The Two Most Important Numbers: Zero and One

R (Chandra) Chandrasekhar

2023-10-31 | 2023-10-31

The unique properties of the numbers zero and one make them mathematically indispensable. In this slow-paced stroll though the ideas streaming out of these two numbers we uncover well-known as well as relatively obscure facts about them. It is hoped that in the process we are enabled to discover how they cement disparate areas of Mathematics.

The shy one

The number one is often implicit in mathematical notation. While we may write $2x$ to denote $2 \times x$, or two multiplied by x, we *do not* write $1x$, even if it is literally correct, because of convention. In instances like this, the number one is implicit, and assumed to be understood by those who know. If you happen to be one of those *not* in the know, here's your chance to join the other side.

When we write a fraction as $\frac{3}{4}$ we mean the decimal 0.75 and matters are clear. But all whole numbers are also fractions with the denominator being 1. So, the fraction $\frac{3}{1}$ is rarely written in that form, even if syntactically correct, because usage dictates that whole numbers are written to stand on their own, as 3, in this case. Again, the 1 in the denominator is assumed to be unobtrusively present: out of sight but *not* out of mind.

When we write 4^2, spoken out as "four squared" we mean the number obtained by multiplying 4 by itself. This nomenclature arose because, if 4 was associated with the *length* of, say, a piece of string, the number "four squared" was used to denote the *area* of a square that had a side of length 4. So, $4^2 = 4 \times 4 = 16$

Likewise, the expression 7^3 or "seven cubed" denoted the volume of a cube of side 7. Beyond the third dimension, this naming scheme faded out, because we cannot percieve dimensions higher than three.

Therefore, 6^4 is spoken as "six raised to the fourth (power)" or "six to the four". In such statements, the number 6 is called the *base* and the number 4 is called the *exponent*.

Following this logic, we might assert that $5^1 = 5$ and that is perfectly correct. But again, convention intrudes to say that we write it simply as 5. *Any number raised to the power of* 1 *equals itself.*

The notation making 1 implicit in these scenarios reduces clutter and simplifies notation. The absence of the implicit 1 might trouble the heart of the sincere young mathematician, but familiarity with these conventions will make for comfort in using them.

Multiplication

Division

Why we cannot divide by zero

Exponentiation

Exponentiation may also be called *taking powers*. It is a short form for repeated multiplication by the *same* number. For example, if we multiply 5 by itself three times, we write it so:

$$5 \times 5 \times 5 = 5^1 \times 5^1 \times 5^1 = 5^{(1+1+1)} = 5^3 = 125 \tag{1}$$

The number 5 is called the *base* and power 3 is called the *exponent*. Notice that $5^1 = 5$. This is the first special attribute of 1 when we exponentiate.

Acknowledgements

Feedback

Please email me your comments and corrections.

A PDF version of this article is available for download here:

https://swanlotus.netlify.app/blogs/the-two-most-important-numbers.pdf