How to think in dB

Eric Bogatin - October 28, 2013

The dB (decibel) is one of the most common units in engineering, yet also one of the most confusing, especially when it comes to manipulating S-parameters. Here is how not to be confused.

The Bel is a fundamentally new unit, which was originally established in 1928 by The Bell System to describe sound levels. This scale has since been generalized to relate to the log of the ratio of any two power levels.

The most important principle of the Bel scale to keep in mind is that it is, without exception, by definition, the ratio of powers. It originally related to sound level powers, but has since been generalized to *all* types of powers.

Sound level power has a huge range. It starts at the threshold of hearing (TOH) with a sound level of about 10^{-12} W/m^2. The onset of pain is about 10W/m^2, and the power level to rupture an eardrum is 1000W/m^2.

The loudest recorded man-made sound is from rocket launches. The Saturn V produced 100W/m^2 of sound level power about 1 mile from the launch pad. It only got larger closer to the launch pad.

This amount of sound power reflecting off the launch pad could have easily damaged the Space Shuttle. This is why a 900,000 gallon per minute of water sound suppression system was installed on the launch pad (Figure 1). Even with this system, the sound level power at the position of the Orbiter was about 100W/m^2.

Figure 1. Sound suppression system for the Space Shuttle, using 900,000 gallons of water per minute.

There are 15 orders of magnitude between the threshold of hearing and the rupture level. When there is such a large dynamic range, it is more convenient to describe the values in terms of the log of the ratio of the power levels to a reference level.

For sound, the TOH is used as the reference. A whisper is a power about 100 times the TOH. This is a sound level of log (100/1) = 2 Bels. A vacuum cleaner is about 10 million times the TOH. Its loudness is $\log(10^7/1) = 7$ Bels.

But on this scale of Bels, the loudness of a Space Shuttle launch at the orbiter is only 14 Bels. For all its awesome power, 14 just doesn't seem like a very large number. To give it a little larger range, rather than referring to the Bel directly, by convention, we've expanded the scale by 10x and use units of 1/10th a Bel, which is called a deciBel, or dB for short.

A dB is always, without exception, by definition, 10x the log of the ratio of the powers: in units of dB = 10 x log(P1/P0). On the dB scale, loudness ranges from 0dB as the TOH up to 140dB for the space shuttle.

Over the past 85 years, the dB scale has been adapted to describe not just the log of the ratio of sound power, but the log of the ratio of *any* two powers. When we use a reference power of 1W of power, we can measure any other power in dB of power. When we use 1mW as the reference power level, we often designate the dB as dBm to identify that the reference power is 1mW.

Now comes the subtle part. When we want to compare two quantities that are not powers, we have to somehow convert them into powers so we can use the dB scale.

Voltage isn't power, but amplitude. To use the dB scale to describe a voltage, we have to convert the voltages into powers, and take the ratio of the powers that are related to the voltages.

For example, the power dissipated in a resistor by a voltage, V1, is $V1^2/R$. We can describe the ratio of the power generated by two voltages if they were across the same resistor, as dB = 10 x log ($P1/P0$) = 10 x log ($V1^2/V0^2$). The resistances, of course, cancel out. We can pull the square terms out of the log and get 10 x log ($V1^2/V0^2$) = 10 x 2 x log($V1/V0$) = 20 x log ($V1/V0$).

Now, we see where the factor of 20 comes from. Whenever we describe the ratio of two things that are not powers, but amplitudes, we use a factor of 20 to get back to the original ratio. The value in dB is really about the ratio of the powers of the two quantities we are comparing. The factor of 20 lets us convert to the log of the ratio of the voltages.

An S-parameter is *always* the ratio of two voltages. This means that to describe an S-parameter in dB, we use the factor of 20 to relate the log of the ratio of the output voltage from some port compared to the input voltage. The power associated with the voltages is V^2 and the value of the S-parameter in dB is = 20 x log (V_{OUT}/V_{IN}).

On the next page, we look at some of the subtle aspects of the dB scale, like how much voltage is 10dBm and do we use 10 or 20 when describing an impedance in the dB scale.

Decibels and S-Parameters

On the previous page, I introduced the dB (decibel) scale to measure the ratio of two powers as dB=10log(P1/P0). I explained why we use a factor of 10 when describing the ratio of powers, but a factor of 20 when describing the ratio of voltages. I also explained that S-parameters, a ratio of voltages, need a factor of 20 when converted into dB.

An S11 with a value of -20 dB means a reflected voltage signal that is $10^{-20/20}$ = 10% the incident voltage signal. Because power scales as $S11^2$, the power reflected is 1% the incident power, which is $10^{-20/10}$ = 1%. This is a pretty good value for most digital interconnect structures.

A value of an output power of -3 dB means that the ratio of output to input power is $10^{-3/10}$ = 0.5. This is what most of us learn early in our engineering career: -3 dB down is a 50% drop. But, this is a 50% drop in the power level. What happens to the voltage level?

A value of -3dB is always a 50% drop in power, but the ratio of output voltage to input voltage is $10^{-3/20}$ = 70%. When S21 is -3dB, the voltage level of the signal coming out is 70% of the voltage going in.

When we refer to the SNR (signal-to-noise ratio), we usually measure it in dB. Without exception, the SNR in dB always refers to the ratio of the power between the signal and noise. When described in dB, the SNR can immediately be converted into the ratio of the signal power to the noise power by SNR_power = $10^{SNR\_dB/10}$.

An SNR of 20 dB means the signal power is 100x the noise power. What is the ratio of the signal voltage to noise voltage? SNR_voltage = $10^{SNR\_voltage/20}$. The same SNR of 20 dB, a signal power 100x the noise power, is at the same time, a signal voltage = 10x the noise voltage.

We often use the dB scale to describe cross talk. An isolation of noise to signal of -60 dB is a noise power that is $10^{-60/10}$ = 1,000,000 times smaller than the signal power. But, the voltage noise is only $10^{-60/20}$ = 1,000 times smaller than the signal voltage.

In some mixed-signal applications, -100dB isolation is needed between the RF receiver and the noisy digital circuits. This means the received noise voltage needs to be 0.001% of the signal voltage. This is a tiny amount of coupling and is very difficult to achieve in practice.

In some applications, impedance is measured in dB. While the reference level of impedance is always 1Ω, do we use a 10x or 20x when converting impedance in Ohms to impedance in dBΩ? Is Ohms an amplitude or a power?

The way to answer this question is by observing how impedance is derived from an S-parameter. In a 1-port measurement, impedance is directly related to the ratio of (1+S11)/(1-S11). In a 2-port S-parameter measurement of impedance, the impedance is roughly = 25Ω×S21.

Counter-intuitively, impedance is considered an amplitude and uses a factor of 20 when converting from Ohms to dB.

An impedance of 0dB is an impedance in Ohms of $10^{0/20} = 10^0 = 1\Omega$. When the impedance is 10mΩ, the value in dB is 20log(0.01/1)=-40 dB. Figure 1 shows the same measured impedance of a decoupling capacitor displayed in impedance and in dB.

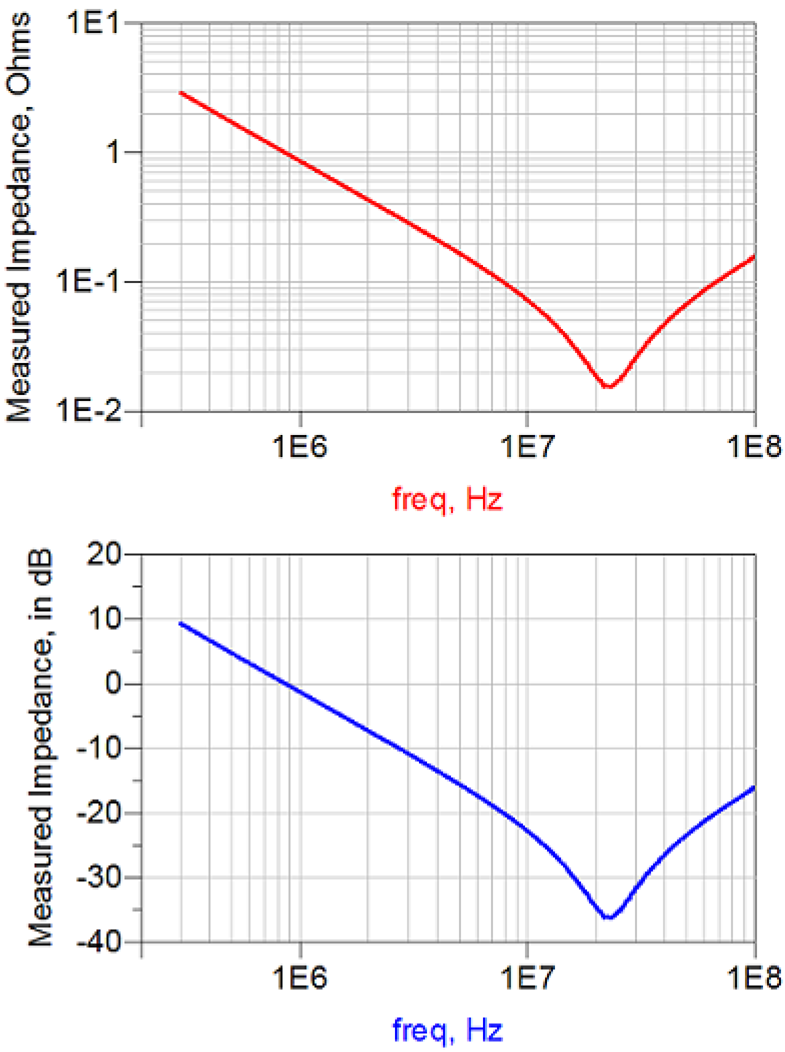

Figure 2. The upper plot shows a measured impedance profile of a capacitor displayed in Ohms with the same measurement in the lower plot, shown in dB.

What else have you encountered that is measured in dB and is it an amplitude or a voltage?