The Wonder That Is Pi

R (Chandra) Chandrasekhar

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This blog began life more than two decades ago, as part of a series of lectures I delivered to very bright first-year engineering students at an Australian university.

The number π (pronounced "pie") has been recognized from time immemorial because its physical significance can be grasped easily: it is the ratio of the circumference of a circle to its diameter. But who would have thought that such an innocent ratio would exercise such endless fascination because of the complexities enfolded into it?

Not surprisingly, some students I met recently wanted to know more about π. Accordingly, I have refreshed and revised my original presentation to better accord with the form and substance of a blog. The online references have also been updated to keep up with a rapidly changing Web.

If there are any errors or omissions, please email me your feedback.

Circumference, diameter and π

The straight line or geodesic is the shortest distance between any two points on a plane, sphere, or other space. The circle is the locus traversed by a moving point that is equidistant from another fixed point on a two-dimensional plane. It is the most symmetrical figure on the plane. The diameter is the name given both to any straight line passing through the centre of the circle—intersecting it at two points—as well as to its length. When we divide the perimeter of circle, more properly called its circumference, C, by its diameter, d, we get the enigmatic constant π, which has a value between 3.141 and 3.142:

$$\frac{C}{d} = \pi. \tag{1}$$

The diameter d is twice the radius r, and substituting for d into Equation (1), we get the well-known school formula:

$$C = \pi d = 2\pi r \approx 2\left[\frac{22}{7}\right] r \approx 6.28r. \tag{2}$$

Note, however, that π is *not exactly equal* to $\frac{22}{7}$. This value is a convenient *rational fraction approximation* for π that serves well in elementary contexts.[1]

You might reasonably wonder whether the ratio of the circumference to the diameter of *any* circle is *always* π. The answer is "Yes", because *all circles are similar*. The ratios of corresponding lengths of similar figures are equal. This idea is also covered in my blog "A tale of two measures: degrees and radians".

The symbol π is the lowercase version of the sixteenth letter of the Greek alphabet. For the history of its use in mathematics, see adoption of the symbol π in Wikipedia.

[1] See "A tale of two measures: degrees and radians".

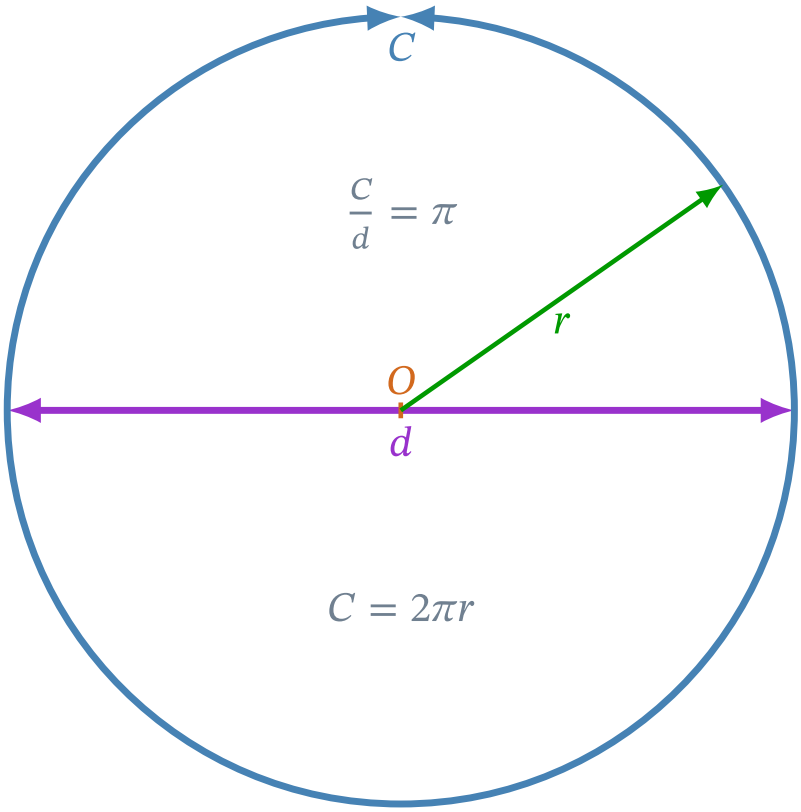

Figure 1: The ratio of the circumference to the diameter of *any* circle is π.

Figure 1 shows the relationships in Equation (1) and Equation (2) pictorially. The circumference of a circle is about 6.28 times its radius. Why this should be so is a secret, a mystery of Nature.

A closer look at π

Pi is both an irrational and a transcendental number. Let us see what each of these appelations mean.

How did we arrive at π = 22/7?

Acknowledgements

Feedback

Please email me your comments and corrections.

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