PHP 2510 Homework 1

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Problem 1

(a) Suppose that there are 100 people in this group, and I pick two people at the same time (no ordering issue). In this case, the probability of having two people as type B will be:

$$\frac{\binom{10}{2}}{\binom{100}{2}}$$

(b) In this scenario, the order matters since there's already one person selected ahead. Let's also assume that the group has 100 people, and the probability of having type A or O will be p(having only A type) + p(having only O type) + p(having both A and O types):

$$\frac{\binom{42}{1} * \binom{41}{1} + [2 * \binom{42}{1} * \binom{44}{1}] + +\binom{44}{1} * \binom{43}{1}}{\binom{100}{1} * \binom{99}{1}}$$

(c) Follow the assumption for previous question, the only group of people that can donate blood to everyone is the O-type people. Therefore, for probability of choosing a person that can't donate blood to everyone is 1-p (O type):

$$1 - \frac{\binom{44}{1}}{\binom{100}{1}}$$

For the probability of a person that can at least donate two types, it will be 1-p (only one type). Therefore, it will be 1-p (AB type):

$$1 - \frac{\binom{4}{1}}{\binom{100}{1}}$$

Problem 2

In the description of the problem, the order of getting heads and tails matters. Therefore, in each time, the probability of getting a head and the probability of getting a tail will be the same, 0.5, respectively. Therefore, for both cases, their probabilities will both be

$$(0.5)^{10}$$

Problem 3

(a) The size of this sample size will be (number of possible outcomes from the first trial)*(number of possible outcomes from second trial):

$$6 * 6 = 36$$

(b) The events can be in the following combination (odd number, odd number), (even number, even number). The following is the list:

(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5), (2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)

(c) In the previous question, there are 18 different elements in total. Given that the number of all possible outcomes is 36, its probability will then be:

$$\frac{18}{36} = \frac{1}{2}$$

Problem 4

(a) The number of possible combinations will then be:

$$25 * 24 = 600$$

(b) The number of possible combinations will then be:

$$25 * 25 = 625$$

Problem 5

(a) Toss a die for twice. The event that I get an even number in the first trial and the event that I get an even number in the second number.

(b) Toss a die for once. The event that I get an odd number in this trial and the event that I get an even number in this trial.

(c) For the first one, it's independent since the probability of I getting even number in both trial will be:

$$\frac{3}{6} * \frac{3}{6}$$
$$= probability(getting\ an\ even\ number\ in\ first\ trial) * probability(getting\ an\ even\ number\ in\ second\ trial)$$

For the second one, since these events are mutually exclusive. It means that two events can't happen simultaneously. Therefore, for probability of the intersection of the two events, it will be 0, which makes those two events not independent to each other.

Problem 6

(a) 250

(b) 253

Heads	Tails
247	253

Figure 1: Table Screenshot from R

(c) The more times the simulation runs, the more it will get closer to the probability of having tails as 0.5. Therefore, the number of getting tails is expected to be close to 50000.

Problem 7

The same concept from the previous problem can be implemented by changing the content of the list and change rep to False:

```
out <- sample(c("Alice", "Bob", "Charlotte", "Dan", "Emily"), 2, rep = F)
table(out)
```

Figure 2: Code Screenshot

```
out
  Dan Emily
    1     1
```

Figure 3: Result Screenshot

Problem 8

(a) The exact answer will be:

$$\frac{\binom{770}{1}}{\binom{1100}{1}} = 0.7$$

The simulation code can be written as follow:

```
T <- 200
sleep_hour <- rep(c("<5", "5-8", ">9"), times=c(275,770,55))
test <- replicate(T, sample(sleep_hour,1))
prop.table(table(test))
```

Figure 4: Code Screenshot

```
test
  5-8    <5    >9
0.755 0.215 0.030
```

Figure 5: Result Screenshot

The sampling time is set to be 200 to simulate a result that is closer to the theoretical value.

(b) The probability of this scenario will be:

$$\frac{p(\ sleeping\ hour \geq 9)}{p\ (\ sleeping\ hour \geq 5)} = \frac{\frac{55}{1100}}{\frac{770}{1100} + \frac{55}{1100}} = \frac{55}{825} = \frac{11}{165}$$