_{pump} = I_{pump} \cdot \tau_{pump} = \frac{\Phi}{ch\nu_{pump}} \cdot \tau_{pump} \tag{I.38}$$

gives an expression for the maximum storage efficiency η_{store} of a laser:

$$\eta_{store} = \frac{F_{ex}}{F_{pump}} < 1 \tag{I.39}$$

An example for the storage efficiency and the according small signal gain (see next section) as a function of pump intensity and Temperature is given in figure I.2.

So we have the following outcomes:

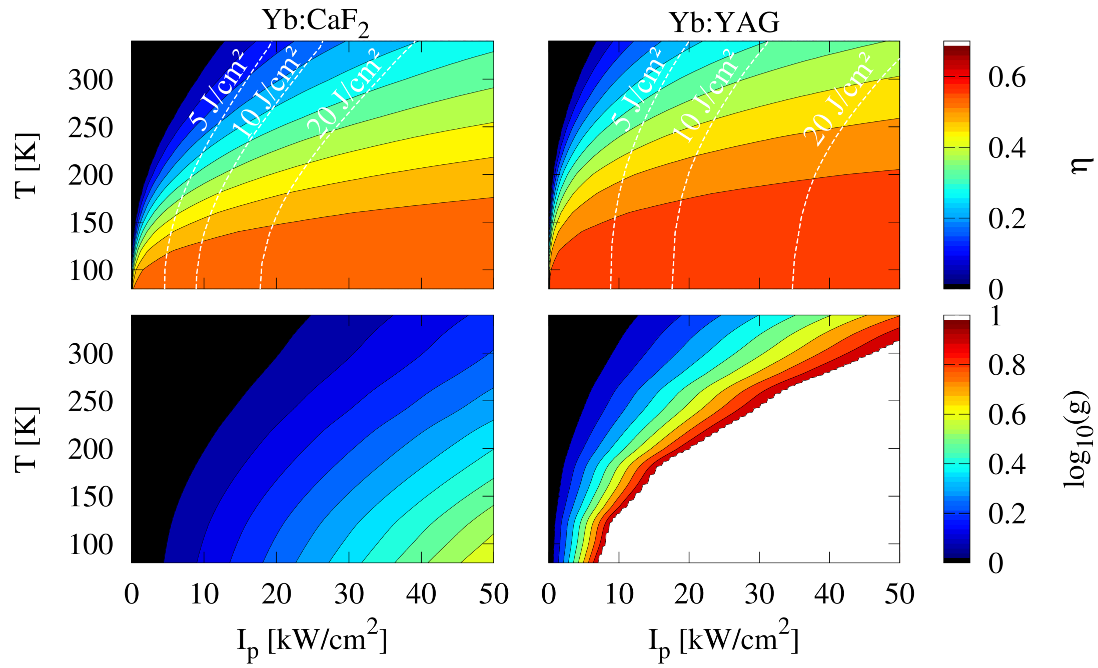

Figure I.2.: Storage efficiency (upper graph) and small signal gain (lower graph) as a function of pump intensity and temperature for Yb:CaF_2 and Yb:YAG in a sample configuration. Pump duration was equal to the corresponding radiative lifetime. The inset shows the corresponding extraction fluencies when extracting with optimum efficiency. Taken from [Kör14].

- efficiency increases with intensity
- efficiency increases by multipass pumping - limited by setup
- efficiency increases for lower temperatures
- efficiency increases for shorter but more intense pump pulses - limited by size of pump engine - compromise: pump duration about equal the fluorescence life time.

So why cant we just increase the pump intensity as far as we would like?

- LIDT: for nanosecond pulses extracting pulses with more than approx. 10J/cm^2 will destroy our optics
- B-Integral: nonlinear effects will occure (more Details later!)
- Brightness of pump: The higher the pump intensity the higher the opening angle will be.

I.3. Amplification

We now will have a closer look on the amplification process itself For this we will use the rate equation in the following form neglecting as well the pump radiation as the fluorescence as this process will be rather on the timescales of nanoseconds:

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = -\sigma_\mathrm{e}(\lambda_\mathrm{l})c\Phi_\mathrm{l}(f_{21}N_2 - f_{11}N_1) \tag{I.40}$$

I.3.1. small signal gain

To estimate the small signal gain we can again use the photon density and again do the same substitutions as for the pump process:

- inversion

$$\beta(t,z) = \frac{N_2(t,z)}{N_{dop}} \tag{I.41}$$
$$N_1 = N_{dop} - N_2 \tag{I.42}$$

- equilibrium inversion (the name will become clearer looking at the final formula)

$$\beta_{eq} = \frac{f_1}{f_1 + f_2} \tag{I.43}$$
$$\beta_{eq} = \frac{\sigma_a}{\sigma_a + \sigma_e} \tag{I.44}$$

Furthermore we will assume, that β is constant over z.

$$\frac{d\Phi}{dz} \cdot c = -\frac{\mathrm{d}N_2}{\mathrm{d}t} \tag{I.45}$$
$$= \sigma_{\mathrm{e}} c \Phi_{\mathrm{l}} (f_{2\mathrm{l}} N_2 - f_{1\mathrm{l}} N_1) \tag{I.46}$$
$$\frac{d\Phi}{dz} = \sigma_{\mathrm{e}} \Phi_{\mathrm{l}} \cdot [(f_{2\mathrm{l}} + f_{1\mathrm{l}}) \cdot N_2 - f_{1\mathrm{l}} N_{dop}] \tag{I.47}$$
$$\frac{d\Phi}{dz} = \sigma_{\mathrm{e}} \Phi_{\mathrm{l}} N_{dop} (f_{2\mathrm{l}} + f_{1\mathrm{l}}) \cdot (\beta - \beta_{eq}) \tag{I.48}$$
$$\rightarrow \Phi = \Phi_0 \cdot e^{\sigma_{\mathrm{e}} N_{dop} (f_{2\mathrm{l}} + f_{1\mathrm{l}}) \cdot (\beta - \beta_{eq}) z} \tag{I.49}$$
$$\rightarrow \Phi = \Phi_0 \cdot e^{\sigma_{\mathrm{e}} N_{dop} f_2 \cdot \frac{f_{2\mathrm{l}} + f_{1\mathrm{l}}}{f_2} \cdot (\beta - \beta_{eq}) z} \tag{I.50}$$
$$\tag{I.51}$$

The expression $\frac{f_1+f_2}{f_2}$ can be transformed in a similar way as done for β_{eq} before, using $E_i = E_j + E_{ZL} - h\nu$:

$$\frac{f_1 + f_2}{f_2} = \frac{\frac{1}{Z_2} e^{-\frac{E_j}{kT}} + \frac{1}{Z_1} e^{-\frac{E_i}{kT}}}{\frac{1}{Z_2} e^{-\frac{E_j}{kT}}} \tag{I.52}$$
$$\frac{f_1 + f_2}{f_2} = \frac{e^{-\frac{E_j}{kT}} + \frac{Z_2}{Z_1} e^{-\frac{E_j}{kT}} e^{-\frac{E_{ZL} - h\nu}{kT}}}{e^{-\frac{E_j}{kT}}} \tag{I.53}$$
$$\frac{f_1 + f_2}{f_2} = \frac{1 + \frac{Z_2}{Z_1} e^{-\frac{E_{ZL} - h\nu}{kT}}}{1} \tag{I.54}$$
$$\frac{f_1 + f_2}{f_2} = \frac{\sigma_e + \frac{Z_2}{Z_1} \sigma_e e^{-\frac{E_{ZL} - h\nu}{kT}}}{\sigma_e} \tag{I.55}$$
$$\frac{f_1 + f_2}{f_2} = \frac{\sigma_e + \sigma_a}{\sigma_e} \tag{I.56}$$
$$\tag{I.57}$$

Now we can further modify our equation:

$$\Phi = \Phi_0 \cdot e^{\sigma_e N_{dop} f_2 \cdot \frac{\sigma_e + \sigma_a}{\sigma_e} \cdot (\beta - \beta_{eq}) z} \tag{I.58}$$

$$\Phi = \Phi_0 \cdot e^{N_{dop} f_2 \cdot (\sigma_e + \sigma_a) \cdot (\beta - \beta_{eq}) z} \tag{I.59}$$

$$\Phi = \Phi_0 \cdot e^{N_{dop} f_2 \cdot (\sigma_e \beta + \sigma_a \beta - \sigma_a) z} \tag{I.60}$$

$$\Phi = \Phi_0 \cdot e^{N_{dop} f_2 \cdot [\sigma_e \beta - \sigma_a (1 - \beta)] z} \tag{I.61}$$

$$\tag{I.62}$$

This also demonstrates, that emission as well as absorption are handled well with these equations. Further equivalent as before for the pump we will assume that f_2 can be approximated with 1.

It can be seen:

- the small signal gain goes exponential with the inversion
- there is only gain, when $\beta > \beta_{eq}$ is fulfilled

I.3.2. saturation fluence

Similar to the pump process one can define a saturation fluence which corresponds to the input fluence of an amplifier for which the small signal gain is reduced by $1/e$. It is given by:

$$F_{sat} = \frac{h\nu}{\sigma_a + \sigma_e} \tag{I.63}$$

As this fluence denotes an operation scheme, which significantly alters the inversion state of the active medium, it is desirable to work at fluencies which are even higher, as this would mean, that the energy is efficiently extracted (the ultimate limit is the laser induced damage threshold). It should be noted, that in an multi-pass amplifier the fluence is added up for every pass, which would allow extraction with higher fluencies than which are applied in a single pass.

Further as soon as one approaches these fluencies, it will no longer be possible to use small signal gain. Hence a more elaborated solution for the amplification got to be used, which will be described in the following section.

note: The derivation is based on using the Frantz-Nodvik (see next section) solution in combination with the rate equation. As this is a bit long, I refrain from showing the derivation here.

I.3.3. Frantz Nodvik

The Frantz - Nodvik algorithm as given in the original paper from 1963 allows for an accurate solution but has to be modified for a three level scheme. We will use the same definition of inversion now as Frantz-Nodvik [FN63]:

$$\Delta = N_2 - N_1 \tag{I.64}$$

This results in a slightly different rate equation:

$$\begin{aligned} \frac{\partial \Delta}{\partial t} &= f_2 \frac{\partial N_2}{\partial t} - f_1 \frac{\partial N_1}{\partial t} \\ &= - c \cdot \sigma_e \phi \cdot \Delta \cdot (f_1 + f_2) \end{aligned} \tag{I.65}$$

Further for the propagating pulse we use the photon transport equation:

$$\frac{\partial \phi}{\partial t} + c \cdot \frac{\partial \phi}{\partial x} = c\sigma_e \phi \Delta \tag{I.66}$$

The solution of this equation system is rather complex, so I do not want to do it here (for interested students: The exact derivation can be found separately in the download section). The final result is:

$$\phi(x,t) = \frac{\phi_0(t-\frac{x}{c})}{1-\left(1-e^{-\sigma_e \int\limits_0^x \Delta_0(x')dx'}\right)\cdot e^{-\frac{c\sigma_e}{1-\beta_{eq}}\cdot \int\limits_{-\infty}^{t-\frac{x}{c}} \phi_0(t')dt'}} \tag{I.67}$$

$$\Delta(x,t) = \frac{\Delta_0(x)\cdot e^{-\sigma_e \int\limits_0^x \Delta_0(x')dx'}}{e^{-\sigma_e \int\limits_0^x \Delta_0(x')dx'} + e^{\frac{c\sigma_e}{1-\beta_{eq}}\cdot \int\limits_{-\infty}^{t-\frac{x}{c}} \phi_0(t')dt'} - 1} \tag{I.68}$$

or in our terms:

$$\phi(t) = \frac{\phi_0(t)}{1-\left(1-e^{-\sigma_e \int\limits_0^{l_{mat}} N_{dop}\frac{\beta_0(x')-\beta_{eq}}{1-\beta_{eq}}dx'}\right)\cdot e^{-\frac{c\sigma_e}{1-\beta_{eq}}\cdot \int\limits_{-\infty}^{t-\frac{x}{c}} \phi_0(t')dt'}} \tag{I.69}$$

$$\beta(x) = \frac{(\beta_0(x)-\beta_{eq})\cdot e^{-\sigma_e \int\limits_0^x N_{dop}\frac{\beta_0(x')-\beta_{eq}}{1-\beta_{eq}}dx'}}{e^{-\sigma_e \int\limits_0^x N_{dop}\frac{\beta_0(x')-\beta_{eq}}{1-\beta_{eq}}dx'} + e^{\frac{c\sigma_e}{1-\beta_{eq}}\cdot \int\limits_{-\infty}^{\infty} \phi_0(t')dt'} - 1} + \beta_{eq} \tag{I.70}$$

For simplification we will now assume a spatially constant inversion and a temporally constant input photon density giving:

$$\phi(t) = \frac{\phi_0}{1-\left(1-e^{-\sigma_e N_{dop}\frac{\beta_0(x')-\beta_{eq}}{1-\beta_{eq}}l_{mat}}\right)\cdot e^{-\frac{c\sigma_e}{1-\beta_{eq}}\cdot\phi_0\cdot(t-\frac{x}{c})}} \tag{I.71}$$

$$\beta(x) = \frac{(\beta_0(x)-\beta_{eq})\cdot e^{-\sigma_e N_{dop}\frac{\beta_0(x')-\beta_{eq}}{1-\beta_{eq}}z}}{e^{-\sigma_e N_{dop}\frac{\beta_0(x')-\beta_{eq}}{1-\beta_{eq}}z} + e^{\frac{c\sigma_e}{1-\beta_{eq}}\cdot\phi_0\tau_p} - 1} + \beta_{eq} \tag{I.72}$$

We can already see, that in the case of Φ_0 very small these equations translate into the ones we already derived for the small signal gain.

Further guesses are hard to make from this complex formula. So one should use this to compute some sample cases. This then allows to make some further statements (again: please use the supplied simulation program to test the statements yourself):

- approaching saturation, the single pass gain is reduced and approaches zero for high input fluencies (complies with energy conservation)

- as long as the gain is not saturated, the input pulse shape is maintained
- in saturation the temporal pulse shape is altered, as the leading edge sees a higher gain then the rest of the pulse (edge steepening). This effect is stronger for:
  - higher gain
  - already steep etched pulse shapes
- the edge steepening should be taken into account for all effects linked to intensity as nonlinear effects and laser induced damage

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