Problem 2.5:

$$\tan(\theta) = \frac{\ell}{z} \qquad \theta = \tan^{-1}\left(\frac{\ell}{z}\right) \ll 1 \Rightarrow \frac{\ell}{z} \ll 1$$

$$r = \sqrt{\ell^2 + z^2} \cong z\sqrt{1+\theta^2}$$

$$= z\left\{1 + \frac{1}{2}\theta^2 - \frac{1}{8}\theta^4 + \cdots\right\}$$

$$\cong z\left\{1 + \frac{1}{2}\theta^2 - \frac{1}{8}\theta^4\right\}$$

$$\mathcal{E}(x,y,z) = \frac{\mathcal{E}_0}{r} e^{ikr} \cong \frac{\mathcal{E}_0}{z} \cdot e^{ikz\left\{1 + \frac{1}{2}\theta^2 - \frac{1}{8}\theta^4\right\}}$$

$$\cong \frac{\mathcal{E}_0}{z} e^{ikz\{1\}} e^{ikz\left\{\frac{1}{2}\theta^2\right\}} \cdot e^{ikz\left\{-\frac{1}{8}\theta^4\right\}}$$

$$e^{ia} \cong 1 \Rightarrow e^{ikz\left\{-\frac{1}{8}\theta^4\right\}} \cong 1$$

$$kz\left\{-\frac{1}{8}\theta^4\right\} = \frac{2\pi}{\lambda} z\left\{-\frac{1}{8}\cdot\frac{\ell^4}{z^4}\right\}$$

$$= \frac{\pi \ell^4}{4\lambda z^3} \ll \pi$$

$$\boxed{\ell^4 \ll 4\lambda z^3} \#$$