Secrets
of
Academic Success

Timeless Principles for Lifelong Learning

R (Chandra) Chandrasekhar

SwanLotus
to help you learn better

DISCLAIMER

This book is published in the belief that it will help those who read and follow it to succeed academically. No claim can be made for guaranteed results.

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[1] Hamlet and David Crystal.

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This book is dedicated to the

HUMBLE STUDENT

within each and every one of us,
who ever thirsts for knowledge,
and who uses that knowledge to serve humanity;

and to all the

EXEMPLARY TEACHERS

who have selflessly illuminated
every stage of learning in our lives.

Epigraph

क्षिप्रं हि मानुषे लोके सिद्धिर्भवति कर्मजा ॥

kṣipraṃ hi mānuṣe loke siddhir bhavati karmajā.

Verily in the world of humans, success is quickly born of action.

श्रीमद् भगवद् गीता ४:१२
Śrīmad Bhagavad Gītā 4:12

கற்க கசடறக் கற்பவை கற்றபின்
நிற்க அதற்கு தக

kar̲ka kacaṭar̲ak kar̲pavai kar̲r̲apin
nir̲ka atar̲ku taka

Learn well whatever is to be learned; then let your conduct befit your learning.

எண்ணென்ப ஏனை எழுத்தென்ப இவ்விரண்டும்
கண்ணென்ப வாழும் உயிர்க்கு

eṇṇenpa ēnai el̲uttenpa ivviraṇṭum
kaṇenpa vāl̲um uyirku

Numbers and the remaining letters—these two—are to living beings like their twin eyes.

திருவள்ளுவர், திருக்குறள் 40:391–392
Tiruval̲l̲uvar, Tirukkur̲al̲ 40:391–392

Preface to the Student

This book has been written to empower you—the student—to succeed in your studies and become self-reliant. Time-tested techniques are presented to enable you to master the irreducible core of all academic learning: reading, writing, and mathematics.

Regardless of where you live, or what you study, whether you are eight or eighty, whether you are in the thick of high school studies, or have retired from the workplace, and whether you simply want to pass your next examination, or dearly want to realize your schoolday dreams of reciting Shakespeare or mastering mathematics—this book is written for you.

In it I share with you practical methods that have worked for me at school and university. I hope that they work for you too.

Learning is lifelong and it is a skill that *can* be cultivated. I hope this book instils in you a love of learning that will endure for life.

Structure of this book

This book has forty-nine chapters, spread over ten parts, covering different aspects of academic success as shown in List 0.1.

Parts of this book

List 0.1: The ten parts of this book.

Browse the table of contents to get an overall idea of what is in this book.

How to read this book

I have tried to make this book as complete and comprehensive as I can. I wanted to write *one* book that would help you right from your schooldays, through university, and beyond. This is that book.

It is a *long* book. In fact, it is *ten* books rolled into one as you might already have noticed. Don't be deterred by its size.

It is replete with techniques, suggestions, and references. You need not read everything at once. Indeed, you might never need to read the whole book from cover to cover at one go. Take a bite at a time, chew it slowly, put it into practice, and assimilate it.

Read and apply what you need first.
Browse the rest.
Repeat the cycle.

Companion website

The companion website for this book is at
https://swanlotus.netlify.app/sas.html

Do visit the website. It contains articles on different academic subjects and an active eclectic blog. You will also find ancillary material there relating to each chapter. What is more, you can post feedback on the site and share with me your thoughts to help improve this book.

The Web is an ever-changing, instantly accessible, almost sentient reservoir of information. Accessing it efficiently and wisely can catapult your academic performance to stratospheric heights.

In Chapter 17, I share with you my knowledge for harnessing the Web and the PC to learn and to do research. The companion website will feature periodically updated versions of this chapter, as the Web and its technologies continue to evolve.

A word of warning, though. Always exercise caution when using the Web: no one guarantees the authenticity, integrity, and factual correctness of the information on offer. You should always cross-validate before accepting what is said.

Unfamiliar words

I have used English words in this book as they occurred to me naturally. I have *not* consciously restricted the vocabulary so that only easy words are used—a practice that is pejoratively known as "dumbing down" the text. I wanted to dunk you directly into the swimming pool of real English because that is where you need to swim, not the shallow wading pool of a beginner's vocabulary. Neither have I written to show off my erudition: my only aim is to help.

What should you do if you come across an unknown word or phrase? You should look it up. How? In a dictionary, whether electronic, or printed on paper.

> ★
>
> Visit the companion website and read the relevant articles and blogs. They tell you how and where to look up the meanings of new words on the Web.

Chapter 30 in this book is devoted to helping you enlarge your vocabulary. Read and follow it: look up new words, learn their meanings, know their etymologies, and use them in speech and writing. In due course, you will enjoy playing the detective game as you search for the meanings of new words and expressions and—after using them—make them your own.

Tone

I speak directly to you, my reader, in this book. I have given you the prescriptions and proscriptions—the dos and don'ts—that have worked for me. My intention has not been to moralize or patronize. I have not adopted the tone of a teacher. Instead, I just wanted to share my experience with you—as an elder brother would with his siblings—in the hope that it might help you. Take my advice or leave it, just the same, but do not take umbrage. No offence is intended.

Repetitive style

I have deliberately *repeated* myself in this book. What has been stated in one chapter *in extenso* is sometimes briefly re-iterated in another. A single concept might be explained in varying depth or detail across several chapters. My purpose in doing so is twofold.

First, repetition emphasizes and embeds an idea in the *memory,* making recall easier. Second, I have tried to make the different chapters *self-contained* to help you read this book a chapter at a time without having to go back and forth across its pages.

Aids to reading

Each chapter begins with a *synopsis* and ends with a *summary*. The synopsis prepares you for what lies ahead. The summary reminds of what you have already read.

Spread throughout the book, including this Preface, are what I have called *starboxes,* an example of which is shown below:

★

A starbox is a box with a central star that highlights some concept that has been presented in the text. This is a starbox.

You may glean the main points of the exposition simply by looking at these visually arresting starboxes. They help you browse quickly. They also aid revision. Even if you read only what the starboxes contain, I hope that you would have gotten something useful from this book.

Some chapters deal with questions and answers, and possibly some commentary on them. Traffic light colour coding has been used to help you identify questions, commentary, and answers; examples are in blue with a star at the end:

- QUESTION

  This is a simulated question.

- COMMENTARY

  The commentary could be about the question, or the answer, or both.

- ANSWER

  This is a simulated answer.

- EXAMPLE: SIMULATED

  This is a simulated example. ★

Abstract, customize, apply, and succeed

Academic success depends on knowing the techniques for efficient and thorough study, applying them diligently and consistently, mastering your subject, and acing your examinations. Success does not come in a can. Nor is there a "success pill" that you can swallow to get instant results. It takes knowledge, effort, practice, patience, and persistence to succeed.

If a technique or idea in this book does not fit like a glove, modify it until it does, and use it to succeed. I have given general principles and explained the reasons behind them. Using those reasons you can extend, refine, or otherwise adapt what has been given until you have fashioned your own comfortable, customized, private manual of study.

★

Abstract. Customize. Apply. Succeed.

It has been said that an ounce of practice is worth more than a ton of theory. Put into use what you find here. Then, and then alone, will your marks improve, your understanding mature, and your keenness for new knowledge grow. Jump on the bandwagon of learning for life and enjoy the ride!

Mathematics

Mathematics is usually the most daunting academic subject. Given its notoriety, I have dedicated one entire part of this book, comprising five chapters, to mathematics. I have also singled out mathematics for special mention in this preface.

> We cannot escape mathematics in our scientific age. So, let us *try* to become more friendly with it. This is the spirit in which I have written about mathematics in this book.

Those who do not need to study mathematics for their examinations can still enjoy its intellectual pleasures. With this in mind, I have written one chapter on the enjoyment of mathematics: it introduces you to my personal, eclectic choice of popular mathematics books that should enrich you if you read them.

For the serious student, there are chapters on overcoming mathophobia, reviewing arithmetic from a mathematically mature standpoint, mastering problem solving, and on coping with university mathematics. I hope this mix caters to all needs and tastes, and that it will lead to a more comfortable relationship with the subject, whatever your age and prior experience.

Discipline neutrality

I have attempted to make this book discipline-neutral. That means it should be useful to you regardless of the subject you study. I have, for instance, drawn questions from different disciplines in Chapter 45, on answering examination questions, so as to address as varied an audience of students as possible.

If there are parts of the book that are difficult to grasp because your background is in the humanities, for example, rather than in the sciences, please send me feedback from the companion website, so that I can improve future editions of this book.

American and British usage

The spelling, terminology, and usage of the English language vary across the Atlantic, if not the world. In this book, the spelling is British and the units are metric. There is no separate American edition. I hope that this does not cause consternation among readers in North and South America.

What is called "primary school" in the UK is called "grade, elementary, and sometimes intermediate school" in the USA. The British "secondary school" corresponds roughly to "middle and high school" in the Americas. "University" is sometimes referred to as "college" or even "school" in the United States. I seek the reader's kind indulgence in making these equivalences mentally while reading the book. Muchas Gracias! Dankeschön! Merci beaucoup! Xièxiè! Dhanyavād! Thank you! ☻

Citations

In academic settings, it is customary to substantiate statements or quotations with *citations* to books or papers where the said fact has been established and accepted as such. On other occasions, you might be referring to a book or website in your writing. To assist the reader locate that book or website, it is proper academic etiquette to provide a citation where the details of the book or website are given.

In this book, citations appear as numbers within square brackets, like so [5], and refer in turn to the books, papers, or websites bearing the same number in the "REFERENCES" section at the back of each chapter.[1]

Above all else …

★

Learn to think for yourself.

R (CHANDRA) CHANDRASEKHAR
JUNE 2021

[1] This book is intended for a general rather than academic audience. Nevertheless it features devices reserved for academic books, like citations and footnotes, to gently introduce the reader to such practices.

Acknowledgements

My first debt of gratitude is to my father, Mr M K Ramachandran. When I was a high school student, my father bought me a book by Eugene M Schwartz [1][1] entitled *How to Double Your Power to Learn*. It was not available in local bookstores at that time, and if I recall correctly, my father cut out a newspaper advertisement and ordered the book by mail shortly after it was first published in the USA.

I devoured the book when it arrived and applied its principles to my school work. *And those methods worked.* I started topping my class consistently from then on. I have imbibed and made my own so many of the ideas Schwartz imparted in his book that it would be well nigh impossible to cite him meticulously in this book each time I have drawn upon his techniques. Suffice it to say that much in this book has been coloured by an idea, or principle, or technique propounded in his book. To Mr Schwartz, I express my grateful thanks in and through this book.

I have also been very fortunate to have been taught by excellent teachers in my formative years. Some were inspiring and memorable, others transmitted their love of their subject to me, while still others had the admirable ability to clarify and lucidly explain the intricacies necessary to master their subjects. If not for them, this book could not have been written.

I would especially like to express my deep gratitude to:

- Mrs Kate Nelligan, YWCA Kindergarten, Penang, Malaysia; Che Zainon Tajuddin, Miss Ngoh Peck Choo, and Brother Gaston, all of La Salle School, Brickfields, Kuala Lumpur, Malaysia;
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- Mr Vincent Sebastian, Mr K K Chellappa, Mr Stanley McMahon Culas, and Mr K Koshy Samuel, all of Saint John's Institution, Kuala Lumpur,

[1] This excellent book sadly appears to be out of print at present.

Malaysia.

In some small measure, I hope to repay their kindness to me by collating all the precious nuggets of knowledge they have given me over time into this volume, and sharing it with students all over the world.

When I was preparing for my GCE O Levels, I chanced to hear on radio a series of programmes entitled ”The Chief Examiner Explains,” one for Science and Mathematics, and the other for English. The Chief Examiners in these two subject areas—for the Cambridge Local Examinations Syndicate in that year—explained how to answer differently phrased questions, and what the examiners were looking for in candidates’ answers. The hints they gave were revelatory to the receptive student. I have distilled what I recall from these programmes, and what I have learned during my subsequent academic life, into this book. To the then Chief Examiner in Science, Mr Bill Kearsey, and to the Chief Examiner in English, whose name now escapes me, my humble gratitude.

From the genesis of this book to its execution now, I have relied upon the work of many.

This book has been typeset using TEX-based open source software, details of which are given in the Colophon at the end. The number of people who have unselfishly given so freely of their time, knowledge, and software runs into legion. Although I do not list them here, I thank them all from the bottom of my heart.

In a way, this book is the logical outgrowth of Chapter 50 entitled “How to Write a Thesis” which was originally written for a seminar that I once presented at the University of Western Australia. Several fellow academics and colleagues critiqued that exposition. I thank Professor Yianni Attikiouzel, Dr Chris deSilva, Dr Mike Alder, Professor Peter Hartmann, Mr Peter Jones, Emeritus Professor David Lindsay, and Dr John Morris for their helpful comments on earlier drafts of the chapter.

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Any errors that remain are my responsibility alone and I seek kind feedback from readers of this book to help me remove them in future editions.

My last and most profound thanks go to my wife, Mrs Jayanthi Chandrasekhar, and my son, Mr Nandakumar Chandrasekhar. Were it not for their unwavering support, understanding, patience, comments, and encouragement over a span of well nigh two decades, a book of this scope and magnitude could not have come about.

REFERENCES

[1] Eugene M Schwartz. *How to Double Your Power to Learn*. New York, USA: Executive Research Institute, 1965.

Part A

You

1

Motivation, Learning, and Success

Synopsis

Studying is the vehicle. Motivation is the fuel. Goal-setting is the map. Success is the destination. This book helps you make the journey to academic success. Undertake it and enjoy the ride!

Cultivate and develop intrinsic motivation. Feed your innate curiosity to probe, experiment, and learn. Once you know the joy of discovery, you will become a lifelong learner.

Academic study is necessary for academic learning. Effective study results in successful learning. Ensure that you truly learn what you study.

Learning has four stages: *naming*, *knowing*, *doing*, and *being*. Each stage feeds into the next. When you reach the *being* stage, you have mastered what you set out to learn. Practice, perseverance, and patience are vital.

Build greater successes upon smaller successes. Finish what you set out to do. Complete tasks on time. Learn from mistakes. Never be afraid to accept that you do not know. Ask questions. Seek answers. Succeed academically. Learn for life.

1.1 Motivation is prime

Motivation fuels success. Whatever your goal in life, you must be motivated before you can achieve it.

> Motivation is a potent mixture of the desire to achieve and the willingness to exert, until success is attained.

1.2 Three types of motivation

Broadly speaking, there are *three* types of motivation [1]:

1. instinctive motivation
2. carrot-and-stick motivation; and
3. intrinsic motivation

Instinctive motivation is driven by biological imperatives like the instinct to survive, the need to assuage hunger, and slake thirst. Think of a drowning man struggling to cling on to dear life and breath and you will get the picture.

Carrot-and-stick motivation is based on reward for a desired outcome and punishment for an undesired outcome. It drives a person to work harder for more pay, or for a gold medal, or for praise from peers. It is also why people get fined for exceeding the speed limit on roads, as a disincentive to repeat offending behaviour. Such motivation, coming as it does from the outside, is *extrinsic*.

Intrinsic motivation or *self-motivation* occurs when the joy of performing an action is its own reward, and justifies the effort lavished on it. No external inducement is necessary.

> Intrinsic motivation is an impulsion from within rather than a compulsion from without.

All creativity—artistic and scientific—is inspired by this inner drive to excel. It is this motivation that fuels collective altruistic endeavours like the Open Source software movement [2, 3] and Wikipedia [4], both of which contribute to the collective good, supplanting commercial gain as the driving force.

Autonomy, mastery, and purpose drive the self-motivated person [1]. Through this book, I seek to *empower* you as a student, give you hints on how to *master* your subjects, and suggest that you develop an interest in *lifelong learning* as a guiding principle.

I have assumed that you are a self-motivated aspirational scholar who wants to succeed academically. If you are not already driven from within, I exhort you to cultivate intrinsic motivation. Others cannot infuse it into you. Neither can they bequeath it to you. Only you can patiently grow it within you.

> Do what you love and love what you do.

The thrill, fulfilment, and joy you receive from intrinsic motivation far exceeds what you get from the carrot and stick variety. It is an expression of the magnificence innate in the human state. It enhances, empowers, ennobles, and enriches.

1.3 Learning versus studying

What is the difference between *learning* and *studying?* They are related words, which are similar enough to be used interchangeably at times, but different enough to be distinguished when occasion demands.

Learning is the goal; studying is the means. When you have learned something, you already know it; when you are studying a subject, you are striving to know it. Learning is natural; studying is deliberate and formal. You learn your first language naturally merely by exposure to it; but you need to formally study a foreign language in order to learn it. You learn a practical skill whereas you study an academic subject. Thus, you learn to walk, swim, ride a bicycle, drive a car, and cook a meal, without studying. But study is required to know mathematics, physics, economics, and other academic disciplines. When you study something, you deliberately apply your mind *with concentration* to learn something. So, there is a distinction between goal and means, between a natural and a deliberate process, and between something practical and something academic.

> You can learn without studying, and study without learning, but you most certainly want to avoid the latter!

1.4 Studying as the core of learning

As a student, you learn by listening, reading, writing, thinking, discussing, revising, and otherwise assimilating knowledge. The core of learning takes place when you study.

> When studying is effective, you have successfully learned what you set out to know.

1.5 Motivation to study

Why study? Why do *you* want to study? Do you *really* want to excel in your studies? It is important that you ask yourself these questions, and answer them honestly to yourself. You will then become acquainted with your attitude toward studies. Your answers will reveal what drives you to acquire and apply new knowledge.

Don't skip this step or gloss over it. Confront and accept the truth, whatever it is. If you do not like your answers, you can work at changing your attitude toward your studies. Self-motivation can be nurtured. But you will never know what to do, if you do not know where you stand.

Your teachers and lecturers can at most infuse interest in different subjects. They cannot inject their enthusiasm into you. That can arise only from within you by a process of resonance.

If you dislike studying and are doing it only to please your parents or someone else, you lack the inner drive to do well. This book might assist you a little, but it is unlikely to make a sea change in your academic life. For that to happen, there must be some deep desire within you that motivates you to do well in your studies.

The drive to excel in your studies must be an impulsion from within, not a compulsion from without.

Regardless of what others think or say, if you *yourself* are convinced that you should study well, you certainly will. Perhaps you have peered into the future, and realize that studying well can lead to plenty of money. Or perhaps you have an ambition to become a professional, like a medical doctor, architect, engineer, scientist, or other professional. Or perhaps you believe that knowledge is power, and would like to become a teacher or professor. Or you might wish to serve society by entering public service or government. Or perhaps, you simply find learning new things a joy in itself.

In all such cases, much study lies ahead of you, and hard work should not put you off. Whatever your motivation for studying, as long as it is innate and not enforced, the ideas in this book will help you to do well. Buckle up and enjoy the ride!

1.6 Curiosity and a sense of wonder

Even as babies, we are equipped with the curiosity to probe, to enquire, to experiment, to discover, to know, to learn, and to wonder.

This sense of wonder, and the accompanying innate curiosity to explore and discover, form the most enduring foundation for a life of learning.

Can you recall from your childhood the thrill of finding out something for the first time? Can you bring to mind the fresh, wide-eyed wonder with which babies look at the world?

All too often, alas, we lose that sense of curiosity as we go through life. A dozen or so years of school education and competitive examinations might have all but replaced that wide-eyed wonder with bleary-eyed confusion. If you find yourself in that boat, the ideas in this book could very well help rejuvenate your learning experience by removing examination anxiety and putting back the fun into learning once more. You will become a self-motivated learner.

1.7 The four stages of learning

I think of learning as a progressive four-stage process involving *naming*, *knowing*, *doing*, and *being*, as shown in Algorithm 1.1. Mastery of any one stage is necessary to progress to the next. And you might be engaged in different stages with different subject areas. Effortless mastery comes with the fourth stage.

The Four Stages of Learning

1. Naming
2. Knowing
3. Doing
4. Being

ALGORITHM 1.1: Learning as a four-stage algorithm.

Regardless of whether you are learning a theoretical subject like accounting, or a laboratory-based subject like chemistry, or a praxis-based subject like cooking, or a skill like swimming, these four stages apply to them all.

1.7.1 Naming

The first stage of learning is *naming*. The first words a baby is taught are names for itself and its parents, names for parts of its own body, names for objects in its surroundings, and so on.

Nouns come before verbs. *We cannot know what we have not named.* In every field of human endeavour, naming precedes all knowledge. Often, this leads to specialized terminology: the medical doctor, the lawyer, the priest, the mathematician, the physicist, the ecologist, the historian, the economist, the builder, the plumber, the midwife, the tailor, the soldier, etc., all have their own jargon.[1]

Each subject you study will have its own nomenclature. Ensure that you master it. Being unsure about word meanings when learning is like erecting a building on shaky foundations: it is condemned to be forever wobbly and unsafe.

[1] Which makes life varied and interesting. ☻

1.7.2 Knowing

Knowing is the second step in learning. Knowing the *definitions* in your subject allows you to further your knowledge of it. An example will make this clear.

EXAMPLE: DEFINITIONS

In physics, the terms *force, power,* and *energy* all have special and precise meanings that are different from the lax, everyday meanings we assign to these words in casual social discourse. The student of physics must not only *know* these special meanings, but must also be able to recall their mathematical definitions and the units in which they are expressed. Knowing encompasses all these. ★

Each and every subject known to man is built upon certain *principles*. Knowing these theoretical underpinnings of your subject is the next step. Again, some examples are helpful.

EXAMPLE: PRINCIPLES

If you are studying physics, there are laws governing force, energy, mass, momentum, etc., that you must become familiar with. If economics is your subject, you need familiarity with how supply and demand affect prices. If you are studying the life sciences, ideas like homoeostasis, the laws of heredity, etc. must become your stock-in-trade. ★

Knowing goes beyond *naming*. It requires a particular idea to be defined, related to other ideas, described both qualitatively, and where applicable, quantitatively, perhaps using a mathematical formula. *Knowing* means becoming thoroughly acquainted with the definitions, principles, and theories governing your subject. It prepares you for *doing*.

1.7.3 Doing

Doing is the third stage of learning. To be useful, all knowledge must be applied. Even in purely theoretical fields, knowing a theory must lead to doing something with that theory, like making a testable prediction from it.

In praxis-based disciplines like surgery, or gymnastics, or nursing, or motorcar repair, *doing* means actually doing something with your hands, based on knowledge you have acquired in the first two stages of learning.

Even in fields like mathematics, where *doing* may simply be putting pencil to paper to prove a theorem, or in computing, where it might be testing a program you have designed, *doing is more active than knowing.* By *doing*, you move from the *noun-phase* to the *verb-phase* of learning.

Practice is paramount to master *doing*. The more you practise, the better you get. If you play a musical instrument, you will know this from your own experience. From cooking to computing, from scuba-diving to surgery, from teaching to tailoring, practice makes you perfect at *doing*.

Practice requires perseverance and patience because you will make mistakes in the beginning. As you patiently persevere and practise more, you will make fewer mistakes until you become well and truly proficient.

★

The *doing* phase of learning is built upon three Ps: practice, perseverance, and patience.

1.7.4 Being

The *being* stage of learning is reached when *knowing* has fused with *doing* to the point where performing the task is effortless. This happens in all fields of human activity.

In integrated skills like swimming or cycling, there is a particular, clearly defined point at which the skill has been mastered. Once you have learned how to ride a bicycle, you cannot suddenly "unlearn" and forget how to ride it. Once you have mastered swimming, you never forget how to swim. *The skill has become part of you.*

This stage of effortless mastery is called *being*. Having reached it, you have mastered whatever you wanted to learn, and made it a part of you. This is the expert state of learning, and once you have attained it in any subject, you should spend your efforts keeping your skills sharp. You could also start learning *other* subjects or skills, expanding your expertise.

The purpose of this book is to assist you to reach this *being* state in the subjects you are supposed to master at high school and university, and to inspire you to keep learning all your life.

1.8 Recapitulating the four stages

In the *naming* stage you get acquainted with a specific branch of learning. In the *knowing* stage you learn the theory. In the *doing* stage you apply the theory to solve problems. In the *being* stage, you have internalized the knowledge so that whenever it is needed, you can apply it effortlessly, because you have mastered *naming*, *knowing*, and *doing*.

1.9 Success

Success means different things to different people. All-round success includes academic success but is not limited to it. In this book, we are concerned principally with *academic success*, which may be translated roughly as *doing well in your studies.*

The principles underlying success are numerous and touch upon almost all of life, as shown in Chapter 11 and other chapters of this book. Here, I want to outline the basics of success and tie them both to motivation and to learning.

In Section 1.5, I have already asked you why you want to study, so that you may fathom your own motivation. I shall assume here that you have a deep-seated personal impulsion to study and that you are not doing it simply to satisfy someone else or to get them off your back.

★

Motivation is vital for success.

1.10 The journey analogy

Before you begin a journey, you must know where you want to go. You also need a vehicle to take you to your destination, and fuel to run that vehicle.

The destination is your definition of success. The vehicle is the process of learning. The fuel for the vehicle is your intrinsic motivation. This book is mostly about the vehicle; you need to supply the fuel and the destination.

1.11 Goal-setting charts the path to success

A journey without a destination is purposeless and degenerates into aimless wandering. A map aids you in getting to your destination. If your destination is success, the map that gets you there is goal-setting. If you do not set clear goals, you cannot know if and when you have succeeded. Let me help you clarify what success and goal-setting can mean to you.

EXAMPLE: CLEAR GOALS

Suppose you define success as "doing well in your studies". What exactly does that mean? Does it mean barely passing all your subjects? Or does it mean getting distinctions in all subjects? Or does it mean something in between?

Does it mean succeeding academically as well as socially? Does it mean a good job and plenty of money after studies? Or does it simply translate into the sheer joy of knowing? How all-rounded do you define success to be? How important to your definition of success are extra-curricular activities like sports, debating, voluntary service, etc.?

There are so many facets to success and you need to choose. It is like choosing one particular route from many possibilities before you begin your journey. ★

If motivation is the fuel, goal-setting is the map. Even if your fuel tank is full, you cannot go anywhere if you do not know where you want to go and/or if you do not know the route to get there. Clearly defined goals accomplish both these ends.

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The first rule of success for any undertaking is to set unambiguous goals and to work to achieve them.

Whether you are setting out on a journey, or are practising for an important tournament, or are studying for an examination, you need a clear picture of what you want to achieve. If you want to reach your destination in five hours, you can choose your means of transport and plan your route accordingly. If you want to be champions in the tournament, you can set aside enough time for practice, and also plan both strategy and tactics depending on who your opponents are. If you are planning on high distinctions in three subjects and credits in four others in your examination, you can apportion your time and effort accordingly.

You require a precise goal toward which you can work rather than a vague "motherhood and apple pie statement" like "I want to do well in my studies."

If you wish to top your class in history, it is important that you clearly define that goal for yourself, so that your entire being—body, will power, conscious mind, and subconscious mind—can all work in concert to achieve it.

Harness both head and heart to reach whatever goal you set for yourself.

1.12 From smaller to greater successes

There is a saying that “Nothing succeeds like success.” I would like to paraphrase and expand it to read:

Nothing motivates like success.
Success begets success.
Smaller successes lead to greater successes.

You need only a small dose of success to inspire you to try harder and do even better the next time. Succeeding in one subject is fuel enough to encourage you to succeed in more subjects. Cultivate a small patch of success and grow it into a vast landholding of success.

Start small. Identify a worthy, *nontrivial* goal that has eluded you in the past but which *seems to be within your grasp*. Ignite your mind with ambition to accomplish what you desire. Apply yourself until you succeed.

Setting a small, achievable goal means that you will neither be discouraged by its difficulty nor overwhelmed by its immensity. Because it is not overly ambitious, you will surely succeed in the end. Because it is not trivial, achieving your goal will give you a sense of accomplishment. The sweat and toil to attain your goal will grow your muscles of patience and perseverance. But the demand on your mental stamina will not be excessive because your task is tractable. If you work at it consistently and never give up, you will accomplish.

Once you have achieved your goal, you will exult in a sense of triumph. *You have succeeded.* That magic sense of accomplishment will spur you on to tackle more demanding tasks and succeed at them. The memory of your victory will form the foundation for further successes in the future. Smaller successes will spawn greater successes until you have a "success mindset". You will then become a person to whom success seems natural and inevitable.

This *divide et impera* or "divide and conquer" is a very powerful approach to solving all sorts of problems. Try it. Use it. Remember it.

Algorithm 1.2 is a summary of this technique of cultivating smaller successes on the road to greater successes.

CULITIVATING SUCCESS

1. Choose a worthwhile goal.
2. Break it down into smaller, achievable tasks.
3. Identify *one* such task.
4. Develop methods to accomplish that task.
5. Apply yourself until you succeed.
6. Repeat the cycle on other tasks until you reach your original overall goal.

ALGORITHM 1.2: Cultivating success.

1.13 Finish what you set out to do

The great Tamil poet Mahakavi Subramania Bharati begins one of his inspiring poems with the line எண்ணிய முடிதல் வேண்டும், transliterated as *eṇṇiya muḍital vēṇḍum*, meaning "Finish what you set out to do." This one line can very well be your personal aphorism of success. Finish what you set out to do.

In the context of building greater successes upon smaller successes, you must not yield to complacency after succeeding at a smaller task. Keep hacking away until you achieve your original overall goal. To be satisfied with something less than what you originally set out to do is to diminish your own potential and deny your own abilities. Take care not to fall into the trap of smug satisfaction.

There is a cliché that some people are great starters but poor finishers. It means that you approach a new task or project with great zest and gusto but your will power, tenacity, and diligence taper off with time so that you end up not completing the project. Sometimes, such people might even delude themselves that their half-finished project *was* what they initially set out to do. This is where goal-setting is vital because it will show you when your project is really done. You cannot then delude yourself in this way.

1.14 Complete on time

Perfectionists attempt to do *everything perfectly* and end up completing nothing on time. This is not acceptable in the academic arena. Nor is it allowed in the real world. The person who habitually misses deadlines will not be employed for long. Needless to say, if you spent all your time in an examination on completing just one question, where you needed to finish five, you would be failed outright in that subject.

★

Develop a balance between finishing on time and doing something perfectly.

1.15 Academic success and its two disguises

Academic success wears two disguises: one is *ignorance;* the other *failure.* Do not shun either if you meet them. But neither should you welcome or seek either. ☻

1.16 Enquiry opens the gates to new knowledge

Ignorance and knowledge are the obverse and reverse of the same coin. Convert ignorance to knowledge by metaphorically turning over the coin. Enquiry is the gateway to all new knowledge. To open it you need to do four things:

First and foremost, never pretend to know something you don't know. Be honest. Be humble enough to admit "I don't know". This magic mantra opens the doorway to new knowledge. Suppress it and you will always remain steeped in ignorance. Admit it and start seeking knowledge: you will find it.

Be precise in identifying what you do not know. If you are vague about your ignorance, you cannot dispel it.

Enquiry

1. Never pretend to know something which you do not know.
2. Identify what you do not know.
3. Ask questions that will dispel your ignorance.
4. Seek answers to your questions.

ALGORITHM 1.3: Enquiry as the gateway to new knowledge.

Ask questions to find out what you do not know. Seek answers. Attain new knowledge. Build upon it. Repeat the cycle. Never be afraid to ask a question to dispel ignorance. There is a joy in finding things out. Become a lifelong learner.

1.17 Mistakes are part of learning

Failure is a stepping stone to success. Its lessons are extremely valuable and I have dedicated Chapter 10 entirely to this subject. Making mistakes is part of learning.

When we do something for the first time, we are very likely to make mistakes. Your first attempts to ride a bicycle probably resulted in quite a few falls. If you have never cooked before, your first culinary creations might not exactly be gastronomic delights. If you have never debated in front of an audience, you could experience the jitters during your maiden speech.

★

Don't get upset if your first try at something is a failure.
C'est la vie.

The important thing is to try—and try again— until you succeed. Here is why.

The human brain is a remarkable organ. In one aspect, it may be viewed as a programmable computer. It is equipped to deal with immense complexity, and what is more, it can be re-wired for any task. This flexibility to learn anything arises from the brain's ability to learn from examples, and from mistakes.

We learn by making mistakes, and by correcting for them, until we get things right.

If you look at a toddler attempting to take his or her first steps, you will find that the infant falls many times and might even cry for a while. But, most importantly, the child makes the attempt *again and again*. By making repeated efforts, the child's brain is gradually re-wired to the rhythm of walking. The first independent and clumsy steps gradually lead to a smooth and confident gait where swing follows stance effortlessly. Practice makes perfect. Most importantly, after mastering walking, the child discards the memories and pain associated with falling.

1.18 From motivation to success

Your journey to academic success will involve much study and learning but its starting point is always motivation and its destination is always success, working in a virtuous circle, as shown in Figure 1.1.

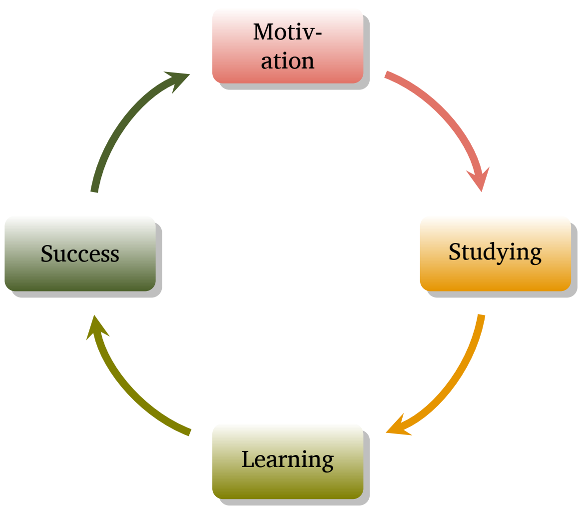

FIGURE 1.1: The virtuous circle of motivation and success. When motivation succeeds, that success itself motivates, giving rise to a virtuous circle. The core of academic learning takes place within this virtuous circle when you study, and learn as a result.

1.19 To explore further

Books on success are among the perennial bestsellers. Some become enduring classics that remain continuously in print for decades. They often dwell upon those aspects of the human mind and spirit that are not always acknowledged or cultivated.

Here is a personal list of books on success that I am familiar with and have found useful. The first book is short and sweet, and small enough to fit into your pocket. It is the eminently practical *The Law of Success* by Paramahansa Yogananda [5]. The second book is the monumental *The Law of Success: In Sixteen Lessons* by Napoleon Hill [6] which was immensely influential when first released and which remains so to the present day. Two other enduring classics I have found inspiring and useful are *The Power of Your Subconscious Mind* by Joseph Murphy [7] and *The Power of Positive Thinking* by Norman Vincent Peale [8].

It would be impractical to list more such books here. A saunter down the “motivation” or “self-help” aisle of your local bookstore will help you identify many more books devoted to the topic. Read and follow those that resonate with you.

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Summary: Motivation, Learning, and Success

- You need motivation to succeed at anything.
- To excel academically, you must be impelled from within; you cannot be compelled from without.
- Effective studying leads to successful learning.
- The four stages of learning are: *naming*, *knowing*, *doing*, and *being*. You have mastered your subject when you reach the last stage.
- Set clear goals to define what academic success means to you.
- Start succeeding at modest tasks. That will motivate you to undertake more difficult ones.
- Success breeds success.
- Finish what you start and finish on time.
- Never pretend to know something when you don't know it.
- Never be afraid to ask questions about something you don't know.
- Seek answers to your questions.
- Expand your knowledge.
- Mistakes are a necessary part of learning.
- Motivation is the fuel. Studying is the vehicle. Clear goals are the map. Success is the destination. Embark on your journey to academic success. Enjoy the ride. Enjoy lifelong learning.

3

Become Your Own Observer-Auditor

Synopsis

When you solve a mathematics problem, check your working every few steps, and correct it if necessary. When you write, review your work, and silently read it to yourself—in your mind—to determine how well it reads. If you do not like what you hear in your mind, rework your writing.

Such detachment while being active requires the mental skill of *detached witnessing*. It is something we humans are capable of. Initially, it will be difficult. But with practice, it will become easier. When it becomes second nature, you have within you, your very own personal friend, guide, and philosopher: *yourself*. I call this *becoming your own observer-auditor*.

This practice will reward you not only in your studies but also in many other circumstances in your life. It will save you from needless strife with those around you. It will induce calmness and introspection before reaction. It will make your speech measured and circumspect. It is mighty helpful during job and promotion interviews. Overall, it will make you a more balanced and successful individual.

One point to beware though. Do not practice this when your attention needs to be laser-focused, as when you are driving on the road. You cannot then relax your attention on the traffic, for a few seconds of mental abstraction could be hazardous to you and others.

As long as you exercise your common sense about when to do it, becoming your own observer-auditor will help you scale undreamed of heights and succeed wildly.

> Self-observation brings man to the realization of the necessity of self-change. And in observing himself a man notices that self-observation itself brings about certain changes in his inner processes. He begins to understand that self-observation is an instrument of self-change, a means of awakening.
>
> *In Search of the Miraculous* [1]
> GEORGE IVANOVICH GURDJIEFF (1872–1947)

3.1 How I became my own Observer-Auditor

The example below is an account of how my desire to reduce errors in mathematical problem solving inadvertently led me to this very useful study paradigm: the *observer-auditor*.

EXAMPLE: HOW I BECAME MY OWN OBSERVER-AUDITOR

When I started doing Mathematics seriously at school—to master the subject and score high marks in it at examinations—I unconsciously developed the ability to become my own observer-auditor.

I wanted to cut down the *number of errors* I made when solving a mathematics problem. I also wanted to catch those errors *as soon as they were made* rather than six pages downstream.

One way was to practise, practise, and practise, until solving a particular type of problem was second nature to me. That was a rote method which cut down the number of errors, but did little to help immunize me against errors when solving new, unfamiliar types of problems. I needed a more generic solution.

To catch errors as soon as they were made, I realized that I would have to halt every few lines and go over the working, before resuming the solution. This was a start-stop-start technique that was a little jerky at first. Some errors still remained, and the time spent in checking was borrowed from time that could have been used for solving.

The reason I still made errors was because I was not *sufficiently detached* from my work to view it with *fresh eyes* when checking it. *I had failed to separate the solver of the problem from the checker of the solution.*

Accordingly, when I paused after every few lines of working, to go over what I had written, I would *detach myself* from it as much as possible, and review my solution with the critical but impartial eye of an instructor or examiner.

I was thus able to catch my errors almost as soon as they were made. By not allowing a few erroneous lines a chance to snowball cumulatively into several erroneous pages, I saved myself valuable time and effort. Some errors still escaped me, but the majority were caught in time and on time.

In this way, I slowly developed into my own internal critic, not only in Mathematics, but also in English, where I was both an observer and an auditor.[1] The prose I had written should not only appear correct to the eye, but should also sound pleasant to the ear.

In the fullness of time, I was able quite literally to split myself into two: one person doing the writing, and the other doing the review, calling a halt when a correction was due. These two activities, which initially were two separate steps, gradually coalesced into one action of writing and almost simultaneous checking and correction. I had developed into my own observer-auditor. ★

★

This splitting of myself into two parts—the writer as the first part and the observer-auditor as the second part—is one of the most rewarding habits I was led to cultivate during my studies.

3.2 *The Reader Over Your Shoulder*

The book, *The Reader Over Your Shoulder: A Handbook for Writers of English Prose*, was first published in 1947. It has been called "...the best book on writing ever published." [2]. I will be looking at this book, among others, in detail in Chapter 35 where we look at "Writing".

As alluded to by the title, the authors—Robert Graves and Alan Hodge—advise the writer to imagine a reader over her/his shoulder, reading what has been written. If that reader could be confused or perplexed by the prose, it is the duty of the writer to amend the text to infuse clarity, brevity, and elegance into it.

This advice is uncannily similar to the observer-auditor whom I am asking you to cultivate as your internal critic, to help polish your expression, whether in Mathematics or English.

[1] The *observer* is the one who *looks critically* while the *auditor* is the one who *listens critically*.

3.3 Importance of splitting yourself into two

As human beings, we have subjective awareness. This means that we are aware of ourselves, and moreover, we are aware that we are aware of ourselves. This faculty is one we all possess.

What I am suggesting here is to further split yourself into an observer-auditor while you perform some other activity. The observer-auditor then observes and corrects whatever errors arise during the performance of the activity. Think of it as *detached witnessing*.

This is *not* an exhortation to become absent-minded, careless, or any such thing. Rather it concentrates your mind efficiently on the task at hand, allowing you to perform it better and perhaps, totally error-free. Think of it as continuous, autonomous, real-time, quality assurance.

3.3.1 Overcoming initial reluctance

Becoming your own observer-auditor might seem strange and even a little mentally unhinged at first. It might remind you of strange folk who talk loudly to themselves, or others who mutter inaudibly with lip movements in an unabashed act of soliloquy, or even worse, of those who hold audible conversations with persons invisible to the rest of us.

Fear not. By becoming your own observer-auditor, you do not risk joining their ranks. This whole habit is an internal attitude of mind. As in all matters, be your own judge. If you find this practice unsettling, eschew it. There is no compulsion: only a reward at the end for those who persevere and master the technique.

3.3.2 Watch your breath

There is an ancient practice dating back centuries when folk who wanted to quieten their minds would sit comfortably and start observing their own breath.

When we breathe we are largely unconscious of the act because it is regulated by the autonomic nervous system. By making conscious what is normally an unconscious act, the ancients could calm and concentrate their minds to great effect.

In watching your breath, you are splitting yourself into two—the one who breathes and the one who watches the breath—to become aware of an unconscious, albeit unitary, act. If you are afraid of becoming your own observer-auditor, try this ancient, calming, meditative practice. If you find it unnerving, do not proceed further. If you find watching your breath helpful, try to extrapolate the sense of observership to your studies, to become your own observer-auditor, and determine if that helps you too.

3.3.3 Caution while driving

Practising to be your own observer-auditor is a powerful technique; so powerful in fact that it could divert your attention from some visually demanding task—like driving on the road—that you might be performing as second nature.

You cannot and must not relax your attention while driving. Then alone can you avoid danger to yourself and others. So, under restricted circumstances like driving, keep your attention on the task at hand. Practise being your own observer-auditor later; not while engaged in activities like driving.

3.4 Relevance to the four stages of learning

Let us see how being your own observer-auditor can help with the the four stages of learning introduced in Section 1.7. The way to apply this paradigm to each stage is clarified in the abstract example below.

Example: Observer-Auditor and the Four Stages of Learning

The first stage is *naming*. When you are learning new nomenclature, ask yourself what mnemonic or other device you can use to remember the new name. After you have familiarized yourself with the new names, test yourself to see how much has stuck with you.

Repeat the test, without revision, after a week or two, and see where you stand. Then, revise, but this time while you revise the names, let the observer also stand apart in your mind and witness the revision. If you do this often enough, you will almost certainly absorb and recall better than before.

The second stage is *knowing*. What facts adhere to the name you have learned? There is scope in this stage of learning for an explicit question-and-answer dialogue between yourself as the learner and yourself as the questioner-observer. The dialogue can, with practice, become a productive conversation in which you condense your newfound facts into a tightly interconnected ball of knowledge that can be pressed into use at will.

For the third stage of *doing*, practise watching yourself answer problems or questions, and track any errors in what has been recalled or written down. As I have already described in Section 3.1, this is how I stumbled upon the observer-auditor paradigm myself. Practise and make it your own. It will yield rich dividends when put to regular use.

The final stage of *being* is actually the autopilot state of your subject mastery. Your observer-auditor will provide oversight for your already effortless mastery. No unconscious errors can then creep in as everything is vetted not once but twice. ★

3.5 Oral Examinations

If you get to do a higher degree at university you will very likely have to defend a thesis you have written before a panel of examiners, and an audience of all-comers.

This encounter can get quite nerve-wracking because it is difficult to anticipate everything that *could* happen during the "oral" or *viva voce*.

It is not uncommon to start off with the jitters until you hit your stride. Then you will field questions with precision and aplomb. If you have practised being your own observer-auditor, this practice can be of great help during your oral examination.

By setting yourself apart from yourself as the candidate, you remain unperturbed by fears. By viewing yourself, your examiners, and your audience from the vantage point of the observer-auditor, you will experience the detachment necessary to acquit yourself well. You might even excel in your oral presentation and get a distinction for it. ☻

3.6 Interviews

Interviews can be quite unsettling, whether for a scholarship, a promotion, or a new job. You are often presented with problems or hypothetical situations and asked how you would handle them.

You can become quite anxious for several reasons. First, you can never fully predict what you are going to be asked. Second, the situation can appear confrontational—like a predator-prey encounter—and that will only dial up your stress level. Third, you might be unaccustomed to thinking on your feet, especially in front of others. Fourth you are in competition with others.

The observer-auditor in you can function as an efficient stress-relief valve in such a situation. You mentally stand apart both from yourself and your panel of interlocutors. Then you focus on the question(s) and formulate your solution in sweet and detached serenity.

By disengaging from the fear-inducing, stress-producing emotions that are common during interviews, you can act—rather than react — with composure, clarity, and concentration. Whether or not you get the scholarship, promotion, or job, reducing your stress levels alone would make the observer-auditor paradigm worth adopting.

3.7 Interactions with others

In debates, discussions, seminars, etc., you will interact with others as an academic co-participant. If you are your own observer-auditor, you will not be carried away by rude behaviour in the heat of the moment, but rather will be in control of yourself, as arguments are fought and won on the basis of ideas and their strength, and not on the loudness of voice or the crassness of speech. Such temperate conduct will win you like-minded friends and also a larger circle of admirers. Overall, you will gain respect and respectability as a civilized and genteel person.

If this polished behaviour is internalized to become your normal conduct, you will be a well-adjusted member of society and a go-to resource person for your family, friends, peers, and other social circles.

3.8 Ethical dividends

The observer-auditor paradigm has applications beyond the academic, embracing much of life itself. Let us consider a contrived example.

Example: Foreknowledge of an Examination Paper

Suppose you saw, lying open and unguarded due to some happenstance, the examination paper for a forthcoming examination. What would you do?

Either curiosity or temptation could instigate you to steal a wee look at what questions the examiners have set. There is an unethical underpinning to this action. It gives you an unfair advantage over all your classmates who did not or could not get the peep you got. Would that trouble you?

The observer-auditor within you would warn you well in advance to steer clear of such tempting revelations as a Faustian bargain that is best avoided. ★

You might say that I am just alluding to that shrill inner voice—*the conscience*—that all of us have. Yes, indeed, I am. That voice of conscience is what I am calling the observer-auditor. Only, the word conscience is used in the context of right and wrong when confronted with a *behavioural* dilemma. The value-laden word "conscience" is not appropriate for ensuring error-free problem solving in mathematics. So, I have instead chosen the more widely applicable, neutral term observer-auditor.

Perhaps by now, you are getting to see that the observer-auditor is not such a cockeyed idea after all, but is something already known to humankind as conscience, the inner moral compass that guides us to behave correctly, as long as we do not ignore it.[2]

★

Perhaps the greatest intangible but inwardly palpable benefit from the observer-auditor paradigm is the approval of your own self.
You will live in harmony and comfort with yourself.
And that is something worth cultivating assiduously.

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[2] Robert Graves and Alan Hodge. *The Reader Over Your Shoulder: A Handbook for Writers of English Prose*. e-book. Rosetta Books, 2018.

SUMMARY: BECOME YOUR OWN OBSERVER-AUDITOR

- To become your own observer-auditor in mathematics do this: as you solve a problem, after every few lines, take a *detached* look at your working and scrutinize it for errors. This way, you stop errors from propagating because you catch them as soon as they are made.

[2] An ignored conscience will often retire into silence.

- Split yourself into two independent personae: one performing the action and the other reviewing it.
- Put enough distance between yourself and the work being checked so that you do not gloss over errors.
- Apply this idea to your written English as well. Ensure that it reads well both silently and aloud.
- Practise becoming your own observer-auditor until it becomes second nature to you.
- The observer-auditor paradigm is useful in academic discussions, oral examinations, and interviews for scholarships and jobs.
- This habit of separating a part of yourself to witness and hear what you do will also steer you toward ethical behaviour.

Part B

Mind

10

Learning from Failure

Synopsis

Failure is part of the learning process and a natural stepping stone to success. Every failure provides valuable feedback on what you are doing wrong, and how it should be corrected. Do not get fixated on failure. Rather concentrate on the necessary corrective action. Learn from failure and then forget failure.

Give up all negative emotional baggage associated with failure—like fear, doubt, and despair—and treat it simply as a feedback mechanism, which is all it is. Let go of all past regrets, shames, sorrows. This clears the way for success.

Give up tension. Relax. Learn from mistakes. Do not repeat them. Give up negativity. Become positive. Keep trying until success is achieved. Practise positive visualization and affirmation, seeing yourself as already successful in your mind's eye. Aspire to and achieve the "being" stage of learning.

Failure is inherent in all academic research. Do not personalize failure by blaming either yourself or your peer reviewers if your manuscript is rejected. Use the review instead to improve your submission, and try again.

A person who never gives up can never fail.

10.1 Failure is a stepping stone to success

What is a chapter on "Failure" doing in a book that has "Success" in its title? Well, failure is a stepping stone to success, that's why. We are indoctrinated during our upbringing that failure is bad and success is good. Yet, there is no one who has succeeded without failing, nor anyone who has always failed, without ever succeeding.

All human learning takes place by trying, failing, and trying again and again, until success is attained. This is how we learn to walk and talk, to swim and cycle, to read and write, to add and multiply. In the long run, the successful person learns as much, if not more, from failure as he or she does from success.

10.2 Failure in early life

Everyone's early life, from infancy onwards, is peppered with failure that impels them to success. I go back to my favourite example of a baby learning to walk. Walking is a tall order when a baby cannot even turn from a supine to a prone position. When that first battle is won, it is time to toddle. Then, when the legs get stronger, it is time to practise standing up. Only then is walking attempted. And each stage brings with it a large number of failures before success is achieved. *But when the baby succeeds, it forgets the failures and keeps on succeeding thereafter.* This too, should be our formula for dealing with failure.

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Learn from failures. Try until you succeed. Then forget past failures.

10.3 Coping with the emotional fallout of failure

If we had the pliable mind of a baby, we will simply forget all past failures the moment we succeed. Unfortunately, as we grow older, we take on the burdens of a personality. Success and failure then become a reflection of who or what we are. We become sensitive to how we are rated by our peers. We care about what others would think of us if we fail. Or we silently crave their acclaim when we succeed. This emotional baggage compounds the stress of failure and enhances the allure of success.

Life is not an Olympic event held once every four years with three medals of different colours given to only the top three competitors. *Life* *is a process of continuous learning.* Once you realize that learning, rather than peer approval or adulation is the goal, your attitude to failure will change.

> If you have learned the lessons from failure, you have actually succeeded.

If you are heartbroken by a failure that shatters a dream, and you need to release the pent up grief, do so in a welter of tears at an appropriate time and place, perhaps in the company of family or a sympathetic, trusted friend. You will feel much better afterward. Having released the emotion, do not allow it to build up again. Say farewell to grief and welcome the next opportunity to succeed. Stay determined.

> Focus your efforts on what you need to do to avoid making the same mistake(s) again.

To assist those who find it difficult to let go, the following sections address both the need to let go and the art of letting go in some detail.

10.4 The need to let go

Any traumatic event leaves its emotional trace in your being. The more you dwell on past trauma, the deeper it gets entrenched in your mind. This applies to anything negative from something physical like slipping on a banana peel, to something mentally anguishing like failing an examination, to something personally devastating like losing a dear member of your family. While you might think that the grief of bereavement is "noble" whereas the pain of failure is "shameful", in reality, the emotional residues lodged in your being from either are just as corrosive to your peace.

> Any stored negative emotion from past events can rob you of your peace at anytime. Only by letting it go can you regain your peace of mind.

I once read that we need to forgive, not so much for the benefit of the other person, as for our own benefit. If you accept the validity of this viewpoint, you will realize that every negative emotion that you capture from an unhappy past event and keep sealed within your heart has the potential to re-emerge and taunt your peace and steal your happiness. By harbouring it and giving it safe haven within your psyche, you are punishing yourself, not the party that you feel aggrieved by. So, why would any logically-minded person cultivate such a punitive and negative habit?

If you find it silly to harbour negativity like this, you are on the road to freedom. Sometimes, you might find it hard to let go, regardless of all the logic in the world. What would you do then? Let me illustrate with an example very relevant to the subject of this chapter.

EXAMPLE: FAILURE IN MATHEMATICS

Suppose you failed an examination in mathematics in high school. The shame and torment arising from it have wedged this event securely within your psyche. Each time you think of mathematics, this unhappy memory would bubble to the top and torment you afresh. Each reminder more deeply entrenches the past failure. With each recurrence, mathematics and failure are associated ever more strongly in your mind. You have begun to fear mathematics and have started doubting you could ever shine in it. One day you begin to believe in the inevitability of failure each time you *think* of mathematics, let alone *take* the subject. How did this pernicious state of affairs come about? *By unthinking repetition of a destructive memory.* ★

★

By repeatedly playing back a poisonous memory you have unwittingly bequeathed it ever greater power to ruin your life.

10.5 The art of letting go

The way out of this tangled mess is to cut the Gordian knot. Ruthlessly cast out all corrosive emotions.

> There is no virtue in tormenting yourself by mindlessly playing back the memories of past shames and failures.

You must let go. Sometimes you will so stubbornly feel that you are right that you will refuse to let go. Indeed, you might view "letting go" as conceding defeat to whatever event or person you have your quibble with. That is utter foolishness. By not letting go, you are embracing defeat. *For your own sake, learn to let go.*

The moment you are willing to let go, you can instantly exorcise and excise the burden of past failure, shame, fear, guilt, sorrow, etc. But you must wholeheartedly, willingly, and sincerely cast out the feeling with a clear attitude that you will not sabotage yourself by letting it recur. Your emotional healing will then be complete.

There are dozens and dozens of self-help books and websites that teach you how to let go of those stored-up negative emotions. Some use the term *releasing* for the act of letting go. You can do it with affirmation and breathing, or by some ritual, or by a concerted act of will, or by potent visualization. Use any one or more of these methods.

> Realize that you are not the emotion. You are not the shame. You are not the fear. You are not the guilt. You are you. Nothing can take away even a smidgen of your precious self.

My intent here is not to school you into how to let go. I am keen to impress upon you the absolute necessity of letting go so that the sweet waters of success may bathe your life. Learn to let go. Unburden yourself. Cast out all fear, doubt, shame, guilt: indeed all negativity. Dissociate yourself from your past failures.

> You are not a failure. You can never be a failure. For the simple reason that the failure is not you. You are you.

10.6 Positivity feeds success

Positivity and negativity are choices you make. They determine whether you habitually succeed or habitually fail. It is not so much the circumstances of your life or upbringing as much as your habits of thought that propel you to success or failure.

Negativity is an accomplice of failure and spawns feelings of fear, doubt, guilt, despair, etc. These in turn undermine your self-confidence and performance, robbing you of success. Each failure reinforces the negativity that led to the failure in the first place. A vicious circle is established: negativity engenders failure, and failure reinforces negativity, as illustrated in Figure 10.1.

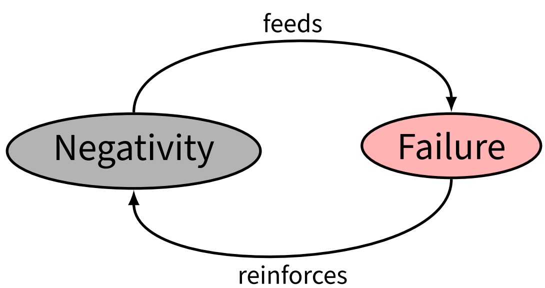

FIGURE 10.1: Negativity feeds failure, which in turn reinforces negativity.

The negativity-failure couple becomes self-sustaining and self-fulfilling. You then accept the label that you are a failure and descend deeper into its dungeon of gloom, despairing ever to succeed.

> Dissociate yourself from all negativity. Then alone will you be ready to embrace success.

The best weapon to disrupt this vicious circle is a sunny disposition. Cultivate a positive attitude of mind. *Believe that what you think becomes your reality sooner or later.* If you practise optimism, you will one day *realize* that you alone are the architect of your destiny. To extricate yourself from the vicious dual stranglehold of negativity and failure, practise positivity, which leads to success, as illustrated in Figure 10.2.

Even if at first, you meet only with failure rather than success, keep on trying. Never lose your positivity. Each time you fail, analyze what

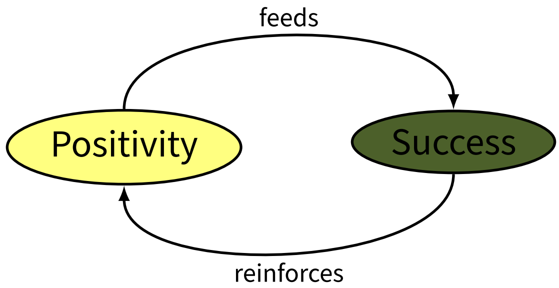

FIGURE 10.2: Positivity feeds success, which in turn reinforces positivity.

went wrong. Failure will then stimulate fresh hope. Correct and retry. Redouble your efforts. Positivity in turn will stimulate renewed application to achieve success. Plough the field of failure to plant the seeds of success. By nourishing those seeds with positivity, you are bound to succeed.

An indomitable will and a habitual positive attitude are the magic ingredients for success in all fields.

10.7 Positive visualization and affirmation

We have already touched upon visualization and affirmation in Chapter 8. It is such an important practice that it bears some repeating here in the context of failure. Apart from putting in the effort needed to master some skill or knowledge, you should also practise positive visualization or affirmation.

Olympic athletes are taught to visualize themselves doing perfectly what they are supposed to do at the Olympic Games. Olympic swimmers or gymnasts will be trained not only to swim or perform gymnastics, but also to *imagine* or *visualize* their swimming or executing the gymnastic routine perfectly. This mind's eye training helps produce Olympic champions.

You can do the same. Work at whatever have set for yourself. At the same time, visualize yourself as *already having perfected* whatever you are trying to master, whether it be doing mathematics, presenting a seminar, breezing through your examinations, or becoming proficient in Web searches. Effort and visualization will together work wonders. Repeated visualization will actualize whatever you are imagining.

Likewise, you can practise affirming with words whatever you are trying to master. Suppose you wish to write flawless English essays: you might try saying "I can write flawless English essays" or "I have mastered English essay writing". Note that you do not use words like "try" or "attempt" in these affirmations. There is no room for doubt. *Instead, you affirm that reality which you wish to actualize.*

★

Your subconscious mind accepts as truth the thought you are feeding it—through visualization or affirmation—and effortlessly manifests it in your life.

Practise positive visualization and affirmation without ceasing until whatever you desire actually comes to pass.

10.8 If at first you fail, try, try again

Failure is not the end of the world. Some of the most successful people in the world have been those who were *undaunted by repeated failure.*

EXAMPLE: THE LEGEND OF ROBERT THE BRUCE

Keep in mind the legend of the Scottish king Robert the Bruce, who while hiding after losing in battle six times, saw a spider attempt to spin a web. It failed six times in succession, but as the fascinated king watched, the spider went on to spin the web successfully on its seventh attempt. He took heart from the spider's example and redoubled his efforts at battle, winning, and reigning as monarch. ★

Alfred, Lord Tennyson ends his famous poem *Ulysses* with these memorable words:

> ...that which we are, we are,
> One equal temper of heroic hearts,
> Made weak by time and fate, but strong in will
> To strive, to seek, to find, and not to yield.

That last line—"To strive, to seek, to find, and not to yield"—was adopted as the motto of the Outward Bound movement [1]. Make it your personal motto as well.

> You cannot fail if you never give up!

10.9 The importance of small successes

Success motivates like nothing else. It is the supreme encouragement. Use little successes to build up larger victories.

Break a large task into smaller ones. Whether it is learning swimming or the anatomy of the brain, break the task up into smaller, more manageable chunks. Then focus on *one* small chunk. Try and try again until you succeed at it. Once you do. move on to another. In this fashion, build up your "bank account" of successes. March confidently from small successes to large ones.

10.10 Life support systems: family and friends

Solace at times of disappointment or failure is a healing emotional balm. It help us to cope and to hope. Family and friends are a life support system who can help lift us from the muddy mire of despair into the bright sunshine of renewed hope and resounding success.

While you should not be an emotional leech on those who love and support you, do not err on the other side and desist from asking help when necessary. The sooner you vanquish failure, the better for everyone. So, seek help early.

Most of all zoom out of the storm clouds of moodiness. Navigating out of confining moods might be the ultimate Houdini trick that you need to master. Use every resource you can access to propel yourself out of pernicious moods. Once you master your moods, you will find success so much more within your grasp, like a trained dog awaiting its master's bidding.

10.11 Patience is indispensable for success

Patience is indispensable for success. Cultivate patience. The dictum to try unceasingly until you succeed is vacuous without patience. There is a famous legend about a yogi called Milarepa who lived in Tibet. He was told by his teacher to build a house. After he had built it, his teacher told him to pull it down, return the rocks and stones to their places of origin, and start again on another house. This was repeated twice. Finally, the teacher asked Milarepa to build a fourth and final building: a nine-storey tower. Milarepa obeyed unflinchingly on each occasion. In the end, having passed all the tests, Milarepa succeeded in his spiritual quest. Patience itself waits upon on such a one, as indeed, does success.

Each time you feel impatient with yourself, your circumstances in life, a problem you cannot master, or a skill that eludes you, remember Milarepa. Become an embodiment of patience. You cannot but succeed.

10.12 Do not repeat the same mistakes

Failure is usually the result of a cascade of errors. But making mistakes is part of learning, and repeating the same mistakes once or twice is part of the learning curve that leads to success. But what you need to guard against is repeating the same mistakes all the time, because that is a symptom that learning has not taken place.

Errors are common in the first three stages of learning: "naming", "knowing", and "doing". Here are some examples to illustrate how mistakes may be made and how to avoid repeating them.

For example in the "naming" stage, you might confuse a bacterium with a virus, because you know that they both can cause illness, but cannot recall how they differ. Such errors are easily corrected by reference to textbooks, or lecture notes, or a reliable Web source.

If you cannot fix in your mind the differences between a virus and bacterium, try making up a mnemonic *to highlight the differences, in terms that have meaning for you.* Test your knowledge a week later and see if you can remember correctly. If not, revisit the issue and try another technique—perhaps a visual one—like a table, an octopus diagram etc., either alone, or along with a mnemonic, as discussed in Chapter 7. Work patiently at it until the gross and subtle differences between a virus and bacterium are safely and durably locked away in your long term memory.

In the "knowing" stage, you might encounter similar difficulties of confusion, or of simply being unable to recall a fact or method, etc. For instance, say you are learning English as a foreign language, and you do not know the correct verb form of the verb "to be" that goes with the first person singular pronoun "I". In short you do not know if it should be "I be", "I am", "I is", or "I are". The best way to overcome this type of knowledge deficit is to *immerse* yourself in a sea of spoken English. Then, you will *know* that it is "I am". It is futile to use logic in such a situation to figure out the answer because language, pronunciation, and spelling are not algorithmically or logically consistent, but rather are artefacts of history and usage.

In the "doing" stage, you might slip many times because of lack of practice. Whether you are learning to skate on ice, or are trying to translate textbook symptoms into a diagnosis in real life on a real patient, or are trying to prove a theorem, there is no guarantee that you will get it right the first time. As always, practice makes perfect.

> ★
>
> Practice until you achieve that ease which comes with mastery.

Practice is the best antidote to failure in the "doing" phase of learning because it makes success second nature. You yourself will know that you know, when you master the "doing" stage of learning.

10.13 Aspire to the "being" stage of learning

The thralldom of failure is palpable in the first three stages of learning. But failure dare not peep at you when you reach the "being" stage. So, aspire always to that expert level. Think of a high-flying eagle effortlessly gliding on the air currents, cruising majestically with nary a flap of the wings. That is the image you should have of the "being" phase of learning. That is mastery. When you attain that, you have indeed succeeded.

> ★
>
> Work hard at any subject until you attain that finesse which is synonymous with the "being" stage of learning.

10.14 Failing gracefully

By milking failures for lessons, you can leverage failures to yield success. Often, the lessons learned during failures are more valuable than the correct answers. In our personal lives, in business, in the laboratory, and in many other circumstances, success is not guaranteed despite our most assiduous efforts.

In such circumstances, fail gracefully and learn the lessons from failure effectively.

"Failing gracefully" is an expression familiar to computer programmers. It is impossible for a programmer to anticipate the multitude of real-life conditions under which a computer program might fail. Accordingly, the program is so written that it will work correctly under certain assumptions, and fail *gracefully* if those assumptions are not met. This means that the program will not crash the computer, or cause loss of data, or interfere with other programs that are running. Most importantly, your hard disk will not be wiped out because a single program has failed.

Failing gracefully is a very constructive attitude toward life in general and learning in particular.

Failure teaches us what to avoid, how to avoid it, why to avoid it, when to avoid it, etc. Failure inspires renewed effort to achieve what has not been attained yet. But failure cannot and should not cause loss of self-esteem, self-loathing, or other self-destructive tendencies. In no case should any failure cause your "hard disk to be erased".

10.15 Failure and research

Research is different from learning in that you are engaged in discovering new knowledge rather than becoming familiar with what is already known. Nothing worthwhile is gained without effort. And much of the effort in research is to keep alive the flame of the quest in the face of repeated failure. While funding is one prerequisite for research, being able to cope sensibly, courageously, and creatively with failure is another. There is no researcher who has never encountered failure in his or her work. Patience, will power, tenacity, and faith in yourself, and in your approach to the research are all essential for success.

Preparing a manuscript for publication and coping with its rejection are discussed in detail in Chapter 49. Whether as a postgraduate student or researcher, never give up when you are attempting to publish a paper.

Many famous scientists who went on to win Nobel Prizes have had their prize-winning papers rejected when they were first submitted. Cultivate a dogged determination to succeed, but do not take rejection of your submissions personally. Neither hate yourself for having failed to publish, nor denounce or denigrate the reviewer(s) for not recommending your manuscript for publication.

> Do not personalize failure. Analyze why you failed objectively. Correct errors. Try again. Repeat until you succeed.

If you dread failure, you cannot do research. If you are hypersensitive to others' criticism of your hypotheses or methods, you are unsuited to research. Become expert at taking failure in your stride, turning criticism to advantage, revising assumptions, fine-tuning methods, taking on board advice from research colleagues and benefactors, and attacking your problem with a new viewpoint, patiently with creativity and intuition. Success in research will be yours.

10.16 Parting words

> Use failure as a pole to vault to success. Be patient. Do not lose hope. Try and try again. Never give up!

References

[1] *Outward Bound*. Wikipedia, the free encyclopedia. URL: http://en.wikipedia.org/wiki/Outward_Bound, visited on 2014-01-15.

Summary: Learning from Failure

- Failure is part of learning. It is a feedback mechanism. You cannot learn without failing.
- Once we have learned to walk, we forget early failures like falling down when learning to walk.

- Forget the failure and focus effort and energy on what needs to be mastered.
- Do not repeat the same errors.
- Give up all negative emotions like fear, doubt, despair associated with failure.
- Negativity and failure are accomplices. Deliberately give up all negativity.
- Cultivate positivity which leads to success.
- Regardless of how many times you have failed, try again and again. Success is bound to be yours.
- Practise positive visualization in which you imagine yourself effortlessly succeeding at whatever you are tying to master. Your visualization will become your reality.
- Positive affirmation that you have already succeeded will likewise manifest the success you seek.
- Do not personalize failure. This is especially necessary in research.
- If your research manuscript is rejected, use the reviewers' feedback to improve your submission and try to get it published once more.
- If you never give up, you can never fail.

Part C

Foundations

12

English as a Language

Synopsis

Language is by far the most distinctive and transformative achievement of humankind. It has allowed us to progress from the cave to the city and to enjoy all the attendant benefits of that evolution.

All languages are instruments of communication. And they are all built upon the triad of grammar, vocabulary and usage. Grammar enforces structure, and through it, unambiguous communication. Vocabulary provides the word-bricks used to build a language. Usage is the influence of style on both grammar and vocabulary, and varies with time and place.

English and Mathematics enjoy widespread acceptance and are the two languages we deal with in this book. English is both spoken and written whereas Mathematics is principally written. Both languages are evolving and expanding within their respective domains, and richly reward those who master them.

The student who wishes to master the English language should listen, speak, read, write, and think in English to gain rapid proficiency in it. Guidelines are given here to help achieve this.

> The limits of my language means the limits of my world.
>
> LUDWIG WITTGENSTEIN, Austrian-British philosopher, (1889–1951)

12.1 The wonder that is language

Language is an instrument of human communication. We use it whenever we listen, speak, read, write, and think. A moment's reflection should convince you that languages are by far[1] the most *distinctive* achievements of human beings.[2]

Language defines our very identity. Creative freedom has allowed languages to evolve and vary with time and clime. The dazzling variety of languages that exist is testament to the endless human talent for invention.

Not only is language the most distinctive of human creations, it is also the most *transformative*.[3] It has allowed us to communicate, compete, cooperate, coalesce, and consolidate into the terrestrial civilization that we are today.

Indeed, if not for language, you would not and could not be reading this book. Like life, breath, fresh air, and sunshine, language is an underappreciated but magnificent bounty we all enjoy.

12.2 English as a universal language

In 1687, Sir Isaac Newton[4] published his three-volume magnum opus, *Philosophiæ Naturalis Principia Mathematica*.[5] It was written in Latin.

The *Principia*—to use its short name—has been called the most important book in the history of science. It transcended the linguistic and national boundaries of its time and was accessible to the intelligentsia of all Europe. This was because Latin was the common language of the learned at that time.

Today, English performs much the same function as did Latin in the past. Not only is it the language of scholarly and scientific discourse, it is also the language of trade and commerce. And it is the common language

[1] Hyperlinks are coloured. This one leads to an online definition. Follow it to enlarge your vocabulary.
[2] The languages used by other creatures are imposed upon them by Nature.
[3] Both English and Mathematics are human languages.
[4] This hyperlink points to a life of Newton, available on the Web. Explore such links to expand your knowledge.
[5] Latin for *Mathematical Principles of Natural Philosophy*.

of international civil aviation, to name just one area where lives literally depend upon correct use of the language.

English is the closest to a universal language that we have at present. Regardless of where you live and what you study, a good command of English will help improve your academic prospects.

12.3 Grammar, vocabulary, and usage

The edifice of language rests on the tripod of *grammar*, *vocabulary*, and *usage*. Together, they confer upon language the structural integrity and functional consistency necessary for clear communication.

12.3.1 Grammar

Grammar is the *foundation* of language. Grammatical rules governing number, tense, case, and gender dictate the *structure* or *syntax* of what you write. The logic behind grammar enforces *correct, unambiguous communication.*

12.3.2 Vocabulary

Words are the *bricks* of language. Each word has its own nuanced meaning, which befits it better for some tasks than others. A good vocabulary affords you the luxury of choosing—from a large array—the best word(s) for your purpose. The *meaning* conveyed by the words you use—*the semantics*—is determined by your mastery of vocabulary. Vocabulary is treated in-depth in Chapter 30.

12.3.3 Usage

Usage is the *style* or *fashion* of the language you use, not unlike architectural features or flourishes on a building. It is less rigid than the rules of grammar, or the meanings of words, and yet, it impacts upon both. It is governed by the times in which the language is spoken and written.

We instantly and instinctively recognize a style, whether in clothing, buildings, print, or language, and can effortlessly distinguish between a dated fashion and a contemporary one. New usage often emerges stealthily by unspoken convention as "the done thing", often flouting existing grammatical rules in the process. Usage is the focus of Chapter 31.

12.4 Learning English the hard and easy ways

Many, for whom English is not their native tongue, go to great lengths to learn its grammar. They learn how to analyze and synthesize sentences. They struggle with active and passive voice. They contend with prepositions and idioms. They memorize the quirks of English spelling. In short, they expend a great deal of time and brainpower for what are often meagre results. This is the hard way to learn English.

The easy way to learn English is the way you learned your mother tongue: by a process of immersion in an environment in which you are literally soaked in the language. You absorb the language osmotically. What you imbibe by listening you express through speech. It seems effortless. And it *is* effortless.

Grammar is secondary to usage. It matters not if you do not know what the subjunctive is. What matters is your ability to understand the spoken word, to speak fluently and confidently, to read with comprehension, and to write clearly and idiomatically. This immersion method takes time, but its results are far greater and more durable. Moreover, the process is so much more fun.

Listen. Speak. Read. Write. Think.

12.4.1 Listening

All languages have spoken variations or dialects. English is no exception, especially in the land of its birth. The English of the south of England is different from that of the Midlands. The Welsh, the Scots, and the Irish have all added their peculiar musical lilts to the language.

The Indians, the West Indians, the Latin Americans, the Germans, the French, the Dutch, the Italians, the Malaysians, the Singaporeans, the Australians, the New Zealanders, the Americans, the Canadians, the Kenyans, the Nigerians, the South Africans, the Egyptians, the Japanese, and the Chinese of Hong Kong have all put their own, unique stamp on the flavour of English they speak. The ubiquity of English has exacted a price: there is no single spoken English; there are only various flavours of the one language.

Take your pick of which flavour of English you would like to learn.[6] Listen to it as spoken by those who have mastered it. Gradually, you will understand their speech. Then, you will be able to speak like them. To help achieve this:

- Tune in to radio broadcasts where the preferred flavour of English is spoken. Listen to the news. Try to follow panel discussions. Listen to reports by journalists of repute.
- View documentaries and news bulletins in English on television or on the Web. There is a treasure trove of material that you can access online.
- Attend lectures, seminars, and discussions conducted in English to which you have ready access. Again, the Web is your treasury.
- Listen to audio books. Many audio books are freely available for listening at LibriVox.
- Watch English movies, with English subtitles, to enable you to tune in to the spoken word and attain proficiency in both listening and reading.
- Watch movies in your mother tongue with subtitles in English to develop comprehension and vocabulary.

12.4.2 Speaking

If listening is one side of the coin, speaking is the other. Start speaking English once you have listened to its sounds for some time. Do not wait until you have become an "expert listener". Language mastery comes with speaking as well as listening.

However hesitant you might be, *speak* in English. You cannot learn swimming while standing on land. You have to get into the water. Likewise, you will never speak English unless you *try* to speak it. Do not be self-conscious. You might speak the language haltingly or ungrammatically at first. Neither despair nor be deterred. There is no shame attached to learning. You will speak it fluently by and by.

Speaking improves with practice. Use every opportunity to converse in English. What others think of you is inconsequential. What you achieve by your own efforts alone matters. For that, keep on speaking English until you can do so naturally and without obvious effort.

[6] Beware of mixed tongues, where English is mixed freely with another language, as you risk speaking neither language in its pure form.

12.4.3 Reading

Once you can understand and speak English, start reading. By listening and speaking, you would already have learned proper pronunciation. It only remains to associate a pattern of letters with a sound. Read aloud. Read poetry. Go through Chapter 34 on Poetry. Progress to silent reading. Above all read with understanding. Part E deals extensively with reading; refer to it off and on.[7]

Just as listening and speaking are the obverse and reverse of the same coin, so too are reading and writing. As you read, so will you write. You must therefore choose good reading material, worthy of emulation.[8] Here are some hints on what to read:

- Good newspapers, whether paper-based or online, are excellent for starters. Alas, many are now behind paywalls. I have found to my dismay that even established newspapers put out material that has mistakes of omission, spelling, usage, etc. And tragically, newspapers nowadays are not necessarily purveyors of fact, but rather of opinion masquerading as fact. Accordingly, I make no recommendations here; use your own judgement.

- National broadcasters like NPR, the BBC, the ABC, and the CBC generally have good standards of English and reportage. They usually feature an eclectic mix of news and opinion, but always beware of bias—national broadcasters are not exempt! By browsing their websites, you will learn not only English but also how to think critically.

- Online and paper-edition current affairs magazines are an excellent source of good English provided you know where to look. You need to choose magazines which have good standards of English writing, and which also offer truthful, relatively unbiased opinions. I will not make any blanket recommendations; quality and probity can vary with time. Search out those that resonate with you and read them.

[7] "On and off" is equally acceptable. ☺

[8] Unfortunately, the democratization ushered in by the Web has meant that there are very many examples online of both of poor and good English. Use a good textbook to guide you to find out which is which.

- Read book reviews in newspapers, like The Times Literary Supplement, and at specialist websites like the Los Angeles Review of Books, Kirkus Reviews, and goodreads. Then read those reviewed books that appeal to you on subject matter and quality. The reviews and essays at these websites are also usually an enriching read in themselves. You could also sign up for newsletters from such sites if you so desire.

- Visit your local library and see if you can borrow reading material suited to your stage of learning English. Ask the librarians for assistance if necessary.

- Shakespeare is the great bard of English. Read him and other renowned writers. Also read good translations of non-English authors. You will expand your habits of thought as you encounter new idioms and exult in the effervescence of fresh viewpoints.

- If you are studying English as a group or in a classroom, find out if you can exchange reading material with your fellow students. Share web links, or books, or periodicals. Enrich one another.

12.4.4 Writing

Writing is both a science and an art.[9] Writing well comes with careful reading and even more careful writing. The first step is to read well and read widely. We are all mimics when it comes to language. What we have read reverberates in our minds and ultimately finds expression as the spoken or written word. Cultivate a love for reading. You will automatically start writing better.

Maintain a diary in English. Recording your thoughts is a helpful practice, and doing so in English would be doubly helpful.

> ★ Practise writing.

The more you write the better you will write. You will become internally sensitive to the flow, the rhythm, and logic of your own words and sentences. In time, you will know whether a sentence rings true as

[9] See Chapter 50 and the paper by Gopen and Swan [1] for more on this.

an expression of good English. Once you hit that "sweet spot" you will fall into a happy relationship with composition in English.

A good and ever expanding vocabulary is essential if what you write is to excite, entertain, rivet, inform, and educate your reader. Become friends with words.

Even if no one but you reads what you write, keep on writing. If you can get an instructor to review and correct your writing, so much the better. Pay attention to repeated patterns of mistakes—whether of spelling or usage—and correct them. Do not be discouraged even if you get poor marks. Never give up, but keep on trying. You *will* write good English eventually.

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Always revise what you have written.

One whole part of this book, Part F, deals with writing. Refer to it as needed.

12.4.5 Thinking

When you have mastered listening, speaking, reading, and writing in English, you might gradually find yourself *thinking* in English. If you do, you have mastered the language. When you *think* in a language, you have become so habituated to it that it becomes second nature to you.

There are some who think more comfortably and effortlessly in a language other than English. That is fine. But you would then be transposing your thoughts from one language to another. Patterns of usage and idiom, vocabulary and syntax, style and grammar, all need to be translated, which is an additional burden. Ideally, therefore, you should be able to think in the language that you are using at any moment.

12.5 Resources

I have assumed that you are able to read and write English, i.e., that you have a working knowledge of the language, however rudimentary. If that is not the case, you may refer to the printed and online resources listed here to get you over that initial hump.

12.5.1 Textbooks and references

If you are looking to learn English from scratch, perhaps the best books to get you up and running are books that embody in their titles the words "English Through Pictures". I used one such book many hundred moons ago but that book is now sadly out of print. ☹

At present, there are two series of books with "English Through Pictures" in their titles. One is the two-volume set by Jonathan Crichton and Pieter Koster [2, 3]. The other is the three-volume set by Richards and Gibson [4–6]. While I have read neither set of books, they seem to have been well received by the community of learners they are intended for.

The above two series should impart enough knowledge on grammar, vocabulary, and usage to someone just starting out with English. If you need further help with English grammar, take a look at the two books by Raymond Murphy [7, 8]. For more help on vocabulary and usage, refer to Chapters 30 and 31.

Surf the Web to find sites that can help you with English. Look for free online courses on learning English. If you learn visually, look for videos on a site like YouTube using a search phrase like "Learn English". The results you see will vary with your location in cyberspace, and the didactic quality might be variable. As with all matters on the Web, be wary, and confirm the correctness of the material you learn from, before you invest time and effort.

12.5.2 Helpful Q & A websites

There are many websites devoted to helping students learn English. Some are *forums* with a searchable *Question and Answer (Q & A)* format. Examples include:

- The English Language Learners Stack Exchange which is a member of the very useful Stack Exchange family of websites, each devoted to a specific subject area or specialist group. Explore these sites well as they are generally authoritative and helpful. This site is a beginner-level, friendly site.
- English Forward is especially useful if you are learning English as a foreign language. You might find volunteer helpers there who speak your native language, and are therefore attuned to your peculiar difficulties of understanding or usage.

- The English Language and Usage website is where more intricate questions on English are asked.

You would need to register before you can participate in most Q & A websites. Each site has its rules for good reason. Acquaint yourself with them and follow them. Benefit and be enriched.

The degree of courtesy extended to newcomers varies with websites.[10] Some sites are more welcoming and friendly than others. Find out those which agree with you. Follow site-etiquette always, but do not get deterred from asking for help just because someone somewhere once growled at you online. ☺

12.6 Looking ahead

This concludes the brief overview of the English language that is one of the two principal foci of this book. In later chapters we look at the skills needed to read discerningly, and to write persuasively.

Take a look at the chapters in Part E and Part F for further aid in the effective use of English. Chapter 30, Improving Your Vocabulary, is particularly relevant. So too is Chapter 31, Mastering Word Usage.

SUMMARY: ENGLISH AS A LANGUAGE

- Language is the most distinctive and transformative human achievement.
- All languages are instruments of communication.
- Grammar, vocabulary, and usage underpin all languages.
- Grammar enforces logic and structure to allow unambiguous communication.
- Vocabulary provides the word-bricks used to build ideas with a language.
- Usage modifies both grammar and vocabulary to some extent, based on the time and place where the language is used.
- English is the closest to a universal language that we have at present.
- The easy and enjoyable way to learn English is to immerse yourself in an environment where it is used.

[10] Sites scattered throughout the Web feature opinionated folk who exercise petty fiefdoms because they consider themselves experts. Just be aware that the Web is a reflection of humankind, warts and all.

- Listening, speaking, reading, writing, and thinking are the five actions we need to perform, in that order, to gain proficiency in English.
- The rank newcomer to English can rely on generously illustrated textbooks of English to gain rapid familiarity and fluency in the language.
- The interested student should judiciously mine the Web to further her or his knowledge of English.

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13

Mathematics as a Language

Synopsis

The two languages we are concerned with in this book—English and Mathematics—enjoy widespread acceptance. While English is both spoken and written, Mathematics is principally written. Both languages are evolving and expanding within their respective domains, and richly reward the student who masters them.

All languages are instruments of communication. And they are all built upon the triad of grammar, vocabulary and usage. Grammar enforces structure, and through it, unambiguous communication. Vocabulary provides the word-bricks used to build a language. Usage is the influence of style on both grammar and vocabulary, and varies with time and place.

Mathematics, being wholly logical, is less forgiving of errors of syntax and semantics. Nevertheless, it may be mastered by the diligent student who will then behold its silent, hidden beauty. This will help establish a lifelong friendship with the subject.

> The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics.
>
> *A Mathematician's Apology*
> G H HARDY(1877–1947)

13.1 Mathematics *vis-a-vis* English

The primary purpose of any language is to facilitate *communication*. Whenever you listen, speak, read, or write, you are engaged in an act of communication. English and Mathematics are two universal languages which we have developed for specific purposes.

We are concerned in this book with both. English as a language has already been discussed in Chapter 12. Mathematics is introduced here as no less a language than English.

While English is one of many languages adopted by different people around the world for daily discourse, Mathematics is, by and large, *a single language*, universally used and understood, without significant variation, serving a specialist function: *the recognition of patterns*. Everything is viewed through the lenses of quantity, symmetry, shape, structure, and beauty[1] to reveal latent and patent patterns.

The fact that Mathematics is also a language should in itself allay any fears that you might have about it being vastly different from a spoken language like English. They are both means by which we may share "stories" of one sort or another.

The only major difference is that Mathematics is not so much spoken or listened to as it is written and read. Mathematics is silent, and much of its grandeur and beauty derives from this quality,[2] but this very reserve deters the student from befriending it easily.

13.2 Grammar, vocabulary, and usage

Much of what we said about English grammar, vocabulary, and usage in Chapter 12, also applies to Mathematics. Whereas English is both written

[1] Mathematical beauty has most often been compared to poetry and music, and ranks with them as among the noblest and most creative achievements of the human mind.

[2] English is like a river exhibiting both boisterous cataracts and placid flows. Mathematics, on the other hand, is like a glacier—silent, powerful, austere, deceptively mobile, and in its own way, breathtakingly beautiful.

and spoken, Mathematics is principally written rather than spoken.[3] Its similarities to English as well as its quirks are outlined below.

13.2.1 Grammar

Mathematics is a symbolic expression of concentrated and undeviating logic. It is therefore unsparing in its grammar, which cannot be flouted if communication is to be unambiguous. Mathematics boasts of a precision unparalleled in the annals of any language.[4]

Let us take a simple example of the addition of two numbers, accessible to all, and see what caveats might arise in its grammatical interpretation.

EXAMPLE: GRAMMAR: INFIX, POSTFIX, AND PREFIX NOTATIONS

Let us add the number 3 to the number 2 like so:

$$2 + 3 = 5 \tag{13.1}$$

This expression with an "=" (equals sign) is a statement of mathematical truth. It says that if we add 3 to 2, the result is 5. That is something all of us can agree upon.

The "+" (plus sign) is, by convention, the symbol we use to denote the action of addition. We can think of 2 and 3 as nouns being acted upon by the verb + to yield the resulting noun 5.

Let us now confine our attention not to the *truth* of Equation (13.1), but merely to the *act* of addition being performed on the left hand side, i.e., to:

$$2 + 3$$

The *notation* of interposing the verb between the two nouns is called *infix* notation. There are two other equally valid conventions to denote addition—*postfix* and *prefix*.

In postfix notation (also called Reverse Polish Notation or RPN), instead of $2 + 3$, we write

$$2\ 3\ +$$

[3]Except in primary school when we *recite* tables of addition and multiplication.

[4]Computer programming languages are a subset of mathematics and therefore exhibit the same unforgiving repudiation of syntactic errors.

where the two nouns come first followed by a verb. Because it acts on *two* numbers, we call addition a *binary* operation.[5]

In prefix notation (also called Polish Notation), we write

$$+\ 2\ 3$$

where the verb comes first followed by the two nouns.

With the latter two notations, the space between the 2 and the 3 might be small enough for the two numbers to be mistaken as 23, when the expression will fail to make sense, because the second number would then be missing. It is therefore no surprise that the infix notation—with less scope for ambiguity—is the one adopted in many contexts.

But the postfix and prefix notations have the major advantage that they are *parenthesis-free*.

For example, in the expression $(5+2)3$, we evaluate the term within parenthesis, namely, $(5+2)$ *first* and then multiply the *result* by 3 to get 21.

Because addition and multiplication are binary operations, in postfix notation, there are two numbers followed by an addition or multiplication sign. We start at the innermost parenthesis and work outwards. So, we write $5\ 2\ +$ and the result from this operation is multiplied by 3. So, the full postfix expression is $5\ 2\ +3\ \times$, which is both unambiguous and parenthesis-free.

Postfix and prefix notations have found use in early electronic calculators [1] and some computer languages such as FORTH [2].

The point to take away is that despite being rigid, mathematical grammar has evolved to allow different accepted notations, all of which work flawlessly. One needs awareness of the context before making pronouncements of grammatical validity. ★

Example: Grammar: order counts

In English, the statement "The cat ate the rat" has the opposite meaning to the statement "The rat ate the cat", derived by transposing the two nouns—subject and object. This is because English is principally a non-inflected[6] language where word order can and does influence meaning.

[5] So too is multiplication.

[6] The non-inflected nouns *cat* and *rat* do not change their forms according to case (subjective or objective), unlike the inflected pronouns *I* (subjective) and *me* (objective).

Consider now two mathematical statements where the two nouns have been transposed:

$$2+3=5$$

and

$$3+2=5$$

Both are valid and true because addition is *commutative*—order does not matter.

But see what happens when we try the same thing with subtraction:

$$2-3=-1$$

and

$$3-2=1$$

Obviously the two statements give different results and are not equivalent. So, subtraction is *non-commutative*—order does matter. Therefore, we cannot always blithely transpose terms in a mathematical expression.

If you have not come across *negative numbers*, the first statement will seem nonsensical to you; and rightly so. But even then, it should be apparent that no sleight of hand will allow us to take away 3 from 2 and be left with something we can still count as we normally do. So, the negative number -1 is a new beast in the mathematical menagerie.

The ever-expanding empire of Mathematics has grown by fits and starts over several centuries, during which people grappled long and hard with "unnatural" ideas like negative and *imaginary numbers*, before they were gradually accepted and allowed into the mathematical fold. ★

13.2.2 Vocabulary

Mathematics is written rather than spoken and consists more of symbols and pictures than words. Its vocabulary is likewise largely symbolic and sometimes pictorial. It is this unfamiliar vocabulary that appears so forbidding to the budding student of the subject and induces what I have called "Mathophobia", which is considered in detail in Chapter 39.

English expands by the addition of new words and phrases to capture novel ideas and situations; see Chapter 30. Mathematics enlarges its domain by adding new mathematical concepts, objects, and their interrelationships, with symbols to boot.

While every English word must be made up of the 26[7] letters of the alphabet, mathematics is composed largely of Hindu-Arabic numerals, English, Greek, and Hebrew letters, and a very large variety of symbols like "+, −, =" etc. Indeed, mathematical symbols may be and are invented according to need. This spontaneous expansion of symbols makes mathematics especially daunting; how does one comprehend symbols that are new and unknown? An example will illuminate.

Example: Vocabulary: some symbols from Set Theory

When the highly philosophical and logical Set Theory [3, 4] was introduced in the late 1800s, it was initially criticized, but later recognized and adopted as being capacious enough to provide the foundations for a magnificent and unshakeable edifice of mathematics which was being rebuilt for the modern age.

Sets were simply given names of uppercase letters of the alphabet like A or B and the elements in them were named with lowercase letters like a or b. Operations between them, however, needed new symbols as tabulated in Table 13.1.

Symbol	Meaning	Example
$\in$	Is an element of	$a \in \mathrm{A}$ a is an element of set A
$\subset$	Is a subset of	$\mathrm{B} \subset \mathrm{A}$ B is a subset of A.
$\varnothing$ or $\emptyset$	The empty set	$\varnothing \subset \mathrm{A}$ or $\emptyset \subset \mathrm{A}$ The empty set is a subset of A
$\cup$	Union of two sets	$\mathrm{A} \cup \mathrm{B}$ The union of sets A and B
$\cap$	Intersection of two sets	$\mathrm{A} \cap \mathrm{B}$ The intersection of sets A and B

Table 13.1: Set-theoretic symbols with meanings and examples. Note that variations are allowed, as in the case of the empty set.

These symbols had to be invented, agreed to, and adopted by practising mathematicians. They are now part of the common vocabulary of mathematics. Necessity and familiarity with these symbols makes friends out of strangers.

[7] Or 52, if you count uppercase as different letters.

The explosion in mathematical vocabulary, to deal with new concepts, means that almost no living mathematician has mastered the entire vocabulary. The price paid for unfettered mathematical expansion has been restricted specialization, not unlike in medicine. ★

13.2.3 Usage

We have seen above that as mathematics evolves—and more elegant, overarching concepts are discovered—new notations are invented to accommodate them. The result is greater conciseness in expression, or to greater power or breadth in dealing with concepts both novel and familiar. New vocabulary naturally leads to new usage.

EXAMPLE: USAGE: SYMBOL FOR THE IMAGINARY NUMBER $\sqrt{-1}$

When mathematicians first tried to solve the quadratic equation

$$x^2 + 1 = 0$$

they rightly concluded that there was no number known at the time that would fit the bill. After much hand-wringing and brain-wrangling, they decided to accept the existence of a new type of number whose square would be -1. Because this number was not real, it was called an *imaginary number*. Accordingly, the symbol i was assigned to represent $+\sqrt{-1}$, which is the positive square root of minus one.[8]

These imaginary numbers later found application in the real world for describing the relationships between currents and volt-ages associated with alternating current (AC). But electrical engineers had already enlisted the symbol i for electrical current. So, a convention was established that electrical engineers would use the symbol j in place of i to describe the unit imaginary number. This new usage avoided the possibility of confusion about what i meant in an electrical context.[9] ★

Usage evolves not only to avoid conflict of symbols, but also to accommodate newly forged mathematical concepts, usually accompanied by the added benefit of a more generic or concise notation.

[8] The Greek letter *iota*, written ι, is also sometimes used in place of i.

[9] However, the symbols j or J also denote current density. Given the finite number of alphabetic letters, we have to rely on context for meaning. But j as a legitimate representation of $\sqrt{-1}$ is here to stay.

EXAMPLE: USAGE: MAXWELL'S EQUATIONS

Vector notation was a new concept born in the late 1800s which led to a powerful unification of disparate mathematical ideas, as well as to a concise and elegant notation.

The great Scottish physicist, James Clerk Maxwell, had summarized the entire phenomenon of electromagnetism in a set of equations known popularly as Maxwell's equations [5].

What may not be equally well known is that Maxwell's original formulation consisted of *twenty* different equations [6]. Using vector notation, these could now now be reduced to *four* magnificently concise, almost poetic, equations couched using the arcane symbology of vector calculus, shown below for their symmetry and beauty alone, and not for comprehension: ☻

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}$$

Once conciseness and power have been gained by a new mathematical usage, rarely, if ever, does one encounter the old usage again. ★

13.3 Beauty in Mathematics

Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry.

"The Study of Mathematics" in *Mysticism and Logic and Other Essays*
BERTRAND RUSSELL (1872–1970)

Recognizing mathematical beauty requires familiarity with the subject and some practice in its discipline.[10] When an elegant and unifying the-

[10]See [7] for an example where even non-mathematicians were found, by objective measurements, to appreciate mathematical beauty.

orem pops out at the end of a series of logical steps, it gives pleasure to the eyes of a mathematician.[11]

$$e^{i\pi} + 1 = 0$$

The above equation, known as Euler's identity, has been described as the most beautiful equation because it embodies in one expression the five most important mathematical quantities. Just as a jeweller has trained his eyes to identify a flawless gem, so too, the dedicated mathematician has accustomed himself to spot mathematical gems.

Beauty resides in abstract mathematics, pursued for its own sake. Equally, it resides in applied mathematics that is used in the real world. Both types of mathematics can and do embody beauty [8] for they spring from the same source. Once you start appreciating the beauty of mathematics, the subject will seem less forbidding and more friendly to you. Develop this friendship and cultivate it for life. You will be enriched.

13.4 Looking ahead

This concludes the brief overview of Mathematics as a language. Refer to the five chapters on Mathematics in Part G for help with with the subject as you progress on your academic journey.

SUMMARY: MATHEMATICS AS A LANGUAGE

- All languages are instruments of communication.
- Grammar, vocabulary, and usage underpin all languages.
- Grammar enforces logic and structure to allow unambiguous communication.
- Vocabulary provides the word-bricks used to build ideas with a language.
- Usage modifies both grammar and vocabulary to some extent, based on the time and place where the language is used.
- While English has a finite alphabet from which its vocabulary is constructed, Mathematics takes recourse to unlimited invented symbols, strung together with logic, to convey its meaning.

[11] Sadly, Euler lost the sight in one eye, and later, in the other eye as well.

- Mathematics also embodies beauty to the beholder who can see it. Befriending the subject in this way will allay the fears that it usually inspires.

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Part D

Study Techniques

29

Moving From Bs to As

Synopsis

Moving from Bs to As is simple: Find out why you are getting Bs. Determine what you should do to get As. Then, do it!

Ask, analyze, discuss, and introspect to find out why you are missing out on As. Three major factors impact your grades: knowledge, examination performance, and the pressure of time.

Slice the orange of knowledge with a different cut. Adopt for your weaker subjects those techniques that have led to A grades—for yourself or for others.

Read questions carefully, understand them correctly, and answer them precisely. Guard against incomplete answers. Practice on past papers.

Start early. Work steadily. Finish unhurriedly. Do not buckle or break under a heavy workload. Prioritize and pace your work. Hand in your best effort. Never give up.

29.1 Why do you want As?

Most people who get to university are more than happy if they manage to pass their examinations, even if only barely! If your motive in attending university is to get a degree that will enable you to settle into a good job and life, just passing your examinations might well suit your purpose admirably.

However, if you are one of those free spirits who is ever in quest of excellence, passing grades alone would not satisfy you. Nay, even Bs would not do. You seek As, or as the expression goes, *straight As*. You want to excel.

The reasons for this ambition could range from the thrill of topping your class each year, through wanting to do research in your chosen field, to a personality trait that simply does not accept anything but a superlative outcome in all your endeavours.

Whatever the underlying motivation, one thing is certain. You want As not Bs. If you are one of these students, read on, for this chapter is written especially for you, whether you are at school or university.

29.2 Why do you get Bs and not As?

To move from Bs to As, the glaringly obvious first question you need to ask is:

★

Why do I get Bs and not As?

What is it about your academic performance that has shackled you to Bs instead of freeing you for As? The key to transforming your grades from Bs to As lies within the varied answers to this overarching question in all its variations. I suggest some approaches below, but they are by no means exhaustive. Use them *and any others* that yield the precious answer-nuggets that you seek.

29.2.1 Ask

Ask more probing and detailed questions. Apply the W^7 framework of Chapter 19—*what, when, where, why, who, which, how*—to devise questions about why you are getting Bs and not As. They will help unearth the reasons why you are stuck at Bs.

Here is a sample checklist for doing this. Use these and other relevant questions to guide you to find the root causes for your lower-than-expected grades.

ASKING QUESTIONS ABOUT POOR PERFORMANCE

- Which? All subjects or only a few?
- Why? Phobias toward any subjects?
- Why? Wobbly subject knowledge, poor examination technique, time pressures, or a combination of these and other factors?
- What? Multiple choice, essay, or practical questions? Why?
- When? Only in examinations or in routine work as well?
- When? Early morning, late afternoon, or evening?
- When? After an illness or other personal issue?
- Where? Only in certain locations or everywhere? Why?
- Who? Any particular instructor or teaching style?

LIST 29.1: Asking questions about poor performance.

29.2.2 Analyze

Another approach is to analyze the data. You could fire up a spreadsheet and fill its rows and columns with hard and soft data relating to your subjects, examination grades, gut feelings before an examination, locations and times of examinations, durations of illnesses, etc.

Include whatever *in your opinion* might be relevant. Look for tell-tale patterns. Try to extract from the data a consistent picture that helps explain your results. Because no two people are the same, the answers you seek will be personal and customized to you.

29.2.3 Discuss

The third approach is to discuss the issue. The opportunity to bounce off ideas—in a discussion with friends or conversation with a mentor—helps greatly. Think of it as a Socratic dialogue with finding clarity as its end.

EXAMPLE: EXAMPLE

Seek out an instructor in one of the subjects where you dearly want to do better but have not. Take a recent assignment or examination paper as a template for your discussion. Fix an appointment for a half hour or so, and discuss candidly why you did not soar higher in your grades. Walk away with something concrete that you can use to remedy your grades. ★

29.2.4 Introspect

There are certain subjective matters that only you can know and that only you can identify as causes for your failings. Reflect. Introspect honestly. Be frank. Be impartial. Do not wince at unpleasant truths—simply confront them matter-of-factly. Do not sabotage yourself with subterfuges and rationalizations. Become your own friend as you do some detective work on your academic performance.

EXAMPLE: EXAMPLE

For example, you might be a video game aficionado who is addicted to gaming. Or you might be spending too much time playing sport or a musical instrument at a time when you should be studying at full throttle for your examinations. Or you might spend too much time watching television or at the movies. Or you might fear a subject, or hate it so much, that you put off studying it. These are highly personal subjective factors known only to you. Identify and overcome them. ★

29.2.5 Conclude

At the end of this soul searching, you should have some clear ideas about what factors are responsible for holding you back in your grades. If you haven't got any clarity, relax and start over, until you do. Remember my advice from Chapter 10:

> ★
>
> Never give up.

29.3 Knowledge versus performance at examinations

Examinations grade your performance. Knowledge underpins your performance. So, it is important that you know for yourself whether it is knowledge, or examination performance, or both, that are dragging you back.

By knowledge, I mean what you have *internalized* in your discipline. If you know which page of which book has what is needed to answer a question, but you simply cannot recall the actual information, then you really lack the knowledge. Knowing where to find out what you need will mark you in later life as an educated person, but it will not transmogrify your Bs into As.

You may lay claim to something as *your own* knowledge only if it is *ready for recall* at the snap of a finger. And that is what counts at examinations.

Performance at examinations makes or mars your grade regardless of how much you know and how well you know it. Develop the ability to gauge for yourself, after the event, whether you have given of your best to any examination that you sit for.

Both knowledge and examination performance are dependent on the how much time you can lavish on them. The pressure of time impacts on both, and ultimately on your grades.

29.4 Moving from Bs to As

Once you have diagnosed the problem, you can dispense the correct remedy. In List 29.2, I have shown the three specific factors we have identified so far.[1] Each is dealt with in greater detail in succeeding sections.

MOVING FROM BS TO AS

1. Knowledge
2. Examination performance
3. Time pressures

LIST 29.2: Moving from Bs to As.

[1] This list is by no means complete. Find out for yourself what ails *your* grades using these ideas as a guide.

29.5 Knowledge

Because you are already a B student, your subject knowledge should be above average. If inadequate knowledge has been identified as a cause, you must home in on what precisely you need to learn better and how.

29.5.1 Wobbly knowledge

Have you mastered all your subjects or is your knowledge still a little wobbly? If you are weak across *different* subjects, it is more your study techniques that need to be improved rather than subject knowledge. Review Part D of this book, and use ideas from it like active reading, mnemonics, formulating your own examination questions, slicing the orange of knowledge with a different cut, efficient time management, regular revision, summaries, group study, etc., to establish your knowledge on an unshakeable foundation.

29.5.2 Weak subjects

What are the subjects you are weak in? Do you get Bs in all subjects or only in a few? Why are you weak in those subjects? Is it because of phobias, or a natural dislike for a particular discipline? Getting rid of phobias is like treating a disease. Just as you cannot claim good health until a basal infection has been eradicated, so also you cannot attain mastery of a subject until your phobias related to it are excised completely.

EXAMPLE: EXAMPLE

Many people have an unhappy relationship with Mathematics and develop mathophobia. In the five chapters in Part G, dedicated to mathematics, I have discussed overcoming mathophobia, explained the rationale behind the rules of arithmetic, emphasized carefulness when solving problems, and touched upon a whole host of other issues. While these are specific to mathematics, the principles in those chapters may be applied to other subjects as well. ★

29.5.3 Cross-fertilization

Do you get As in any subjects? Do you know why? Can you apply those same factors to your weaker subjects, to move from Bs to As? This approach is excellent if you have all As, except for one or two Bs which you want to convert into As as well.

If you are into group study, and have friends in your group who have got As in the subject of interest, and who are unselfish enough to share how they did so, find out what they did and apply those principles in your own work.

29.5.4 Slicing the orange of knowledge with a different cut

When you revise, use the technique of "slicing the orange of knowledge with a different cut" introduced earlier. Integrate your knowledge of a topic by linking it to other knowledge in that subject by purposely adopting a different perspective from your textbook or lecture notes. Chapter 19 and the companion website have several examples of this valuable study technique. It can help you master what has eluded you in the past.

29.6 Examination performance

Knowledge alone does not get you good grades. Examinations are the ultimate arbiter. They are so important that Part H of this book is devoted to examinations in their different aspects. Browse through and revise the chapters therein to refresh your memory on how to do well in examinations.

Do you perform badly only during examinations or do you get poor grades in assignments, term papers/projects, and tests as well? Is your performance stymied by time constraints? Do you panic? Is it because of poor time management? If so, read Chapter 21 and follow the suggestions there.

29.6.1 Misapprehension

Misunderstanding a question means that you invariably answer poorly and bag low marks for it. Diagnose why you do this.

Do you *consistently* misunderstand questions? If so, *why?* Is it due to an undue haste to write answers, brought on by nervousness? Or have you dallied for too long on one question, and squandered on it the time meant for other questions, leading to panic? Are you oppressed by the pressure of time?

> Learn to read a question unhurriedly.

Let the question speak to you. Listen to it. Be silent and attentive. Cut out all mental chatter. Do not interrupt what the question is saying to you by putting together a preliminary answer in your mind while reading it.

You will have all the time later on to respond to the question. Read **??** and Chapters 36 and 41 and Section 36.8 carefully and follow the suggestions there. In a nutshell: read carefully, understand correctly, and answer precisely.

29.6.2 Incomplete answers

Related to misunderstanding a question is answering it incompletely. You might think that you have answered a question fully when in fact you have carelessly left out some relevant details. It becomes perplexing when you get part of the marks instead of the expected full marks for that question.

I have given examples of unintended partial answers in Chapter 45: see for instance Section 45.8.1. Learn from these cases to be more careful when answering questions. Understand the question correctly and answer it completely and precisely.

29.6.3 Post-examination analysis

After an examination, compare your answers with the model answers. Identify what omitted facts or comments cost you marks. Take care not to repeat the same type of error in future. If you still cannot divine why you lost marks, seek a meeting with your instructor to find out why. Conduct your own "post-mortems" after examinations until you become expert at avoiding whatever costs you marks.

29.6.4 Not attempting the requisite number of questions

★

Attempt enough questions and all parts even if only partially.

Even a poor answer is better than none. Partial answers qualify for partial marks. An answer in point form is better than a blank line. You will get some marks and those marks will add up to a better grade in the end. Don't be like the dinner guest who preferred not show up than be late. The rules of an examination are different. Play by them.

29.6.5 Practicals and orals

Praxis-based subjects such as music and the performing arts, as well as the natural sciences, medicine, dentistry, etc., entail practical examinations as well as theoretical ones. You might also have oral or *viva voce* examinations in almost any subject.

Are you a student whose theoretical knowledge is sound, but who becomes nervous when confronted with a practical or oral test? Do your fingers tremble when you have to set up a practical experiment, or play some music, or examine a patient as part of an examination? Are you put off by having to face an examiner in a *viva voce* examination?

If so, you need to work the panic and nervousness out of your system. Becoming fearless takes time and practice. Be patient. Be persistent.

EXAMPLE: EXAMPLE

Suppose you are told to stand near a whiteboard, pen in hand, and asked to explain something to a group of examiners seated at a table. Your knees are weak and wobbly. Your mouth is dry. Words form in your head but not in your mouth. What should you do?

Take one or two deep breaths to calm yourself. Then think that you are a king and that the seated professors are your subjects. Have utter confidence in your knowledge and launch forth on your answer.

If the “king-subject” fantasy does not work for you, imagine something more outrageous: that you are an invincible lion and that the seated examiners are mice. Or use another analogy.

Visualize yourself in a position of utter dominance and mastery and start answering. You will find the results nothing short of miraculous.

One cautionary point though. Practise this mental attitude only during your oral examination. *Do not make hubris your habit.* In later life, even if you are the acknowledged world leader in your field of knowledge, let humility always be your ornament. ★

29.6.6 Time and place

If you know your material but get poor grades at examinations, analyze the pattern of examinations at which you did not do well. Do time and place have anything to do with it?

Do you do badly only at examinations held late in the day to allow working students to attend? Do *you* do badly at such examinations because you are tired after a hard day's work? Or is it afternoon examinations that bring out the worst in you, drowsy as you are after eating lunch? Did you have two examinations held back to back that left you too exhausted to do well in the second? Or was it only examinations held after you were ill and a bit under the weather? By analyzing the patterns, you can put your finger on the cause, its degree of seriousness, and its remedy.

Is a particular examination venue associated with poorer grades than other venues? Did it have to do with toilets or temperatures? Are you superstitious? Do you feel less comfortable writing an examination at one venue than at others? Is your angst rooted in something physical or is it purely mental, based perhaps on past experiences?

Time and place of examination should *never* influence your grades. You cannot choose either and should be immune to variations in both. Refer to Chapter 5 to overcome mental obstacles like this.

29.6.7 Adequate practice

> Answer at least one relevant past paper in each subject under timed examination conditions.

Do not wait for your instructor to get you going. Do it yourself. Lay your hands on a past paper, sit sequestered, and write the exam while timing yourself. After answering, get hold of a model answer script and grade yourself. Know where you stand, and how soundly, *before* the examination. If the results of your self-assessment are dismal, identify your weaknesses and practise until you eliminate them. See Chapter 43 for details.

29.6.8 Formulate your own examination questions

Pretend that you are your own examiner. Frame questions. Then answer them under examination conditions. Finally correct your answers and grade yourself. You will see clearly where you need to improve.

Do this frequently in each subject you wish to improve, as you study it. Keep these questions and answers for revision prior to the examination. The more you practise, the better you will become. Practise until examinations become a routine non-event in your mind.

29.7 Time pressures

The pressure of time pursues us all throughout our lives. Sometimes, we are acutely aware of it, as during examinations; at other times, not as much.

Overcome the panic brought on by the pressure of time during examinations by practising on enough past papers, by disciplining your allocation of time to each question according to the marks it is worth, by attempting the requisite number of questions, and by attempting all parts of answered questions, even if only briefly.

In the succeeding sections, I consider time pressures outside the scenario of examinations, and suggest how you should cope with them.

29.7.1 Procrastination and missed deadlines

Do you procrastinate? If so, work at eradicating this undesirable habit. See especially Chapters 2 and 21. Are you continually assessed in any subject? Do you miss deadlines for submission of term papers and assignments? If so, do you lose marks on that count? How serious a problem is it for you? Acknowledge and address missed deadlines and lost marks if they become habitual.

★

Begin early. Work steadily. Finish unhurriedly. Fulfil deadlines. ☻

29.7.2 Competing demands for time

You are a whole person and need time for family, rest, relaxation, recreation, chores like cooking, washing, etc., apart from academic tasks. Apportion your time sensibly.

Do a *time audit* of how you spend your time during a typical weekday, a typical weekend, and a typical week. Refer to Chapter 21, and especially Section 21.19 for examples and guidelines. If you find hobbies or recreation taking up large chunks of time, re-schedule your activities, according more time and a higher priority to your studies.

29.7.3 Workload

Even if you topped your school earlier, on entering an elite school or university, you might find yourself in competition with a student cohort that is so good that just keeping up takes all you can give it.[2]

Apart from the excellence of your fellow students, your elite institution itself might adopt the Spartan technique of winnowing out the weaklings by imposing a punishing—perhaps impossible—workload. Unless you are highly motivated, what started out as a much anticipated, joyous intellectual adventure could morph into a nightmarish Sisyphean imposition from which there is no escape. You simply have no time to finish all that you are required to do.

If you are in this situation, brace yourself. Do not buckle. Do not grieve. Do not pity yourself. Do not complain. Do not neglect your health or sleep. Simply and manfully confront the task. Affirm that you *will* accomplish. Take steps to succeed. As long as you do not give in you *will* prevail. Remember that you are in an elite institution in the first place because you *already* have excelled. What you have done before, you can do again.

Prioritize your work. Identify what is core or compulsory work. Distinguish it from the optional or elective material. Fulfil the core tasks first. Then attend to the optional work. Strictly enforce guillotines of time. Plan how much time you can give to each task and give it that much and no more. After you have dealt with the lot, give any spare time to whatever you think needs doing better.

Once you have got on top of your work, you will find the going so much easier. Without relenting, keep on top until you graduate.

29.8 In a nutshell

Grade repair from Bs to As is possible and highly likely if you identify the causes of your B grades and take corrective action. There are basically three major areas that you must address: knowledge, examination performance, and time pressures.

If all three have been tackled, and you are still lugging some B grades, you need to take a deep breath, put some metaphorical distance between yourself and your studies, and take a careful objective look at why it is still happening. Once you have found out *why* ask *what* to home in on remedial action.

[2] The movie *The Paper Chase* and the related TV series illustrates this in the case of law students who enter a top-notch US law school and find that they need to devise means like group study to cope with its demands.

Learning to diagnose and treat your own academic problems this way, on your own steam, is the surest way to prepare yourself for lifelong learning.

SUMMARY: MOVING FROM BS TO AS

- Finding out why you get Bs rather than As is fundamental to improving your grades. Use the W^7 framework for this.
- Analyze the facts surrounding your examination grades and put it into a table or spreadsheet to discover patterns that might explain why, when, or where you do badly.
- Fix an appointment with an instructor and discuss a recent term paper or test—where you scored lower than expected—to discover causes and design cures.
- Apply the study methods given elsewhere in this book to strengthen your grasp of the material in subjects where your ready-knowledge is weak.
- Flush subject-phobias out of your system.
- Read carefully. Understand correctly. Answer precisely. Apply these dicta to routine as well as examination questions.
- Overcome panic at practicals and orals by visualizing and adopting a supremely knowledgeable dominant standpoint.
- Eradicate procrastination. Begin early. Work steadily. Finish unhurriedly.
- Manage your time sensibly. Accord and enforce priorities to all tasks.
- Move from Bs to As and keep on getting straight As.

Part E

Reading

34

Poetry

Synopsis

Poetry is a significant part of human thought and heritage. Your reading in any language is complete only when it encompasses the poetic masterworks of that language.

Cultivate an appreciation for poetry as it nurtures memory and creativity. Reading a poem aloud can be delightfully soothing because of its inherent rhythm and rhyme. Let poetry work its magic on you. Read the great poets who live on in their poems. Imbibe their inspiration. Emulate their mastery of language. Improve your vocabulary. Perfect the spoken and written word.

Above all else, learn to enjoy poetry, and thereby to appreciate beauty. You will be enriched and educated, and become a more cultured, refined, and sensitive human being.

> A poet is, before anything else, a person who is passionately in love with language.
>
> *The Complete Works of W H Auden: Prose, Volume II: 1939–1948*
> W H Auden, Anglo-American poet, (1907–1973)

34.1 Why Poetry?

Why is there a chapter on Poetry in a book devoted to helping you succeed academically? The reasons are several and I will list just a few here.

The first and most glib reason is that *I love poetry.* Call it an idiosyncrasy if you will. This chapter is accordingly more personal and opinionated than the rest of the book.

Secondly, most of us learn a new language using rhyme and song—just recall the nursery rhymes of your childhood—and poetry, at least in its more ancient forms, is rooted in metre and often in rhyme.

Thirdly, much of the enduring literature of the world—like the *Rāmāyaṇa,* the *Mahābhārata,* the *Iliad,* the *Odyssey,* the *Rubaiyat of Omar Khayyam,* Goethe's *Faust,* and Dante's *Divine Comedy,* to name but a few—is in poetic form.

Fourth, given its innate rhyme and cadence, melody and beat, poetry set to tune as song, is *memorable,* as evidenced by religious hymns, martial songs, national anthems, folk songs, etc.

Moreover, when you are trying to master a new language, your reading is never really complete until you have recited and relished poetry in that language.

Lastly, poetry is profoundly evocative and often exposes beauty that might otherwise have lain unseen. And appreciation of beauty is part of what makes you an educated person.

34.2 "The best words in their best order"

The English poet Samuel Taylor Coleridge, famed author of the fabulous poem *Kubla Khan,* called poetry "the best words in their best order" [1]. When you wish to enrich your feel for a language, read its best poetry. Your command of words and phrases, your feel for rhyme and diction, your sense of what is right and what is wrong when using the language, will all improve.

Whether or not you become inspired and confident enough to pen your own lines of rhyme, I very much encourage you to *recite* the great poems you can lay your hands on. Your command of the language will improve immeasurably. Your confidence in using it will grow in leaps and bounds. You will also slowly learn to appreciate, in poetry, the beauty of some of the most abstract structures man has built with his intelligence.[1] Round off your reading every day with a little poetry. It will be like a truly satisfying dessert at the end of a meal!

34.3 Poetic distinctions

Poetry is language at its finest. There is an economy of expression, a richness of feeling, a profundity of thought, a musical lilt or rhyme, a measured cadence when read aloud, that endears poetry to all lovers of language. A simple phrase from Shakespeare like "Sleep that knits up the ravell'd sleeve of care"[2] is frugal, yet richly expressive and evocative.

★

If you have not recited poetry before, I heartily recommend that you try it.

Poetry enriches thought, feeling, and expression. Reciting poetry is exhilarating and educational. Not only would you be improving your vocabulary, you will be learning how to write better from the masters of the language in which the poem is written. Indeed, you might even learn some nonsense words!

34.4 Lewis Carroll's *Jabberwocky:* evocative nonsense

The English clergyman, mathematician, author, and poet, Lewis Carroll, wrote, as part of his book *Through the Looking-Glass, and What Alice Found There,* a poem called *Jabberwocky* [2, 3] that contains *nonsense words in English.*

For some strange reason, it was a poem that we had to learn as secondary school students of English Literature in Singapore. Our teacher, a diminutive New Zealander called Ms Elizabeth Phillipp, read the poem

[1] In this sense, I think Poetry, Music, and Mathematics represent the *greatest abstract* achievements of humankind.

[2] Even though this quotation is from the play *Macbeth*, II:2:48, I consider it consummately poetic, in the Coleridgean sense.

aloud in class. When she came to the penultimate verse, given below, she glided across the front of the classroom with arms outstretched like a joyous bird in flight, exultantly reciting the verse with such gusto, that you felt that the nonsense words *really had meaning*—and that meaning was simply *joy!*

> "And has thou slain the Jabberwock?
> Come to my arms, my beamish boy!
> O frabjous day! Callooh! Callay!"
> He chortled in his joy.

Even today, when I say to myself "O frabjous day! Callooh! Callay!" I cannot help feeling a throb of joy in my heart. Such is the evocative power of poetry, and why it has held sway over humankind since dim beginnings in history around nightly, communal campfires.

Interestingly, nonsense or not, *Jabberwocky* has been translated into other languages, and if you have not read it yet, do read the version with annotations by Martin Gardner [3, pp 191–197]: you will gain so much more from his book.

34.5 Alliteration, rhyme, and rhythm in Poe's *The Raven*

I am an avid fan of the works of the tragic American littérateur, Edgar Allan Poe, who wrote some of the finest detective short stories, and also penned some very memorable poems. Of these, I once heard his poem *The Raven* on a chilly Hallowe'en night in Toronto, Canada, many years ago, sonorously read over the radio by broadcaster Allan McFee, on a Canadian Broadcasting Corporation programme called *Eclectic Circus*.

It left an indelible impression on me: for the first time in my life, I experienced the entrancing power of alliteration, rhyme, and rhythm in poetry. To give you a feel of what it was like, I quote below two stanzas from the poem:

> Open here I flung the shutter, when, with many a flirt and flutter,
> In there stepped a stately Raven of the saintly days of yore.
> Not the least obeisance made he; not a minute stopped or stayed he;
> But, with mien of lord or lady, perched above my chamber door—
> Perched upon a bust of Pallas just above my chamber door—

Perched, and sat, and nothing more.

Then this ebony bird beguiling my sad fancy into smiling,
By the grave and stern decorum of the countenance it wore,
"Though thy crest be shorn and shaven, thou," I said, "art sure no craven,
Ghastly grim and ancient Raven wandering from the Nightly shore—
Tell me what thy lordly name is on the Night's Plutonian shore!"
Quoth the Raven, "Nevermore."

It is interesting that the poem tells a story that is easily comprehended; the poesy does not intrude to obscure the narrative. Yet, from start to finish, you will be drawn in by the alliteration and the rhythmic trochaic octameter [4] that lulls and mesmerises. Poetry has this ability to engage and enthral both reason and emotion and thus contribute to bicameral brain development.[3]

If you have not already read it, do read Poe's *The Raven* in full. Many websites have analyses that will help you understand what the rhyme and rhythm are. Better still, hear it read by one who has a rich and resonant voice, who relishes poetry, who reads with feeling, and who has mastered the English tongue. You will then be drawn into the captivating web of poetry—of "the best words in their best order."

34.6 A selection of my favourite verses

Most readers of poetry have their favourite poems, and I am no exception.[4] I will now quote verses from some of my favourite poems and say a few words about each.

34.6.1 Wordsworth's *Daffodils*

William Wordsworth was one of a group of poets, including Coleridge, who lived in the English Lake District and who wrote romantic poetry. I particularly like and remember his poem *Daffodils*, sometimes titled, *I Wandered Lonely as a Cloud,* from its first line. Here are its first and last verses:

[3] This is just my hunch. I must confess that I have no hard evidence whatsoever for this assertion!

[4] I urge you, dear reader, to make up your own list of favourite poems, with the names of the poets, and the titles and first lines of the poems.

I wandered lonely as a cloud
That floats on high o'er vales and hills,
When all at once I saw a crowd,
A host, of golden daffodils;
Beside the lake, beneath the trees,
Fluttering and dancing in the breeze.

...

For oft, when on my couch I lie
In vacant or in pensive mood,
They flash upon that inward eye
Which is the bliss of solitude;
And then my heart with pleasure fills,
And dances with the daffodils.

Extreme joy or extreme sorrow seem to be the most frequent stimuli for poetry. In this case, as in *Jabberwocky,* it is joy. Note the craftsmanship with which Wordsworth has described what might, to you and me, have been a casual and nondescript walk in the outdoors. By freezing the poetic word-picture bequeathed him by the Muse, he has immortalized a specific scene for all time.

34.6.2 Longfellow's *A Psalm of Life*

Henry Wadsworth Longfellow was an American romantic poet and Harvard professor. I particularly like his poem, *A Psalm of Life,* which is a philosophical reflection on life itself. The poet's advice, given in the selection of verses below, seems almost tailor-made for the earnest student, and hence is doubly appropriate for inclusion in this chapter:

Art is long, and Time is fleeting,
    And our hearts, though stout and brave,
Still, like muffled drums, are beating
    Funeral marches to the grave.

In the world's broad field of battle,
    In the bivouac of Life,
Be not like dumb, driven cattle!
    Be a hero in the strife!

Trust no Future, howe'er pleasant!
    Let the dead Past bury its dead!
Act,—act in the living Present!
    Heart within, and God o'erhead!

Lives of great men all remind us
    We can make our lives sublime,
And, departing, leave behind us
    Footprints on the sands of time;

Footprints, that perhaps another,
    Sailing o'er life's solemn main,
A forlorn and shipwrecked brother,
    Seeing, shall take heart again.

Let us, then, be up and doing,
    With a heart for any fate;
Still achieving, still pursuing,
    Learn to labor and to wait.

> The sincere student who puts into practice what is advised in this poem cannot but succeed.

Indeed, it is possible that such an assiduous student would one day accomplish so much in this world as to leave behind "footprints on the sands of time". It requires application, courage, perseverance, and patience.

If ever you are discouraged or melancholic or depressed because of some event that has taken the wind out of your sails, read Longfellow's *A Psalm of Life* and redeem your courage, enthusiasm, focus, and energy.

34.6.3 Coleridge's *Kubla Khan*

I started this chapter with a quote from Coleridge and alluded to his poem *Kubla Khan*. Here are a few lines from the start and middle of that poem to give you an idea of his deft turn of phrase and choice use of words that leaves echoes in the mind:

In Xanadu did Kubla Khan

A stately pleasure-dome decree:
Where Alph, the sacred river, ran
Through caverns measureless to man
    Down to a sunless sea.

…

The shadow of the dome of pleasure
    Floated midway on the waves;
Where was heard the mingled measure
    From the fountain and the caves.
It was a miracle of rare device,
A sunny pleasure-dome with caves of ice!

34.6.4 Gray's *Elegy Written in a Country Churchyard*

I could not conclude this section on my favourite poems without an excerpt from Thomas Gray's *Elegy Written in a Country Churchyard*. This poem has attracted attention since its first publication in 1751, and its merits and defects have been relentlessly analyzed, although successive generations of scholars have been unable to decide conclusively what these are. Despite the absence of scholarly consensus, the poem has uniformly enjoyed immense popularity throughout its history. It is often counted among the best-loved or most-quoted poems in the English language.

So, what is it about Gray's *Elegy* that commands such enduring mass appeal? I will let you savour a few of my favourite verses and let *you* decide what makes it special for you:

The curfew tolls the knell of parting day,
    The lowing herd wind slowly o'er the lea,
The ploughman homeward plods his weary way,
    And leaves the world to darkness and to me.

Now fades the glimmering landscape on the sight,
    And all the air a solemn stillness holds,
Save where the beetle wheels his droning flight,
    And drowsy tinklings lull the distant folds:

…

Full many a gem of purest ray serene,
 The dark unfathom'd caves of ocean bear:
Full many a flower is born to blush unseen,
 And waste its sweetness on the desert air.

…

Far from the madding crowd's ignoble strife,
 Their sober wishes never learn'd to stray;
Along the cool sequester'd vale of life
 They kept the noiseless tenour of their way.

These few disjointed verses do not do justice to the whole poem, which I urge you to read aloud and in full when you find time. Despite being an elegy with its focus on death, the poem exercises a *soothing magic* on me each time I read it.

Did you feel its magic? It is not as important to analyze whether it is alliteration, or rhyme, or rhythm, or lyricism, or felicity of expression, or thematic integrity, or some other literary device, that contributes to this magic, as it is to *experience* the magic.

In my humble opinion, a poem must exercise its peculiar magic on you. When it does, you resonate with the poem and the poet.

★

Reciting a poem aloud often releases its magic, if at first you cannot feel it.
To experience this magic is the most satisfying feeling arising from reciting poetry.
Try it. Absorb it. Imbibe it. Feel it. Live it.

34.7 Shakespeare's *Sonnet 18*

The inimitable English playwright, William Shakespeare, although better known for his plays, also wrote poems. The former are considered prose, although some of them rise effortlessly to poetic heights. Among his poems, his sonnets—poems of fourteen lines—are well known, and *Sonnet 18* is considered to be Shakespeare's finest. It deals with beauty, Nature, change, death, poetry, and immortality within the brief compass of its fourteen lines:

Shall I compare thee to a summer's day?
    Thou art more lovely and more temperate:
Rough winds do shake the darling buds of May,[5]
    And summer's lease hath all too short a date:

Sometime too hot the eye of heaven shines,
    And often is his gold complexion dimm'd;
And every fair from fair sometime declines,
    By chance or nature's changing course untrimm'd;

But thy eternal summer shall not fade
    Nor lose possession of that fair thou owest;
Nor shall Death brag thou wander'st in his shade,
    When in eternal lines to time thou growest:

So long as men can breathe or eyes can see,
So long lives this, and this gives life to thee.

To understand and appreciate this sonnet better, refer to the online paraphrase by Amanda Mabillard [8]. It is interesting that the pronoun "thee" in this poem is cryptic and the person to whom it refers remains unnamed. Whom or what exactly Shakespeare was immortalizing in these lines therefore remains enigmatic. Nevertheless, the very fact that you are reading these lines *now* is evidence of the truth of the last two lines of the poem!

34.8 Humour in poetry: Edward Lear and Ogden Nash

Even though poetry often treats of serious and sombre subjects like life and death, there is still room in it for humour, laughter, and comic relief. Such poetry is sometimes called *light verse*. Among the humorous poets in English, I have enjoyed the works of Edward Lear and Ogden Nash and would like to share a sampling of their work with you.

Edward Lear was an English poet and artist. His illustrations of his own poems adds another dimension of charm and wit to them. Like Lewis

[5]Shakespeare had a great knack for coining unforgettable phrases that endure far beyond their time and context: *The Darling Buds of May* was the title of a popular Yorkshire Television drama series of the same name based on the eponymous novel by H E Bates [5, 6] who took it from Shakespeare. In Shakespeare's day this line was spelt "Rough windes do shake the darling buds of Maie," [7, p 60].

Carroll, he too introduced nonsense words into the English language, some of which have now been lexically legitimized, i.e., inducted into the language. Lear was a master of the *limerick* and I give below one of them from his *Complete Nonsense* [9, p 73] which has a hilarious accompanying illustration:

There was an Old Man, on whose nose,
Most birds of the air could repose;
  But they all flew away,
  At the closing of day,
Which relieved that Old Man and his nose.

Ogden Nash was an American comic poet and author. He was a master of the funny pun and wrote a lot of punny fun! I love his short poem entitled *The Llama*:

The one-l lama,
He's a priest.
The two-l llama,
He's a beast.
And I will bet
A silk pajama
There isn't any
Three-l lllama.[6]

34.9 One last poem

I will conclude this selection of poems with one I have written. While it is no masterpiece, I thought it might encourage you to write your own poem if I showed you one of my own. In that spirit, here it is:

Balance

What purpose in an infant's chuckle?
In spring's first rose or winter's 'flake?
Does Mother Earth convulse and quake,
In mirth or wrath to buckle and break?

Pray tell me now, "Is life in vain?"

[6]Apparently, Ogden Nash appended this footnote to this poem: "The author's attention has been called to a type of conflagration known as a three-alarmer. Pooh." [10].

Each newborn babe someday must die.
Each joy must find its doleful mate,
That life and death e'er alternate.

Each rising sun and setting moon,
To cyclic time do softly croon,
Like whirling Earth, life too revolves,
Seeking rest in ceaseless quest.

Newton's Third has said it all:
"Balance is the Beat of Life"
If Purpose sought is Purpose found,
Then Balance be her name profound.

34.10 Anthologies of poems

Poetry is usually published as an *anthology*[7]—or collection of poems—either from a single author or from different authors. I have been particularly delighted with two such anthologies which have now attained vintage status.

The first is Francis Turner Palgrave's *The Golden Treasury of the Best Songs and Lyrical Poems in the English Language* [11]. It is a pleasure to dip into even if all you can manage is to turn to a page at random and read what is there. You will befriend many poems there and will return repeatedly to the book to renew old friendships and make new ones.

The second anthology is Morris Edmund Speare's *A Pocket Book of Verse* [12] which is another eclectic selection. It offers a wide choice of poems, and dipping into it is again a most pleasurable reading experience.

Since both my favourite anthologies are dated editions, you might not be able to find them still in print or at your local library or bookstore. In that case, there is a slim, inexpensive, and readily available third anthology that is more brief, but which still contains most of the poems mentioned here. It is Philip Smith's compilation entitled *100 Best Loved Poems* [13].

Visit your school, university, or community library and browse the shelves on poetry. See if any book there appeals to your taste and fancy.

[7]Literally, a collection of flowers.

If so, check the book out and read it. Does it help you develop a taste for poetry?

If you do not have convenient access to libraries, you might wish to surf the Web searching for websites dedicated to poetry. You will find sites devoted to popularizing poetry, sites celebrating specific poets, sites with an academic bent for poetry, sites on well known quotations from poems, and so on. As always, use your judgement when assessing websites for quality, reliability, and accuracy before you rely on their content.

34.11 The benefits of reading and reciting poetry

I hope this roundup of poems which I have enjoyed has whetted your appetite for poetry. If you are new to poetry, you should read the book by Edward Hirsch entitled *How to Read a Poem: And Fall in Love with Poetry*.

When you cultivate a love for poetry, your vocabulary will be enlarged spontaneously. Your ability to read and to speak English will likewise improve. You will understand when to pause and when to stress when you speak. This skill will prove immensely useful when making oral presentations, whether as part of an assignment, or at a PhD oral defence examination, or when delivering a paper at a conference.

Reciting poetry and enjoying it will make you a wordsmith who is both relaxed and exacting in the use of a language. You will become a master of the spoken and the written word.

Poems exercise and strengthen memory. Their innate expressiveness and rhythm makes poems memorable and memorizable. As you repeatedly recite your favourite poems, you will find their words and phrases reverberating in your mind at unexpected moments. Or you might find that as you walk, your stride and stance keep beat with a poem's metre. Indeed, once you have internalized a poem, you will at times find recollections of it flooding your mind unbidden from the secret vaults of memory. You will be pleasantly surprised to one day learn that you have memorized the poem almost unconsciously.

Reciting poetry helps exercise and strengthen your memory. And an active and powerful memory helps you succeed academically.

Reading poetry can also be your ideal pick-me-up when you feel down and out. By immersing yourself in a cheery or cheerful mindscape conjured up by the majesty and magic of a great poem, you might instantly snap out of a gloomy mood and find yourself refreshed and renewed, ready for more work.

In our result-driven society, most things are unfortunately measured by their monetary value. So, there will be naysayers who will ask, “Will poetry teach me to make more money?”, or “Will reading poetry help me get a better-paying job?”. Do not be deterred by such questions. There is more to life than just making money. Utilitarian cynicism never gave birth to genius or greatness. Do not stymie creativity, motivation, and enjoyment by putting money before everything else. Be true to knowledge and to art. You will then find money effortlessly following both.

Poetry induces creativity. When you recite a poem, you are resonating with the creativity of the poet when he or she wrote the poem. When you immerse yourself in the atmosphere and imagery of a poem, you are likewise accessing that secret domain of creative genius that bequeathed the poem. If you write your own poems, you will experience at first hand what it feels like to have the creative, poetic juices flowing.

Creativity cannot be taught. It must be captured through resonance with Creativity itself. And Poetry gives you a magnificent gateway to Creativity. Treasure it. Use it.

Reciting poetry will help you become a better human being. Above all else, by reciting poetry you would be enjoying yourself! And ultimately, joy is what we are all seeking.

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SUMMARY: POETRY

- Poetry represents the finest expression in any language.
- Some poems are profoundly evocative.
- Others are almost hypnotic given their alliteration, rhythm, and rhyme.
- Poetry has the ability to soothe and lull, to inspire and uplift, to encourage and refresh.
- Develop a taste for poetry. Read poets who appeal to you and address your needs. Develop a kinship with poems that speak to you.
- Read poetry aloud and allow it to work its magic on you.
- By reading and reciting poems, you will effortlessly enhance your vocabulary, and master the spoken and written word.
- Poems exercise and strengthen memory. A memory that is active and powerful will help you succeed academically in all your subjects.
- Poetry puts you in touch with the creativity of the poet and it can thereby stimulate your own creativity.
- Poetry helps appreciation of beauty. You will become a better educated, more cultured, more refined, and more sensitive human being as a result.

Part F

Writing

36

Essay Writing

Synopsis

Essay writing is both an art and a craft. First read the question carefully and understand it correctly. Then, and only then, attempt to answer it.

Every essay has three parts: the *Introduction*, the *Body*, and the *Conclusion*.

You should write an essay in four stages: *jot*, *sort*, *write*, and *revise*.

First jot every idea that occurs to you in response to the essay question. Do not censor anything. Do not sort ideas. Do not dismiss an idea as being silly. Simply write down everything.

Then sort your ideas. Cluster them under several large headings. Discard ideas that do not fit in with the theme of the essay. Merge repetitive ideas into a single idea. Aggregate and link to achieve coherence and continuity. Develop a solid outline.

Next, write your essay using your outline. Pay special attention to the introduction and the conclusion. Practise writing the fair copy at once, without a rough draft. Infuse fluency, integrity, substance, and elegance into your essay.

Revise. Check that there are no missing words or sentences. Correct errors of grammar, spelling, and usage. Polish up the style. Unify the essay. Make sense. Stay on message. Pass up your best effort.

36.1 The craft of essay writing

Essay writing is a required skill for long-answer and essay questions in examinations. It is also a core competence in many professions and vocations. Economists, lawyers, journalists, bureaucrats, policy-analysts, researchers, academics, politicians, and many others put into practice the skills of essay writing in their daily working lives.

Essay writing demands creative thinking tempered by disciplined expression. An essay serves to communicate, to educate, to entertain, to enshrine the writer's opinion, and above all, to persuade the reader—at the very least to think, and at the very best to adopt the writer's viewpoint.

An essay is a refined instrument of intellectual persuasion. It demonstrates that the pen is indeed often mightier than the sword.[1] A good essay is the hallmark of a literate and cultured upbringing. Essay writing is a craft well worth learning and honing for life.

We have already covered the fundamentals of good writing in Chapter 35. Writing skills are acquired patiently over time by careful reading and even more careful writing. In this chapter, we focus on the skill of writing good essays in some detail. I believe that the fine craft of essay writing can be learned systematically by the eager and intelligent student.

In this age of instant and impatient communication, where learned discourse has given way to limited length SMS messages and "tweets", essay writing is sadly a vanishing craft. It is my hope that through this chapter, the skill of essay writing, may in some small measure, live on a little longer through future generations of students.

36.2 Essay writing conditions

The conditions under which an essay is written determine what you can and cannot do in the process. For example, under examination conditions, you must write your essay by hand, without the aid of reference material. When you write a routine non-examination essay you are usually permitted to type your work on a PC and also to consult references. How you write your essay would be determined by which of these two very different sets of conditions you have to work under. This issue is

[1] See the model essay in Chapter 37.

dealt with later in Sections 36.14 and 36.15. I will assume the more stringent examination conditions here.

36.3 General knowledge

You need ideas to write an essay. To get ideas, you obviously need to listen and to read. You must also understand what you have read. Then you should integrate what you have read with your previous knowledge. Thus, when you are called upon to discuss or debate a topic, you will have a ready fund of knowledge that you can draw upon to throw light on the discussion.

If you are bereft of ideas, you will balk when asked to write an essay. Hence the importance of having *sound general knowledge* built up regularly through reading, watching educational documentaries, debates and discussions, surfing the Web, and other means.

36.4 Consult references

In addition to general knowledge, you need specialized or expert knowledge for each essay you write. You must be conversant with the relevant facts. For this, you should consult references.[2] Each exercise in essay writing will then contribute to your overall store of ready knowledge.

You should always research your topic thoroughly and get alternative viewpoints. Look up unfamiliar words and usage in a dictionary. Master necessary jargon. Consult a thesaurus to substitute one word for another so that you do not sound overly repetitive.[3] Your general knowledge will improve as will your intellectual range, vocabulary, and standard of expression.

★

Use online and printed reference materials liberally at all stages of essay writing and cite them appropriately.

[2] Obviously, you cannot do this in an examination!

[3] This is called *elegant variation*. I dispute H W Fowler's contention [1, p 243] that elegant variation is a stylistic trap into which "second rate writers" and "young writers" fall. Apply it until you get good at it.

36.5 Do not plagiarize

Plagiarism is a scourge encouraged by the ready availability of information and knowledge via the World Wide Web. I have warned against plagiarism in Chapter 2 and do so again here and in Chapters 17, 35 and 48 to 50.

We must all fight plagiarism. It might seem so very tempting to cut something from some Web page and paste it into your essay and pass it off as your own work. *Do not do so.* Use the Web only as a source of facts.[4] If you need to quote someone else's work or words, do so with proper attribution.

★

Plagiarism is academic cancer.
Never pass off another's work as your own.

36.6 Construct your essay

Like speaking before an audience, writing an essay can and does inspire fear. Many are daunted by the prospect of putting their thoughts down on paper. It is true that an essay does not pop out of your head ready-made, although you might wish that it did! Rather, you have to *construct* or *compose* your essay with care and patience. There are time-tested methods for accomplishing this deliberate act. We first look at the structure of an essay before discussing methods of constructing one.

36.7 The three parts of an essay

Like a musical composition, an essay is a *composition* in a language, that meets certain requirements. Composing a piece of music and writing an essay are both creative acts, and both have to obey certain rules of composition. Like a musical piece, an essay usually deals with a *single subject* or *theme*. It has three parts: the introduction, the body, and the conclusion.

36.7.1 Introduction

The *Introduction* eases the reader into the subject. If there were no introduction, and you delved straight into the depths of your subject, it

[4]Of course, after confirming the reliability of your online sources.

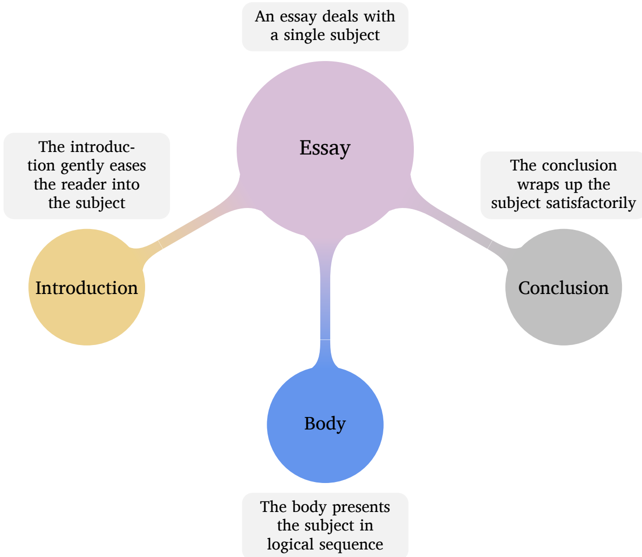

FIGURE 36.1: The three parts of an essay.

would be like plunging your reader into a cold and deep pool of water without warning. Your introduction is intended to gradually and gently acclimatize your reader, so that encountering the subject is a pleasant and comfortable experience rather than a harsh and intimidating one.

> You gently induct the reader into the subject matter of your essay through the *Introduction*.

36.7.2 Body

The *Body* of the essay deals with the subject matter proper. Here you carefully weave your thoughts and arguments together so that a coherent and convincing picture emerges rather than a perplexing muddle. Use logic to sequence your ideas. State assumptions first rather than last. Allow later ideas to rest upon the foundation of earlier ideas. Above all, be persuasive. Imagine that you are building a wall, brick by brick, using ideas and words, to serve some definite purpose

> The substance of your essay resides in the *Body*.

36.7.3 Conclusion

The *Conclusion* is a bit like dessert at the end of a meal. It provides finality and satisfaction. When we speak, we usually lower our pitch at the end of a sentence to indicate a conclusion. If we raised the pitch instead, it would be interpreted as a question. Your conclusion should "lower the pitch" of your writing, rather than raise it. It should neatly and convincingly wrap up your ideas, together with any opinion you have one way or the other.

The *Conclusion* is not the place to raise new ideas or start a fresh dialectic or polemic discussion. It should partake of the concluding qualities of a conversation that is about to end, or a piece of music that reaches its finale. It should give your reader the satisfaction of time well spent reading what you have written.

> The *Conclusion* is the finale of your essay.

36.8 Reading, understanding, and answering

> Read carefully.
> Understand correctly.
> Answer precisely.

Reading is the zeroth step in answering any question in any academic discipline. The essay question is no exception. Time spent reading the question carefully is time well spent. Fight the temptation to hurl yourself hurriedly into writing your essay, especially during examinations when time is scarce.

Reading and understanding the question is a systematic process. It requires familiarity with certain *task words* which encode what you are expected to accomplish in your answer. These task words are identified and defined briefly in Table 36.1, which is repeated in Chapter 45 as Table 45.1.

TASK WORD	MEANING
STATE	Write down
ENUMERATE	List one by one
OUTLINE	Present the main points, in order, without the detail
DESCRIBE	Paint a word picture
EXPLAIN	Clearly give the meaning and/or reasons for something using jargon-free language
ACCOUNT FOR	Give the causative reasons for something or some situation
DISCUSS	Look at both sides of an issue
COMPARE AND CONTRAST	Show the similarities and differences
ANALYZE	Break the subject down to its causes and constituent parts

TABLE 36.1: Commonly used *task words* or *instruction words* and their meanings.

Reading a question carefully is like participating politely in a conversation. When the other person is speaking, you neither interrupt nor interject. You simply listen. Reading a question carefully means listening to what the question has to tell you. It is pure listening. Your turn to respond will come later. For now listen to the question with all the attention and concentration you can muster. Remain silent yourself during this process. This prescription is so important that I repeat it in Sections 29.6.1 and 41.5.1.

36.9 The four stages of essay writing

We are now ready to look at essay writing proper. Writing an essay is a four-stage exercise—*jot*, *sort*, *write*, *revise*—as shown in Algorithm 36.2 and illustrated in Figure 36.2.

Until you are practised in the art of essay writing, you might find it a chore to remember the names of the four stages. I want to digress for a moment here with an example on how to remember them using mnemonics.

Read and Understand the Question

1. Read the question carefully.
2. Look out for task words, underline them, and pay special attention to them.
3. Understand the question correctly.
4. Test your understanding by re-reading the question.
5. Commence answering only when you are satisfied that you know exactly what is required of your answer.

ALGORITHM 36.1: Reading and understanding the question.

The Four Stages of Writing and Essay

1. Jot
2. Sort
3. Write
4. Revise

ALGORITHM 36.2: The four stages of writing an essay.

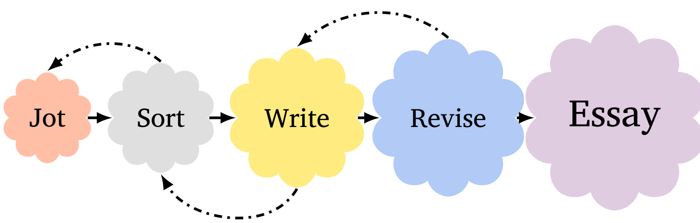

FIGURE 36.2: The four stages of writing an essay. Writing is not a linear process but rather a cyclic one. The dash-dotted feedback between successive stages highlights this aspect.

Example: Mnemonic for Jot, Sort, Write, Revise

Suppose you find it difficult to remember the four stages written as an ordered list as in Algorithm 36.2 or in the flow diagram of Figure 36.2. Are there other devices to make these four stages more memorable or memorizable? Can we apply the principles of Chapter 7 to an actual case like this? Yes, indeed.

If you are one who loves the terse and the succinct, you might want to use a *visual mnemonic* for the four stages: perhaps the *acronym JSWR*. You might then try something like Figure 36.3. Imagine *JSWR* as chocolate letters decorating an orange cake.

If something visual does not appeal to you, try this mnemonic:

> *Just Someone Who Reads.*
> *Jot, Sort, Write, Revise.*

I think it is simple and memorable enough. Try anything until you find that something which works for you. ★

FIGURE 36.3: A visual mnemonic for the four stages of essay writing. Along with the acronym, the playful font and arresting colours could better help you memorize the four stages.

36.9.1 Feedback

Although I have divided the process of essay writing into four stages, there is bound to be cycling or feedback between successive stages. For example, an idea might pop up at the sorting stage that did not occur to you in the jotting stage. Or you might want to write an extra section to make your point more clearly, while revising the essay. That is perfectly all right.

We are not linear machines moving like conveyor belts on a factory floor. We are creative beings. And creativity itself is a highly complex, nonlinear process, with feedback. So relax if you find one stage of writing spilling over into another. The stages are mere guidelines given to you until you master the art of the essay. Once you do that and graduate to the *being* stage of essay writing, you do not need any crutch. You would write effortlessly, flawlessly, fluently, and persuasively, as it would have become natural to you.

36.10 Jot

You should first *jot* down your thoughts as they occur to you, *without sorting or censoring them.* Give vent to whatever occurs to you and record it. Do not classify these inchoate ideas in any way. Whether profound or trivial, whether sober or ridiculous, whether inspired or pedestrian, just allow your thoughts to bubble up and record them in point form. Do not sequence your ideas either. Save that for the *sort* stage.

The ideas that come to you might take the form of single words, or phrases, or whole sentences, or even mental images. Again, do not sort or filter. Simply record. If a memorable phrase comes ready-made to you, record it. Do not worry about its meaning or correctness at this stage. You are a mere scribe for the fount of creativity that is spurting within you.

If you have a mental block or starting trouble, you need to kick start your mind motor. I have over the years found several methods helpful in stimulating the flow of thoughts. These tools are briefly described below.

36.10.1 Tool: W^7

The W^7 framework introduced in Section 19.5 is an excellent stimulator of ideas. Using your subject as the central hub, ask these questions pertaining to it: *who, what, when, where, why, which, how.* Use the answers to galvanize ideas. This method works for a very wide range of subjects. Practise until you become adept at using it.

36.10.2 Tool: PMI

The medical doctor, author, and originator of the term *lateral thinking,* Edward de Bono, has written several books on thinking. In one of them, *Serious Creativity: Using the Power of Lateral Thinking to Create New Ideas* [2], he suggests analyzing any topic by looking at the *plus, minus* and *interesting* aspects, shortened to the acronym PMI. This is especially useful in analytical and decision-making contexts, and is a technique that can prove useful not only academically but also in later life.

36.10.3 Tool: PEMS

Yet another road leading to the oasis of ideas is labelled PEMS. View the topic from its *political, economic, moral,* and *social* aspects. This technique is useful for topics in the humanities. If you work in another discipline such as medicine or the law, make up your own idea-triggers and their applicable acronyms.

36.10.4 Tool: Visualization

Visualization is a powerful way of using your mind and I have talked about it in Section 8.9. Visualize the situation, period, activity, etc., with such intensity that it becomes real to you mentally, and gives rise to ideas spontaneously. It might sound like magic, and perhaps, in a way, it is. Try it to experience its effects for yourself.

36.10.5 Summary: Jot

Let the ideas bubble up. Do not censor. Do not filter. Do not sort. Simply record. A summary of the *jot* stage of writing an essay is given in Algorithm 36.3.

JOT

1. Jot all ideas that occur to you in response to the question.
2. Do not sequence or censor any.
3. Apply one or more of W^7, PMI, PEMS, visualization, etc., to release mental blocks and get ideas flowing again.
4. Stop when you run out of ideas or have too many.

ALGORITHM 36.3: The *jot* stage of essay writing.

36.11 Sort

Once you have jotted your ideas, you should *sort* what you have written, discarding what is irrelevant or repetitive, and re-arranging and threading your different ideas together into one integral idea-garland. This is an important stage where your disparate thoughts are made to cohere around a central theme and confer integrity and identity on your essay. Order your ideas so that what comes first leads to what comes later, rather than the other way round. Coherence, continuity, and fluency are the watchwords.

36.11.1 Aggregating ideas

Identify major headings and group your jotted ideas under them. I call this aggregating ideas.

Imagine you are making a dough out of flour. You sieve, sift, add water, aggregate, and knead until you get rid of all the lumps and end up with a unified, smooth, non-friable, pliable, elastic dough.

The jumbled jotted points are your flour. The outline is your dough. Sieve and discard what you do not need and sift and order what you do keep. The process that converts flour to dough is the *sort* stage of writing your essay. You must master it to write good essays. You are the architect of your essay laying out and arranging and re-arranging your thoughts until you are satisfied with the design.

If one or two points you have jotted are worth mentioning, but they do not otherwise fit into the general pattern, decide whether to retain or reject them. If you decide to keep them, take care to blend them well into the general flow of ideas. Your dough-outline should not be lumpy or have foreign matter sticking out of it that clearly does not belong there.

★

Aggregating ideas confers coherence.

36.11.2 Linking ideas

Link your headings to ensure a smooth flow. Another analogy helps here. Imagine now that you are making a chain, forging its links together. Each link is connected to the one before it and also to the one after it.

Likewise, each sorted idea in your essay should be linked to its predecessor and to its successor. Re-arrange and re-order your ideas until you are satisfied with the continuity and flow.

★

Linking ideas confers fluency.

36.11.3 Housekeeping

Attend now to whatever words or phrases you are unsure of. Check their meaning and usage if in doubt. The *sort* stage is a bit like an aircraft in the taxiway, waiting for takeoff clearance. You are not moving yet.

Nevertheless you are ready to gain high speed on the runway and lift off the ground. Use this quiet period, before the hurly burly of writing to check everything, not unlike what an aircraft cabin crew do before takeoff.

36.11.4 The outline

Your outline should not embody abrupt jumps in thought except when you move from one part to another in the three-part structure of the essay as shown in List 36.1.

THE THREE PARTS OF AN ESSAY

1. **Introduction:** leads the reader gently to what you are going to say.
2. **Body:** contains the substance of what you have to say.
3. **Conclusion:** wraps up what you have said and gives a note of finality to your work.

LIST 36.1: The three component parts of an essay and their essential functions. Use this to *sort* your jottings into a working outline.

★

The end result of the *sort* stage is a detailed *outline* in ordered point form.

Your essay gains shape, wholeness, continuity, and fluency from your outline. The better your outline, the easier your next task of writing becomes.

36.11.5 Summary: Sort

Aggregate. Impose order on your ideas during the *sort* stage. Remove duplicates. Discard irrelevancies. Group similar ideas together.

Link. Use logic and reason to propel the sequence of ideas. Chain ideas together. Re-arrange and re-order until you are satisfied with the flow of ideas.

During the *sort* stage, your essay becomes a coherent and persuasive document with a definite goal. You are forging the instrument that becomes your essay from the ore of your jottings. Metaphorically, if your *jot* stage gave you a mixture of letters, you form meaningful anagrams from that jumble during the *sort* stage, a summary of which is given in Algorithm 36.4.

Sort

1. Go through your jottings and group related ideas together.
2. Discard those that are irrelevant to your theme.
3. Merge repetitions and duplicates.
4. List the unique points you intend to use.
5. Group the relevant points from step 4 above under each major heading of your outline.
6. Read the outline and check to confirm that the ideas flow logically and smoothly.
7. Re-arrange your outline and re-order your points as necessary until integrity and fluency are achieved.

ALGORITHM 36.4: The *sort* stage of essay writing.

36.12 Write

Having jotted and sorted, you should now *write* your essay—furiously if need be in an examination!—to put on paper what you have in your head and in your outline. The skeleton from the outline is clothed in the flesh of words, phrases, idioms, and sentences, and vivified as your essay. Put into practice the suggestions given in Chapter 35 to write as best you can.

36.12.1 Fair copy not rough draft

The usual practice is to write a rough draft, revise it, and then transcribe it to produce a fair copy that is submitted. Because you are writing *twice* in the process, you are actually wasting time—something that you should avoid in an examination. Imagine how much more time you would have if you wrote the fair copy at once.

There is another drawback in writing a rough draft. Since it is only a rough draft, you might feel free to scratch out what you have written—because you are not wholly satisfied with it—and start again. Indeed, you might find yourself *repeatedly* deleting what you have written and starting again. You can waste a lot of time and effort with almost nothing to show for it if you suffer from this *starting trouble syndrome*. Again, you can ill afford this in an examination. To circumvent these problems, do not write a rough draft.

Write your final fair copy at once.

You will save time in an examination. Practice until you are able to write the fair copy effortlessly. Mean business when you write. It is an examination lifesaver.

36.12.2 Writing and revising in tandem

Just as each of us has a unique voice, so too, each of us has a unique writing style. You might be one of those who has a flood of ideas and needs to *write* non-stop until that flood has abated. You would prefer to *write* first and *revise* later.

Or you might like to *write* one paragraph at a time and then read and *revise* it before writing some more. This latter style is useful when you have to write your essay by hand, as in an examination. You can amend something you have written by cancelling it and rewriting in the blank space below it if you write and revise in small instalments. You cannot do this if you write your whole essay first and then find that you need to revise a paragraph somewhere in the middle, where it is sandwiched between other paragraphs. Discover what is your most comfortable writing style and stick with it.

If you are writing the essay under non-examination conditions, and using a PC, you could write your essay in a non-sequential manner. In an examination, however, you must handwrite your essay from start to finish, in the sequence *Introduction*, *Body*, and *Conclusion*. This should be easily accomplished because your outline follows the same order. I will assume the more stringent examination conditions here.

36.12.3 The Introduction

If you have jotted and sorted your ideas, your mental pump is primed to write. Stick to your outline. Start writing *at once*. Do not delay. Do not dilly dally. Introduce the reader gracefully to your subject and point of view. Always lead from the known to unknown. First and last impressions count a great deal. So, write an impressive *Introduction*.

36.12.4 The Body

When you write your *Body*, you should be progressing smoothly from one idea to another in your outline, like someone climbing a ladder rung by rung. Use the guidelines in sources such as Gopen and Swan [3] and Lindsay [4] to help you write your body. Pay particular attention to paragraphing.

Paragraphs represent breaks in ideas. They correspond to pauses in conversation or scene changes in plays and movies. Ensure that your paragraphs are neither too long nor too short. Do not squash several ideas into a single paragraph. The text will then appear dense and uninviting, and your poor reader might have to backtrack to take stock of everything you are saying. Order different ideas into separate paragraphs.

Your paragraphs should flow smoothly from one to the other. Link the last sentence of one paragraph to the first sentence of the next to achieve this. Do this habitually and it will become effortless.

The topic sentence appears at or near the beginning of a paragraph and sets its tone and content. Write your sentences so that you get to the substance or topic quickly. Do not waffle. Adopt a healthy mix of sentence length: avoid a cluster of long or short sentences.

Use verbs liberally: they add vim and verve to your writing. Use elegant variation skilfully to avoid using the same word several times in the same sentence or in successive sentences. Improve your vocabulary and use synonyms to do the work. Consult a thesaurus if possible.

Lead the reader from the known to the unknown. Too many new ideas introduced willy nilly may lead to confusion. Too many old ideas repeated endlessly will lead to boredom. Aim to keep your reader constructively engaged: challenged and interested, but neither confused nor bored.

36.12.5 The Conclusion

Wrap up your whole essay in your *Conclusion*. Avoid repetition. Ensure that the *Conclusion* is in accord with both the *Introduction* and the *Body* of the essay. There should be no conflict between the three. Do not introduce new ideas or questions here. Pack a punch into it as last impressions tend to linger. Inject finality into the *Conclusion* so that the reader knows that you have well and truly finished with the topic.

36.12.6 Writing correctly the first time

If you find that you cannot write a whole sentence without deleting something in it, you need to change your habit. Practise the art of putting your thoughts on paper correctly, clearly, and coherently *the first time*.

EXAMPLE: AN EXERCISE IN WRITING

Take any topic. Jot and sort your thoughts. Then write five lines as part of an *Introduction* and another five as part of a *Conclusion*. You are not writing an entire essay: simply the first and last fragments. If you do this often enough, you will slowly start writing fluently without scratching out what you have written. You will write without hesitation. You will not backtrack. You will accustom your mind to the linear flow an essay demands. Practise until you are proficient in this art. ★

36.12.7 Summary: Write

Write correctly, clearly, and persuasively. Use verbs rather than nouns. Introduce important ideas at the beginning of the sentence rather than at the end. Insert a new paragraph whenever there is a transition of topic. Employ elegant variation. Write a little and then revise it before writing again, if that helps you write better.

WRITE

1. Lead the reader from the known to the unknown.
2. Prefer active to passive voice, and verbs to nouns.
3. Put important ideas at the beginning of the sentence.
4. Break different ideas into different paragraphs.
5. Link paragraphs so that your writing flows smoothly.
6. Avoid clusters of long or short sentences.
7. Use synonyms to circumvent tedious repetition.
8. Practise writing the fair copy at once.
9. Write and revise incrementally if it helps.

ALGORITHM 36.5: The *write* stage of essay writing.

36.13 Revise

The final stage is to *revise* what you have written for elegance and eloquence. Correct errors of grammar, usage, spelling, and punctuation. Do not omit this stage. In the frenzied writing typical of examinations, it is all too easy to skip a word or misspell it, so much so that what you have written does not make sense, even to you!

Exercise deliberate care when you proof-read and edit what you have written. Rather than claiming ownership of your writing, adopt an impartial attitude, as if reading someone else's work, because errors show up better then. Locate and correct your errors before your examiner does! Only then is your essay complete.

36.13.1 Grammar and usage

Some claim that the English language does not have a grammar like other languages. Others claim that usage prevails over grammar. In any case, the globalization of English and its use on the Web are changing the face of English very rapidly. My prescription is simple. Write as correctly as you can for your intended audience. I do not recommend grammar checking programs. See Chapters 12 and 35 for more on grammar and usage.

Omissions of words, phrases, or sentences can happen in the heat of an examination. No one else but you can know what is missing. Read carefully and insert the missing the words during the *revise* phase.

36.13.2 Spelling

There are two types of spelling error. One is mis-spelling a word to yield another word that does not exist. That word will be flagged as a spelling error if you are working on a PC with a spellchecker program.

However, if your mistyped word happens to be a valid word in the language, then you will both distort the meaning of what you have written, and also be unable to detect the error using a spellchecker.

You should therefore learn to scan what you have written and be able to detect quickly both obviously mis-spelled words and also valid words which were mistyped and have led to unintended meanings.

Example: Types of spelling error

Suppose you have a tendency to type "teh" instead of "the". This transposition of characters is common but the error is easily picked up by a spellchecker or other program. You only need to instruct the program to replace every occurrence of "teh" with "the" and you are done.

Here is an example of the second type of error: If instead of typing “Visible light has wavelengths from 400 to 700 nanometres”, you typed “Visible light has wavelengths form 400 to 700 nanometres”, your sentence would make no sense and the error in it would not be picked up by a spellchecker.

Finally, if you are writing an examination essay, it is unlikely that you would write “teh” for “the”. But you could very well write “recieve” instead of the correct “receive”. Prepare yourself beforehand by memorizing correct spellings for such easily mis-spelled words. ★

36.13.3 Punctuation

The punctuation marks from comma to full-stop indicate progressively longer pauses. Punctuate your sentences with commas, semi-colons, colons, and full-stops to increase readability. Imagine that you are reading what you have written to someone else, and use these punctuation marks whenever you would naturally pause either for breath, or to make your meaning clear.

36.13.4 Summary: Revise

However attractive it might seem, do not ever succumb to the temptation to pass up your work without revising it. If you skip this step, your work is bound to have errors that will cost you marks. Looking over your work and correcting it is not a luxury or after-thought. It is part and parcel of essay writing.

REVISE

1. Read or re-read carefully what you have written as if it were written by someone else.
2. Look out for missing words or sentences and insert them.
3. Make sure that what you have written makes sense and can be easily understood.
4. Look out for spelling mistakes, grammatical errors, wrong usage, etc.
5. Pay special attention to fluency. Rearrange paragraphs or rewrite sentences to achieve this if need be.
6. Correct whatever seems amiss.

LIST 36.2: The *revise* stage of essay writing.

36.14 Handwritten essays versus those typed on a PC

Except in an examination, where you literally have to put pen to paper, much essay writing may now be done using a word processing program on a PC.[5] I have covered the use of word processing systems in Chapter 17 and suggest that you review the material there. Typing an essay on a PC absolves you somewhat of responsibility for spelling errors—especially if you are a poor typist—because of the ready availability of programs to check spelling. Do not, however, become spelling-agnostic as a result. You are still accountable for your spelling in an examination!

One of the greatest advantages of using a PC is that you can shuffle entire paragraphs from one part of your essay to another to improve continuity and fluency. This is one tremendous advantage you have over the students of yesteryear. They either had to laboriously re-write whole pages, or they had to *literally cut and paste* their paragraphs using scissors and glue rather than the keystrokes you are familiar with!

Another difference between an essay typed on a PC and a handwritten one is the presence of corrections, deletions, etc. on the latter. While a profusion of such corrections is undesirable, they are generally unavoidable, especially in the heat and fury of an examination. Ensure that your handwritten essay does not contain too many arrowed detours between crossed out paragraphs, and that it is generally neat and legible.

36.15 Non-examination and examination conditions

There are basically two types of time limitations under which you have to write an essay. One is measured in days or weeks; the other in minutes or hours. The former, I have called non-examination conditions; the latter obviously represent examination conditions.

I have recommended writing your essay in full at one go at the very first attempt. This is simply to prepare you for examinations where you might not have enough time to write a rough draft and then a fair copy. There is a danger that this suggestion might be conveniently misinterpreted by the lazy student. So, let me clarify.

[5] In a chapter entitled "Pen and Computer", Fowler [5] gives his views on these two modes of writing.

> Under examination conditions, when time is limited, write your essay in full at the first attempt, but revise it thereafter.

When you are not stretched for time, and especially if you have the luxury of writing your essay on your PC, you are very likely to write the essay in *several drafts* and in *several sittings.*

For example, you might write it once. Then during revision, you might rearrange your ideas and paragraphs. You might on subsequent reading, change your mind and improve the structure once more. This iterative improvement of your essay is natural and should take place every time you write under non-examination conditions.

> Under non-examination conditions, write and revise your draft any number of times until you are satisfied with the final result.

To sum up, the first draft is never adequate. You need to revise it. This might entail, among other things, rewriting it, transposing paragraphs, deleting sentences, improving clarity or diction, etc. All of these belong to the *revise* stage of essay writing. Just as you should never hand in a mathematics problem without checking it, so also you should never hand in an essay without revising it.

> Never hand in an essay without revising it.

36.16 Timing your work

If you have two weeks notice to write and hand in your essay, my advice is to start early and pace your work so that your essay is completed two or so days *before* the deadline. How you pace your work is something you must manage yourself.

If you are in an examination and have three hours to complete your essay, your deadline is a lot shorter and all four stages of essay writing must fit within that three-hour limit. Again, budget your time across the four stages of essay writing. Allocate the largest chunk of time for writing but do not overshoot it. Ensure that you give enough time to the *revise* stage to read, edit, and polish up your essay.

36.17 Using references

Develop and nurture the ability to identify relevant and reliable references to consult while writing your essay. This depends on topic and subject. Likewise, you should cultivate the ability to appraise online material, and gauge it for its reliability and accuracy. Consult online resources like Wikipedia [6], Dictionary.com [7], Thesaurus.com [8] etc., as you work on the PC on your essay. You will find your accuracy, productivity, and efficiency improve as you do so.

You will not be expected to provide a properly cited bibliography for a handwritten essay under examination conditions. Non-examination essays are not exempt though. You should learn the art of researching facts, identifying suitable references, citing them correctly, and producing an accurate bibliography when you type your essay on a PC. This applies equally well to printed as to online references. See Chapters 47 and 49 for more details.

36.18 Model essays

Model essays are a boon so long as you realize that there are no "correct" or "best" answers ever to an essay question. There have been and continue to be a rich variety of excellent essayists in the English language. It is true that their style and usage would reflect the period of their work, but their acuity of thought and elegance of expression still shine through the essays that they have left behind.

Essayists whose writings I have found worthy of emulation include Ralph Waldo Emerson, Aldous Huxley, Henry David Thoreau, Edgar Allan Poe, and Ambrose Bierce, to name just a few. Many contemporary journalists write fine essays and blogs.[6] Read essayists whose work you find interesting and whose style you would like to emulate.

When I was a student, I found two books of model essays very useful. Both are, I believe, sadly out of print now. One is Macmillan's *Simple Es-*

[6]The Web-based shorter cousin of the essay is the We*b log* that has been abbreviated to *blog*.

says with their Outlines and the other, *One Hundred and One English Essays* by Waryam Singh Malhi.[7]

You might find other books on model essays more suited to your clime and time. I encourage you to read not only books of model essays, or anthologies of exemplary essays, but also any good writing that you find in newspapers, magazines, learned journals, on the Web as blogs, etc. All of these can guide and inspire you to write good essays.

The next chapter, Chapter 37, is devoted to a model essay written by me, in answer to a self-drafted, but plausible, essay question. The model essay illustrates in specific detail the processes that I have outlined in this chapter for writing an essay. I would recommend that, at least once, you read Chapter 37 immediately after reading this chapter so that you are able to integrate the theoretical and practical aspects of essay writing at one reading.

36.19 To explore further

I have given a comprehensive list of references related to writing and vocabulary in Chapters 30, 35 and 50. Some of those are relevant to the subject of this chapter as well.

For more detailed guidance on writing essays, I would highly recommend *A Student's Writing Guide: How to Plan and Write Successful Essays* [9] by Taylor. It is contemporary, admirably well-written, and gives excellent advice. You might also like to take a look at Starkey's *How to Write Great Essays* [10]. Alastair Fowler gives wide-ranging advice on writing and, if you have the requisite academic maturity, you will find his book *How to Write* [5] very helpful indeed. Sorenson's comprehensive manual, *Webster's New World Student Writing Handbook* [11] has many prescriptions to algorithmize writing; use it if you find that it agrees with your style of writing. And last, but not least, even though its subtitle states that it is meant for students of literature, the book *How to write critical essays: A guide for students of literature* [12] by Pirie offers some eminently *practical generic advice* that would benefit students writing essays in all disciplines.

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SUMMARY: ESSAY WRITING

- Read carefully and understand the question correctly before attempting to answer it precisely.
- Cultivate general knowledge to facilitate essay writing.
- Learn to identify authoritative references, whether they be printed or online, and use them liberally.
- Never plagiarize.
- Acknowledge others’ work by proper citation and attribution.

- An essay has three parts:
  1. Introduction
  2. Body
  3. Conclusion
- Essay writing has four stages:
  1. Jot
  2. Sort
  3. Write
  4. Revise
- At the jot stage, do not sort or censor your ideas. Simply write down all that occurs to you in response to the question.
- At the sort stage, group like ideas together. Discard irrelevant ideas. Identify major headings under which to group the jotted points. Link headings to unify the essay and for a smooth flow of ideas. Generate a solid outline.
- Plan your *Introduction* and *Conclusion* carefully.
- Write the fair copy of the entire essay at one go when preparing for examinations.
- Revise what you have written for fluency, relevance, continuity, grammar, spelling, accepted usage etc.
- Read, revise, and rearrange what you have written until you are satisfied with it.
- Under non-examination conditions, make the most of Web-based reference material and the efficiency and productivity gained by typing your essays on the PC.
- Study model essays for technique, style, usage, etc. Emulate what you find worthy.
- Read books devoted to the art of the essay.

Part G

Mathematics

38

Enjoying Mathematics

Synopsis

Mathematics pervades all aspects of our lives. Because the fruits of mathematics are so enjoyable, mathematics itself should be fun. Yet, for many, the subject is alas anything but fun. The purpose of this chapter is to help you enjoy mathematics by meeting it away from classrooms and examinations.

Numbers and letters are equally natural to us when we are young. But over time, mathematics becomes a formidable school subject.

One way to restore an amicable relationship with mathematics is to read gentle introductions to the subject. Popular mathematics books are ideal in this respect and we are fortunate to have a wealth of such books written by gifted expositors. Some are recommended in this chapter. The companion website will reflect a larger compilation. I encourage you to read at least three popular mathematics books that appeal to your taste.

Recreational mathematics books are also an excellent, non-threatening entry into the fascinating world of mathematics and its logic. You should look for puzzle books that suit your taste and that are readily available to you.

Mathematics ceases to be formidable when you meet it within your comfort zone. Then, you may gradually expand your domain of mathematical comfort, mastering a little at a time. In due course, you will recognize how much fun it is and start enjoying mathematics naturally.

38.1 Mathematics and civilization

Our scientific civilization is steeped in mathematics. Every time you use a mobile phone, you are applying mathematics. Every time you watch television, mathematics is in action. Every time you pay at a supermarket cash register or withdraw money from an automatic teller machine, you are enjoying the fruits of mathematics. If you travel by air, take a bus, listen to music, or surf the Web, you are riding a magic carpet of mathematics. The list is endless.

If mathematics is so useful, if it is so pervasive, and if its fruits are so enjoyable, surely, mathematics itself must be fun! *Yet, for many alas, mathematics is anything but fun.*

This chapter is an attempt to help you put the fun back into mathematics. It is especially for you if you have a reluctance to engage with the subject or count yourself among the mathematically challenged. Numbers are part of our common intellectual heritage no less than letters. One way to reclaim that heritage is by reading popular books on mathematics. I share with you a list of books that I have found enjoyable, in the hope that you too can start enjoying mathematics by reading them.

38.2 Numbers and Letters

If you think long and hard about what unique aspect of our nature distinguishes human beings from other species, you might justifiably conclude that it is our ability to abstract, idealize, generalize, and apply. For this we need symbols. *We are manipulators of symbols.*[1] Indeed, it is our ability to make and understand symbols that has led to our civilization.

Our aural symbols range from spoken languages to music. And our visual symbols range from stick drawings of human beings to great art, from scrawls to calligraphy, from alphabet and ideograph to binary computer code and Unicode.

The sounds we articulate as infants slowly take shape as the words of our native language. And just as we are taught to point and name, so also are we taught to count and name. Letters and numbers; numbers and letters: these are the twin braid of symbols we encounter throughout our lives.

[1]Users of tools and manipulators of symbols, to be more precise.

எண்ணென்ப ஏனை எழுத்தென்ப இவ்விரண்டும்
கண்ணென்ப வாழும் உயிர்க்கு

eṇṇenpa ēnai eḻuttenpa ivviraṇṭum
kaṇenpa vāḻum uyirku

Numbers and letters are to human beings like their twin eyes.

So said the Tamil philosopher-poet Tiruvaḻḻuvar in Chapter 40 Verse 392 of his famous compilation, the Tirukkuṟaḷ.

We are equipped at birth with a sense of numeracy no less than a sense of literacy. To argue otherwise is to deny our bilingual intellectual heritage of numbers and letters, and possibly our bicameral brain. If you need any convincing about our inbuilt sense of numeracy, read Stanislas Dehaene's *The Number Sense: How the Mind Creates Mathematics* [1] and Brian Butterworth's *The Mathematical Brain* [2]. You will doubt no more. Indeed, it has been claimed that "Mathematics *is* our other native language." [3, p 3]. To use Tiruvaḻḻuvar's metaphor, would you not far prefer binocular vision to Cyclopean perception?

Euclid is reputed to have said that there is no royal road to geometry [4]. I venture to suggest, however, that mathematics may be sweetened by popular books on the subject so that it becomes increasingly enjoyable.

38.3 Popular mathematics books

Textbooks of mathematics are often restrained and concise. They are meant to instruct the student, impart new skills to him or her, and exercise those skills through drills and problems. Several modern textbooks of mathematics have lightened and brightened their approach to the subject with generous use of colour, sidebars to emphasize points, boxes to highlight formulae, etc., but being textbooks, they cannot afford to be too conversational.

So, if you are a student in search of a really *friendly* mathematics book, you might need to consult a good *popular mathematics* book, dealing with the topics that you are interested in.

Popular mathematics books promote enjoyment of mathematics.

From the last decade of the twentieth century onward, we have been treated to a wealth of popular mathematics books written by gifted authors: professional mathematicians, historians of mathematics, engineers, scientists, and journalists. They are written mostly in readable English, rather than the dense hieroglyphs of symbolic mathematics. Most of them require no more than high school algebra and trigonometry to understand and enjoy, give or take a little calculus.

Some of these books recount the historical development of important ideas. Others present familiar concepts in a new light, weaving different strands into a unified tapestry. Still others will open your eyes to applications that you might not have even remotely anticipated. All of them will enchant you with mathematics that is friendly, useful, fascinating, and enjoyable.

The most painless way to get re-acquainted with the pleasures of mathematics and to start enjoying it afresh is to delve into such books. Some are a breeze to read. Others will demand attention and effort, but in return will reward you with insight and understanding.

Your re-acquaintance with mathematics takes place without the pressure of syllabus or semester, assignment or examination, grade or rank. It is almost like pursuing a hobby.

> When learning becomes fun, you are relaxed and absorb more, understand better, and enjoy learning.

Here then is my reading list, sorted by author, of recommended popular mathematics books. This selection is necessarily personal, brief, and incomplete: many excellent books have been left out. Refer to the companion website of this book for a more complete and up to date list of recommended popular mathematics texts. I start off with a troika of books on my "must read" list.

38.3.1 Strogatz

Steven Strogatz is a mathematician who is also a gifted popularizer of the subject through his books, the radio, and the press. He has playfully titled one of his books *The Joy of x: A Guided Tour of Math, from One to Infinity*. I have chosen to highlight it first because it has "joy" in its title just like this chapter!

The book consists of bite-sized chapters that may be read out of sequence if desired, even though the book itself is organized according to the major branches of mathematics. Strogatz has a knack for presenting material so expertly that it is often largely self-evident: the reader needs no convincing then. His tone is not didactic. Rather, he comes across as a fellow traveller co-exploring the world of mathematics. This reduces the distance between author and reader, making the subject that much more easily assimilated. The book is witty and generously illustrated, with a narrative that is punctuated by interesting real-life stories. For the serious student intent on further exploration, there are copious notes for each chapter at the end of the book. All in all, a joy to read! And read it you must!

38.3.2 Bellos

Alex Bellos has written a wide-ranging and admirable book having the sumptuous title *Here's Looking at Euclid: From Counting Ants to Games of Chance—An Awe-Inspiring Journey Through the World of Numbers* [6]. If you have not read it yet, an intellectual treat awaits you.

Bellos is a journalist educated in mathematics and philosophy. He brings to his book the combined strengths of his métiers. The rigour of the mathematician and the insight of the philosopher are delightfully blended with the breezy style of the journalist with a keen eye for the noteworthy and the unusual. The book effortlessly draws in and charms the reader. His writing is peppered with colourful anecdotes, filled with amazing facts, and lightly laced with touches of subtle humour.

If ever you thought that mathematics was intimidating or dull, read this book and you will come away thinking otherwise. And if you dread mathematics, this book will be an icebreaker, perhaps even sweet medicine to cure your malady forever! If you are at school or university, I would exhort you to read this book as an essential part of your education.

38.3.3 Gullberg

The third "must read" book on my list is Jan Gullberg's *Mathematics: From the Birth of Numbers* [7]. The author is—believe it or not—a *surgeon,* and penning the almost 1,100-page volume must have been a true labour of love for him. From it, you will learn that mathematics is neither boring nor stodgy, but an intellectually piquant and refreshingly effervescent *human endeavour.*

The book is engrossing, encyclopaedic, rigorous, clear, accurate, up to date, rich in history, complete with examples, delightfully illustrated, reader-friendly, and on occasion, even humorous. What more could you ask? To top it all off, the book is moderately priced for such a weighty tome. Own it if you can, but do read it regardless! It is a gem.

38.3.4 Dunham

I now come to other books on my list. I have enjoyed the exposition, pace, and style in the popular mathematics books written by mathematician and historian William Dunham. His first book, *Journey Through Genius: The Great Theorems of Mathematics* [8] is a leisurely stroll amidst the great theorems. Proofs of these are given in a less formal but no less rigorous manner, with enough historical and biographical background to convey a realistic idea of how mathematics actually evolved.

When you recognize that the best mathematicians of bygone centuries grappled long and hard with such newfangled ideas as negative numbers, complex numbers, and irrational numbers, you will be less disheartened if these very same numbers have taxed your own understanding at school, where you have weeks rather than years to assimilate new concepts.

Dunham's second book, *The Mathematical Universe: An Alphabetical Journey Through the Great Proofs, Problems, and Personalities* [9] is his personal "A to Z" of mathematics and is similar in flavour to his first book. The third book, *The Calculus Gallery: Masterpieces from Newton to Lebesgue* [10] is devoted to the evolutionary history of the calculus and the contributions of "the masters" who created the "masterpieces". All three books are engaging reads and convey clearly the *human* nature of the mathematical enterprise.

38.3.5 The Kaplans

Robert and Ellen Kaplan are two of the founders of The Math Circle, a mathematics school that teaches the *enjoyment of mathematics* and that is open to anyone of any age [11]. The first book, by Robert Kaplan, is about zero and is intriguingly titled, *The Nothing That Is: A Natural History of Zero* [12]. The second, by both Robert Kaplan and Ellen Kaplan, is called *The Art of the Infinite: The Pleasures of Mathematics* [13].

To cover zero and infinity within the span of two popular books is a very tall order indeed, but the Kaplans manage it breathtakingly. Their books are a heady mixture of mathematics, science, history, literature, philosophy, music, and a wry reflection on the human state. To understand everything they have written would require some effort from you, and that is not because of the mathematics alone![2] The fluent ease with which they pack mathematical truths among their often lyrical prose is stunning. If ever you wanted to see magic in mathematics, you might just have found it in these two books.

38.3.6 Maor

Infinity captures the mathematical imagination like no other concept. The next author whose books I have relished is the mathematical historian, Eli Maor. He too has written a book on infinity entitled *To Infinity and Beyond: A Cultural History of the Infinite* [14]. His other books deal with the perennially intriguing Pythagorean theorem [15], the mathematically ubiquitous number e [16], and trigonometry [17]. I have found his books carefully written and easy to read. Do take a dip into them to discover the richly layered tradition embedded within each of these mathematical morsels.

38.3.7 Pickover, Nahin, Banks, Ghazalé, Stein, and Kline

I now come to books that are a little more heavy-duty in terms of mathematical demands on the reader, but they are well-written and make for rewarding reading, even if you need to skip the mathematically intense portions.

Clifford A Pickover is a prolific author who has written very widely on mathematics and science. Some of his books are dazzling and expose little known nuggets of mathematical truth or history. Three of his books that I have enjoyed have as their subjects chaos [19], numbers [18], and structure [20]. His popular book on calculus is colourfully titled *Calculus and Pizza: A Cookbook for the Hungry Mind* [21] and is packed with interesting exercises. If you like to browse and dip into a book here and there, reading whatever takes your fancy, many of Pickover's books would suit you very well.

[2]Hint: have a dictionary handy!

Paul J Nahin is an electrical engineer who has written on central mathematical ideas like the imaginary unit i [22], probability [23], and optimization [24]. His later book *Dr Euler's Fabulous Formula: Cures Many Mathematical Ills* [25] is a sequel to his first book *An Imaginary Tale: The Story of* $\sqrt{-1}$ [22]. Beware that these books contain a healthy smattering of formulae. So, if symbols deter you, skip them and just read the English text.

Robert B Banks is another engineer who has written interesting popular mathematics books with equally interesting titles like *Towing Icebergs, Falling Dominoes, and Other Adventures in Applied Mathematics* [26] and *Slicing Pizzas, Racing Turtles, and Further Adventures in Appled Mathematics* [27]. Because they are not single-themed, you can read the chapters out of sequence if you wish. These books also have a fair share of symbols that you might have to gloss over.

By now, you should have cottoned on to the fact that when engineers write popular mathematics, their books are likely to be generously seasoned with formulae and equations. The next author, Midhat J Ghazalé, is also an engineer and his books follow the same pattern! His two books *Gnomon: From Pharaohs to Fractals* [28] and *Number: From Ahmes to Cantor* [29] are both entertaining and well illustrated.

Sherman K Stein is a mathematician who has written two insightful popular mathematics books. His *Mathematics: The Man-Made Universe* [30] is wide-ranging and systematic. The other book, *Strength in Numbers: Discovering the Joy and Power of Mathematics in Everyday Life* [31] is more conversational in tone while addressing the difficulties faced by students of mathematics. These books demand effort from you if you are to get the most out of them.

Morris Kline is another mathematician who has a gift for exposition. Although his books were written for a more sedate age, they are timeless in their relevance. His aptly named *Mathematics for the Nonmathematician* [32] was originally meant as a text for Liberal Arts students in American universities. It is a gentle introduction to serious, everyday mathematics. His other book *Mathematics and the Physical World* [33] can well serve as a soft launch into mathematical physics.

38.3.8 Hogben and Sawyer

The next three books are sugar-coated introductions to mathematics at secondary and high school level. These books are a rare blend of popular mathematics and auxiliary textbook. The approach is always from the concrete to the abstract, and profound ideas are introduced through well-chosen and carefully explained examples. And because mastering mathematics requires practice, these books also contain problems and their solutions, where appropriate. They will appeal more to the school student than to the causal reader.

The first recommended book is zoologist Lancelot Hogben's *Mathematics For The Million: How To Master The Magic Of Numbers* [34]. Its six hundred odd pages contain a panoramic survey of mathematics covering numbers, geometry, algebra, trigonometry, logarithms, probability, and more. The author has very thoughtfully collected formulae and facts under the heading "Things to Memorize" which will prove immensely helpful, especially if you are a student appearing for calculator-free tests and mathematics competitions.

The other two recommended books are both by the mathematician W W Sawyer. The first, *Mathematician's Delight* [35] covers much the same ground as Hogben's book [34]. The second, *Prelude to Mathematics* [36] is pitched at a slightly higher level and even covers some material you are likely to encounter only in first year at university. Sawyer makes very few assumptions about prior knowledge, writes clearly and directly, and shows the links between the topic at hand and the different branches of mathematics, giving you the big picture—something which a textbook might not always do.

These three books are classics which are not too expensive to own. They are from a less hurried, more patient era, when teachers took great pains to ensure clarity and absence of ambiguity in their expositions. All three books have aged well since their first publication. The fact that each has remained in print for more than half a century is in itself a testament to their collective worth.

If you are a student, read these books, not as a spectator, but as a participant, working through the text and exercises at a comfortable pace. You will well and truly start *enjoying* mathematics so much so that you might even forget that you are *learning* it at the same time!

38.4 Recreational mathematics

Learning should be fun. Recreational mathematics, which is the solving of mazes, puzzles, games, riddles, and conundra, is another painless and extremely enjoyable way of building up and flexing your mathematical muscles.

From the construction of magic squares to the solution of how a person would transport a fox, a chicken, and some sticks across a river on a boat that can hold only two, recreational mathematics has uniquely attracted and fascinated humankind since ancient times. Number puzzles like Sudoku and geometric brain-twisters like Rubik's Cube and Origami should also be familiar to you. What is it about these puzzles and games that so captures the imagination and interest?

It is the *promise of a solution* at the end of the quest. And of having *fun* in the process. Mathematical puzzles generally do not require specialized knowledge but they *do* require logical thinking. There is an unwritten compact between puzzle-maker and puzzle-solver that you will need only basic logic to solve the problem. And if all else fails, there is always trial and error. Mathematics appears to be both enjoyable and within our grasp when viewed through the lens of puzzles and games.

Solving puzzles does not so much teach you mathematics as inculcate *mathematical thinking.* You need a methodical and logical approach to solving puzzles, and practice at this art, by trial and error if nothing else, helps develop habits of systematic rational thinking.

> Solving puzzles is fun learning.

You proceed at your own pace, taking days if need be to solve a puzzle. You will experience at first hand the thrill of the "Aha!" moment and the intellectual triumph it brings, when after protracted effort, you have finally cracked a problem. This is when *mastery* begins, and you will know it yourself when you experience it. Mastery follows enjoyment.

38.5 Recreational mathematics books

There are so many good recreational mathematics books that recommending a list here would not do justice to them at all. I will instead list the names of authors who have fascinated me with their books on recreational mathematics and include a short sampling of titles here. Bear in mind that some authors of recreational mathematics books have also written popular mathematics books.

The pre-eminent mathematical puzzlemaster of the twentieth century must be Martin Gardner[3] who used to author the "Mathematical Games" column in *Scientific American* for twenty five years. Collections of Gardner's puzzles and games have also been published as books and you might like to whet you appetite by starting off with his "colossal" book *The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems* [37].

The philosopher, mathematician, logician, and magician, Raymond Smullyan has written several absorbing puzzle books with memorable titles such as *To Mock a Mockingbird: And Other Logic Puzzles Including an Amazing Adventure in Combinatory Logic* [38], *This Book Needs No Title* [39], and *The Riddle of Scheherazade, and Other Amazing Puzzles, Ancient and Modern* [40]. If you love logic puzzles, these books will engross you.

The book *Problem Solving Through Recreational Mathematics* [41] by Bonnie Averbach and Orin Chein harnesses the fun in solving puzzles to assist in problem solving, which is the core subject of Chapter 41. It contains solved sample problems and sets of practice problems. Another book with a varied collection of solved problems, featuring unhurried discussions, is Hans Rademacher and Otto Toeplitz's *The Enjoyment of Mathematics* [42]. Both these books will be especially useful if you are a student at school wishing to improve your mathematics.

Browse your local library, or bookshop, or search the Web to uncover more from the treasure heap of recreational mathematics books and websites.

38.6 Wrapping up

I hope that you will read at least the first three of the recommended books. I also hope that you will improve your relationship with mathematics in the process.

[3]Whom you have already encountered in Chapter 34 in connection with the poem *Jabberwocky*.

Mathematics is more about *how to think* than about *how to calculate.* Acquainting yourself with mathematics enables you to hone your thinking skills.

That alone is reason enough to enjoy mathematics because you are effortlessly sharpening your mind.

One fact that will strike you as you read books on the subject is that mathematics is about much more than numbers and arithmetic; much more than rules and manipulation. Mathematics draws on our intuitions about time and space and ties them together with logic. It is about structure, pattern, quantity, relationships, order, classification, abstraction, generalization, extension, application, logic, proof, and rigour. If after exposure to friendly books, you are still apprehensive about mathematics, the next chapter will perhaps help resolve your angst.

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SUMMARY: ENJOYING MATHEMATICS

- Mathematics underpins our scientific civilization and makes our life enjoyable.
- Human beings have a natural aptitude for numeracy just as they have a natural aptitude for literacy.
- Popular mathematics books written by good authors can help you understand mathematics in a fun way.
- Solving mathematical games and puzzles, mazes and riddles, is another enjoyable means to develop mathematical skills.
- Read the recommended books and others like them to become more comfortable with mathematics and to start enjoying it.

39

Overcoming Mathophobia

Synopsis

Unpleasant experiences with arithmetic in early learning often lead to a dread of mathematics as a whole later on.

Rote learning in elementary school robs mathematics of its prime strength: reason founded on logic. Memorization replaces understanding, paving the way for confusion.

Mathematics demands evolving viewpoints and changing skills at different stages of learning the subject, and this again contributes to discomfort with the discipline.

Entrenched misconceptions and their attendant fears seal the lid, making *mathophobia*—a dread of mathematics—the norm rather than the exception.

A threefold approach is suggested to overcome this:

Misconceptions: Correct common misconceptions.

Fears: Vanquish the fears of arithmetic, of symbols, and of asking questions.

Mistakes: Identify frequent mistakes and correct them systematically until they disappear.

39.1 Mathophobia

In my lexicon, *mathophobia* denotes a dread of mathematics. The express purpose of this chapter is to help you overcome that apprehension. I start off by looking at why so many suffer from mathophobia, and deduce that it is partly due to the nature of the subject itself, and partly due to the way it is taught in primary school.

39.2 Why mathophobia?

Among all the subjects taught at school, it is mathematics that inspires the most widespread fear. Those who excel in language, or history, or economics, or biology, or any of a myriad other subjects often proudly profess to a fear or loathing of mathematics. The style and pace of early mathematics education is often to blame, relying as it does on rote rather than reason, thereby robbing mathematics of its prime strength: reason founded on logic. Instead of understanding leading to an appreciation of its beauty, memorization of mathematics leads to uncomprehending confusion.

But there is a deeper underlying cause for why mathematics is probably the least favourite subject of most students. It is the ever expanding nature of the subject itself. Mathematics is not something that you master, and which stays put thereafter. It keeps changing as your education evolves and presents different challenges at each stage.

★

The facets of mathematics seem inexhaustible.

Simply knowing this can help assuage mathophobia. I will now take you on my personal tour of the ever expanding nature of mathematics and what it holds for the student.

39.2.1 Arithmetic

In the beginning, mathematics seems well within our grasp. *Counting* is easy, even if a little tedious. And *addition* is really a form of counting and thus poses no particular problems.

Then comes *subtraction,* which is often the very first stumbling block. When we take away something from itself, what do we get? The introduction of *zero,* which stands for *nothing* is the first profound conceptual leap we have to take, mostly on faith. And with zero comes one stricture: you can never divide by it!

Even if we accept zero without a murmur, *negative numbers* pose the next hurdle. When told that $4-6=-2$ we militate against the unnaturalness of the whole exercise. Later, when we encounter bank loans and reconcile negative numbers with sums owed, the disquiet is eased for a while.

The next difficulty is with *division,* which is really repeated subtraction. *Fractions* with their strange notation, with *least common multiples* and *greatest common divisors*, are often enough to frighten the young student away from mathematics for good. Imagine being asked to do *mental arithmetic* on fractions!

39.2.2 Algebra

If you stayed the course and survived these ordeals in primary school, you would have been rewarded with *equations.* Here *alphabetic symbols* invade the mathematical domain and whatever reassurance you felt from meeting numbers alone in mathematics vanishes.

The symbol x has been the reason for generations of students being put off mathematics for life. Because mathematics keeps on adding to its alphabet of symbols, with no end in sight, mastering it seems an almost Sisyphean task. And who knew that you write $2x$ but not $1x$ because that was written simply as x?

If you have survived $x-2=3$ to get $x=5$, you have made it through middle school and are to be congratulated. But just as you got your second wind and your confidence back, you encounter $x^2+1=0$ and the whole cycle repeats itself.

Mathematics is voracious: each new tier of advance brings with it new domains, new concepts, new symbols, and new relationships. If you went on to master *complex numbers* you would recognize i as the positive square root of -1, whatever that may mean![1]

39.2.3 Calculus

Before you finish high school, you would meet the *calculus,* sometimes called the infinitesimal calculus, which comprises *integration* and *differ-*

[1] j if you are an electrical engineer and do not want to confuse current with the positive square root of -1!

entiation. This is arguably one of the great triumphs of the human mind and of mathematics.

Irregular areas can be computed using integration, assuming only the well-known formula for the area of a rectangle, and by invoking a magical device called a *limit*. Likewise, rates of continuously changing quantities can be found from differentiation, using the rise-on-run definition of a slope, and again using the notion of a limit. The philosophical difficulty with these techniques is that we are dealing with *vanishingly small quantities that cannot vanish* so as not to break the taboo of dividing by zero.

Even if you can accept and live with the "infinitesimal but not zero" idea, you might balk at the concept of *infinity*. Limits and infinity are often paired. And while zero is a number, infinity is not. It is a profound concept whose full import has not been realized yet. And mathematicians themselves are not certain about how many flavours of infinity there really are!

39.2.4 Proof

Entry into university often represents the transition from purely manipulative mathematics to conceptual forays into mathematical thought. If your comfort zone lay in "doing" mathematics, you now have to add *proof* to your armamentarium. Non-Euclidean geometries, harmonic analysis, algebraic structures, and other mathematical abstractions will also keep you company at university.

Mathematics is ever expansive. New concepts, definitions, symbols, relations, proofs, and theorems are being created all the time. Mathematics is a genie that has come out of the bottle and cannot be put back. It is this characteristic that makes it so endlessly fascinating to professional mathematicians and so dauntingly frustrating to the rest of us!

★

Every stage of mathematics makes fresh demands on you. If you understand this, you can cheerfully coexist with the mathematics that lies within your comfort zone. You will then start enjoying it.

39.3 Mathophobia can be overcome

Mathophobia might have been with you since primary school. Or you might have lost your mathematical nerve later on. Cheer up! There are many keeping company with you! And mathophobia *can* be overcome.

The purpose of Chapter 38 was to demonstrate that mathematics is not some fierce beast out to gobble you up, but rather something beautiful and enjoyable that pervades nature and our lives. Popular mathematics books show how enchanting mathematics can be and you were encouraged to read a few of them. If you have thereby shed some of your reluctance to engage with mathematics, you should now be ready to confront any residual fears head on.

Because fears often arise from wrong ideas, dispelling some widespread misconceptions about mathematics will itself go a long way toward overcoming mathophobia. A cocktail of major fears that assail those who are mathematically challenged is tackled next. I then look at mistakes and their place in learning. Finally, I share with you what has worked for me in coping with and surmounting mistakes in mathematics, in the hope it might help you too.

39.4 Misconceptions about mathematics

Misconceptions about mathematics abound. They arise most often as a result of a person's first exposure to mathematics in primary or elementary school. dispelling them makes mathematics more comfortable as a subject of study.

39.4.1 Arithmetic alone is not mathematics

For many, mathematics is synonymous with arithmetic. Although numbers are still vital to mathematics, the latter day subject has grown a great deal away from its early moorings in numbers and arithmetic.[2]

Mathematics encompasses arithmetic *and a whole lot more*. You would have found this out from the books recommended in Chapter 38. Mathematics is the native tongue of Nature. Time and space, arithmetic and geometry, words and pictures—all feature in mathematics. At the risk of oversimplification, one could claim:

[2]Indeed, the highly abstract and symbolic subject of contemporary *pure mathematics* would hardly be recognizable as being mathematics to a layman, and possibly even to numerate scientists and engineers.

Mathematics is the science of patterns and the study of the logical and quantitative relationships between various mathematical objects expressed through symbols and pictures.

39.4.2 Mental arithmetic is not a prerequisite

The early fixation with arithmetic is often carried over into the gentle art of mental arithmetic, especially in primary or elementary school, where students are taught to perform computations in their head, and later tested on that skill.

Proficiency in mental arithmetic comes easily to some, but others find it difficult. Indeed, I venture to postulate from personal knowledge that many professional mathematicians are *not* skilled in mental arithmetic.

While mental arithmetic might assist in developing memory and nurturing visualization, it is not a prerequisite for mathematical competence.

39.4.3 A mathematician is not a calculating machine

One stumbling block for most people is that they are unable to add or subtract without making mistakes—even with paper and pencil. While computational accuracy might have been a necessary attribute for the mathematicians of earlier centuries, it is an anachronistic expectation in our age of calculators and computers.

Don't get me wrong. Being able to do arithmetic without making mistakes *is* a very useful mathematical skill. The ability to do error-free numerical calculations (such as a hand-held calculator effortlessly provides) is, however, not a prerequisite for becoming a competent mathematician.

> To do well in mathematics, you need imagination, the faculty for recognizing patterns, and the capacity for abstract and logical thought, more than the ability to perform calculations fast and correctly.

39.5 Vanquishing a cocktail of fears

Having dealt with misconceptions, let us now turn to fears. Before you can prosper in mathematics, you need to banish the cocktail of fears that generate mathophobia. These include fear of numbers and arithmetic, fear of symbols, fear of appearing stupid in front of others.[3] Each of these fears is a stumbling block toward academic success as a whole. You need to vanquish them all if you would travel the road to mathematical and academic success. Let us look at each in turn.

39.6 Fear of numbers and arithmetic

Numbers should not cause us to panic for the simple reason that we cannot escape them. So, rather than fear them, we have to get used to them. From the time shown by clocks to the values of our bank balances, numbers are ubiquitous and are here to stay.

Fear of numbers arises mainly from fear of arithmetic. The review in Chapter 40—of numbers and arithmetic from a more mature vantage point—should help expunge those fears.

Rote learning of addition and multiplication tables might some day become history, but at present, they are part of primary education. Fear of arithmetic—the inability to add and multiply, not to mention subtract and divide—is usually caused in early school by an inability to memorize or correctly recall tables of addition and multiplication.

It is not so much *understanding* as *memory* that needs to be buttressed. You need a decent memory to master these tables. But you also need a good memory to master spelling. And English spelling is frightfully arbitrary compared to the addition and multiplication tables. If you can memorize English spelling, these tables should be a breeze.

[3] There could be other fears as well but I will focus on these three here.

Start with the multiplication tables for two, five, ten, and nine. *Observe the patterns in each*. Then go on to the other single digit numbers. Flash cards or some other technique touched upon in Chapter 7 might help you. The trick is to progress *steadily* even if a little slowly. Do not regress. Always progress. Once you have learned by heart the tables for addition and multiplication, they will be at your beck and call for life. That is the benefit and power of memory.

If you are allowed to do so, use a calculator by all means. But if you have to perform paper and pencil calculations, or mental arithmetic, you still need to master those tables at least enough to get by.

If you still make mistakes, take a look at Section 39.10.1. Try to single-step through your calculations to find out where you *consistently* make mistakes. Then, practice until you conquer the type of error you have identified.

EXAMPLE: EXAMPLE

The multiplication tables for two, five and ten are very easy to memorize. All multiples of 2 are even. Likewise, multiples of 5 all end with 5 or 0, and multiples of 10 all end with 0. So, if you compute a product involving 10 and get 3 as the last digit, you *know* that you have an error. You can then backtrack and correct it. Develop this knack of sensing and correcting errors before they snowball. ★

Finally, as mentioned in Section 39.4.1, arithmetic alone is not all of mathematics. While numbers and arithmetic descended from our notion of *time,* shapes and geometry are derived from our sense of *space*. And *logic* is their underlying connecting thread.

This view of mathematics alone should reassure you enough to cease fearing numbers. Even if numbers seem formidable, you might fall into a more friendly relation with the mathematics of space or of logic. Mastering them would, by and by, give you confidence enough to master numbers as well. Once that primal fear is cast out, you can develop a friendly acquaintance with numbers and matters numerical—not to mention shapes and matters geometrical—away from the pressure of classrooms and examinations.

39.7 Fear of symbols

Next to fear of numbers, the fear of symbols paralyzes most people who are uncomfortable with mathematics. Even those who are comfortable with numbers are sometimes put off when the familiar letters of the Latin alphabet, or worse, the relatively unfamiliar Greek alphabet, leave their respective linguistic domains to invade the realm of mathematics.

Although letter symbols like x arose as stand-ins for *unknown fixed quantities* in the context of solving equations in algebra, over time they acquired a status akin to numbers. Thereafter they came to be used for a multiplicity of ever more abstract purposes, like representing *variable quantities,* and abstract objects like *mathematical functions,* etc.

Like a *blank tile* in a game of *Scrabble*™ [1], a letter symbol by its very indefiniteness came to subserve a variety of roles, and thereby helped unleash the mathematical imagination from its preoccupation with numbers.

As your mathematical knowledge evolves, you will find that knowing the symbols for the ten digits and the four arithmetic operations alone is not enough. You need to learn and use many more symbols. But there is no reason to get alarmed. Learn each new symbol, its meaning, and uses, as you go along. An example helps here.

EXAMPLE: EXAMPLE

Perhaps you have suddenly found out that there is a new symbol written as $\sqrt{\ }$ or $\surd$ or √. Let us apply the four stages of learning to it.

Naming: It is called the *square root* symbol. It is a stylized letter r which was a short form for the word *radix* or *root*.

Knowing: The expression $\sqrt{2}$ is an abbreviation for that non-negative number, which when multiplied by itself, gives 2.

Doing: This means $\sqrt{2} \times \sqrt{2} = 2$. Also, $(-\sqrt{2}) \times (-\sqrt{2}) = 2$. Moreover, the symbol $\sqrt[3]{\ }$ stands for *cube root* and $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$.

Being: ☻

The words square and cube arise from the days when geometry was more developed than algebra. Repeated multiplication was then denoted by the areas of squares and the volumes of cubes. Naming the symbol, recognizing it, familiarity with its etymology, understanding its meaning, and using it should allay any fears you have about this symbol. ★

39.7.1 Symbols in mathematics

Mathematics is yet *another language* with its own symbols and alphabet, which constitute its orthography. Special meanings are attached to symbols borrowed from *other* alphabets like the Latin and the Greek. And the set of mathematical symbols is expanding all the time, almost like vocabularies for spoken languages. Mathematicians keep inventing newer and newer symbols to express newer and increasingly abstract ideas and their relationships.

Mathematics has today evolved into so many different branches that even professional mathematicians can scarcely hope to understand *all* of mathematics or cope with *all* its symbols and contexts. And you certainly cannot be expected to know the symbols of higher mathematics if you have never encountered them before.

★

Suffice it if you know the symbols which you are expected to know.

39.7.2 Symbol plus context gives meaning

Mathematicians are inventing newer and newer mathematical objects all the time. Therefore, existing mathematical symbols are under strain to express these newer mathematical objects, their meanings, and their inter-relationships.

Therefore, the *same* mathematical symbol is often *overloaded* with different meanings or functions in different mathematical *contexts*. As long as you are familiar with both the symbol and its context, all you need do is to remember what its meaning is in that context. You will then be able to make sense of the *seeming* symbolic gibberish of mathematics. Again, a simple example will illuminate.

EXAMPLE: EXAMPLE

The symbol x is the standard way to denote an *unknown quantity* and is used for that purpose in equations. In the *equation* $x - 5 = 3$, if we add five to each side, we get $x = 8$. Here x stood in for an *unknown quantity* in an equation, which when solved, revealed its value to be the *fixed* value 8.

However, in the *function* $y = x - 5$, the symbol x serves the role of a *variable* which can take on *different* values. It is not a substitute for an unknown fixed quantity. And the value of x is not fixed at 8 either. Indeed, the value of x is not fixed at all; it varies, and it can take on the value 8 just as well as any other value. As x varies, so does y. So, when $x = 0$, $y = -5$; when $x = 5$, $y = 0$, when $x = 8$, $y = 3$, and so on.

This is a simple example of the same symbol x performing different roles in different contexts with different meanings in each. ★

39.7.3 Secret language must be learned

Hark back to your childhood, when you and your friends used a secret language to communicate when you played. Someone who was outside your circle would not be able to understand what you were talking about until they too were introduced to your secret language. It is the same with mathematics.

★

You need to learn the secret language to understand the meaning.

Once you understand this basic fact, you will pay attention to symbols and notation, to meaning and context in mathematics, and be comfortable with mathematical language expressed in and through mathematical signs and symbols.

EXAMPLE: EXAMPLE

As an example of how arcane and terse symbolic mathematics can get, I give below, without any explanation, the modern differential vector calculus version of Maxwell's four famous equations of electromagnetism:[4]

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{39.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{39.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{39.3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{39.4}$$

There is no need to get alarmed if you do not know what these various symbols mean or even what they are called. As I said before, consider it

[4]Named after the eminent Scottish theoretical physicist and mathematician, James Clerk Maxwell (1831–1879).

a secret language that must be learned. Suffice it if you become aware that mathematics as a language is so powerful and concise that it allows us to summarize in four elegant equations the *entire* corpus of human knowledge about classical electromagnetism, which gives us light and heat, radio and television, computers and mobile phones. ★

39.8 Fear of asking questions

Are you afraid to ask questions because you fear that others might think that you are stupid? None of us is born omniscient. We all learn as we go along. We learn by imitation. We learn by practice. We learn by doing, failing, doing again, repeating the cycle, until we can do whatever we want to *without* failing.

★

We learn by asking questions.

Give up self-consciousness when you are learning. If something is unclear to you, ask your teacher, or lecturer, or tutor to explain it to you. Let nothing prevent you from understanding your subject better.

★

Never be afraid to ask a question because you fear that others will think that you are stupid.

The really stupid ones are those who keep their ignorance concealed, even from themselves, and who later flounder in their examinations. Not only for mathematics, but for all your subjects, follow the suggestions in Algorithm 39.1.

Learning by Asking Questions

1. Do not hesitate to ask questions.
2. Ask questions boldly but courteously.
3. Keep asking questions until you understand.

Algorithm 39.1: Learning by asking questions.

39.9 Mistakes in general

It is natural to dread making mistakes, especially in examinations, where mistakes cost marks, and grades, and possibly even scholarships, and thereby a chance at a better education, and a more prosperous life. It is therefore natural to count mistakes among your enemies to a brighter future. However, while making the same mistake over and over again can and should be avoided, mistakes per se are not all bad.

39.9.1 Mistakes help learning

Do not resent making mistakes in the beginning. Mistakes are friends in disguise. We human beings learn by making mistakes. No newborn has the innate ability to walk, or talk, or calculate. All these are learned skills, and we make very many mistakes before becoming proficient in them. The feedback we get from mistakes allows us to develop our skills so that we eventually become adept at these tasks.

To expect to do something expertly the very first time is therefore not only unrealistic, it also runs counter to the mechanism by which we learn.

★

Making mistakes lays the foundation for learning.

39.10 Mistakes in mathematics

Making mistakes in mathematics is, as we have seen, a natural part of learning the subject. Those who transcend their mistakes go on to revel in mathematics. Those who are discouraged by their mistakes and refuse to outgrow them end up disliking mathematics.

Mathematics is often touted as the *only* subject in which you can score full marks. Perhaps you have classmates who *do* score full marks in it. Yet, you might be frustrated that you keep on making mistakes in mathematics despite your best efforts.

Can anything be done about this? Can you too get full marks in mathematics? Indeed, you can. After the initial learning phase where mistakes are inevitable, it is possible to graduate to a level of proficiency where mistakes are the exception rather than the rule. You simply need to cultivate the right habits to achieve this.

39.10.1 Coping with mistakes in mathematics

★

You need patience to cope with mistakes in mathematics.

Over the years, I have developed a step-by-step questionnaire for overcoming mathematical mistakes, shown in List 39.1. It arose out of the application of the W^7 framework introduced in Chapter 19. Examples of common mistakes in mathematics are discussed later, in Section 41.6.

QUESTIONS TO ASK ABOUT MISTAKES

- Have I made a mistake?
- Where is it?
- Why did I make it?
- How to correct it?
- How not to repeat it?

LIST 39.1: Questions to ask about mistakes.

39.10.2 Have I made a mistake?

The hardest question to answer is *whether* you have made a mistake. There is no guardian angel perched above your shoulders whispering to you every time you make a mistake. You have to perform that function yourself. The entire trick to mastery is to become your own guardian angel in this sense—your own censor or witness.

Develop that knack, or intuition, or sixth sense to *know* that you have made a mistake, *when* you make a mistake. It is paramount to your success. It comes with practice. It is a sensitivity that alerts you that all is not well. It is a bit like discovering that you have lost your way when you are surrounded by unfamiliar scenery. Or like feeling under the weather before you fall ill. It is a premonition that something is wrong. In Section 41.11 I strongly advise you to develop this skill and suggest how it may be done.

The sooner you become aware of a possible mistake, the faster you can retrace your steps and rectify it. You save time and conserve effort. Your target should be to become aware of a mistake almost as soon as you make it.

39.10.3 Where is it?

Assuming that you *have* made a mistake, the next step is to *discover* it. Mathematical mistakes cascade. A mistake made upstream in your solution will affect the downstream result. To locate your mistake, you must find where you *first* made it. An example will illustrate what I mean.

EXAMPLE: EXAMPLE

Assume that you are asked to work through some algebra to show that $a = b$, whatever a and b may be. If you made a mistake early on, you will find that just when you hoped the magic equation $a = b$ would pop up in your working, something else showed up instead.

To your dismay, you conclude that you have made a mistake somewhere. You then patiently trudge upstream, going through your working to find that *first* erroneous step where you *introduced* the mistake. *This is where you made the mistake.* Everything before will seem sound.

Correct that first mistake, and all your intermediate results should change, and you should end up with that magic equation $a = b$. If despite this, you are still unable to elicit that elusive equation, then you need to repeat your mistake tracing routine for one or more residual mistakes. Once all mistakes have been ferreted out, the sought after equation will come tumbling out, naturally without stress or duress. ★

It is easy to know exactly where you went wrong when others point out your mistakes to you. It might be a teacher in class, a tutor at a tutorial, a marked examination script, etc.

But if you have to detect your mistakes on your own, you need help. It could be a worked example or model answer that shows you how to solve that type of problem correctly. Mathematics texts are replete with worked examples.[5] Compare your working with that of the model answer. You will rapidly identify *the very line* where you *first* went wrong. You will develop the virtue of patience—as a beneficial side effect—as you persist in this practice. ☻

39.10.4 Why did I make it?

Once you have *recognized* your mistake and *located* it, your next task is to *analyze* it. Why did you make that particular mistake?

[5] Consult suitable books or media like CDs or DVDs, or even reputable websites for assistance. See Chapter 17, and the companion website for this book, for a selection of up-to-date, reputable links.

There could be many reasons. Did you misunderstand the question? Were you confused about a mathematical concept? Did you apply the wrong method? Did you make a careless mistake in technique? Were you rushed for time? Carefully and unhurriedly analyze the causes underlying your errors. Become sharp at this task.

Only you can know *why* you made that mistake. No one else can. No one else knows what particular doubt or confusion or misapprehension is bothering you.

★

By nailing down the cause of your error, you are taking the first step to eliminating all such errors in future.

Let me give you a contrived example on how to analyze your mistakes so that you may overcome them in future.

EXAMPLE: EXAMPLE

Suppose you are doing algebra and you have a particular habit of writing your number 5 and the uppercase letter S in almost identical fashion. Suppose also that you problem involved both 5 and S. In your haste to complete your work, you confused one for the other and generated an error. You later had to backtrack and swap S for 5 or vice versa. After that you got the correct answer.

To avoid repeating this type of error, you need to improve your handwriting so that 5 and S are distinguishable clearly even during the heat of an examination. It might seem a little incredible that coping with mistakes in mathematics could lead to a prescription for better handwriting, but it is so. Your analysis of why you made an error holds the key to never repeating it. ★

39.10.5 How to correct it?

Successful analysis of mistakes automatically leads to knowledge of how to correct them. Rectify the mistake and resume working through the problem until you get the right answer. If there are any more errors, recognize, locate, analyze, and correct *each* of them as you go along. Chapter 41 has several examples of common mistakes and how to correct them: do refer to it as well. With practice, you will be able to spot your errors without any aids.

39.10.6 How not to repeat it?

Prevention is better than cure. To not repeat the same mistake leads to rapid learning and assured success. If you do not analyze why you made the mistake, you might be condemned to repeat it many times more.

Analyze. Analyze. Analyze.

That analysis is your insurance against further recurrence.

Develop your own internal guardian angel or censor. Let a part of your mind witness what you are writing or working out all the time. If something appears amiss, it will shout out loud within you. Take heed. Look for the mistake. Correct it and proceed.

Each time you finish a set of problems, go through your working as if you were the instructor marking it. Take a look at how efficiently you tackled the problem. Criticize your own working. Streamline your steps. Develop a keen eye for common mistakes that you make. Mercilessly track down every hint of sloppiness that could lead to mistakes, and rectify it.

39.10.7 Algorithm for correcting mistakes

The answers to the questions in List 39.1 helped me develop my own recipe for coping with mistakes in mathematics, shown in Algorithm 39.2. I share it here with you in the hope that it will help you move from mistakes to mastery.

COPING WITH MISTAKES IN MATHEMATICS

Recognize that you have made a mistake.
Locate the mistake.
Analyze why you made the mistake.
Correct the mistake.
Repeat the cycle until there are no mistakes.

ALGORITHM 39.2: Algorithm for coping with mistakes in mathematics.

After a while, you will have a good understanding of the types of mistakes you make and where your techniques are weak.

> Become familiar with your areas of weakness, whether these are due to poor understanding, or careless technique, and spend more time mastering that area.

Solve more problems. Test your own knowledge by quizzing yourself. Purposely vary a question which has caused you to stumble, and see if you can work through that new question without stumbling again.

39.11 Conquering mathophobia

To help conquer mathophobia, we have dealt with several misconceptions and an assortment of fears pertaining to mathematics. While mistakes are an essential part of learning, repeating the same mistakes is counterproductive. It is only when mistakes are outgrown that mastery is achieved.

The first thing to do, if you suffer from mathematical angst, is to take stock of what makes you falter mathematically. If you cannot do this alone, seek the advice and guidance of a competent mathematics teacher, or lecturer, or tutor, or even a brilliant fellow student.

Become aware of your weaknesses. Strive to strengthen yourself in those areas. There is no silver bullet to get the right answer the first time or always. The only way is to understand the theory, practise the techniques, check your work, and correct your errors, repeating this cycle until mastery is achieved. Once you have mastered a small area of mathematics using this method, you are unlikely to stumble again because of a problem in that area.

If you patiently use this technique, shown in Algorithm 39.3, to redress every mathematical weakness you have, your mathophobia will vanish tracelessly. In its place, you will feel a joyous anticipation each time you are confronted by a delectable mathematical problem, and you will feel a confidence-reinforcing thrill of success whenever you solve it.

Just as you cannot swim if you fear water, so also you cannot excel in mathematics if you fear it.

> Mastery arises from enjoyment after fear is overcome.

CONQUERING MATHOPHOBIA

1. Become aware of where you falter; seek help if necessary.
2. a) Understand the theory,
   b) Practise the techniques,
   c) Check your work, and
   d) Correct your errors.
3. Repeat the above cycle until mastery is achieved.
4. Move on to another area of weakness and repeat.
5. Enjoy mathematics and master it.

ALGORITHM 39.3: Conquering mathophobia.

It is therefore vital to conquer mathophobia—and the misconceptions and cocktail of fears that contribute to it—completely before attempting to master mathematics.

REFERENCES

[1] *Scrabble*. Wikipedia, the free encyclopedia. URL: http://en.wikipedia.org/wiki/Scrabble, visited on 2013-05-10.

SUMMARY: OVERCOMING MATHOPHOBIA

- Mathophobia is a dread of mathematics fed by several misconceptions and fears.
- Early experiences with mathematics often puts people off the subject for life.
- The ever-expanding nature of mathematics means that new demands are made of the student at each level of learning. So, mathematics does not appeal to students like other subjects do.
- Arithmetic alone is not mathematics. You might find geometry or logic easier than arithmetic and they too are part of mathematics.
- To master mathematics you need not be a human calculating machine. Instead you should be capable of abstract logical thought.
- The ability to imagine, recognize patterns, and think logically are more important to mathematicians than the ability to do mental arithmetic.

- You need not be a human calculator to be a mathematician.
- Learn the addition and multiplication tables slowly and steadily, just as you learn English spelling, which is more arbitrary and hence obviously more demanding.
- Do not fear mathematical symbols. The meaning of each symbol is dependent on its context. Learn the secret language of the symbols that you need to know.
- Mathematics has an insatiable hunger for symbols, fuelled by its ever expanding scope. Become familiar with whatever signs and symbols you need to know rather than worry about all those you do not understand.
- Never let fear of being labelled stupid by others deter you from asking questions. Ask questions until your understanding is clear and unshakeable.
- Do not fear mistakes. Instead learn from them. Do not repeat the same mistakes.
- Seek help to identify your weak areas in mathematics and work to fortify them. For each area, patiently discover where you go wrong and why. Correct the mistakes. Practise until you get it perfect. Repeat for other weaknesses. Conquer mathophobia. Master mathematics. Enjoy the subject.

40

Arithmetic Revisited

Synopsis

Primary school mathematics is exclusively arithmetic. For most of us, arithmetic is simply memorizing tables of addition and multiplication along with the rules of the four basic operations. In this chapter, we re-visit the arithmetical foundations of mathematics and discuss not only the "how" but also the "why" of what we were taught.

The number system and place and face values are first introduced. Addition, multiplication, subtraction, and division of whole numbers are all covered in some detail. Multiplication and division of fractions precede addition and subtraction. Prime numbers, the greatest common divisor, and the least common multiple are also featured.

Some mathematical shortcuts as well as references to several good books on the topic are given. The interested student is encouraged to develop skills in fast calculation. Memory, concentration, and visualization will all be improved in the process. The chapter concludes with an overview of decimals.

40.1 The importance of arithmetic

We first encounter mathematics through numbers: that strange alphabet of ten symbols starting with none and ending with nine, capable of expressing quantities succinctly. Elementary school mathematics is almost entirely *arithmetic:* the addition, subtraction, multiplication, and division of whole numbers and fractions, with perhaps a square root or two thrown in.

Whilst arithmetic is not the whole of mathematics, it is the one aspect of mathematics that we can never evade or escape. From balancing cheque books, through calculating bills at supermarkets or restaurants, to estimating monthly expenses, arithmetic figures in our lives ubiquitously. It is most important, therefore, that our foundation in arithmetic be solid and sound.

In this age of calculators and computers—not to speak of mobile phones that can double up as calculators—why am I giving such importance to arithmetic? Speedy and accurate mental arithmetic is a necessary professional skill for accountants, but is it really necessary for the rest of us? I venture to answer in the affirmative, and here's why.

For many of us, our first encounter with arithmetic at elementary school was neither happy nor comfortable. That lack of computational fluency leaves its mark in later life as a diffidence or dread toward mathematics as a whole. If only we could revisit the past and strengthen those first, shaky arithmetical foundations into solid ones, we would gain new understanding and confidence, and start enjoying mathematics. In the process we will exorcise, once and for all, the demon of *mathophobia,* the dread of mathematics—discussed in Chapter 39.

Mastery of arithmetic is a pre-requisite if you wish to enter a college or university in the United States. Many standardized American tertiary entrance tests[1] like the SAT [1], GMAT [2], GRE [3], etc., have quantitative sections in which the candidate must work out numerical and other mathematical problems without any aids. These tests are designed to filter out those who lack fundamental literacy and numeracy. Hence they are calculator-free, and that makes sense.

Becoming proficient in arithmetic builds up your mental muscles as well. Memorizing addition and multiplication tables might seem like

[1] Several other countries also have similar entrance tests in place.

tedious drudgery, but they are still worthwhile exercises in training the memory, developing concentration, and practising visualization. And memory, concentration, and visualization are core skills, not only for all-round academic success, but also for success in life as a whole.

For all these reasons, and more, you would be much benefited if you would devote time and effort to redevelop your arithmetic skills. And there is no shortage of popular books dedicated to “speed arithmetic” or “lightning calculation” [4–8]. Most importantly, many of these books feature interesting bylanes for improving your arithmetic skills that are far more enticing and engrossing than the main road of rote memorization learned in primary school. I will touch upon some of these more enjoyable techniques later in this chapter.

To help you better understand, apply, and retain such techniques, I will first rapidly review numbers and arithmetic operations by means of examples, using a question-answer-commentary format that is also used elsewhere —in Chapters 37 and 45—in this book. Because I have tried to explain *why* you do *what* you have been taught to do, I hope the succeeding sections help you better comprehend numbers and the arithmetic operations that you learned by rote in early school.

40.2 Numbers and their meaning

Our *decimal* number system uses only *ten* digits to represent *all possible* numbers. The symbols for each digit are shown in Table 40.1.

DIGIT	NUMBER
0	zero
1	one
2	two
3	three
4	four

DIGIT	NUMBER
5	five
6	six
7	seven
8	eight
9	nine

TABLE 40.1: Our number system uses only *ten symbols* from zero to nine to represent *every* possible number. A single digit can only represent a single value from zero to nine. *Zero represents none or nothing.*

★

The single most important aspect of our number system is that we have a symbol for nothing, written as 0.

It is clear that a single digit can only represent a single number from zero to nine. How then do we write the number ten? Because we have only ten symbols—zero to nine—and because *each digit* can occupy only *one* place in a number, it is obvious that we have to use at least *two* digits to represent the number ten.

This brings us to the second most important aspect of our number system: the idea of *positional notation*. When we write a number, not only does *the value of the digit in isolation* matter, *the value of the digit in context* also matters. The actual value of a digit in a number is determined by both the digit alone and also by where that digit occurs within the number.

Face Value and Place Value

Each digit in any number has:

a face value which equals the value of the digit in isolation; and

a place value which depends on the position of that digit within the number.

Returning to the number ten, we write it using positional notation as 10. In this scheme of numbering, the rightmost digit is called the *units*[2] digit and its place value is always equal to its face value, as given in Table 40.1.

The second rightmost digit is called the *tens* digit. The place value of any digit in this position is its face value multiplied by ten. In the representation for ten, the digit 1 occupying this place has a face value of one and a place value of ten, since $1 \times 10 = 10$.

★

In writing 10 for ten, we have moved from having ten units to having one ten and zero units. This is the essence of the decimal number system.

[2]Unit means one. Sometimes the units position is also called the "ones" position.

Each place to the left after the units digit is one higher power of ten, namely, ten, hundred, thousand, etc. This positional notation gives us a way of writing *every possible whole number* concisely, unambiguously, and uniquely.[3] We now look at some examples concerning numbers using the question-answer-commentary format.

40.2.1 Example: Difference between 10 and 01

QUESTION

If 10 represents the number ten, what does the number 01 represent?

ANSWER

The number 01 has one in the units position and zero in the tens position. This number therefore has no tens and one unit and its value is simply one. Formally, its value is $(0\times10)+(1\times1) = 0+1 = 1$. Hence, the number 01 is the same as the number 1.

COMMENTARY

The following facts have emerged from this example:

ZERO, ONE, AND TEN

1. Any number added to 0 remains unchanged.
2. Any number multiplied by 0 gives 0.
3. Any number multiplied by 1 remains unchanged.
4. 01 and 10 are different numbers.
5. $01 = 1$ meaning that *we may omit leading zeros* in a whole number.
6. $10 \neq 1$ meaning that *we may not omit trailing zeros* in a whole number.

The special properties of zero with addition, and one with multiplication, confer on them the status of the *neutral* or *identity element* with those respective operations. This important and powerful concept appears and reappears in different guises and contexts in the upper reaches of mathematics.

[3] Strictly speaking, $0.999\cdots = 1$, and in this sense, uniqueness is lost. So we can represent any number in at most *two* forms.

40.2.2 Example: Face and place values

QUESTION

What are the face and place values of each digit in the number 345?

ANSWER

The number 345 has three digits. We start from the rightmost digit which is the units digit. The place and face values of the digit 5 are both equal to five, because $5 \times 1 = 5$. The digit 4 occupies the tens position and has a place value of forty, given by $4 \times 10 = 40$, and a face value of four. The digit 3 occupies the hundreds location and has a place value of three hundred, because $3 \times 100 = 300$, and a face value of three. This information is summarized in the table below:

DIGIT	FACE VALUE	POSITION	MEANING	PLACE VALUE
5	5	Units	5×1	5
4	4	Tens	4×10	40
3	3	Hundreds	3×100	300

COMMENTARY

What we are really saying here is that

$$\begin{aligned} 345 &= 300 + 40 + 5 \\ &= (3 \times 100) + (4 \times 10) + (5 \times 1) \\ &= (3 \times 10^2) + (4 \times 10^1) + (5 \times 10^0) \end{aligned} \tag{40.1}$$

The positional notation of our numbers is a shorthand for the sum written out as the right hand side of equation (40.1). *It is vital that you understand the meaning of this shorthand.*[4]

[4]Referring to numbers by means of sums is neither far-fetched nor modern nor unique to the decimal number system with its positional notation. The French say *quatre-vingt-dix-cinq* meaning "four-twenty-ten-five" for the number ninety-five, doing the arithmetic, $(4 \times 20) + 10 + 5 = 80 + 10 + 5 = 95$, implicitly when they speak. And an ancient language like Sanskrit has a synonym for the number 19 as *ekonaviṃśati* meaning "twenty deficient by one".

40.3 Addition and multiplication

Addition and multiplication are easier operations to understand than subtraction and division. Addition of whole numbers is simply counting, and multiplication is repeated addition. We now consider an example each of addition and multiplication, paying special attention to the steps involved, and to face and place values.

When we add or multiply two numbers, we write them so that their respective units digits are aligned so as to honour place values. The actual operations of addition and multiplication, however, are themselves carried out only on the face values of the respective digits. Let us work out an example of addition—perhaps the most used arithmetic operation of all—to demonstrate this.[5]

40.3.1 Example: Addition of two numbers

QUESTION

Add 25 to 336.

ANSWER

When we add, we honour place values by writing the units digits of one number under the units digit of the second number. Note that 25 has no explicit hundreds digit. *But the implicit, unwritten hundreds digit is* 0 because zero hundreds is exactly zero, as we have seen before. To emphasize that the missing hundreds digit is indeed zero, I have written it in red below. However, because leading zeros in whole numbers may be omitted, we do not write them down in routine calculations. *When we add, we start from the right and move to the left:*[6]

$$\begin{array}{rccc} & \color{red}{0} & 2 & 5 \\ + & 3 & 3 & 6 \\ \hline & 3 & 6 & 1 \\ \hline \end{array}$$

[5] Computers can only add. All other arithmetic operations are reduced to flavours of addition when performed on computers.

[6] Even though we read numbers out from left to right, we add them from right to left.

Commentary

When we add the units digits 5 and 6 we get 11. Our decimal system does not allow us to write eleven units as eleven units—as we have already seen with ten—because that would occupy *two* digit places and would defeat the scheme of face and place values.

The solution is to write the eleven as *one ten and one unit*. The one ten then becomes the digit 1 that is carried over to the left—the tens place—and when added to 2 and 3 gives a sum of 6. The 0 added to the 3 in the hundreds place gives a sum of 3. Because there are no more digits, we stop, and the number at the bottom, 361, is the required *sum*.

When we add two numbers, we are only adding the face values of the two digits in any column. The place value is taken care of by *alignment* of digits, and by *carrying* a digit over to the next column on the left when the current column sum equals or exceeds ten.

Regardless of how large the two numbers that we add are, *we only need to know how to add the face values of two digits*. The ingenious method of writing the digits in a particular order and assigning them place values according to position takes care of the actual values of the sum.

The addition of *two* numbers is a systematic process. Any such systematic, step-by-step procedure to accomplish a goal, with definite starting and stopping points, is called an *algorithm*. The algorithm for the addition of two numbers, each having one or more digits is given in Algorithm 40.1.

Adding Two Numbers

1. Align the two numbers to be added in two rows, one on top of the other, with the units digit of one above the units digit of the other.
2. Starting from the right, add the digits in each column. Treat any missing digits as zeros. If the column sum equals ten or more, carry its tens digit and add it to the next column on the left.
3. Keep repeating the above steps for the next column to the left, until all columns have been summed; then stop.
4. The result is the sum.

Algorithm 40.1: Addition of two numbers.

The upshot from this example is that when we add two numbers, however large, we only need to know how to add *two* individual digits *as* individual digits, using their face values. We need not worry about place value so long as we faithfully follow the algorithm.

When we look later at more efficient methods of addition or multiplication, depending on the algorithm, we might be required to bear in mind both the face and place values of the digits.

40.3.2 Example: Multiplication of two numbers

QUESTION

Multiply 719 by 862.

ANSWER

The number 719 is the *multiplicand,* 862 is the *multiplier,* and 619778 is the *product.* As with addition, we align the multiplicand and multiplier to preserve place value, and proceed from right to left as shown below:

$$\begin{array}{rrrrrrrl}
 & & & & 7 & 1 & 9 & \leftarrow \text{multiplicand} \\
 & & & \times & 8 & 6 & 2 & \leftarrow \text{multiplier} \\
\cline{4-7}
 & & & 1 & 4 & 3 & 8 & \leftarrow \text{multiply by 2} \\
 & & 4 & 3 & 1 & 4 & 0 & \leftarrow \text{multiply by 6} \\
+ & 5 & 7 & 5 & 2 & 0 & 0 & \leftarrow \text{multiply by 8} \\
\cline{2-7}
 & 6 & 1 & 9 & 7 & 7 & 8 & \leftarrow \text{product} \\
\cline{2-7}
\end{array}$$

We do the multiplication in three stages: one for each digit of the multiplier. The first line of products is the result of multiplying 719 by 2 where we start from the right and move left and multiply each of the digits 9, 1, and 7 in that order by 2.

The second line of products represents the multiplication of 719 by the second digit of the multiplier, namely 6. Note that we start writing the product shifted one place to the left. I have deliberately shown a zero in red to emphasize the reason behind why we do this. We are actually multiplying by 60 and the terminal zero in the units place denotes this fact. We often omit writing down this zero, but its presence explains the reason for shifting one place to the left when writing the products of the second line.

Likewise, the third line represents the multiplication of 719 by the third digit of the multiplier, 8. Here the place value of the digit 8 is actually 800. Again, the two red zeros denote this fact and explain why the products of the third line are shifted two places to the right.

The product is the sum of the three sub-products in the three lines and is 619778. Algorithm 40.2 summarizes these steps for the general case.

Multiplying Two Numbers

1. Write the multiplicand on top and the multiplier below it, aligning the digits by place value of each.
2. Multiply the digits of the multiplicand in turn, from right to left, by the units or rightmost digit of the multiplier, carrying to the left when the product exceeds nine, and write the answers aligning units digit with units digit.
3. Move one digit to the left in the multiplier and repeat step 2, writing down the product shifted one place to the left.
4. Repeat step 3 until all the digits in the multiplier are exhausted.
5. Treat all missing digits in columns as zeros. Add all the digit columns starting from the right. If the column sum exceeds nine, carry the tens digit to the left.
6. Move one column left and repeat step 5 until all columns are exhausted; then stop.
7. The resulting number is the required product.

Algorithm 40.2: Multiplication of two numbers.

Commentary

We again see that even though we are dealing with the product of two three-digit numbers, all we are ever doing is multiplying two single-digit numbers at any one time: a relatively undemanding task. Moreover, the discipline of aligning the numbers all through the computation relieves us of having to pay separate attention to preserve place value. Indeed, the zeros shown in red may be omitted so long as you shift one place to the left at each step.

★

It is most important that you understand and practise the discipline of aligning digits when you do calculations.

40.4 Subtraction and negative numbers

While addition and multiplication are intuitively easy to understand, subtraction poses some difficulties. We can take away 3 from 5, but we cannot meaningfully take away 5 from 3. The reason is that we do not have enough in 3 to take 5 away from it.

To overcome this difficulty, we introduce a prefix, or sign, to numbers. Ordinary numbers have a positive sign + as prefix and negative numbers have − as a prefix. The + prefix is usually dropped when we write a number; but the − prefix has to be retained because it is the very reason signed numbers have been introduced. With this enlargement in our scheme of numbers, we have

$$5-3=+2=2 \tag{40.2}$$

$$3-5=-2 \tag{40.3}$$

Notice that we may drop the positive sign as prefix in equation 40.2, in accord with convention, whereas we *must* show it in equation 40.3. This new notation, and new species of numbers, allows us to perform subtraction on whole numbers unimpeded by considerations of which number is the larger of the two.

The number from which we take away is called the *minuend;* the number that is taken away is called the *subtrahend;* and the result is called the *difference.*

How did we get $3-5=-2$ in equation 40.3? There are different ways of explaining this operation, at different levels of mathematical sophistication. I am going to use the most basic explanation. We actually subtract the smaller number from the larger, i.e., 3 from 5, and append a negative sign to the result. *In other words, we swap the minuend and subtrahend and append a negative sign to the difference.* You might wish to think of it as $-(5-3)=-2$.[7]

Addition and multiplication exhibit symmetries: $3+5=5+3$ and $3\times 5=5\times 3$. However, as we have seen, $5-3\neq 3-5$ because $2\neq -2$. Apart from the need for signed numbers, this is another important respect in which subtraction differs from addition and multiplication. Operations

[7] If you are interested in going into greater depth on the various roles performed by the − sign in mathematics, do take a look at Appendix C of Stein's fascinating and highly readable book *Mathematics: The Man-Made Universe* [9].

like addition and multiplication that obey this symmetry with respect to operands are said to be *commutative. Subtraction is not commutative.*

The algorithm for subtraction is similar to that for addition except that we *borrow* rather than *carry*. If the minuend is smaller than the subtrahend, we swap them around, perform the subtraction, and append a negative sign to the difference.

40.4.1 Example: subtraction of two numbers

QUESTION

Take away 837 from 1425.

ANSWER

We first check to see if the minuend is larger than or equal to the subtrahend. In our case, it is; so we proceed as usual.

We write the minuend on the top row and the subtrahend below it, aligning the units digit of each so that they are directly below each other. Leading zeros are omitted although we show them in red below for emphasizing a teaching point.

$$\begin{array}{rcccc l} & 1 & 4 & 2 & 5 & \leftarrow \text{minuend} \\ - & 0 & 8 & 3 & 7 & \leftarrow \text{subtrahend} \\ \hline & 0 & 5 & 8 & 8 & \leftarrow \text{difference} \\ \hline \end{array}$$

Starting at the units ends of both numbers, we realize that $5 < 7$, i.e., 5 is less than 7 and we cannot take away 7 from 5. You might be tempted to write -2 as the answer using signed numbers, but we do not do that here because there are more digits in the minuend from which we can *borrow*.

We borrow one from the digit 2 in the tens place in the minuend. This immediately gives us ten units at the units position in the minuend. We add these ten units to the 5 that is already there, in effect giving us 15 units, from which we take away 7 units to get 8 units.

The same process is repeated for the tens place where the 2 has now become 1. *Do not forget to decrement the number from which we have borrowed.*[8] Because 3 cannot be taken away from 1 we again borrow from

[8]This second item of bookkeeping, decrementing, is almost always the reason why more mistakes are made with subtraction than with addition. Of course, there is also the negative sign in the answer, which must not be forgotten, if the minuend is smaller than subtrahend.

the next digit to the left, 4. In this case, we get 10 tens and the 4 is now decremented to 3. The 10 tens added to the 1 already there now becomes 11 tens from which we subtract 3 tens to give an answer of 8 tens.

Next, we have 3 hundreds from which to take away 5 hundreds. Borrowing again, we have 13 hundreds. Taking away 5 from 13 gives us 8. Because we have decremented the thousands place, the last column is really $0-0=0$. The difference is 837. Algorithm 40.3 gives the general procedure.

SUBTRACTING ONE NUMBER FROM ANOTHER

1. First check to see if the minuend is larger than the subtrahend. If it is, proceed to the next step. Otherwise, swap the minuend and the subtrahend, and call them by their new names henceforth.
2. Write the minuend at the top, and the subtrahend at the bottom, aligning the units digits of each.
3. Starting at the rightmost units column, If the bottom digit is smaller than or equal to than the top digit, subtract the bottom digit from the top digit and write it down. If the bottom digit is larger than the top digit, borrow one from the top digit in the next column to the left, add ten to the current digit in the top row, and subtract the bottom digit from it and write it down.
4. Move to the next column. Decrement the top digit in the next column, if a borrow had taken place in the previous step. Repeat step 3.
5. Stop when all columns have been exhausted.
6. Affix a negative sign if minuend and subtrahend were swapped in step 1. Otherwise, do nothing. The result is the required difference.

ALGORITHM 40.3: Subtracting one number from another.

COMMENTARY

Because subtraction—with its borrow-decrement cycle—requires more mental bookkeeping, most people find it harder. Mistakes are more likely in subtraction. If you repeat the subtraction, in order to check it, you are likely to repeat any mistakes you have already made.

★

You should always check your answer by another method.

In this case, we can add the difference to the subtrahend to see if it gives the minuend. This is done below:

$$\begin{array}{rcccc} & & 8 & 3 & 7 \\ + & & 5 & 8 & 8 \\ \hline & 1 & 4 & 2 & 5 \\ \hline \end{array}$$

It is left as an exercise for you to confirm that the sum is indeed equal to the minuend, 1425.

Suppose the problem had instead been $425 - 837$. What would your answer have been? In this case, you would observe that $425 < 837$. So, you should swap minuend with subtrahend and calculate 837−425. Then you should affix a negative sign to your answer so that your difference would be −412. Work it out and confirm that it is indeed so.

40.5 Subtraction by complementary addition

Computers only add. So, how do they subtract? They do so using *complementary addition* to achieve subtraction. Computers use the binary number system to accomplish this, and that is something we will skip here. Instead, I will introduce you to subtraction by complementary addition using the decimal number system.

First we need to define the *nine's complement* of single digits. This is simply the number obtained by subtracting the digit from 9. The nine's complements of the ten single digits are shown in Table 40.2.

Digit	9's Complement
0	9
1	8
2	7
3	6
4	5

Digit	9's Complement
5	4
6	3
7	2
8	1
9	0

Table 40.2: Nine's complements of the digits from zero to nine.

What exactly does the nine's complement mean? Consider the number 5276. Its nine's complement is 4723. You can work it out digit by digit in any order. Observe that $9999 - 5276 = 4723$. For a number with three digits, we subtract it from 999, and so on.

The ten's complement of a number is one more than its nine's complement. The ten's complement of 5276 is $4723 + 1 = 4724$. Again, the meaning of the ten's complement is easily found out by observing that $4724 = 9999 - 5276 + 1 = 10000 - 5276$. So, the tens' complement is obtained by subtracting the number from a power of ten having as many zeros as the number and a one in front of all those zeros. We now look at an example on how to use the ten's complement to subtract one number from another.

40.5.1 Example: subtraction by complementary addition

QUESTION

Take away 837 from 1425 using complementary addition.

ANSWER

With complementary addition, we do not worry about whether the minuend is larger than the subtrahend or the other way round. However, we add leading zeros, if necessary, to make the number of digits in the minuend and subtrahend equal.

We wish to add the minuend to the ten's complement of the subtrahend. We first write 837 as 0837 so that both minuend and subtrahend have four digits. By inspection, or reference to Table 40.2, the nine's complement of 0837 is 9162. Adding one to this number, we get the ten's complement, 9163.

We now add 1425 to 9163 as shown below.

$$\begin{array}{ccccc l} & 1 & 4 & 2 & 5 & \leftarrow \text{minuend} \\ + & 9 & 1 & 6 & 3 & \leftarrow \text{nine's complement of subtrahend plus one} \\ \hline 1 & 0 & 5 & 8 & 8 & \leftarrow \text{penultimate step} \\ & & 5 & 8 & 8 & \leftarrow \text{drop leading one and all leading zeros after it} \\ \hline \end{array}$$

The answer is 588 which we have already found by direct subtraction in Section 40.4.1.

COMMENTARY

Why do we discard the leading digit 1? Let us recall that the ten's complement of 0837 is really 10000−0837. By adding this number to 1425 we

Subtraction by Complementary Addition

1. Inspect minuend and subtrahend. If they have an equal number of digits, do nothing. Otherwise pad the smaller number with as many leading zeros as necessary so that both numbers have the same number of digits.
2. Form the ten's complement of the subtrahend by taking its nine's complement and adding one to it.
3. Add the minuend and the ten's complement of subtrahend.
4. a) If the answer has a leading one, drop it. Strip any leading zeros. What remains is the required difference.
   b) If the leading digit is not one, add a leading zero. It signifies a negative sign, which is prepended to the ten's complement of the remaining digits, which is the required difference.

Algorithm 40.4: Subtraction by complementary addition.

are really computing $1425+10000-0837 = 10000-(1425-0837)$. Dropping the leading one is effectively taking away 10000 from the answer and gives us the difference we are seeking.

Let us now consider a case where the minuend is smaller than the subtrahend: $425-837$, again from Section 40.4.1. In this case, we follow the same principles. Because both minuend and subtrahend are three digits long, we do not pad leading zeros to either. The nine's complement of 837 is 162 and its ten's complement is 163. Now, $425+163 = 588 = 0588$. Note that we have deliberately added a leading zero to the sum. We do this whenever the leading digit is not a 1.

In complementary addition, whenever the sum has a leading zero, it tells us that the answer is negative. And the answer itself is the tens' complement of 588 which is $411+1 = 412$ with a negative sign, i.e., $425-837 = -412$ as already noted in Section 40.4.1.

40.6 Division

Addition and multiplication are arithmetically similar operations, as are subtraction and division. Just as multiplication is repeated addition, so also is division repeated subtraction. Consider for example the case of 6 divided by 2. We know that $6 \div 2 = 3$. In this expression, 6 is the *dividend,* 2 is the *divisor,* and 3 is the *quotient.* We can amplify the division as repeated subtraction so:

$6-2=4$	First lot of two
$4-2=2$	Second lot of two
$2-2=0$	Third lot of two

Observe that the difference between 6 and 2, namely 4, in the first step becomes the minuend or number from which to subtract in the second step. We stop subtracting when the difference at any step is smaller than the divisor. The difference in the last subtraction step is called the *remainder*. It is shown in red and is zero as expected because 6 is divisible by 2. The quotient is the number of times we repeated the subtraction operation before we stopped. In this case, the quotient is 3.

When you first encounter division, you would be told that there is one prohibition: *you cannot divide by zero*. If you have ever wondered why, but never been told why, perhaps the next example might clarify matters for you.

40.6.1 Example: Why division by zero is not allowed

QUESTION

Explain why division by zero is not permitted.

SIMPLE ANSWER

As we have already seen, division is repeated subtraction. Consider now what happens when we divide 6 by 0. Following the same repeated subtraction algorithm, we have:

$6-0=6$	First lot of zero
$6-0=6$	Second lot of zero
$6-0=6$	Third lot of zero
$\dots$	$\dots$

It is clear that repeated subtraction of zero—from six or any number for that matter—is a process that will never end because we end up with exactly what we started off with. We run the risk of never stopping the process of repeated subtraction.[9] Recall that an algorithm must have a definite stopping point. A process that never ends is not an algorithm.

[9]Indeed, we could say that there are infinitely many lots of zero in any natural number. But that is precisely the problem: infinity is not a number, but the label we give to something endless. Because division by zero is entangled with the idea of infinity, which is not a number, we do not define division by zero.

Because the division algorithm fails when we divide by zero, we do not permit division by zero.

ANSWER FOR THOSE WHO KNOW SIMPLE EQUATIONS

We know that any number multiplied by zero gives us zero. Therefore the equation

$$5 \times 0 = 4 \times 0$$

is true. If we allowed division by zero, we could divide both sides of the above equation by zero and end up with the obviously false statement $5 = 4$. In order to avoid such nonsense, division by zero is disallowed. The rules and conventions of mathematics are there to maintain and enforce consistency and logic above all else.

ANSWER FOR THOSE WITH STILL MORE MATHEMATICAL MATURITY

In the real number field, zero is the additive identity. It is also the only number without a multiplicative inverse. Since division by zero is multiplication by the multiplicative inverse, division by zero does not exist. In other words, division by zero is not defined.

COMMENTARY

It is important to be clear that whereas 0 cannot be a *divisor* it can very well be a *dividend*. Nothing divided into 2 or 5 or one million parts is still nothing!

40.6.2 Example: Long division

I shall introduce long division by way of an example of multiplication, albeit employing a shortcut. This is one of a large number of special cases for specific numbers[10] where we end up dividing rather than multiplying.

QUESTION

Multiply 567 by 25 using the fact that $25 = \frac{100}{4}$.

[10] That are the bread-and-butter of "lightning calculation" techniques.

PRE-COMMENTARY

The method proposed here will only work for the multiplier 25 in particular. So, the answer below is specific to this number.

ANSWER

We are told that $25 = \frac{100}{4}$. This means that multiplying 567 by 25 is the same as multiplying 567 by 100 and then dividing the product by 4. You should be knowing that multiplication by 100 is accomplished by adding two zeros to the right end of the number. So, we note that $567 \times 100 = 56700$ *mentally* and then divide 56700 by 4 using *long division* as shown below:

$$
\begin{array}{r l}
14175 & \leftarrow \text{quotient or result of division} \\
4\overline{)56700} & \\
-\underline{4}| & \\
16| & \\
-\underline{16}\downarrow & \\
07 & \\
-\underline{04} & \\
30 & \\
-\underline{28} & \\
20 & \\
\underline{20} & \\
{\color{red}0} & \leftarrow \text{remainder is zero;} \\
 & \ \text{we stop dividing when we run out of digits}
\end{array}
$$

COMMENTARY

Note that while we start from the *right* for multiplication, we start from the *left* for division. Also, whereas we *add* consecutive products in multiplication, we *subtract* consecutive products in division. These opposite operations arise because multiplication is repeated addition and division is repeated subtraction. Moreover, addition and subtraction are themselves opposite or *inverse* operations.

During division, we bring down the next digit to the right in the dividend for the next division step. This is shown, for example, by the downward arrow for the digit 7 in the dividend. Also, when we run out of digits to the right, the result of the final subtraction is the remainder. In our case, it is zero and is shown in red. If we had got a non-zero remainder, it would signal am error in calculation, because the product of two whole numbers, 567 and 25 must itself be a whole number and cannot contain any fractions. The generic procedure for long division is shown in Algorithm 40.5.

Long Division Algorithm

1. Start at the *leftmost* digit of the dividend and divide it by the divisor.
2. Multiply the quotient digit by the divisor and subtract this product from the dividend digits *at each stage.*
3. Proceed to the right, one or more digits at a time, and repeat the division, multiplications, and subtraction steps.
4. Assemble the quotient, digit by digit, at the top, aligning it with the dividend.
5. Stop when there are no more digits at the right end of the dividend.
6. The result of the last subtraction is the remainder; the number at the top is the quotient.
7. Quote the answer as a quotient and a remainder.

Algorithm 40.5: Long division algorithm.

If you found the division of 56700 by 4 too involved, you could always try direct multiplication. Indeed, because you should check your answer by another method, you should try to get the same result by direct multiplication.

40.7 Fast arithmetic

The “lightning calculation” books I have mentioned previously feature a number of special cases and special tricks, but they are all based on sound algebraic reasoning. Here is a sampling of some fast arithmetical tricks.

40.7.1 Multiplication by 11

One especially simple trick is multiplication by 11. Because $11 = 10 + 1$, multiplying by 11 reduces to adding zero to the original number and adding that number to the original number. So, $351 \times 11 = 3510 + 351 = 3861$.

If we wrote the number down as 0351 and added every digit to its right neighbour and wrote it down, except the last, which is written down unchanged, we get 3861 which is exactly what we got before. By reducing the "multiply by ten and then add the original number" routine to something that can be done *in place,* multiplication by 11 suddenly becomes even easier. Bear in mind that the leading zero helps prevent mistakes.

40.7.2 Squaring a two-digit number ending in 5

Another trick is squaring two-digit numbers that end in 5. Suppose you wanted to compute 75^2 fast. The method is to know that the last two digits of the square are always going to be 25. You then multiply the tens digit by a number one greater than itself. In this case, we compute $7 \times (7+1) = 7 \times 8 = 56$. The answer is assembled as 5625.

Does it work with three-digit numbers? Yes, but you would have to multiply the number formed by the first two digits with a number one greater than itself, and that might not be such a trivial computation. For example, with 175^2, we work out $17 \times 18 = 306$ and prepend it to 25 to get 30625. As always, the "magic" behind this algorithm can be revealed by algebraic analysis of what is going on.

40.7.3 Square roots of perfect squares

Extracting square roots by hand is tedious work, especially when you have a calculator nearby. However, if you are given a perfect square, you can guess its square root quite quickly by estimation and guessing. Suppose you are given 289, told it is a perfect square, and asked to find its square root. The first thing to do is to make the number of digits even by prepending zeros if necessary. In this case, we have $289 = 0289$. Each pair of two digits contributes one digit to the square root. So, our square root is a two-digit number.

Then we look at the first two digits and see what number squared would equal 02 or less. Obviously, that number is 1. We then have the next two digits 89. Here, it helps to know the squares of the first ten digits and the digits that they end in:

Because 89 ends in 9, Table 40.3 gives us two candidates for the second digit of the root: 3 or 7. We could compute the squares of both 13 and 17 or we could guess and try out 17 first. Indeed, $17^2 = 289$.

Digit	Square
0	00
1	01
2	04
3	09
4	16

Digit	Square
5	25
6	36
7	49
8	64
9	81

Table 40.3: The squares of the first ten digits, shown as two digit numbers. If a square ends in 0, its root must also end in 0. If a square ends in 9, its root might end in either 3 or 7, and so on. This pattern of terminal digits of squares is useful to know for calculator-free tests like the quantitative sections of the SAT, GMAT, GRE, etc.

40.7.4 "Speed mathematics" books

The book on the Trachtenberg system [5] has an interesting way to add multiple rows of numbers, in a columnwise independent manner, with error checking. You might wish to check it out. Indeed, all the books I have mentioned [4–8] are worth a browse: seriously study in them whatever takes your fancy and master it. You would then enjoy working with numbers and will count them among your friends. Your powers of memory, concentration, and visualization will also improve.

40.8 Fractions

A fraction is simply one number divided by another number, written in a manner to show that division explicitly. It is composed of three parts: a *numerator,* written at the top which is the dividend, a *denominator,*[11] written at the bottom, which is the divisor, and a line separating the two to indicate division. A fraction may be written in many similar but slightly different ways. Two thirds is written so:

$$\frac{2}{3} = \tfrac{2}{3} = {}^2\!/_3 = 2/3$$

Fractions are often a source of difficulty in early arithmetic. One reason for this problem is that unlike whole numbers, which can be expressed in only one form, a fraction has infinitely many aliases. The following chain of equalities shows that half can be expressed in a bewil-

[11] An easy mnemonic is to remember that numerator is upstairs, and denominator is downstairs.

dering variety of ways:

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \dots$$

All we did above was to multiply both numerator and denominator by 2 to get the next "version" of the fraction. And that is not all:

$$\frac{3}{6} = \frac{5}{10} = \frac{1}{2} = \dots$$

Although are all valid representations, or *equivalent fractions,* $\frac{1}{2}$ is that unique version of this fraction reduced to its *lowest terms,* meaning that numerator and denominator do not share any common factors, i.e., they are *relatively prime.*

40.9 Multiplication and division of fractions

Apart from the confusion caused by different guises for the same fraction, arithmetic with fractions poses its own challenges. Addition and subtraction of fractions, even though they are taught first, are computationally more demanding than multiplication and division. They can also be more confusing. Part of the problem lies with ideas like *least common multiple* (LCM) and *greatest common divisor* (GCD)—also called *highest common factor* (HCF)—learned early in school at an age when such concepts might have been a little hard to grasp. *You should learn multiplication and division of fractions first.* And that is the order we will follow here.

40.9.1 Example: Multiplication of two fractions

QUESTION

Multiply $\frac{2}{3}$ by $\frac{1}{4}$.

ANSWER

We multiply the two numerators together and write them as the new numerator, and do likewise for the denominators.

$$\frac{2}{3} \times \frac{1}{4} = \frac{2 \times 1}{3 \times 4} = \frac{2}{12} = \frac{1}{6}$$

COMMENTARY

Reducing the answer to its lowest terms is generally good form, even if not explicitly asked for in the question. We are now ready to state the algorithm for multiplying two fractions.

MULTIPLYING TWO FRACTIONS

1. Multiply both numerators and write it down as the numerator of the product.
2. Multiply both denominators and write it down as the denominator of the product.
3. Reduce the resulting fraction to its lowest terms where possible.

ALGORITHM 40.6: Multiplying two fractions.

40.9.2 Example: Division of one fraction by another

Division is the *inverse* of multiplication. Nowhere is this seen more clearly than with fractions. First we need to understand the notion of a *reciprocal.* All numbers, other than zero, including fractions, have a reciprocal. Simply stated, for any fraction or whole, non-zero number, its reciprocal is that number which when multiplied by the original number will give a product of 1.

Any number divided by one results in itself. So, we may equally validly write the number 4 as $\frac{4}{1}$. In this way, anything we say about fractions will also apply equally well to whole numbers. What is the reciprocal of 4? It is simply that number which when *multiplied* by 4 will give 1. We know from before that

$$4 \div 4 = \frac{4}{4} = \frac{4}{1} \times \frac{1}{4} = 1$$

So, the reciprocal of 4 is $\frac{1}{4}$.

★

The reciprocal of any non-zero number is one divided by that number.

A convenient way of getting reciprocals is to invert the fraction or exchange the numerator and denominator.

We are now ready to consider division of one fraction by another. Let us consider the same two fractions as before for our example.

QUESTION

Divide $\frac{2}{3}$ by $\frac{1}{4}$.

ANSWER

To divide by any fraction, we simply multiply by the reciprocal of that fraction. More formally, we multiply the "dividend fraction" by the reciprocal of the "divisor fraction". The reciprocal of $\frac{1}{4}$ is $\frac{4}{1} = 4$. So, to divide $\frac{2}{3}$ by $\frac{1}{4}$, we simply compute

$$\frac{\left[\frac{2}{3}\right]}{\left[\frac{1}{4}\right]} = \frac{2}{3} \times \frac{4}{1} = \frac{2 \times 4}{3 \times 1} = \frac{8}{3} = 2\frac{2}{3}$$

COMMENTARY

Note that division by a number smaller than 1 leads to a quotient that is larger than the dividend. This is meaningful if you think about it. The steps for dividing one fraction by another are given in Algorithm 40.7.

DIVIDING ONE FRACTION BY ANOTHER

1. Take the reciprocal of the "divisor fraction" by inverting it, i.e., swapping its numerator and denominator.
2. Multiply the "dividend fraction" by this new fraction.
3. Reduce the resulting fraction to its lowest terms if possible.

ALGORITHM 40.7: Dividing one fraction by another.

We have encountered some useful operations with the number one that are worth summarizing in List 40.1 because they will prove useful in later study.

40.9.3 The logical basis for multiplying fractions

Every fraction represents a division, and every division is a multiplication by the reciprocal. *So, every fraction represents the multiplication of the numerator by the reciprocal of the denominator.* This is why multiplication and division of fractions are simpler and more natural than addition and subtraction.

Operations with the Number One

1. Multiplication of any number by 1 yields the original number.
2. Division of any number by 1 yields the original number.
3. The reciprocal of any non-zero number is obtained by dividing 1 by that number.

List 40.1: Operations with the number one.

The multiplication and division of several fractions really amounts to *multiplying several numbers together*. As long as we follow the rules governing the multiplication of several numbers together, we are on solid logical foundations. In practice, multiplying a sequence of such fractions amounts to multiplying all their numerators together and all their denominators together and expressing the results as a fraction, usually reduced to its lowest terms.

40.10 Addition and subtraction of fractions

Addition and subtraction of fractions are altogether another matter. Suppose we simplistically assume that, following multiplication, to add two fractions, we should sum the numerators and divide it by the sum of the denominators.

Let us see where that gets us. Adding two halves would then not give us one whole, but rather give us one half again:

$$\frac{1}{2}+\frac{1}{2}=\frac{1+1}{2+2}=\frac{2}{4}=\frac{1}{2} \quad \leftarrow \text{Wrong! Two halves cannot equal one half!}$$

What appears naïvely justified has led to nonsense. Mathematics is based on logic. All its operations are jealously guarded by rules that maintain its logical rigour. So, it is worth stating at the very beginning what the addition and subtraction of fractions *are not*.

WRONG

$$\frac{2}{3}+\frac{1}{4}=\frac{2+1}{3+4} \quad \leftarrow \text{Nonsense!}$$
$$\frac{2}{3}-\frac{1}{4}=\frac{2-1}{3-4} \quad \leftarrow \text{Nonsense!}$$

Adding fractions is really like adding several multiplication operations together. While multiplication *distributes* over addition, i.e., $5\times(2+3) = (5\times 2)+(5\times 3)$, there is no corresponding property for addition over multiplication.

To add or subtract fractions, you must get their *lowest common denominator.* To understand how to get to that number, we make a little detour to consider prime numbers and related topics.

40.10.1 Prime numbers

Any number other than one that has only two factors, namely 1 and itself, is called a *prime number.* All other numbers are called *composite numbers.* The first ten prime numbers are $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$. All even numbers other than 2 are composite.

There is a very important theorem in mathematics, called *the fundamental theorem of arithmetic,* that states that any number can be expressed as a *unique* product of prime factors, save for the order of the factors themselves. Just so that you are clear what this means, let me explain with an example. The number 6 is composite, being the product of two prime factors, 2 and 3. So, $6 = 2\times 3 = 3\times 2$. The *order* of the factors does not matter; what matters is that they are *unique:* one 2 and one 3. The number 6 has no other prime factors.

Prime factors are essential to understanding the GCD and the LCM, which in turn are required for adding and subtracting fractions.

40.10.2 Greatest Common Divisor (GCD) and Least Common Multiple (LCM)

We have encountered these terms before in Section 40.9. Bear in mind that these two definitions apply only to *two* numbers. The GCD of, say, 2 and 6, is easily seen to be 2. We write $\gcd(2,6)=2$. Likewise, the LCM by inspection is 6, written $\operatorname{lcm}(2,6)=6$.

If we have more than two numbers, the GCDs and LCMs are obtained by chaining. Suppose there are three numbers, a, b, and c. Then

$$\gcd(a,b,c) = \gcd(a,\gcd(b,c))$$
$$\operatorname{lcm}(a,b,c) = \operatorname{lcm}(a,\operatorname{lcm}(b,c))$$

The order of a, b, and c does not matter and the same results will be obtained if we interchanged a with b etc.

There is an easy pictorial method—using Venn diagrams for multisets—for finding the GCD and LCM that is helpful if we have *two or three* numbers to consider. It is best illustrated by an example.

QUESTION

Find the GCD and LCM of 16, 30, and 24

ANSWER

We will write each number in terms of its unique prime factors so:

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$
$$30 = 2 \times 3 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

We assign the prime factors of 16, 30, and 24 to the multisets P_{16}, P_{30}, and P_{24} respectively and depict their Venn diagram representation in Figure 40.1.

COMMENTARY

With reference to Figure 40.1, confirm for yourself that the products of the numbers in each of the multisets P_{16}, P_{30}, and P_{24} do indeed give 16, 30 and 24 respectively.

The Venn diagram representation of Figure 40.1 allows us to determine by inspection the GCDs and LCMs of any two of the three numbers as well, if we confine our attention to those two numbers alone as outlined in Algorithm 40.8.

Relying as it does on a visual representation, this method is not robust and should be used *only* when the GCD and/or LCM of *two or three* numbers are required. It should not be used for four or more numbers as the Venn diagram representation can then get confusing.

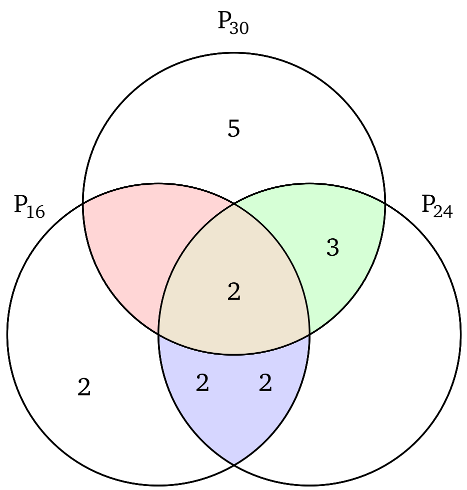

FIGURE 40.1: The GCD and LCM of two or three numbers can be found visually quite easily from a Venn diagram representation of multisets showing the prime factors of each number. The three numbers are 16, 30, and 24 and their prime factors are shown in the multisets P_{16}, P_{30}, and P_{24} respectively. Multisets allow repetition of elements, in this case prime factors. The product of the prime factors in the intersection of all three sets is 2; it is the GCD. The product of the prime factors in the union of all three sets is 240 and it is the LCM. Because zero and one are not prime numbers, this pictorial method will fail when all the given numbers are relatively prime, or when one or two of the given numbers are zero.

GCD AND LCM OF TWO OR THREE NUMBERS

1. Decompose each number into its prime factors.
2. Draw two, or three, circles to represent Venn diagrams of the multisets of the prime factors of each number.
3. Find out which prime factors go into which intersections between the pairs or triples of sets and write them there.
4. Write all remaining prime factors in the non-intersecting regions of the respective sets.
5. The GCD is the product of the prime factors in the intersection of all the sets.
6. The LCM is the product of the prime factors in the union of all the sets.

ALGORITHM 40.8: GCD and LCM of *two or three* numbers.

Suppose that two numbers had no factors in common. Then there would be no number in the intersection of their two prime factor multisets and they would be *relatively prime*. In such a case, their GCD is 1 and their LCM is their product.

The pictorial method, while being quick and serviceable for finding the least common denominator for fraction problems, will fail in certain cases, as noted above. We record the results for those cases here, without explanation, so that this treatment is complete and accurate. Let a and b be two positive whole numbers. Then,

$$\operatorname{lcm}(a,0) \triangleq \operatorname{lcm}(b,0) \triangleq 0 \tag{40.4}$$

$$\gcd(a,0) = a \tag{40.5}$$

$$\gcd(b,0) = b \tag{40.6}$$

$$\gcd(a,b) = 1 \iff a \text{ and } b \text{ are relatively prime} \tag{40.7}$$

$$\operatorname{lcm}(a,b) = \frac{ab}{\gcd(a,b)} \tag{40.8}$$

In plain English, we *define* the LCM of two numbers to be zero if one of them is zero. The GCD of two numbers, one of which is zero and the other non-zero, is the value of the non-zero number. The GCD of two *relatively prime* numbers is one; and conversely, if the GCD of two numbers is one, they are relatively prime. Finally, equation (40.8) relates the LCM and GCD of two positive numbers.

40.10.3 The lowest common denominator

We are now ready to determine the lowest common denominator of two or three fractions. We simply need to find the LCM of their denominators. This is also their lowest common denominator.

When we have a number of fractions to be added or subtracted, we must express them in an equivalent form, or alias, in which the denominator is the lowest common denominator for all the fractions. We may then add or subtract the respective numerators, write them above the single common denominator, and get a correct result. *We leave the least common denominator unchanged.*

40.10.4 Example: Adding two fractions

QUESTION

Add $\frac{1}{3}$ to $\frac{1}{6}$.

ANSWER

By inspection, 6 is a multiple of 3. Therefore, $\operatorname{lcm}(3,6) = 6$. We now need to express $\frac{1}{3}$ as a fraction with denominator 6. Again, it is easy to see by inspection that the numerator must be 2. So

$$\begin{aligned}\frac{1}{3} + \frac{1}{6} &= \frac{1 \times 2}{3 \times 2} + \frac{1 \times 1}{6 \times 1} \\ &= \frac{2}{6} + \frac{1}{6} \\ &= \frac{1+2}{6} \\ &= \frac{3}{6} \\ &= \frac{1}{2}\end{aligned}$$

COMMENTARY

If we could not find the LCM by inspection, we would have to find it by a formal method such as outlined above. Moreover, if we could not see that $\frac{1}{3} = \frac{2}{6}$, we could divide the new, lowest common denominator by the old and multiply it by the old numerator to get the new numerator. That is another way of saying $(6 \div 3) \times 1 = 2$.

Subtraction of fractions is similar: we still derive the least common denominator but now subtract one numerator from another. It is left as a trivial exercise for you to show that $\frac{1}{3} - \frac{1}{6} = \frac{1}{6}$.

★

When all fractions have been expressed in equivalent forms, with a common denominator, we then add or subtract the numerators, as required, and divide the result by that single common denominator.

40.10.5 What to do if you cannot find the LCM?

There may be occasions, where because of panic, or forgetfulness, or lack of time, or some other reason, you are not able to find the LCM. Do not panic. There is a long-winded way of adding and subtracting that never fails. The price you pay is that you need to calculate more carefully to avoid mistakes.

Adding or Subtracting Fractions

1. Find the LCM of all the individual denominators.
2. Express each fraction in an equivalent form with the LCM as its denominator.
3. Add or subtract the numerators as required, keeping the common denominator unchanged.
4. The result is the required sum or difference.
5. Reduce the result to lowest terms where possible.

Algorithm 40.9: Adding or subtracting fractions.

As before, let us compute

$$\frac{1}{3}+\frac{1}{6}.$$

Instead of computing the LCM, we could simply multiply 3 and 6, to get 18, and use it as the common denominator, although it is not the least such. The LCM we had from before was 6; and 18 is thrice that—so it will do the job.

We next need to express each fraction in terms of this common denominator by multiplying both numerator and denominator by the same factor, as shown below.

$$\begin{aligned}\frac{1}{3}+\frac{1}{6} &= \frac{1\times 6}{3\times 6}+\frac{1\times 3}{6\times 3}\\ &= \frac{6}{18}+\frac{3}{18}\\ &= \frac{6+3}{18}\\ &= \frac{9}{18}\\ &= \frac{1}{2}\end{aligned}$$

This is the brute force method because we did not find the LCM but the answer is the same as before. If ever you are deterred by having to compute the LCM for a problem requiring addition or subtraction of fractions, you can always simply multiply the denominators together and use that as the common denominator, as we have done.

Beware, though, that you would be dealing with much larger numbers than if you used the LCM, and that you would have to reduce your answer to the lowest terms anyway. Subtraction of fractions may be done similarly to addition.

40.11 Decimals

A *decimal fraction* or simply *decimal* is a fraction whose denominator is 10 or a power of 10. A number like $2\frac{1}{2}$ may be re-written as $2\frac{5}{10} = 2.5$. Numbers to the *right* of the decimal point are successively *divided* by 10 or one of its powers. Compare this with numbers to the *left* of the decimal point which are successively *multiplied* by powers of 10. The decimal point serves as a "border" to separate the whole number portion from the fractional portion of any mixed number.

The number 827.931 represents the following sum

$$\begin{aligned} 827.931 &= 800 + 20 + 7 + \tfrac{9}{10} + \tfrac{3}{100} + \tfrac{1}{1000} \\ &= 8 \cdot 10^2 + 20 \cdot 10^1 + 7 \cdot 10^0 + 9 \cdot 10^{-1} + 3 \cdot 10^{-2} + 1 \cdot 10^{-3} \end{aligned}$$

The last line in the above expression shows the beautiful consistency in the decimal nomenclature that has been adopted.[12]

Numbers with fractions are written with a decimal point and are accommodated without extra effort by aligning the decimal points to preserve place values. With this device, we may now represent not only whole numbers, but also numbers with fractional parts as decimal numbers in which the digits have place values.[13] This concludes our brief foray into the very bowels of mathematics: fundamental arithmetic.

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[1] *SAT—College Board*. The Most Widely Used College Admission Exam. URL: http://sat.collegeboard.org/home, visited on 2011-10-19.

[2] *The Official GMAT Website*. URL: http://www.mba.com/, visited on 2011-10-19.

[3] *GRE*. Educational Testing Service. URL: http://www.ets.org/gre, visited on 2011-10-19.

[12] If you have not encountered the index or power notation just yet, you certainly will, later in your studies.

[13] *Irrational numbers* also have decimal representations, but they are neither finite nor recurring.

[4] Henry Sticker. *How To Calculate Quickly*. Dover, 1955.

[5] Jakow Trachtenberg. *The Trachtenberg Speed System of Basic Mathematics*. Trans. by Ann Cutler and Rudolph McShane. London: Souvenir Press, 1962. Reprinted 2011.

[6] Gerard W Kelly. *Short-Cut Math*. Dover, 1984.

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[8] Arthur Benjamin and Michael Brant Shermer. *Mathemagics: How to Look Like a Genius Without Really Trying*. Los Angeles: Lowell House, 1993.

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SUMMARY: ARITHMETIC REVISITED

- Arithmetic has fundamental importance both in mathematics and in our daily lives.
- The decimal number system that we use has 0 as a symbol for nothing.
- Digits in decimal numbers have a position-independent face value, and a position-dependent place value.
- Addition and multiplication are accomplished from right to left, digit-by-digit, after careful alignment of the units digits of all numbers. Subtraction and division proceed likewise, but with subtraction taking the place of addition in their respective algorithms.
- Fractions are multiplied by multiplying numerators together and denominators together and dividing the former by the latter.
- To divide one fraction by another, multiply the formed by the reciprocal of the latter.
- The numbers zero and one hold a unique place among numbers: adding zero to any numbers leaves it unchanged; multiplying any number by one also leaves it unchanged.
- Multiplying and dividing fractions is easier than adding or subtracting fractions.

- The product of two fractions is the new fraction obtained by multiplying the respective numerators and denominators.
- To add or subtract two fractions you need to find the LCM and use it as denominator before adding or subtracting the numerators.
- The greatest common divisor (GCD) and the least common multiple (LCM) of two or three numbers may be conveniently found by using Venn diagrams depicting the prime factors of the multisets of these numbers.
- Mixed numbers may also be expressed as decimals by inserting a decimal point at the correct place and performing the four arithmetic operations on them, respecting alignment of decimal points.
- Some examples of fast arithmetic are given along with references to several books devoted to the topic.

41

Solving Mathematical Problems

Synopsis

Systematic problem solving is perhaps *the* core mathematical skill.

My personal, deceptively simple, three-step algorithm for problem solving is to read the question carefully, understand it correctly, and answer it precisely.

Write down every step. Check your working every few steps. Verify your answer by another method. Become adept at discovering and correcting your errors on your own.

Common mistakes in manipulative mathematics like sign changes, missing terms, mathematical formulation of word problems, forgetting one part of the answer, etc., are also discussed in this chapter, with examples.

A discussion of common misconceptions, due to wrong extrapolations from known to new areas in mathematics, concludes the chapter.

41.1 From primary to high school

The transition from primary to secondary school brings with it greater challenges across all subjects. In primary school, learning is centred around fun and discovery. In secondary and high school, it becomes a more serious pursuit. The subject matter is more advanced, books weigh more, homework is routinely assigned, and tests and assessments are more frequent.

But paradoxically, some subjects become more interesting and enjoyable in secondary school, and mathematics is one of them. The reason for this is simple: you would be reaping the harvest of the hard work done in primary school.

You would already have memorized the addition and multiplication tables and gained enough competence in basic arithmetic. You would have crossed intellectual chasms like the transition from arithmetic to algebra, from word problems to mathematical formulations, from planar to solid geometry, and so on. You would have encountered sufficient variety in mathematical nomenclature and notation to be unfazed by new ideas and symbols. You would also have developed a taste for solving a large variety of problems and started savouring the confidence and joy of success.

Most of all, you would have learned that mathematics is not a spectator sport; it can be mastered only by solving problems.

41.2 Expanding mathematical horizons

After the first six or seven years of formal schooling, your encounters with mathematics would have taken on a slightly different complexion. The changes you would have encountered in middle and high school include:

1. Extension of the concept of number to include negative, irrational, and complex numbers.

2. Use of alphabetic letters in mathematical expressions to represent fixed but unknown numerical quantities to solve equations.

3. Use of alphabetic letters to represent variables rather than fixed unknowns in functions.

4. Solution of a wide variety of non-trivial word problems that require a sizeable mathematical arsenal for their solution.

5. Introduction to calculus and the techniques of integration and differentiation to solve practical problems.

6. Use of set theory as the common leitmotif for mathematics henceforward.

You mathematical horizons have expanded, becoming more abstract, more general, and more powerful. The cost is that you need to keep abreast of an ever-expanding subject that appears to mutate in appearance and function to cope with its new domains.

In all of this change, though, there is one constant: *problem solving.* It is the mainstay of mathematics and is something that you need to master. The purpose of this chapter is to help you master mathematics at high school level and beyond by developing the core mathematical skill of problem solving.

41.3 Problem solving

Most of us deal only with the *computational* or *manipulative* aspects of mathematics to solve clearly defined problems. Even scientists and engineers, who apply mathematics daily in their professional lives, usually find themselves more as problem solvers than as explorers in search of new beasts in the mathematical jungle.

41.4 Books on problem solving

Mathematical problem solving has established methods. It is a subject that merits its own book, and it would be unrealistic to go into details here. There are many excellent books on problem solving that can greatly enrich your mathematical abilities. I give below a small sampling of such books as I am acquainted with. Do read them.

41.4.1 Pólya

The modern progenitor of problem solving books is the perennial and venerable *How to Solve It: A New Aspect of Mathematical Method* by George Pólya [1]. This gem of a book is divided into three parts and has a crisp, no-nonsense, almost algorithmic approach that will reward you with insights not only into mathematical problem solving, but also generic problem solving.

Pólya divides mathematics problems into two classes: *"problems to find"* and *"problems to prove"*. If you have time to read only one book on problem solving, let it be this; and if you cannot read all three parts, at least read the short, second part in full, and dip into the other two when you find the time.

41.4.2 Zeitz

If you find Pólya's book too abstract, and are in search of a more concrete—but no less effective—introduction to problem solving, I recommend Zeitz's *The Art and Craft of Problem Solving* [2]. It is an excellent and very readable text which covers a wide array of problems. If you have difficulties with mathematical problems, do yourself a favour by *reading and working through* Zeitz's book.

41.4.3 Briggs

The third recommended book on problem solving is *Ants, Bikes, and Clocks: Problem Solving for Undergraduates* [3] by William L Briggs. It too draws on Pólya's ground-breaking work on problem solving and features an eclectic selection of problems with careful explanations and exemplary solutions. You should literally follow the author's wise advice: "*Remember that very little is gained by reading the solution to a problem before seriously attempting to solve it.*" [3, p 3].[1] It encapsulates the fact that mathematics is not a spectator sport.

41.4.4 Other books

Problem Solving Strategies for Efficient and Elegant Solutions by Posamentier and Krulik [4] bears the subtitle *A Resource for the Mathematics Teacher* but do not be put off by that. The book is sufficiently detailed to give hands on assistance to the enthusiastic student. Its ten chapters suggest a wealth of problem solving strategies that the high school and undergraduate mathematics student could profit from immensely. Another fine and comprehensive book on generic problem solving methods is *How to Solve Mathematical Problems* by Wickelgren [5].

[1] Original, author's italics, not mine.

41.4.5 Competition books

Mathematics competitions, such as national and international Olympiads, are a never-ending source of new and demanding problems that will challenge the gifted high school student and undergraduate. Many excellent books have been written to prepare students for these prestigious competitions.

First among these is one written by the eminent mathematician and Fields medallist, Terence Tao, entitled *Solving Mathematical Problems: A Personal Perspective* [6]. This is the second edition of a book that was first written before the author became a professional mathematician. It is rare for a mathematician of such eminence to address a book specifically to the high school student on a subject as generic as problem solving. The author takes us on a guided tour of his thought processes as he discusses different problems, and this is perhaps the most valuable feature of the book. The author has also written an interesting blog on solving mathematical problems [7] which contains some very helpful Web links. Both book and blog are highly recommended.

Problem Solving Strategies by Engel [8] is another fine book devoted to problem solving geared to students preparing for mathematics competitions. There is also an excellent series of books published by the Australian Mathematics Trust in their *Enrichment Series* [9–12]. These books are very reader-friendly and avoid the perplexing terseness of traditional mathematics texts. They sometimes cover the underlying theory before elucidating the solution and, on occasion, demonstrate dead-end approaches, and how to recover from them.

Two other competition-based—but mathematically more demanding—texts are the *The Red Book of Mathematical Problems* [13] and its companion *The Green Book of Mathematical Problems* [14] by Hardy and Williams.

41.5 Algorithm for problem solving

My personal algorithm for problem solving is deceptively simple. It consists of three steps shown in Algorithm 41.1:

Of course, this applies not only to mathematics but to all your academic subjects. Once you have assimilated this algorithm into your system and made it your own, it will prove very serviceable in all your studies. It is also invaluable in examinations.

PROBLEM-SOLVING

1. Read the question carefully.
2. Understand the question correctly.
3. Answer the question precisely.

ALGORITHM 41.1: Algorithm for solving problems.

Applying the algorithm is perhaps a little more involved than at first meets the eye. You need to give time, space, and importance to *each* step. You cannot tumble from one to another but must progress at a deliberate and dignified pace from one step to the other. Let me explain further.

41.5.1 Read the question carefully

The first step in the problem solving algorithm is to read the question carefully. Let the import of the question sink in. Reading a question is like listening to someone else speak. You do not interject or speak when the other person is speaking. Your only job is to listen. So too with reading questions. Give the question time to speak to you.

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When you read a question listen to what it has to say while remaining silent yourself.

A computer analogy will help here. When you read, you are in *input mode*. When you write, you are in *output mode*. In between, when you attempt to understand what is required from you, you are in *processing mode*. Keep each of these steps separate. Do not conflate reading with understanding or with writing.

Do not let your mind wander and think about the answer *while* reading the question. If you do, you are likely to misunderstand or misinterpret the question because your attention is not wholly centred on reading it. Develop the discipline of attentive and deliberate reading so that no carelessness can slip in to confound or cloud your understanding.

41.5.2 Understand the question correctly

Having read the question, deliberate on what you have read. This is the understanding part of the problem solving algorithm. All the mathematical knowledge that you have patiently acquired will now be applied. An example will help here.

QUESTION

Solve the quadratic equation $x^2 + 3x - 28 = 0$ by *completing the square.*

PRE-COMMENTARY

Whether or not the words are italicized as shown in the question, they should be *highlighted in your mind* when you mull over the question and try to understand it. You should then go through something like this in your mind:

1. Why is the given equation a quadratic equation?
2. What does completing the square mean?
3. Is there an algorithm or formula to be applied?
4. How might I apply it?

The technique for completing the square is outlined in Algorithm 41.2.

SOLVING A QUADRATIC BY COMPLETING THE SQUARE

1. Express the quadratic equation in the form *q(x) = 0.*
2. Move the constant term to the right of the equation.
3. Divide all terms by the coefficient of x^2.
4. Now halve the coefficient of the term containing *x.*
5. Square it.
6. Add it to *both* sides of the equation.
7. Rewrite the terms on the left of the equation as a perfect square of the form $(x - k)^2$.
8. Take square roots on both sides and solve for *x* taking into account both positive and negative square roots.

ALGORITHM 41.2: Solving a quadratic by completing the square.

NOTE

I have set out the thought process in great detail above simply to illustrate what is involved in the understanding step. But if you really know your stuff, it would all come tumbling out in the twinkling of an eye, as indeed it should.

ANSWER

$$\begin{aligned}
x^2 + 3x - 28 &= 0\\
x^2 + 3x &= 28\\
x^2 + 3x + \left(\frac{3}{2}\right)^2 &= 28 + \left(\frac{3}{2}\right)^2\\
\left(x + \frac{3}{2}\right)^2 &= 28 + \frac{9}{4}\\
\left(x + \frac{3}{2}\right) &= \pm\sqrt{\frac{121}{4}}\\
x &= \pm\frac{11}{2} - \frac{3}{2}\\
x &= -7 \text{ or } x = 4
\end{aligned}$$

COMMENTARY

Your very first instinctive reaction on *seeing* the equation would have been to recognize it as an easily factorized quadratic that could be solved by factorization like this:

$$\begin{aligned}
x^2 + 3x - 28 &= 0\\
(x + 7)(x - 4) &= 0\\
x &= -7 \text{ or } x = 4
\end{aligned}$$

But doing so would *not* be answering the question. So resist that impulse. If you had freedom to choose the method of solution, simple factorization such as the above would indeed be the method of choice. But it cannot be applied in this case, except to check your answer. You *have to* use the method of completing the square because the question asks you to. Do not jump hastily to answer the question before understanding it fully.

Moreover, *any* quadratic $ax^2 + bx + c = 0$ may be solved using the *quadratic formula* which gives

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Even though this formula itself was derived using the method of completing the square, it too may *not* be used in the answer. It is vital to understand the question clearly so that it is answered precisely.

41.5.3 Answer the question precisely

You have just seen an example of what it means to answer the question precisely. Mathematics, more than other subjects, exacts a discipline of thought and expression from you. It is in answering questions that all your carefully cultivated good mathematical habits will come to the fore.

In the succeeding sections, I will outline some of these good habits that you should nurture. But before that, to provide some context, let us review some all too common mistakes in mathematics.

41.6 Common mistakes in Mathematics

Certain mathematical manipulations, routinely used for problem solving, are more prone than others to make us slip. These *common mistakes in mathematics* could also be some of those that *you* most commonly make when solving problems. Know these latter errors like the back of your hand. Classify them. Then take steps to weed them out so that your personal error list becomes null. A number of these common mistakes in mathematics are listed and discussed below.

41.6.1 Misunderstanding questions and answering them wrongly

Maybe you have been asked to find x but you have found y instead, which was not asked for. Or you might have misunderstood a word problem and got the solution by division, when it should have been multiplication. Or you might consistently leave out part of an answer in your haste to complete the problem. There could be a dizzying array of reasons for this type of error. One example of this sort of error was illustrated in Section 41.5.2.

You should patiently and carefully sift out what went wrong, where, and why, and how to correct it. Practise until you become expert at troubleshooting your own errors. Prevention is better than cure.

Spend time in reading the question carefully as suggested in Section 41.5.1 and in understanding it clearly as outlined in Section 41.5.2. Attempt an answer only after both these stages have been completed.

41.6.2 Sign changes in algebraic manipulations

Negative signs and operations involving terms like $-(-x)$, for example, trap many students. Sometimes, you might have forgotten to reverse an inequality after multiplying both sides by a negative number. Again, classifying your errors by cause and type will help you iron out the wrong mathematical habits from your system and set you on the road to success. Let us review some of these errors in slow motion.

Algebraic manipulations usually involve solving equations or making one variable the subject of a formula. In both cases, we need to *transpose* or move terms from one side of the equals sign ($=$) to the other. To achieve this, we perform *identical* operations on *both* sides of the equation.[2] Here are some examples.

QUESTION

Solve the *linear* equation $5x - 2 = 2x - 11$.

PRE-COMMENTARY

A linear equation is one in which the highest power of x that occurs is one. This particular equation has terms involving x on both sides of the equals sign. Solving requires moving terms so that those containing x are all on one side and constants all on the other. Think of weighing fruit on a two pan balance with the fruit on one pan and standard weights on the other.

[2] Imagine a two-pan weighing balance in equilibrium. Adding or taking away the *same* weight from *both* pans will still preserve equilibrium.

ANSWER

We gather all terms involving x on one side and all the numbers, or constants, on the other side of the equals sign so:

$$\begin{aligned}
5x-2&=2x-11 && \text{; subtract } 2x \text{ from both sides}\\
5x-2-2x&=2x-11-2x\\
3x-2&=-11 && \text{; add 2 to both sides}\\
3x-2+2&=-11+2\\
3x&=-9 && \text{; divide both sides by 3}\\
x&=\frac{-9}{3}\\
&=-3
\end{aligned}$$

COMMENTARY

You would have noticed above that subtracting $2x$ from both sides has the net effect of *moving the* $2x$ *from one side of the equals sign to the other, while at the same time changing its sign.* Likewise for adding 2 to both sides. This observation gives rise to the rule that when we move a term from one side of an equation to the other, we must change its sign. Beware though: this sign change applies *only* to terms that are added or subtracted and *not* to terms that are multiplied or divided.

QUESTION

Solve the equation

$$x^2+5x-3=2x^2+3.$$

PRE-COMMENTARY

This is a *quadratic* equation. We expect it to have two distinct real or complex roots or a single repeated real root. In any case, we again need to rearrange the terms so that we have an expression where the left hand side (LHS) has all the algebraic terms and the right hand side (RHS) is equal to zero. This involves bringing the $2x^2+3$ to the left hand side.

OOPS! WRONG ANSWER

Suppose we write:

$$\begin{aligned} x^2+5x-3 &= 2x^2+3 \\ x^2+5x-3-2x^2+3 &= 0 \quad \leftarrow \text{Can you spot the error here?} \end{aligned}$$

PRE-COMMENTARY CONTINUED

We remembered to change the sign of the $2x^2$ but forgot to change the sign of the 3 to -3. *This is a very common mistake.* You must avoid it by being forewarned, by practice, and by being habitually careful.

If you are perceptive, you might have already realized that the term in x^2 is going to be negative when we move terms from right to left. You might feel more comfortable with a term in x^2 that is positive. This is a purely human phenomenon: it has nothing to do with mathematics. And mathematics gives you the freedom to do what you are most comfortable doing.

This means that we could also equally correctly move the terms from the left to the right if we so desire and set the left hand side of the equation to zero. The choice is ours. Only bear in mind that the sign of *each* transposed term is changed from + to − or vice versa during transposition.

ANSWER: PART 1

$$\begin{aligned} x^2+5x-3 &= 2x^2+3 \\ 0 &= 2x^2+3-x^2-5x-(-3) \quad \text{; swap right and left} \\ 2x^2-x^2-5x+3+3 &= 0 \end{aligned}$$

COMMENTARY

Even if we mechanically change signs of transposed terms as a practical manipulative technique, we should remember that we are in reality adding the same term to both sides, or subtracting it from both sides. The mumbo jumbo of solving equations is thereby tethered to its logical basis.

Note also that $-(-3) = +3 = 3$. These are established conventions of mathematics that allow us to manipulate negative numbers without altering the results for positive numbers that we are already familiar with. The rules are simple:

(a) *the sign of a positive number is not necessarily shown but do not forget its existence.* So, the x^2 term has an implicit $+$ sign that must be negated during transposition.

(b) Also, *the negative of a negative number is positive* by convention. Utility has again defined this convention: it is so because it just works.

(c) Likewise, $+(-3) = -(+3) = -(3) = -3$. This too is a convention adopted because it works, and is consistent with everything that existed before it historically.

(d) We have also swapped the left and right hand sides in the third line of the equation sequence above so that the *right hand side is now equal to zero.* This is allowed because $a = b$ is logically equivalent to $b = a$ as a consequence of the meaning of "equality" represented by the "$=$" sign.

Why did we swap sides for the zero? Because most right-handed people seem more comfortable solving an equation whose right hand side is zero. Remember that mathematics has nothing to do with it. This is a purely human convenience allowed to us by mathematics.

ANSWER: PART 2

Continuing, we have:

$$\begin{aligned} x^2 + 5x - 3 &= 2x^2 + 3 \\ 0 &= 2x^2 + 3 - x^2 - 5x - (-3) \\ 2x^2 - x^2 - 5x + 3 + 3 &= 0 \\ x^2(2-1) - 5x + 6 &= 0 \\ x^2 - 5x + 6 &= 0 \end{aligned}$$

COMMENTARY

By now you should have realized that $x^2 = 1x^2$. By mathematical convention we leave out the constant coefficient 1 when writing algebraic expressions. The equation $x^2 - 5x + 6 = 0$ may be solved by any number of techniques. It is not my intention to go into them here. But the roots of this equation are $x = 2$ or $x = 3$, both of which can be checked by direct substitution into the original quadratic equation.

Let me reiterate that sign changes only apply to cases where we *add or subtract* identical terms from both sides of an equation. *No sign change is involved when we divide or multiply both sides of an equation by the same number.*

QUESTION

Solve the equation $-4y = 24$.

ANSWER

The standard way of solving this equation is to divide both sides by the *coefficient* of y, which in this case is -4. So, the solution proceeds as shown:

$$\begin{aligned}
-4y &= 24 && \text{; divide both sides by} -4\\
\frac{-4y}{-4} &= \frac{24}{-4}\\
y &= \frac{24}{-4}\\
&= -6
\end{aligned}$$

COMMENTARY

Note that we divide *both sides* by -4; we do *not* change its sign. It is only in the context of addition and subtraction that we change the signs; we *do not* change signs when multiplying or dividing. You should also be knowing that the product or quotient of two negative numbers is positive. That is why $\frac{-4y}{-4} = y$.

41.6.3 Inequalities

Inequalities behave differently from equalities. When we move a term from one side of an inequality to the other, the standard sign changes still apply. *But when we divide or multiply both sides by a negative number the inequality sign is reversed.*

EXAMPLE: EXAMPLE

Consider:

$$\begin{aligned} 5 &> 3 \quad \text{when multiplied or divided by } -1 \text{ becomes} \\ -5 &< -3 \end{aligned}$$

When manipulating inequalities, it is easy to forget this and run into errors. Beware! ★

41.6.4 Missing terms

When solving a problem, you might fail to write down one or more terms at some point in the written solution. From that point onward, whatever you write will be erroneous. While examiners will generally give you marks for your working, at least until the point where terms were omitted, you would still definitely lose some marks.

No mathematics is involved in this type of error. It is a purely mechanical or clerical error of failing to write down all terms in a calculation or solution. Only human beings make this sort of careless error.

To avoid this type of error, *check your work carefully every few steps.* If you make this a habit, you will trace your omitted term almost as soon as you omit it. You can then backtrack quickly and correct for the omission and complete the problem error-free.

★

Be careful.
Check your answer every few steps.
Practice.
Avoid omissions.

41.6.5 Reformulating word problems into mathematics

In the real world, problems are not stated as equations or inequalities. They are stated in words and we need to reformulate them mathematically. In the process, mistakes may be made. Let us start with a trivial problem.

QUESTION

How many apples are *left over* when 11 apples are divided among 5 people?

COMMENTARY

This is clearly a division problem where we are asked for the *remainder* not the quotient.

Because most division is a quest for the quotient, we might wrongly state the answer as 2 rather than the required remainder which is 1. As always, *care in reading and understanding the question* will help avoid this mistake. You cannot answer a question correctly if you misread it or misunderstand it.

Take time to read carefully and understand correctly before answering any question. This advice applies especially to word problems in mathematics.

41.6.6 Confusion with symbols and notation

Mathematical symbols and notation are exacting and can trap the unwary. A single example involving matrices and determinants is given below, but the principle being illustrated applies across all of mathematics. You cannot afford to be sloppy when reading, understanding, or writing mathematics because the meaning can change. You need to be fastidious and precise.

EXAMPLE: EXAMPLE

A rectangular array of numbers written between two enclosing parentheses or brackets represents a mathematical object called a *matrix*. Uppercase letters are customarily used to designate matrices. For example, a simple *square matrix* of two rows and two columns, named A, is shown below, enclosed in parentheses and also in brackets, as both forms are admissible:

$$\mathrm{A} = \begin{pmatrix} 1 & 3 \\ 4 & 8 \end{pmatrix} = \begin{bmatrix} 1 & 3 \\ 4 & 8 \end{bmatrix}$$

Matrices have their own rules for the four arithmetic operations, underscoring their separate identity as *distinct mathematical objects.*

Associated with any *square* matrix, A, is a related entity, called the *determinant* of the matrix A, written $\det \mathrm{A}$. This is denoted by the *same* rectangular array of numbers as the matrix, but enclosed this time by a pair of vertical lines, rather than parentheses or brackets as shown below:

$$\det \mathrm{A} = \begin{vmatrix} 1 & 3 \\ 4 & 8 \end{vmatrix} = 8 - 12 = -4.$$

While this change in notation from matrix to determinant is visually minimal—replacement of parentheses or brackets with vertical straight lines—the change in meaning is profound. *Whereas the matrix is a distinct mathematical object, the determinant of a matrix is but a single number.* ★

41.6.7 Confusion with inverse operations

Mathematics has inverse operations. Subtraction is the inverse of addition. So, if you add b to a and then subtract b from the result, you end up with the a that you started off with. Multiplication and division are inverse operations. Differentiation and integration are inverse operations, as are exponentiation and taking logarithms.

In primary school there might have been confusion between *direct proportion* and *inverse proportion.* If you never got over that confusion, you are likely to carry over your confusion as you encounter inverse operations in other areas of mathematics.

Sometimes, due to pressure of time, or examination panic, or some other reason, you might perform the inverse operation to the one asked for. In such cases, you might need to spend more time to slowly learn and assimilate the difference between, for example, *integrating* x^2 with respect to x and *differentiating* x^2 with respect to x.

★

Do not confuse an operation with its inverse. Familiarity and practice are a panacea for a host of mathematical maladies.

41.6.8 Forgetting one part of the complete answer

We have seen that a quadratic equation generally gives *two* valid answers as the solution. Unless you are dealing with a real repeated root, if your working showed only one answer, you are missing something. Pay attention to such problems. The same error can appear in answers to questions that are in slightly different guise.

QUESTION

Solve $x^2 - 4 = 0$.

HURRIED ANSWER

You might "see" the answer $x = 2$ leaping at you even as you read the question.

COMMENTARY

While correct, it is not the whole answer. Always solve equations using systematic methods. That way, you will not forget any valid answers that "did not leap at you."

CORRECT ANSWER

The proper way to solve this equation is to factorize the quadratic as $x^2 - 4 = (x+2)(x-2) = 0$ which clearly has the two solutions $x = \pm 2$. Alternatively, you could say $x^2 - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm\sqrt{4} = \pm 2$.

COMMENTARY

There is a convention that the square root of a number by definition denotes the positive square root. So, it is necessary to append the $\pm$ sign to denote both the positive and negative square roots. The same principles apply to questions like finding the cube roots of unity, etc.

EXAMPLE: EXAMPLE

Still another example arises when you are asked to do indefinite integration or antidifferentiation. Say you are asked evaluate

$$\int x^2 \, \mathrm{d}x$$

and being familiar with integrating polynomials, you immediately write

$$\int x^2 \, \mathrm{d}x = \frac{x^3}{3} \qquad \leftarrow \text{Can you see what is missing here?}$$

If you have been on your toes you would know that the arbitrary *constant of integration,* usually written as C was left out above. So, the complete solution is

$$\int x^2 \, \mathrm{d}x = \frac{x^3}{3} + C$$

If you miss parts of answers, it costs you marks, and could rob you of a perfect 100% score in your mathematics paper. ★

★

Practice until you always write down the complete answer as a reflex action.

41.6.9 Enumeration problems

Problems involving choice and chance are notorious for trapping the unwary student. This is usually because you have not enumerated *all* possibilities to answer correctly.

QUESTION

Two unbiased coins are tossed. What are the chances of two heads, two tails, and one head and one tail?

HURRIED ANSWER

Because there are three outcomes, you might be tempted to answer that each outcome has a probability of $\frac{1}{3}$, but you would be *wrong.*

ANSWER

The proper approach is to *enumerate all four* possibilities: HH, HT, TH, TT (where H and T stand for a head and tail respectively). It then becomes clear that two heads and two tails both have a chance of $\frac{1}{4}$ each whereas, one head and one tail can occur in *two ways* and therefore has a chance of occurring of $\frac{2}{4} = \frac{1}{2}$.

41.6.10 Wrong approach

Sometimes, you might have taken a wrong approach to solving a particular problem and enter the “desert that leads nowhere.” This is especially true if you are taking more advanced mathematics units or if you are preparing for a mathematics competition. In such cases, it is very helpful if you have been faced with that particular *type* of problem before. The most efficient approach will then be known to you and you will zip along to the right answer.

If you are not in an examination situation, attempt the question using the most sensible and logical approach. If it leads nowhere, try another angle. Do this until you crack the problem with an "aha" or "Eureka moment" [15]. Then when you see the same type of problem in an examination, you will not waste precious time exploring futile approaches.

★

Solve as wide a range of problems as your time and syllabus permit so that your mathematical arsenal is varied, efficient, and effective.

We have seen an example of heads and tails above, where although enumeration of the four events is simple, it could still trap the unwary student. Now, I will give another example, again from combinatorics and probability, where the number of possibilities is far greater than four, and where adopting the wrong approach could cost minutes and marks.

QUESTION

There are three white balls, three red balls, and three black balls in an urn. What is the probability that if you pick three balls, one after another, they are of at least two colours?

PRE-COMMENTARY

To answer this question, you could start enumerating the ways in which the picked balls would be of two colours, and then of three colours and then add the probabilities of all these mutually exclusive events. And you would have to do this for all colour combinations. That direct approach is not wrong, but it is tedious because of combinatorial explosion, prone to error, and time consuming. Is there a better way?

As it happens, there is. We do not look at the desired event, but at its *complement*. The event we want to avoid is one where all the three balls are of one colour. And we need to do this for *each* of the three colours because they are mutually exclusive events. Note that all three balls are not picked at once, but rather one after another.

ANSWER

First, let us consider the white balls. We assume that one white ball is indistinguishable from another. To start with, there are three white balls and nine balls in all. The probability that the first ball is white is therefore $\frac{3}{9} = \frac{1}{3}$.

Now there are two white balls and a total of eight balls left in the urn. The probability of picking a second white ball is $\frac{2}{8} = \frac{1}{4}$. Finally, we have one white ball and seven balls in all left. The probability of picking the third white ball is therefore $\frac{1}{7}$. Since the three events are independent, their joint probability is their product, which is

$$\frac{1}{3} \times \frac{1}{4} \times \frac{1}{7} = \frac{1}{84}$$

But we have so far considered only the case of the white balls. We have two more cases that occur with identical probabilities: three red balls and three black balls. And, of course, each of these three events is mutually exclusive. So, their probabilities of occurring may be added, to give a total probability of three balls of the same colour of

$$\frac{1}{84} + \frac{1}{84} + \frac{1}{84} = 3 \times \frac{1}{84} = \frac{1}{28}$$

But we are not done yet! This is the probability of the complementary event. The event we are after has a probability of one minus the probability of the complementary event. So, the probability that the three balls are of at least two different colours is $1 - \frac{1}{28} = \frac{27}{28}$.

41.6.11 Units in applied mathematics

Perhaps you are asked for the answer in m^2 (square metres) but you worked your answer in mm^2 (square millimetres) and forgot to convert the answer to the correct units at the end. You will only lose a few marks for this, but they might deprive you of a distinction. So, be careful.

Another unit-related bugbear is interconversion between frequency in Hz (hertz) and angular frequency in radians per second. Angular measure in radians and degrees is yet another trap for the unwary. Again, analysis of where and how often you go wrong will help you overcome these sorts of errors.

41.6.12 Plain carelessness

Carelessness robs you of marks.

★

Make carefulness a habit.

Become deliberately careful when solving quantitative problems in mathematics, physics, chemistry, etc. This habit alone will help you score high marks in all quantitative subjects: the effort and time are well worth the investment because of multiple returns across different subjects.

41.7 Misconceptions arising from wrong extrapolations

Many misconceptions arise from wrongly extrapolating early ideas from arithmetic and applying them blindly to other mathematical objects encountered later in the study of mathematics.

First, you must become aware that a very large variety of mathematical *objects* exist beyond the natural numbers you first encountered. Second, there is also an equally large repertoire of *operations* beyond the addition, subtraction, multiplication, and division of early arithmetic.

Unthinking application of "rules" from arithmetic to these more involved objects and operations can and often does result in errors of understanding which propagate as errors in the solution of mathematical problems. A few of these misconceptions are outlined in this section.

41.7.1 $\sin(A+B)$ does not equal $\sin A + \sin B$

EXAMPLE: EXAMPLE

Multiplication is *distributive* over addition. What this means is that, for example,

$$5 \times (4+1) = 5(4+1) = (5 \times 4) + (5 \times 1) = 20 + 5 = 25 = 5 \times 5$$

This is a property of real numbers and the two binary operations of multiplication and addition. ★

When you first encounter the trigonometric functions, you might be tempted to extrapolate from this experience to claim that

$$\sin(A+B) = \sin A + \sin B \quad \leftarrow \text{erroneous statement}$$

but this is wrong.

Why is the above statement erroneous? Because, in the first place, sin is a *function* or mathematical rule that, given one value, assigns to it another. It is not a binary operation like addition or multiplication in which two numbers are added or multiplied to yield a third. Second, there is no logical justification for applying distributivity in this case because sin is a non-linear function.[3] So, the above statement is wrong. The correct version is:

$$\sin(\mathrm{A}+\mathrm{B}) = \sin\mathrm{A}\cos\mathrm{B} + \cos\mathrm{A}\sin\mathrm{B} \quad \leftarrow \text{correct statement}$$

The same applies to other non-linear functions such as taking logarithms, square roots, etc. So, $\log a + \log b \neq \log(a+b)$. The correct equation is

$$\log a + \log b = \log ab.$$

Moreover, $\sqrt{3}+\sqrt{2} \neq \sqrt{3+2}$; indeed, $\sqrt{3}+\sqrt{2}$ cannot be simplified further.

41.7.2 Non-commutative multiplication

Addition and multiplication are *commutative* for the real numbers. This means that if a and b are any real numbers, we have

$$\begin{aligned} a+b &= b+a \\ a\times b &= b\times a \quad \text{or equivalently} \quad ab = ba \end{aligned}$$

However, multiplication of other mathematical objects need not be commutative.

[3]See Figure 45.1 in Chapter 45 for a graph or pictorial representation of $\sin x$.

EXAMPLE: EXAMPLE

A and B are two matrices whose products AB and BA both exist as shown below.[4]

$$
\begin{aligned}
\mathrm{A} &= \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} \quad \text{and} \quad \mathrm{B} = \begin{bmatrix} 4 & 5 \\ 6 & 1 \end{bmatrix} \\
\mathrm{AB} &= \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 26 & 13 \\ 76 & 43 \end{bmatrix} \quad \text{whereas} \\
\mathrm{BA} &= \begin{bmatrix} 4 & 5 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 43 & 52 \\ 19 & 26 \end{bmatrix} \quad \text{and so} \\
\mathrm{AB} &\neq \mathrm{BA}.
\end{aligned}
$$

Hence, matrix multiplication is not necessarily commutative. ★

41.7.3 One vector cannot be divided by another vector

The two-dimensional Cartesian co-ordinate plane, where a pair of numbers represent a point on the plane, is a fertile ground for growing increasingly sophisticated mathematical ideas. It can be used to represent the value of the independent and corresponding dependent variable of a single-variable function so that a *graph* of the function may be plotted.

A point on the coordinate plane[5] can also represent a *complex number*. In this case, the point (a, b) represents, by convention, the number $a + ib$ where a is the real part and b is the imaginary part of the complex number, both a and b are real numbers, and the symbol i stands for the imaginary unit, or imaginary number, defined by $i^2 = -1$.

If that sounds confusing enough, try this: the point (a, b) can also be pressed into service to represent a two-dimensional *vector*[6] $\boldsymbol{v}$ whose x- and y-components are a and b respectively.

While mathematicians have squeezed all the juice there is out of the co-ordinate plane, and the resulting interplay of ideas gives rise to rich new possibilities, such use of the co-ordinate plane can also contribute to misconceptions.

[4] See Section 41.13 for an explanation on how to multiply matrices.
[5] Called an *Argand diagram* in this context.
[6] A vector is a mathematical object, that for our purposes, has direction and magnitude.

EXAMPLE

For example, suppose that (a, b) and (c, d) are two complex numbers, we can properly divide one by the other so:

$$\begin{aligned}\frac{a+ib}{c+id} &= \frac{(a+ib)(c-id)}{(c+id)(c-id)}\\ &= \frac{1}{c^2+d^2}\left[(ac+bd)+i(bc-ad)\right] \quad \bigstar\end{aligned}$$

You might then be tempted to think that if (a, b) and (c, d) represent two vectors instead of complex numbers, you could equally well divide one vector by the other. But you would be wrong.[7]

So, while cross-application of concepts from one area and context of mathematics to another can be very creative and useful, you need to be careful and knowledgeable enough to ensure correct reuse of previously learned concepts or operations in a new context.

41.8 Write down every step

When you work out a mathematical problem, *write down every step* as you go along. There are two reasons for this prescription.

Firstly, there is a chain of logical reasoning that can be traced back, link by link, if you write down every step. It is proof that you can reason logically. It is also proof that you have not copied the answer from a neighbouring student in an examination, or during routine homework.

★

The sequence of steps is often as important as the correct answer in terms of marks awarded in an examination.

Secondly, when you check your answer, which you should always do, you have the opportunity to scan *every* step along the way. This helps you detect errors at the *earliest* possible step and correct your work. If you make a mistake in a mental step that is not written down, there is no way to trace your mistake when you check your working and answer. If

[7] It took the Irish mathematician, physicist, astronomer, William Rowan Hamilton (1805–1865), ten years of concerted effort to devise a method for "dividing vectors", and even though he did not succeed, in the process, he invented *quaternions* and a non-commutative division algebra [16].

you write down every step, however, you can always trace an erroneous step when checking.

> ★
>
> Do not skip writing down steps in the solution of a problem, even if you can easily do them in your head.

41.9 Check your answer

Once you have worked through a problem you should check your answer. Develop the habit of doing so by following Algorithm 41.3.

> **CHECK YOUR ANSWERS**
>
> 1. Check each step as you write it down. Be quick. Do not let the checking drag down your speed of work. Deliberate care at this stage will scotch many a careless mistake.
> 2. After finishing all your questions, check each answer for correctness. If possible, use a different method as explained in Section 41.10.
> 3. Never hand in your answers without checking them first.

ALGORITHM 41.3: Check your answers.

The additional time spent on checking is well worth it. Once you cultivate the habit of routinely checking your answers in normal homework problems and drill sets, you will find that checking your answers becomes second nature to you: automatic, and very efficient. Checking time will come down. This is especially useful in examinations where minutes matter.

41.10 Verify by another method

If you can, you should check your answer using a different method from the one you used to get your answer in the first place. A trite example will help here.

EXAMPLE: EXAMPLE

Suppose you are asked to solve the equation $5x + 2 = 7$. Your normal sequence of steps will be

$$\begin{aligned} 5x + 2 &= 7 \\ 5x &= 7 - 2 \\ &= 5 \\ x &= \frac{5}{5} \\ &= 1 \end{aligned}$$

where I have laid out the steps in greater detail than necessary simply to illustrate a point.

As you write your solution, you should to go through the steps and check that the correct operation has been performed at each step.

However, one drawback with using this approach for checking your answer is that a mistake might remain undetected during checking because you are basically retracing the *same* sequence of operations that led to the mistake in the first place.

★

It is all too easy to make the same mistake twice.

If, instead, you checked your answer by a *different method,* you might obtain a faster, and more robust verification of your answer than by simply repeating the same steps or scanning them for errors.

One way of doing it for the above example is by *direct substitution.* Use the value of x obtained by solving the equation and substitute it directly into the *original* equation. If that equation holds true, your solution is correct. In our case, substituting $x = 1$ into $5x + 2 = 7$ gives $5(1) + 2 = 5 + 2 = 7$ thus directly verifying the correctness of our solution. ★

★

Always check your answer by another method if you can.

41.11 Discover your errors on your own

This is another excellent mathematical habit that has stood me in very good stead. By repeated practice, seek to discover your own errors on your own. That way, you get to correct them before your teacher or examiner. I have touched upon this skill in Section 39.10.2 as well.

One example of discovering your own errors unaided is given in Section 41.14. Here, we look at yet other techniques.

Start off by working out on your own several problems that have already been solved in your textbook, *but do not look at the worked solutions.* Just note the problem, close the book, and start working it out. After you think you have got your solution, open your textbook and compare your working with that shown in the book.

Do not simply check the answer, and if yours tallies with the book's, close the book and go away. Instead make it your business to compare *each step* of your working with that shown in your textbook. That way, you develop a sense of what you need to show and what you can leave out when writing your solution step-by-step.[8] You will also learn elegant mathematical exposition from your book if you study it this way.

EXAMPLE: EXAMPLE

If you find an error, look at how and why it originated. For example, you might find that you consistently make errors when solving quadratic equations in which the coefficient of x^2 is negative. You then have two options:

(a) Perform a sign change by multiplying both sides of the equation by -1, and *then* solving the resulting equation; or

(b) Practice solving such equations with negative coefficients for x^2 until perfect.

Whichever way you go, you would have identified your own particular tendency for weakness in mathematical manipulation, and corrected it. ★

[8]Always keep in mind the advice given in Section 41.8. If in doubt, as a rule, write down a step rather than omit it.

41.12 Surd manipulation

A *surd* is an *irrational number*[9] embodying the square root sign $\surd$. Surds often inspire angst among students,leading to frequent errors. Quite apart from the unusualness of the symbol, questions involving surds are a favourite in many mathematics examinations.

When you are asked to simplify an expression containing a surd, you must remember the *difference of two squares identity* $(a+b)(a-b) \equiv a^2 - b^2$. Often, when the surd is removed from the denominator through this device, it is called *rationalizing the denominator*.

QUESTION

Rationalize the denominator in

$$\frac{4-2\sqrt{5}}{1-5\sqrt{2}}.$$

PRE-COMMENTARY

Look at the *denominator*. That is where the simplification takes place. This denominator has the form $a - b\sqrt{c}$. The difference of two squares identity tells us that we need to multiply the denominator (and numerator) by $a + b\sqrt{c}$.

ANSWER

To rationalize the denominator, we multiply *both* numerator and denominator by the same term, which in this case is $1+5\sqrt{2}$. Note the sign change in the term compared to the given denominator. We then have

$$\begin{aligned}\frac{4-2\sqrt{5}}{1-5\sqrt{2}} &= \frac{(4-2\sqrt{5})(1+5\sqrt{2})}{(1-5\sqrt{2})(1+5\sqrt{2})}\\ &= \frac{4+20\sqrt{2}-2\sqrt{5}-10\sqrt{10}}{1^2-(5\sqrt{2})^2}\end{aligned}$$

MID-ANSWER COMMENTARY

Pause awhile to note that $\sqrt{a}\sqrt{b} = \sqrt{ab}$.

[9]See, for example, Julian Havil's book*The Irrationals: A Story of the Numbers You Can't Count On* [17] and Jospeh J Rotman's book *Journey into Mathematics: An Introduction to Proofs*[18, pp 180–181].

ANSWER CONTINUED

Simplifying, we get

$$\begin{aligned}\frac{4-2\sqrt{5}}{1-5\sqrt{2}} &= \frac{2(2+10\sqrt{2}-\sqrt{5}-5\sqrt{10}}{1-25(2)} \\ &= -\left(\tfrac{2}{49}\right)(2+10\sqrt{2}-\sqrt{5}-5\sqrt{10})\end{aligned}$$

POST-COMMENTARY

Note that the numerator cannot be simplified further. Do not waste your time trying to. It is only rationalizing the denominator that is required. This was important in the days before computers and calculators because it is easier to divide by an integer than by a surd. This practice is now enshrined as a relic of those times, principally in examination questions. ☻

41.13 Mnemonics

Mnemonics are little aids to memory that you make up as you go along to remember new facts. We have discussed them extensively in Chapter 7. Here I will just mention two mathematical mnemonics—one for trigonometry and the other for matrix multiplication—that I have found useful.

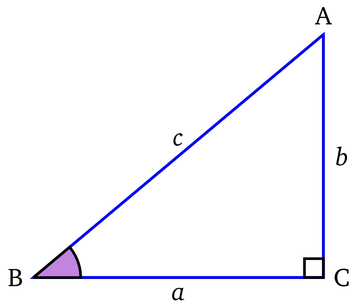
EXAMPLE: EXAMPLE

In trigonometry, we deal with right-angled triangles and the ratios of their sides. The hypotenuse is the longest side, opposite the right angle. The other two sides are either opposite or adjacent to the angle in question. When I was at school, we were taught mnemonics like *son of Hamlet* to denote the letters *s, o,* and *h,* to denote in turn that *sine* is *opposite* over *hypotenuse.*

It was not so much about whether Hamlet had a son! It was about memorizing a trigonometric ratio as shown in Figure 41.1. Make up your own mnemonics; the more funny or absurd the better, because you will remember them all the more. ☻ ★

EXAMPLE: EXAMPLE

Now for the second example. When we deal with matrix multiplication, we need to multiply *pairs of numbers* and add the lot. Assuming that the



son of Hamlet

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \mathrm{B} = \frac{b}{c}$$

FIGURE 41.1: Mnemonic for the sin trigonometric function. For the angle B, the *opposite* side is b and the *hypotenuse* is c and $\sin \mathrm{B} = \frac{b}{c}$. The first letters of each word in the *son of Hamlet* mnemonic are *s, o,* and *h,* corresponding respectively to *sine, opposite* and *hypotenuse.*

matrices are *conformable*—i.e., they can indeed be multiplied in the order given—which pairs of numbers are to be multiplied is often an issue.

At school, I was taught a visual or pictorial mnemonic. It is best *seen* rather than *explained*. Referring to Figure 41.2, there are two matrices

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 28 & 14 \\ 79 & 44 \end{bmatrix}$$

FIGURE 41.2: A visual mnemonic for matrix multiplication. The matrix elements in the two orange ellipses are multiplied together and added to give the circled element in the first row and first column of the product matrix. Note the orientation of the two ellipses and you will understand conformable matrices. The like coloured numbers are multiplied together and the products added so: $(1 \times 7) + (2 \times 9) + (3 \times 1) = 7 + 18 + 3 = 28$.

on the left, which when multiplied together *in the given order,* result in the matrix on the right.

In the mnemonic, visualize an ellipse with its major axis horizontal for the first row of the leftmost matrix and an ellipse with its major axis vertical for first column of the middle matrix. The number of elements enclosed by the two ellipses must be equal if the matrices are conformable.

Then multiply the similarly coloured numbers in each of the two ellipses and add the resulting products to produce a single number. This result is the circled entry in the product matrix on the right. This step is repeated for each row-column pair so that we get four entries or elements in the product matrix.

Once you master this visual mnemonic, you will never be at a loss when multiplying matrices again. Moreover, you will know in the blink of an eye whether or not two given matrices can be multiplied together in a particular order. ★

41.14 Working backwards

There may be occasions when you are faced with a problem but have absolutely no idea of how to proceed. You should make a genuine effort to view the problem from different angles, and rack your brain to see if you have encountered a similar or inverse problem before. But suppose all your hair-tearing efforts come to null, what should you do?

Moreover, sometimes, you *do* have an idea how to solve a problem but find that your solution is incorrect. Try as you might, you cannot locate that vital step where you made that first mistake, and you are then stuck. Again, you need a way out of the cul de sac.

Before you run off to consult the worked solution (if there is one), or ask the mathematics genius in your class, or seek your mathematics teacher, or lecturer, or tutor, there is one thing you might try: *work backwards, if the problem allows you to.*

When working backwards, you will encounter a particular step at which things will just pop out and you will understand what approach you should take to solve the problem from first principles. Or you might grasp in a flash where you made that first mistake. If all this sounds vague and waffly, let me give you a very simple example to illustrate my meaning.

QUESTION

A number is added to 1 and multiplied by 5. It is then squared and the result is 625. What is the original number?

COMMENTARY

The traditional approach is to let x stand for the original number and to construct the equation $[5(x+1)]^2 = 625$ and solve for x. But you might have made a mistake and forgotten to square the number 5, and instead written down the *wrong* equation $5(x+1)^2 = 625$. Then try as you might to trace your error, it is as if you have hit a brick wall. One way around this impasse is to work backwards. Indeed, that is what you would have done in the days before you learned algebra and used it to solve equations arising from word problems.

Working backwards in this problem means that we *undo* every operation, starting from the last and working backwards to the first, until we arrive at the original number. So, we first take the *square root* of 625 to get $\sqrt{625} = 25$. Then we *divide* 25 by 5 to get $\frac{25}{5} = 5$. Finally, we *subtract* 1 from 5 to get the original number as $5 - 1 = 4$.

Somewhere along the way, it would have dawned on you that you should have included the number 5 when squaring the algebraic expression above. You would then have pulled yourself up by your own bootstraps because you would have detected and corrected your error *on your own.*

Note one other point: solving the problem by working backwards may be a legitimate way to solve this problem, but your mathematical sophistication or understanding will not grow thereby. For example, did you notice above that $-\sqrt{625} = -25$ is equally another solution, but one we missed?

Such subtleties pass us by when we use a more elementary method, but they are unlikely to be overlooked when systematic algebraic methods are employed. So, I recommend working backwards *more as a fillip* in helping you detect your own errors *unaided,* especially when you first encounter new mathematical techniques. Otherwise, it is better simply to ferret out your errors by careful examination of your working.

ANSWER

Just for completeness, the correct algebraic solution with *two* possibilities for the original number is given below:

$$\begin{aligned}
[5(x+1)]^2 &= 625 \\
25(x^2+2x+1) &= 625 && \text{; divide both sides by 25} \\
x^2+2x+1 &= 25 \\
x^2+2x-24 &= 0 && \text{; (the LHS can be factorized by inspection)} \\
(x+6)(x-4) &= 0 \\
x = -6 \quad &\text{or} \quad x = 4
\end{aligned}$$

41.15 Beware the graphics calculator

The graphics calculator is now part of the allowed apparatus of the mathematical student, just like the protractor and compasses, or the book of logarithms, or the slide rule were in earlier times. The graphics calculator, however, is a mixed blessing.

It can often give you a misleading picture about the graphical solution of an equation. Because of limited display size and resolution, it might not be possible to see the whole graph at all points of interest on a scale that is comfortable to view. Sometimes, to circumvent this limitation, only part of a graph might be displayed, but this might give a deceptive picture of the whole graph, especially if the axes are not labelled or displayed to scale. Asymptotes might also not be displayed accurately.

Furthermore, rounding errors and finite precision arithmetic could lead to further misconceptions, especially if you are studying rational numbers, and all you see are their decimal representations to a finite number of decimal places. It is important that you become familiar with these and other pitfalls in the results a graphing calculator displays, before you rely unquestioningly on it. Beware the graphics calculator!

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SUMMARY: SOLVING MATHEMATICAL PROBLEMS

- Read the question carefully.
- Understand the question correctly.
- Answer the question precisely.
- Practice solving problems until you become adept at that art.
- Write down every step of your working when you solve a mathematical problem.
- Make checking your steps *as you write down your solution* a reflex action.
- Always check your answers.

- Whenever possible check your answer by using a different method from the one you used to solve the problem.
- Try to detect your own errors unaided by making deliberate and frequent checking of your answers a habit.
- Use the difference of two squares to rationalize denominators containing surds.
- Common mistakes in mathematics that you should avoid include:
  - Forgetting sign changes during algebraic manipulation
  - Missing terms
  - Forgetting to invert inequalities during manipulation
  - Wrongly reformulating word problems into mathematics
  - Wrongly applying inverse operations
  - Forgetting one part of an answer
  - Enumerating cases incorrectly
  - Wrongly approaching a problem so that solution is either tedious or elusive
- Avoid errors arising from wrongly applying rules from earlier mathematical knowledge to new areas where they might not apply.
- Make up your own mnemonics to remember new facts and methods
- Work backwards if the problem is too hard or if you are at an impasse in finding the solution, but only rarely.
- Become aware of the pitfalls of using a graphics calculator.

42

Mathematics at University

Synopsis

Mathematics at university can be difficult for several reasons: weak foundations in high school, increase in abstraction and generality of subject matter, change in emphasis from finding something to proving something, etc.

Specifically, the high school student entering university is somewhat abruptly confronted by four changes:

1. Calculus yields to analysis;
2. Algebra is succeeded by abstract algebra;
3. Geometry metamorphoses into topology; and
4. Logic and proof take centre stage.

Each of these challenges is discussed briefly, and suitable books are suggested to help overcome them.

In addition, books are recommended to help set right shaky high school mathematics, to help ease the transition to university mathematics at the undergraduate and advanced undergraduate levels, and to get an overall picture of mathematics as a whole.

A few mathematical software suites—which are discussed in greater detail at the book's companion website—are also mentioned.

42.1 The high school to university transition

The transition from high school to university education is nowhere near as steep or as keenly felt as in mathematics. It has been compared to a culture shock [1]. It is like leaving a familiar, comfortable country to enter a new, foreign one. It is hard to believe that the same subject could take on such a change in complexion and appear so different. Not surprisingly, many competent students experience a jolt when they first encounter university level mathematics.

Most high school programmes prepare the typical student to be sufficiently numerate and skilled in solving the quantitative problems that he or she is likely to encounter in working life. High school mathematics features an eclectic mix of problem-solving techniques covering the traditional areas of arithmetic, algebra, geometry, trigonometry, co-ordinate geometry, functions, calculus, matrices, probability, and statistics.[1]

At university though, mathematics demands greater abstraction and generalization, increased rigour, an expanded terminology, and facility in proving mathematical statements. Undergraduate mathematics is quite a different beast from the high school variety which you have been accustomed to. Table 42.1 shows some selective differences between high school and university mathematics.

It is the burden of this chapter to outline the major changes in the transition from high school to university mathematics and to point you to well-written textbooks that can help ease this transition.

42.2 The importance of high school mathematics

The mathematics you learn between the ninth and twelfth years of school lays the quantitative foundation for all your future studies. Unfortunately, due to lack of time, or lack of interest, or an uninspiring teacher, or some other reason, you might not have mastered all that you should have in those formative years.

The pace at university is often far brisker than at high school. You will be left by the wayside if your high school mathematics is itself somewhat shaky. If you happen to be such a student, take heart. There are a great many like you. Many of them would not even be aware of their deficiency. Knowing your weakness is already a great start. And you

[1] Professional mathematicians recognize Statistics as a discipline separate from Mathematics.

HIGH SCHOOL	UNIVERSITY
Skill in arithmetic, geometric, and algebraic manipulations	Understanding ever newer abstract mathematical objects and their logical inter-relationships
Solving "problems to find" [2] in the context of applications	Solving "problems to prove" [2] based on axiomatic foundations and logical rigour
Emphasis on solving systems of algebraic equations	Understanding algebraic structures and their properties
Calculus, founded on geometric and intuitive notions, and applied to solving practical problems	Analysis as the theoretical foundation of calculus based on arithmetic and logical precision rather than geometry and intuition, and studied in its own right
Vectors as directed line segments obeying the parallelogram law of addition	Vectors as elements of an algebraic structure called a vector space obeying the applicable axioms

TABLE 42.1: Selective comparison between high school and university mathematics both in emphasis and content.

need to remedy those faltering foundations speedily so that you are not left mathematically shell-shocked in your first year at university.

42.2.1 Recommended texts

There are many excellent textbooks specifically designed to correct poor knowledge of high school mathematics, especially for those who have left high school and are pursuing tertiary education.

Of these, I would especially recommend a very student-friendly text called *Maths: A Student's Survival Guide* by Jenny Olive [3]. It is designed to set right wobbly high school mathematical foundations, especially for first-year university students. Through a series of self-tests, it assists you in identifying your weak areas so that you can do efficient "spot repairs". It also highlights common errors and explains manipulative techniques well. Read it, work through it, and strengthen your weak areas in high school mathematics.

Another excellent book, very much along the same lines, with sections titled "Test Yourself" and "Where Now?" is *Bridging the Gap to University*

Mathematics by Martin Gould and Edward Hurst [4]. I thoroughly recommend you to read and work through it systematically as preparation for entering university.

There is no greater feeling of triumph than to re-visit and master a previously misunderstood or partially understood area of mathematics. Heal yourself of mathematical ills preferably *before* entering university by understanding concepts, identifying relevant approaches, and applying them to solve problems. If you are already in first year at university, repair your shaky foundations just the same, and the sooner the better.

42.3 Mathematical transitions at university

Mathematics units at university are compulsory for undergraduate students enrolled in many different courses of study, such as engineering, the natural sciences, business, economics, psychology, etc. These students will be applying mathematical ideas and techniques in their respective home disciplines after graduation. Therefore, they need to do well in their prescribed mathematics units, but because they are not majoring in mathematics, they often cannot lavish sufficient time or effort on it. Yet they cannot entirely neglect it either, for to do so might imperil getting their degree.

It is to these students that this chapter is addressed.[2] As I see it, there are four major differences between high school and university mathematics:

1. Calculus yields to analysis;
2. Algebra is succeeded by abstract algebra;
3. Geometry metamorphoses into topology; and
4. Mathematical logic and proofs take centre stage, rather than being an occasional interlude: "problems to find" [2] give way to the more abstract variety of "problems to prove" [2].

Each of these changes is outlined briefly below along with recommendations of textbooks that will help ease these transitions.

[2] If you are taking a degree in mathematics, you might want to consult the book *How to Study for a Mathematics Degree* by Lara Alcock [5]. If you are contemplating a postgraduate degree in mathematics do read Steven G Krantz's book *A Mathematician's survival guide: Graduate school and early career development* [6].

42.4 From calculus to analysis

At high school you were taught how to differentiate and integrate. You were exposed to all sorts of tricks and special techniques to compute specific integrals, especially those involving inverse trigonometric functions, and so on. If you revelled in mastering and applying such techniques, you might find that what succeeds high school *calculus* at university is a horse of an entirely different colour, called *analysis*.

You might even be alarmed that rather than routine problems to solve for the value of some quantity, or simply to work through and demonstrate a fact in a straightforward fashion, you are now required to prove theorems: something that requires a different mindset and skill set.

42.4.1 Calculus versus Analysis

High school calculus appeals to intuition and the visual sense through geometric ideas like slopes and areas. Analysis on the other hand is logically precise and uses arithmetic as the basis for deriving results. Intuition has given way to logical precision, and pictures have yielded to symbols. The infinitesimals of calculus have given way to the limits and infinite sums of analysis. All this takes some getting used to.

42.4.2 Recommended texts

Several texts have been written to highlight the differences between calculus and analysis and to ease the transition from the one to the other. Anthony Gardiner's book *Infinite Processes: Background to Analysis* [7] gives an eminently readable account of the rationale for analysis that is accessible to someone familiar with calculus. This book has been slightly expanded, given a new title, and reprinted in an inexpensive edition [8]: read it if you can.

Another very readable and fascinating recountal of the reasons why analysis *had to arise* is given in David M Bressoud's text *A Radical Approach to Real Analysis* [9]. The time spent reading it will be well rewarded with a quantum jump in your mathematical understanding and maturity.

Stephen Abbott's lucid *Understanding Analysis* [10] is another text which gives clear, comprehensible motivations and explanations without losing rigour.

Other, more recent books that bridge the calculus-analysis divide include the texts by Ghorpade and Limaye [11], Brannan [12], Beyer [13] and Schinazi [14].

My favourite text on introductory analysis is Maxwell Rosenlicht's *Introduction to Analysis* [15]. It is a marvel of clarity that is not too expensive to own. Read and work through it if you can.

42.4.3 Clarifying doubts

If you have a specific but recalcitrant doubt, you might need to consult more than one book before the "eureka moment" or epiphany occurs. Often it is the earlier texts with their careful expositions and more leisurely pace that facilitate such understanding. Let me illustrate with an example.

EXAMPLE: EXAMPLE

The logically rigorous—but intuitively far from obvious—*epsilon-delta definition of a limit* is often considered one of the most difficult-to-grasp concepts that the beginning undergraduate has to contend with. It goes like this:

For a real-valued function f of a single real variable, defined on an open interval containing a, with real L, ε, and δ,

$$\lim_{x \to a} f(x) = \mathrm{L} \iff \forall \varepsilon > 0 \; \exists \; \delta > 0 : \forall x \; (0 < |x - a| < \delta \; \Rightarrow \; |f(x) - \mathrm{L}| < \varepsilon).$$

You might be alarmed by the unfamiliar symbolic language in which it is couched. But even if you understood that, you might still not know why it is expressed the way it is, which—if truth be told—is rather contrived. You would then need to do some digging to find out why this specially crafted definition became necessary. For instance, we explicitly exclude the case of $x = a$ in the above definition through the condition $0 < |x - a|$. Thus $f(a)$ need not even be defined!

It is beyond the scope of this book to venture further, but if you are confused or curious, I recommend you to read one of the most careful and thorough explanations of this definition that I have seen. It appears in section 4.3, pp 128–140, of the book *Bridge To Abstract Mathematics: Mathematical Proof and Structures* by Ronald P Morash [16], which is also available online.[3]

Another text that does an admirable job of explaining this definition is George R Exner's *Inside Calculus* [17, chapter 1]. ★

42.5 From algebra to abstract algebra

Your exposure to *algebra* in high school was likely restricted to solving linear and quadratic equations. You also probably solved systems of linear equations. Like a bloodhound sent off on a scent, you were given the equations and told to get the solutions. Methods and *results* mattered. The underlying *patterns* really did not. It was fiercely result-oriented.

At university, it is the patterns that matter more in algebra. Abstraction and generalization allow similarities to be discerned across large swathes of seemingly unconnected mathematical tracts. Such a panoramic overview allows connections to be made, say, between a force vector and a system of linear equations that would not otherwise have become apparent.

This superb power of generalization has come to rule mathematics, and all your future encounters will require at least a nodding acquaintance with the ideas of *abstract algebra.* Perhaps through a course entitled "Introduction to Algebraic Structures" you would meet the first incarnation of this behemoth of abstraction and generalization.

42.5.1 Getting a qualitative feel

You should first get an overall feel for what abstract algebra is about, where it is heading, how its component parts fit together, what its principal applications are, and why it is important. Such a qualitative appreciation might be all you need to get motivated about the subject area and approach it with enthusiasm.

42.5.2 Recommended texts

Abstract algebra is well served by many good introductory textbooks. My first recommendation is Charles C Pinter's classic text *A Book of Abstract Algebra*, now available in an inexpensive reprint edition [18]. Read it first and you will understand all other texts better. Get yourself a copy if you can.

[3]Incidentally, this book also covers the art of writing mathematical proofs, which we look at later in this chapter in Section 42.7.2.

Frederick M Goodman [19] has written an unusually reader-friendly text that is available online for download. The first chapter entitled "Algebraic Themes" provides a panoramic view of the purpose and scope of abstract algebra. Do read at least this first chapter, which weighs in at more than eighty pages![4] You will grasp the subject far better after that.

Other excellent starting points giving gentle introductions to the area include Bergen's *A Concrete Approach to Abstract Algebra: From the Integers to the Insolvability of the Quintic* [20], Cameron's *Introduction to Algebra* [21] and Smith's *Introduction to Abstract Algebra* [22].

After these, you can take your pick from the wealth of standard introductory texts such as Gilbert and Nicholson [23], Dummit and Foote [24], Hungerford [25], etc.

42.5.3 Applying the four stages of learning

In previous chapters I have identified the four stages of learning. I have also emphasized the importance of discovering a good text to study from, the importance of summarizing your ideas using an Octopus diagram, and the vital necessity to test your understanding of the subject every so often. I will conclude this section with an example on how to apply these ideas to the study of the algebraic structures called *groups*.

EXAMPLE: EXAMPLE

Groups are among the most basic of algebraic structures. They form the foundation for much of mathematics. Yet, when confronted with the definition of a group as a non-empty set with a binary operation that is closed, associative, has a neutral or identity element, and for which each element has an inverse, you might feel a little disoriented.

As the noted mathematician W W Sawyer has observed, the most difficult question for the student who first encounters groups is "What is the *purpose* of groups?" [26, p 201]. It is not so much the definitions or hierarchy of structures arising from groups that is so daunting as their purpose and apparent pluripotent nature.

You just *might* have first heard of groups in regard to crystallography, or the symmetry of snowflakes, or in some other concrete context.

A cardinal principle of all learning is to proceed from the known to the unknown. If symmetry is the only idea that you can associate with

[4]Read it in instalments if need be ☻.

groups, let symmetry be your starting point for studying groups. Like a dog seeking a buried bone, unearth good books that explain the links between symmetry and groups.

For this purpose, you could try reading one or more of the following books: Armstrong [27], McWeeny [28], Rosen [29], Farmer [30], Carter [31], Tapp [32], Budden [33], or Goodman [19]. Once you have consulted them and grown more knowledgeable about groups in general, you are ready for the second phase.

To make the abstract more concrete, draw an Octopus or Venn diagram showing the relationships between different types of groups, and between different algebraic structures like groups, rings, vector spaces, etc.[5] Make notes of how and where they are applied. Use the idea of *"slicing the orange of knowledge with a different cut"* to study groups. Hark back to the books you have read to check your understanding and make corrections if necessary.

Having gone through the *naming* and *knowing* phases, you should now embark on the *doing* phase of learning. Start on your prescribed text or one of those recommended above, and work through the text and problems. This is the only way to permanently cement your newfound knowledge into what you already thoroughly know. ★

42.6 From geometry to topology

The third mathematical transition is from *geometry* to *topology*. The convenient and comfortable proofs about congruent triangles in high school are suddenly supplanted at university by the notion of a space with set-theoretic notation attached to it. Measurable lengths and angles have given way to ideas like neighbourhoods and balls, and notions of compactness and connectedness.

Again, the level of abstraction has been ratcheted up one notch. And again, you might feel a little at sea as you encounter these changes for the first time. And by now, you would have figured out what my prescription would be. Rather than give something specific, let me generalize.

The algorithm given here is an example of how to go about learning some unfamiliar mathematics at university. However, it is generic enough to apply to most learning you might undertake. In a nutshell, to

[5]Consult Chapter 19 if necessary to refresh your memory on study techniques.

LEARNING MATHEMATICS AT UNIVERSITY

1. Be clear about what you wish to learn.
2. Identify what you already know.
3. Find some books *you can understand* that start with what you already know and slowly but surely take you to what you wish to know.
4. Read those books.
5. Spend some time getting a *qualitative* feel for the new knowledge.
6. Test your understanding.
7. Read your prescribed texts and lecture notes.
8. Work through representative problems.

ALGORITHM 42.1: Algorithm for learning mathematics at university.

succeed academically, you should become self-reliant in your learning. Identify what needs to be done and how to go about it. Then do it.

42.6.1 Recommended texts

The best introduction to topology that I have come across is *Introduction to Topology And Modern Analysis* by George F Simmons [34]. It covers more than topology and is written by a lucid mathematical expositor. I heartily recommend that you read it.

Other reader-friendly books on topology include *Geometry and Topology* by Reid and Szendrői [35], *Principles of Topology* by Croom [36], and *Topological Spaces: From Distance to Neighborhood* by Buskes and van Rooij [37].

Another book on topology, freely available online as an electronic book, is *Topology Without Tears* by Sidney A Morris [38].[6]

42.7 Logic and proof

We now come to the fourth major transition between high school and university mathematics: logic and proof. Both of them, together with set theory, relations, and functions constitute the foundation of contemporary mathematics. Set theory sits astride the border between mathematics and philosophy. Logic is solidly a science that obeys definite laws. And

[6]If you wish to print a paper copy of this electronic book, you should email the author, seeking his permission.

proof writing is an art and a science based on logic and set theory: so, you need to know both in order to write proofs.

A high school student usually has more than a nodding acquaintance with set theory and logic, but often finds writing anything beyond the most elementary of proofs a rather difficult task. In the succeeding sections, we briefly consider logic, proof and disproof, and recommend suitable texts for those in need of help.

42.7.1 Logic

Mathematics rests on the foundation of logic and set theory. Not only do mathematicians use logic, even computers use an algebra of logic called *Boolean algebra* to perform computations. The rules of logic, usually first introduced in high school mathematics, achieve greater prominence at university. In this regard, it is instructive to distinguish between two "types" of logic: real world logic and mathematical logic.

EXAMPLE: EXAMPLE

The television animation character, Bart Simpson, is reputed to have said "Remember, you can always find East by staring directly at the sun."

Is this statement correct? It is, if you stare at the *rising* sun. What happens if you stare at the *setting* sun? Well, in that case, you will be looking west rather than east. But you can still *find* east because it would be right behind you. But what happens if the sun is right *overhead* at noon? In that case, you would be hard put to tell which way is east.

This type of logical analysis has meaning in the real world and draws attention to the conditional nature of what may be asserted as true. In other words, the truth of a statement varies according to context.

In *mathematical logic,* however, we are in a restricted and artificial universe rather than a real one. When a statement is *given* to be true, it is *taken* to be unconditionally true. We do not question its validity, but instead use its truth to establish the truth or falsity of *other* statements.

If we are told that all bears have black fur, we do not contest that statement by bringing up the example of brown grizzly bears or white polar bears. We just take the given statement at face value and use it to *deduce* other truths using standard rules.

A statement of mathematical truth so deduced is called a *theorem* and the entire edifice of mathematics is built with the bricks and mortar of theorem laid upon solid theorem. Logic and proof are siblings. We use logic to prove theorems. ★

42.7.2 Proof

Mathematical proof is exacting. The rigorous, unyielding rules of logic must be satisfied before a proof can be accepted. The methods of proof are varied, but they are all logically consistent.[7]

Understanding proofs is taxing. And writing proofs is even more demanding. You need *knowledge*. And you need *patience* to single-step through axioms and previously proved theorems when you are proving a theorem yourself. You also need the *creative spark* to blaze your way to an original proof.

The mathematician who writes proofs becomes simultaneously both an artist and a scientist. Knowledge, patience, experience, inspiration, intuition, and many other factors conspire to make proof-writing among the most formidable of creative intellectual endeavours.

The point being made here is that abstract mathematics in general, and writing proofs in particular, is an entirely different kettle of fish from the everyday computational variety that we have been exposed to. Many competent students of mathematics, raised on high school diets of manipulation and problem-solving, find the transition to constructing proofs at university somewhat beyond the range of their abilities.

42.7.3 Disproof

It is easier to disprove something than to prove it. This is because a single example that disproves a mathematical proposition constitutes an adequate disproof. A million examples of the truth of a proposition, however, would not constitute an adequate proof.

This *asymmetry between proof and disproof* is something that might baffle the student who is learning the art of mathematical proof and disproof. Again, studying books devoted to the subject will engender the necessary mathematical maturity.

42.7.4 Recommended texts

Because proof writing is both art and science, you should systematically and rigorously study some good books on *how to construct proofs*. If you

[7] Constructing mathematical proofs is a specialized subject, outside the scope of this book.

had the time to read only one book on mathematical proofs, I would recommend Garnier and Taylor's *100% Mathematical Proof* [39]. It is clear, concise, and accessible. Work though it.

Other suitable books on the subject are those by Solow [40], Cupillari [41], Velleman [42], Aigner, Ziegler and Hofmann [43], Beck and Geoghegan [44], and Oberste-Vorth, Mouzakitis and Lawrence [45].

If you are a visual thinker, an aficionado of recreational mathematics, or prefer pictures to words, you might find the two volumes by Nelsen entitled *Proofs Without Words: Exercises in Visual Thinking* [46] and *Proofs Without Words II: More Exercises in Visual Thinking* [47] engrossing to view.

A freely available online electronic book, covering several topics on the foundations of mathematics, and concentrating on logic and proof, is *A Gentle Introduction to the Art of Mathematics* [48] by Joseph Fields. It is easy to understand and written in a delightful style. Given its free availability, it should be *the* one book on logic and proof that you should read if you do not have access to any others.

42.7.5 Closing thoughts on proofs

It is fitting to close this section on logic and proofs with a quotation from an author whose book I have recommended:

> It is a basic principle in the study of mathematics, and one too seldom emphasized, that a proof is not really understood until the stage is reached at which one can grasp it as a whole and see it as a single idea. In achieving this end, much more is necessary than merely following the individual steps in the reasoning. This is only the beginning. A proof should be chewed, swallowed, and digested, and this process of assimilation should not be abandoned until it yields a full comprehension of the overall pattern of thought.
>
> George F Simmons
>
> *Introduction to Topology And Modern Analysis* [34, p xi]

42.8 Undergraduate mathematics

The typical undergraduate mathematics programme runs the risk of segmenting the subject into disjointed topics, taught perhaps by different lecturers, at different times, and with different student cohorts.

We have already touched upon the foundations that a beginning undergraduate needs to master: sets, relations, functions, logic, proof etc.

Sometimes even these basics are not presented in a unified fashion, but rather piecemeal, as and when required.

If you are on the receiving end of such pedagogy, you might be yearning for some books that give an integrated treatment of the subject. Typically, these texts introduce "mathematical thinking" or "mathematical writing" to ensure that the foundations of mathematics are not left untaught. Such books are recommended next.

42.8.1 Recommended texts

Kevin Houston's admirably clear and concise book, *How to Think Like a Mathematician: A Companion to Undergraduate Mathematics* [49], is designed to ease the passage from high school to university mathematics. It introduces you to mathematical thinking. The chapters are snappy, cohere tightly around single ideas, and are amply endowed with examples and exercises. Read it if you have trouble coping with mathematics at university. Other texts with a similar aim include those by Ulrich Daepp and Pamela Gorkin [50] and Carol Schumacher [51].

Randall B Maddox has written a broad-based text that covers the basics not only of the foundations of mathematics but also of algebra. It is called *A Transition to Abstract Mathematics: Mathematical Thinking and Writing* [52] and could well serve as a reference for the serious undergraduate studying mathematics.

Another text that provides a broad overview and acts as a bridge to university mathematics is *A Transition to Advanced Mathematics: A Survey Course* by Johnston and McAllister [53]. It covers mathematical logic, abstract algebra, number theory, real analysis, probability, statistics, graph theory, and complex analysis.

42.9 Advanced undergraduate mathematics

Most universities routinely put their science and engineering undergraduates through two or more years of fairly demanding mathematics units. This enables advanced undergraduates and beginning graduate students to glide easily into subject areas that are mathematically intensive.

Despite such careful preparation, many an advanced undergraduate in science or engineering has been appalled to find that some of the mathematics that was studied barely two years ago now seems strangely unfamiliar. If you are in such a predicament, you need mathematical first-aid in the form of a refresher course or book. Two excellent texts that help achieve this are recommended next.

42.9.1 Recommended texts

The first book is S M Blinder's *Guide to Essential Math: A Review for Physics, Chemistry and Engineering Students* [54]. It is a student-friendly text written explicitly to help advanced undergraduates, especially in engineering and the natural sciences, who are in a quandary about the mathematics they are expected to know. It covers a lot of ground in a concise but clear fashion and the author's impish sense of humour makes it a delight to read.

The second book is Thomas A Garrity's consolingly titled *All the Mathematics You Missed: But Need to Know for Graduate School* [55]. It proceeds through an enormous volume of material at a lively pace. Goals are clearly stated at the beginning of each chapter and the crisp explanations are neither spare nor excessive, but rather just right. It is also a superb reference to have handy when you are vague or hazy about some definition or theorem.

42.10 Surveys

Many gifted writers of mathematics have recognized the need to impart to students a feel for the subject as a whole, rather than its parts. Their effort has generated an entire genre of general surveys of mathematics. We now call attention to some outstanding broad-spectrum texts in this area.

42.10.1 Recommended texts

If you are looking for a clear, systematic survey of mathematics, you couldn't do better than to browse and read the encyclopaedic volume entitled *Mathematics: Its Content, Methods, and Meaning*, edited by Alexandrov, Kolmogorov and Lavrent'ev [56]. This book is three volumes bound as one and is an inexpensive republication of a translation from the Russian. It does not contain exercises and is therefore not a textbook proper. However, it gives a rare bird's eye view of mathematics, both elementary and advanced, in a concise and comprehensive fashion and thus functions as a valuable reference. It is a gem worth owning, and dipping into now and then. Your time spent with it will be amply rewarded.

The second text gives a sweeping panoramic perspective of mathematics including its philosophical implications. It is *Mathematics: Form and Function* by Saunders Mac Lane [57]. The book has a historical and motivational flavour and is therefore not as laconic as a typical mathematics book. But be prepared for some heavy reading and thinking!

Another wide-ranging text is the classic by Courant and Robbins with the title *What is Mathematics: An Elementary Approach to Ideas and Methods* [58]. It is a charming, browsable overview of mathematics as a whole, and contains examples and exercises.

A remarkable multi-author survey that is breathtaking in its scope, and yet comprehensible and coherent, is *The Princeton Companion to Mathematics*, edited by Timothy Gowers, June Barrow-Green and Imre Leader [59]. It is a panoramic snapshot of mathematics as it exists today, and runs to over a thousand pages. It is destined to attain the status of a venerable classic with the passage of time. If you have access, do refer to it when you need to, and browse it for whatever takes your fancy. You will benefit a great deal. Own a copy if you can afford it! ☻

42.11 Computers, the Web, and mathematics

We have grown so used to computers and computing that it might come as a surprise that mathematics, physics, and electrical engineering were around at a time when computers were not. It is these parent disciplines that helped deliver the computer baby.

We have now come full circle: the computer and the nebulous network of information and knowledge called the Web are now powerful tools for learning and doing mathematics, physics, and engineering. The child has grown to minister to its parents.

42.11.1 Computer Algebra Systems (CAS)

Many undergraduate mathematics programmes include introductions to software for doing mathematics. Much of it is written in proprietary languages having their own syntax with which you need to familiarize yourself.

Mathematica®, [60] Maple™, [61] and MATLAB® [62] are three major commercial, proprietary software compilations that allow mathematics to be performed on computers. The convenient—if sometimes misleading—appellation *Computer Algebra System (CAS),* is often used to describe one or more of these.

There is also a free, open source compilation called Sage [63] that aims for the same capabilities as these three. GNU Octave [64] is yet another free, open source software compilation intended for numerical computation.

Statistical analysis is an area naturally suited to computerization and SPSS [65] is a well-known commercial, proprietary statistical software suite. An open source package that provides somewhat similar functionality is the R project for statistical computing and graphics [66].

Because the nature of software and the Web is constantly changing, more is not said about any of these packages in this book except to direct you to their respective Web pages, so that you might acquaint yourself with the capabilities of the latest version of each software compilation.

42.11.2 Applications of mathematical computing

Numerical solution of problems, like differential equations that have no closed form solution, is one obvious application of computers in mathematics, physics, and engineering.

Computer graphics is another. It injects a new degree of realism into the interactive display of geometrical objects which may be rotated, translated, and zoomed, to enable better visualization.

EXAMPLE: EXAMPLE

The *Klein bottle* is a particularly interesting geometrical object that is not easy to visualize. The companion website to this book has suggestions on how to search the Web. Use those hints to locate and display online images of the Klein bottle.

Note the use of colour, shading, and perspective for a realistic simulation of the object. Pay special attention to sites that permit interactive display of the object to allow it to be examined from different visual perspectives. This will give you an idea of the power of computer graphics to simulate objects. ★

42.11.3 Useful websites

This book's companion website has links to many useful websites on mathematics. Again, given the changing nature of the Web and its content, more is not said about these here except to refer you to the companion website.

42.12 The elephant of mathematics

I thought it would be fitting to end this pentad of chapters on mathematics with the question "What is Mathematics?" And, rather than attempt to answer it directly, let me use a metaphor.

There is an instructive fable about six blind men who attempted to characterize an elephant by touching it [67–69]. Because they could not see, they had no way of apprehending the entire elephant. Whatever each person learned by touching a part of the elephant did not encompass the whole. The elephant was thus described differently as a snake, a spear, a fan, a tree, a wall, and a rope. Their analogies were personally and locally valid but globally misleading.

As with the elephant in the fable, so too with the "elephant of mathematics". It is perceived differently by different people, and perhaps there is no one living at present who can see all of it at once. But ask almost anyone, and they will rightly associate mathematics with numbers. That is as it should be.

★

Mathematics embodies the quantitative aspect of human knowledge even as language enshrines the qualitative aspect.

But mathematics is about a lot more than numbers. I will leave you to ponder and answer for yourself what exactly it is.

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SUMMARY: MATHEMATICS AT UNIVERSITY

- The transition from high school to university mathematics poses several challenges.
- Strengthen weak high school foundations by consulting the suggested references and working through them.
- There are four major changes in mathematics between high school and university:
  1. Calculus to analysis;
  2. Algebra to abstract algebra;
  3. Geometry to topology; and
  4. Logic and proof becoming increasingly important.

  Consult the recommended texts to help ease these transitions.
- Books have been recommended to help you with undergraduate mathematics, advanced undergraduate mathematics, and with appreciating mathematics as a whole. Consult those relevant to your needs.

https://www.youtube.com/watch?v=O85OWBJ2ayo

Part H

Examinations

45

Answering Examination Questions

Synopsis

Read the instructions and the questions deliberately, understand them clearly, and follow them to the letter, respecting the clock.

Only the correct answer matters in multiple-choice questions. Work through them briskly. Use common sense, estimation, logic, etc., to quickly home in on the correct answer. Do not dither on "sticky" questions. Guess intelligently by eliminating wrong answers and best-guessing the right answer.

Task words are guideposts to what an examiner seeks in an answer. Familiarize yourself with commonly used task words and practise writing answers that are pertinent to the task words in the question. Do not omit what is necessary. Exclude what is unnecessary. If time is short, it is better to answer in point form than to leave a question unanswered.

Read widely. Write often. Practise until you are comfortable answering essay questions. The best preparation for practical examinations is regular and relaxed attendance at laboratories or other practical sessions. Do not skip practicals. The only preparation for oral examinations is mastery of your subject. Do not panic. Be calm. Be self-possessed. Answer oral questions clearly, carefully, and to the best of your ability.

Strive for the *Zen* state in answering examination questions by harmonizing your answers with the examiner's expectations.

45.1 Examinations

Examinations are all about questions, answers, and marks. And they are supposed to test your knowledge on both *recall* and *understanding*. We have looked at the preliminaries surrounding examinations in previous chapters. Examination preparation has been dealt with in Chapter 43, and examination technique was the subject of Chapter 44. Here, we grapple with the real nitty gritty of how to answer different *types* of examination questions.

Some written examinations feature multiple-choice questions. Others might consist of conventional short- and long-answer questions, and essay-type questions. Practical examinations and oral examinations demand their own specific question-answering skills.

Being able to answer examination questions is a core skill that any serious, academically inclined student must master. Questions from different subjects, model answers, and copious commentary are interspersed throughout this chapter as concrete examples to help you learn the art of answering different types of examination questions.

45.2 Rubric or instructions

The word *rubric* denotes the instructions to a candidate at the top of an examination paper.[1] For example, you might be asked to answer *all* questions from Part A of the paper, *any two* questions from Part B of the paper, and *any three* questions from Part C of the paper.

Paying attention to and clearly understanding the rubric allows you to carry out the instructions precisely and allows you to score top marks.

★

Do not skimp on reading the rubric: read it carefully several times, and make sure you understand and follow it exactly.

The word *rubric* comes from the Latin root for the colour *red*. So, the rubric is the writing in red. Whether or not the instructions on your examination paper are in red, give them as much importance as you would if they *were* written in red!

[1]Current academic usage, especially in the discipline of Education, associates this word with a scoring tool; that is *not* the usage adopted in this book.

45.3 Multiple-choice questions

We first consider multiple-choice questions. These can test both recall and understanding. Multiple-choice questions are easier than conventional questions because several answers are already given and you only have to choose the correct one. A multiple-choice question is *digital* in that your answer can only be right or wrong. There are no shades of grey; no in-between marks; no special consideration for partial working, etc. It is all or nothing. Your answers might even be processed by machine rather than marked by a human being.

> ★ Getting the right answer is everything in a multiple-choice question.

45.3.1 Homing in on the correct answer

There are two ways to home in on the correct answer in a multiple-choice question:

(1) Identify the correct answer because you know it. This is the *direct* method.

(2) Eliminate wrong answers one by one until you are left with the correct answer. This is the *indirect* method.[2]

Most multiple-choice examinations leave you panting for more time. So, be quick and correct. Knowing when and whether to apply the direct or the indirect method will save you time. This is a skill you must build up by repeated exposure to a large number of multiple-choice questions. Time yourself as you work through entire multiple-choice examinations from previous years to gain experience.

45.3.2 Are you penalized for a wrong answer?

Multiple-choice examinations usually adopt one of two ways of dealing with wrong answers:

[2]This reminds me of Sherlock Holmes' inimitable statement, "How often have I said to you that when you have eliminated the impossible, whatever remains, however improbable, must be the truth?"—Arthur Conan Doyle, *The Sign of Four*.

(a) Marks are deducted for a wrong answer: *you are penalized if you guess wrong.*[3]

(b) Zero marks are awarded for a wrong answer: *you penalize yourself if you do not guess when you do not know*.

Make sure you know which of these two regimes your examination follows.

45.3.3 The guessing game

You might find yourself compelled to *guess* the answer even though the correct answer is staring at you in the face, because you do not know—or are unsure about—the right answer. What should you do then?

> If you are not penalized for a wrong answer, guess by all means.

Otherwise, you might be better off skipping the question, leaving it for later, so that you can spend precious time on other questions that you *can* answer. And, there are two types of guessing:

1. *intelligent guessing,* which relies on logic, your knowledge of the subject, your degree of recall, your ability to eliminate obviously wrong answers, etc.; and

2. *random guessing,* which is plain gambling; you could just as well toss a coin, or roll a die, or pick an answer at random.[4]

45.3.4 Intelligent guessing

Suppose you find yourself in the unenviable position of not knowing the correct answer and yet being able to identify some wrong answers. You might then have no alternative but to play the guessing game. You should then guess intelligently rather than randomly.

[3] I suspect that this is done to compensate for the correct answer being already available, and to deter those who guess blindly.

[4] I am personally dismayed that examinations have come to this pass, where pure gambling could substitute for knowledge, but that is unfortunately our *Zeitgeist.*

> Intelligent guessing means eliminating the obviously wrong answers, and choosing—to the best of your knowledge—the right one from among those that remain.

It is a sensible strategy to adopt when you are unsure about the right answer, and are not penalized for wrong answers. Here is how it works.

Suppose there are five answers of which you are certain three are obviously wrong. You then have two answers from which to choose the correct one. So, the probability that you are right has increased from an original value of $\frac{1}{5} = 0.2$ to a value of $\frac{1}{2} = 0.5$ after elimination of the wrong answers. You have thereby *more than doubled* your chance of getting the correct answer. Of course, even then, you have a 50% chance of being wrong; but before, you had an 80% chance of being wrong. So, if you have to guess, at least ensure that the odds are not stacked against you. Use judicious elimination to help you. Practise intelligent guessing.

45.3.5 Avoid random guessing

If you have no clue about any of the answers, and if you are not penalized for wrong answers, and if you are an inveterate chance-taker, the thrill of gambling might tempt you to guess randomly.

> Do not guess randomly if a wrong answer could lose you marks.

Omitting an answer gets you zero marks, but gambling on an answer might lose you marks. Even if you must be a risk-taker, be sensible when you guess.

45.3.6 Respect the clock

Stick very strictly to the time per mark explained in Section 44.6. Move briskly from question to question. Do not dither. Do not dally. Do not tarry. If a multiple-choice question takes more time that it should, your approach is probably wrong. Do not waste time on irrelevant details but rather use your common sense, ability to estimate, and logic, to home in on the correct answer in minimum time.

If a question proves "sticky", do not lavish time on it. Rather go on to the next question and return to the "sticky" one later on, if time permits. Most multiple-choice examinations are structured so that you would struggle to finish all the questions within the given time.

Maximize your marks by spending time on those questions that will earn you marks.

Take a look at the examples that follow to guide you in answering multiple-choice questions.

45.3.7 Example: Pattern recognition

QUESTION

Which is the odd one out?

(a) Orange

(b) Peach

(c) Chocolate

(d) Apricot

(e) Blue

ANSWER

Item (e)

COMMENTARY

This is a deceptively simple question that does not require any *doing*. There is no computation involved. There are no complicated relationships to sift through. Yet, there is a twist, not unlike a detective novel.

The obvious pattern is that the first four are items of *food,* while the fifth is a *colour,* and obviously the odd one out. But, if you have not noticed yet, *all* the items in the list are *also names of colours.* I point this out so that you become aware that patterns might exist beyond the blatantly obvious and this observation might help you sometime.

Just so that you are acquainted with the possibility of ambiguity in this type of question, let me amend the above question into this new question:

AMBIGUOUS QUESTION

Which is the odd one out?

(a) Orange

(b) Peach

(c) Chocolate

(d) Apricot

(e) Starfruit

COMMENTARY ON AMBIGUOUS QUESTION

If you take a look at the amended, ambiguous question, you will see that the only change has been the inclusion of “Starfruit” as Item (e). Even if you do not know what a starfruit is, you are justified in hazarding a guess that it is a fruit, which indeed it is.

But this simple change has introduced ambiguity. To sift the odd one out, you are basically dividing the group into two classes. If you choose to classify on the basis of fruit and non-fruit, the third is not a fruit; all the others are. On the other hand, if you classify on the basis of the name of a colour, the first four are all the names of legitimate colours; the fifth is not. *So the answer could be either Item (c) or Item (e).*

So, what should you do in an examination?

You should hope that you do not ever get such an ambiguous question. Ambiguity and fuzziness have no place in multiple-choice questions, and you can only hope that your examiners are skilled enough to craft crisp, clear, and unambiguous multiple-choice questions. In the unlikely event that one or two of these ambiguous ones slip through, you can only hope that the post-examination examiner’s board meeting will award marks for either answer, and not punish candidates for no fault of their own.

45.3.8 Example: Mathematics

QUESTION

The number of solutions for the equation $x - \sin x = 0$ is

(a) 3

(b) 1

(c) 2

(d) Cannot tell without more information

(e) Infinite

PRE-COMMENTARY

Don't peep at the answer below while I walk you through the solution of this question.

First, recognize that this is a *transcendental equation;* not an *algebraic one.* So, you need not vainly attempt to solve it algebraically. That leaves us two options: *numerical* solution or *graphical* solution.

Numerical solution is time-consuming and is a waste of time because we are not asked for the *actual* solution(s); only the *number* of solutions.

For a graphical solution, we would need to plot $x - \sin(x)$ and find the number of times the curve crosses or touches the x axis. Notice, however, that $x - \sin x = 0$ may be re-written as $x = \sin x$. Because both $f(x) = x$ and $f(x) = \sin(x)$ are familiar curves, it is easier to plot these two curves and determine how many times they osculate or intersect, if at all.

Before doing that, though, I will consider some of the answers given above, which might trip the unwary.

Item (d) is a very good red herring. Since the sine function is defined for all real values, you might be tempted to think wrongly that absence of domain information makes this question unanswerable. Not true.

Item (e) is another red herring. Since both functions, $f(x) = x$ and $f(x) = \sin(x)$, are defined for all reals, you might be misled into thinking that there is an infinity of solutions. Again not true.

The *range* of $\sin x$ is restricted to $[-1, 1]$. And for this range, we need only consider x in $[-1, 1]$ in the graph $y = x$. In other words, any solutions *must* lie in the interval $[-1, 1]$.

A sketch of the two graphs is shown in Figure 45.1. As already noted we only need the domain $[-1, 1]$; a larger domain is shown in the figure to convince you visually that solutions cannot lie outside $[-1, 1]$. From Figure 45.1, there is only a *single* solution and it is obviously at $x = 0$.

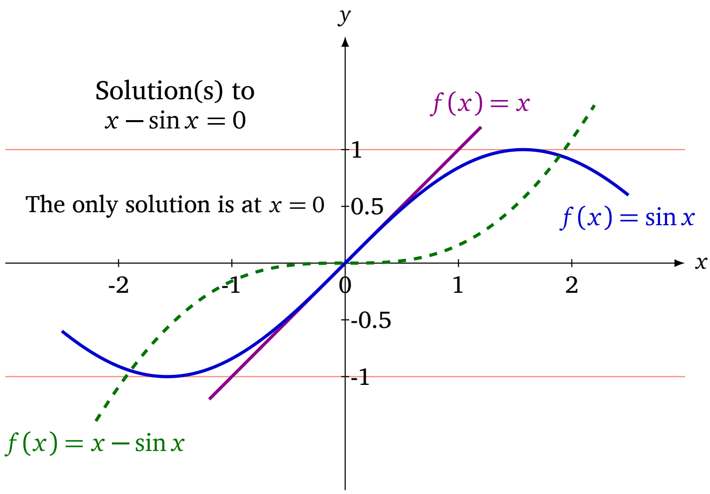

FIGURE 45.1: The solutions of $x = \sin x$ are the points of intersection, if any, of the two graphs $y = x$ and $y = \sin x$. The two curves intersect at $(0, 0)$ which is obviously a solution. Recall that for small x, $\sin x \approx x$ but for larger x the graph of $\sin x$ falls away from the $y = x$ line. It is clear that there are no further solutions in $[-1, 1]$ and that there is only *one* solution. The graph of $x - \sin(x)$ is also shown as a dashed curve for completeness and it is clear that it crosses the x-axis but once.

The sketch you make in an examination need not be print quality as shown here, but simply sufficient to lead you to the correct answer. Is the sketch absolutely necessary? Yes, a graphical solution necessitates a sketch.

If you are innately curious, you might wonder what the graph of $x - \sin(x)$ looks like. To cater to your inquiring mind, I have also shown that graph in Figure 45.1. Clearly it crosses the x-axis at $x = 0$ and nowhere else.

ANSWER

Item (b)

POST-COMMENTARY

Before leaving this question, let us briefly look at the number of solutions to $x - \cos x = 0$. Similar comments apply as before and again the solution is best seen graphically. There is a single solution for x in the interval $[0, 1]$ as seen in Figure 45.2

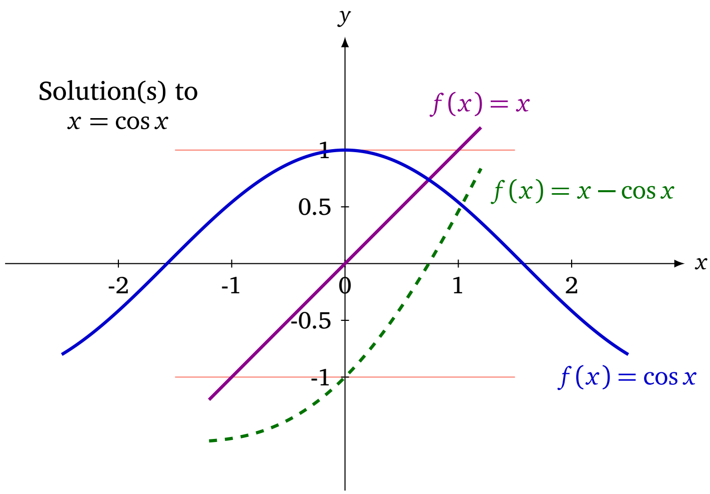

FIGURE 45.2: The single solution of $x = \cos x$ lies in the interval $[0, 1]$. It is the x-co-ordinate of the intersection of $y = x$ and $y = \cos(x)$, or equivalently, the value of the zero crossing of $y = x - \cos(x)$.

45.3.9 Example: Logic

QUESTION

The Taj Mahal was built by Shah Jahan, the fifth emperor of the Mughal dynasty, founded by Babur, who was succeeded by Humayun, the father of Akbar. Which of the following is true?

(a) Shah Jahan was the son of Akbar

(b) Shah Jahan was the great-grandson of Akbar

(c) Shah Jahan was the grandson of Akbar

(d) Impossible to deduce from the given information

(e) None of the above

ANSWER

Item (c)

COMMENTARY

This might appear like a History question, but it is essentially a test of logical deduction. The question clearly gives the names of the first three Mughal emperors as Babur, Humayun, and Akbar. The fourth is missing and not necessary to answer this question. *Do not waste time trying to remember who succeeded Akbar as the fourth Mughal emperor.* Simply use "X" as a placeholder for this emperor in the following list:

(i) Babur

(ii) Humayun

(iii) Akbar

(iv) X

(v) Shah Jahan

Shah Jahan was the grandson of Akbar.

45.3.10 Example: Chemistry

QUESTION

What type of chemical reaction is rusting?

(a) esterification

(b) double decomposition

(c) polymerization

(d) oxidation-reduction

(e) acid-base neutralization

ANSWER

Item (d)

COMMENTARY

This question tests domain-specific knowledge, of chemistry in this case. You either know the answer or you don't. However, if you are only partly prepared and are willing to hazard a guess, can that guess be refined intelligently?

Acid-base neutralization is clearly out as rusting takes place in the absence of acids and bases. Polymerization is associated mainly with organic compounds, as is esterification. Double decomposition is a kind of chemical "ping pong" match which rusting clearly is not. That leaves oxidation-reduction as the answer.

45.3.11 Example: Computer Science

QUESTION

The defining characteristics of an *algorithm* are:

(a) it is series of computational steps

(b) it defines the keywords in the ALGOL programming language

(c) it is a series of computational steps guaranteed to stop

(d) it is an arithmetic operation like addition and subtraction

(e) it is a novel way of taking logarithms of numbers

ANSWER

Item (c)

COMMENTARY

This is a straightforward question that tests domain-specific knowledge in computer science. The definition of an algorithm is given by Item (c) above. Notice the hilarious red herrings as well!

45.3.12 Example: Arithmetical Computation

Numeracy tests of arithmetical computation are parts of many multiple-choice examinations. In many of these questions, erroneous answers arising from commonly made errors are usually also given as possible answers. You have to be very wary, especially if marks are deducted for wrong answers.

QUESTION

A cyclist covers 6 km in half an hour. After resting for another half hour, the cyclist covers an additional 6 km in one hour. The average speed of the cyclist is:

(a) 12 km/h

(b) 6 km/h

(c) 8 km/h

(d) 18 km/h

(e) 9 km/h

PRE-COMMENTARY

Each of the above numbers represents either a correct or commonly erroneous way of computing average speed. I will first work out the correct answer.

Average speed is defined as *total distance* covered divided by the *total time* taken to cover it. In our case, the total distance is 12 km and the total time taken is two hours, including resting time, giving an average speed of 6 km/h.

One incorrect method is to add the total distance of 12 km and divide it by the time taken for each segment which is 1.5 h giving a wrong average speed of 8 km/h. This ignores the resting time. Another error is to take the speed of the first segment as the average speed, in this case 12 km/h. Yet another error is to compute 12 km/h for the first segment and 6 km/h for the second and averaging the two to get 9 km/h. The final error is to add the speeds of the two segments together to get 18 km/h. All answers seem plausible. Moreover, each erroneous method is keyed to one answer. Beware! The unprepared student will be trapped, especially if marks are deducted for wrong answers.

ANSWER

Item (b)

45.3.13 Example: Estimation

Working the answer out to the end often consumes a lot of time. Judicious estimation can save time because you do have a list of answers to choose from. Perhaps, in such multiple-choice questions as nowhere else, specific technique in answering helps you score more marks.

QUESTION

What is the number closest to the number of days in three years?

(a) 600

(b) 3,000

(c) 1,000

(d) 10,000

(e) 2,000

ANSWER

Item (c)

COMMENTARY

Clearly, you can work out that 365 multiplied by 3 equals 1095. Most often, questions like these will be posed in examinations where calculators are not allowed. So, the paper and pencil calculation will take some time unless you know shortcuts in arithmetic. But if you want to whizz past this question in minimum time, you need to estimate, and estimate sensibly.

One year is 365 days which is more than 333 days. So, three years must exceed 999 days. Therefore the correct answer exceeds both 600 and 1000 days, but will be less than 2,000, 3,000, and 10,000 days. Of these, 1,000 is closest to the real number.

One further point. You might think yourself meticulous by considering leap years, but you would be foolish to do so. First, you could choose three years which do not fall within a leap year band. Second, you only need an estimate. So, do not fall into the trap of accounting for the *single extra day* of a leap year, and thereby lose marks or time!

45.4 The Essay Question

Writing essays has already been covered in Chapter 36. Look at that chapter again to refresh your memory on how to write essays. A model essay has been given in Chapter 37. You should study it *very carefully* to see how the theory of constructing essays has been put into practice in writing one. Also cast your eyes and mind back on Chapters 35 and 50. All of these chapters deal with the craft of writing. It is a skill that you learn for life rather than simply for an examination.

The only difference between writing essays generally and writing them in an examination is that, in the latter case, you have a time deadline and are denied access to reference materials like a dictionary, thesaurus, encyclopedia, online Web resources, etc. So, you must make do with what you have in your head to accomplish the best that you can.

Read widely. Write often. Practise until you are comfortable answering essay questions. Apart from constant exposure to new ideas, and constant polishing up of your writing skills, there is not much else you can do to prepare yourself for this type of question.

45.5 Short- and Long-Answer Questions

The short- and long-answer questions are simply the examination version of the drill and assignment questions you would have routinely answered as part of your coursework. They too are meant to test recall and understanding. There is nothing special about them in an examination except that you might be pressed for time and might be forced to answer them in point form. This is more acceptable for the short-answer questions than for the long-answer questions. In any case, *it is better to answer in point form than to leave a question or part of a question unanswered.*

The short- and long-answer questions tend to be rather subject-specific. Listing a few sample questions and model answers would not be as useful as dissecting the wording of *different types* of examination questions and giving answers and commentaries to them. In large measure, the commentaries would be independent of the subject and therefore more useful. This is exactly what is done in the rest of this chapter where I have endeavoured to feature questions from various disciplines of study.

45.6 Types of Examination Questions

Examination questions can and do test all four stages of learning: *naming, knowing, doing, and being,* as explained in Chapter 1. At one end of this spectrum, we have the *naming-type* question that tests *recall;* at the other end of the spectrum, we have the *knowing-doing-being-type* question that tests *understanding.* You need to be proficient in dealing with questions from one end of this spectrum to the other in order to be a roaring success at examinations.

45.7 Task words or instruction words

Individual questions are worded in different ways to test different aspects of the candidate's knowledge. In the succeeding sections, we shall take a look at the most common words in which examination questions are couched, and identify what the examiners are looking for in each case. I call these tell-tale words *task words* or *instruction words,* first introduced in Section 36.8 of Chapter 36.

★

Task words are guideposts to what an examiner seeks in an answer.

There is an implicit code for each of these words which, when known, clarifies exactly what the examiner is looking for. This code is shown in Table 45.1 below. In the succeeding sections, we take each of these task words in turn, and analyze example questions and answers in each case.

Using the clues spelt out by the task words, practise writing answers that are pertinent to the question. Calibrate detail and depth, explanation and exposition, to the task word. Do not omit what is necessary. Exclude what is unnecessary.

We consider a variety of questions from different disciplines, each featuring a different task word, with model answer and elaborate commentary, in the succeeding sections.

45.8 State

The simplest type of examination question begins "State …" where the dots or ellipsis represents some fact or law or definition. Such questions simply ask you *to state a fact.* No embellishment is needed. No comparison is needed. No analysis is needed. No explanation is needed.

TASK WORD	MEANING
STATE	Write down
ENUMERATE	List one by one
OUTLINE	Present the main points, in order, without the detail
DESCRIBE	Paint a word picture
EXPLAIN	Clearly give the meaning and/or reasons for something using jargon-free language
ACCOUNT FOR	Give the causative reasons for something or some situation
DISCUSS	Look at both sides of an issue
COMPARE AND CONTRAST	Show the similarities and differences
ANALYZE	Break the subject down to its causes and constituent parts

TABLE 45.1: Commonly used *task words* or *instruction words* and their meanings.

Simply write down the required fact completely and correctly.

You should score full marks on such questions. "State" questions test the naming stage of learning and are often the first part of a multi-part question that gently eases into the knowing and doing stages in later parts of the same question.

45.8.1 Example: Boyle's law

Let us consider a simple "State" question from physics:

QUESTION

State Boyle's law.

HURRIED ANSWER

You might rush off and write immediately *"Pressure is inversely proportional to volume"*. You might even write down the applicable equation, in this case, $pV = k$, which in plain English means that the product of pressure and volume is a constant.

COMMENTARY

I will now analyze that answer and show you why it is *incomplete*. The following important points were omitted from the answer:

1. We are dealing with a *gas*.[5]
2. Moreover it is a *fixed mass* of gas.
3. The gas is at *constant temperature*.
4. The gas is an *ideal gas* in which effects like intermolecular forces, which occur in a real gas, are neglected.

CORRECT ANSWER

So, the correct answer should be: *"In a fixed mass of an ideal gas at constant temperature, the pressure is inversely proportional to the volume."*

POST-COMMENTARY

Now, for the clincher. How does one remember all these caveats as they apply to Boyle's law? Worse still, what if the question were about Charles' law or Gay-Lussac's law, both of which also apply to ideal gases?

This is where mathematical equations come to the rescue. Usually, the laws of physics are statements in words of some quantitative relationship that describes how Nature behaves, at least to a good first approximation. Once you realize that it is an *ideal gas* that you are dealing with, it is easy to press into service the correct equation. The *ideal gas equation* is

$$pV = nRT$$

where the symbols have these meanings:

SYMBOL	MEANING
p	pressure
V	volume
n	number of *moles* of the gas, representing the mass
R	constant of proportionality called the *ideal gas constant*
T	temperature in kelvins

[5] This relationship does not hold for a liquid, or solid, or other state of matter like a plasma or Bose-Einstein condensate.

Pay particular attention to the fact that since R is itself a constant, when n and T are constant, i.e., mass and temperature are fixed, the right hand side nRT is also constant. So the value of k in $p\mathrm{V} = k$ is actually the value of nRT, and explains the need for two of the caveats attendant upon Boyle's law mentioned above.

This simple equation, $p\mathrm{V} = n\mathrm{RT}$, can be used to check the completeness of *your statement* of Boyle's Law, or Charles' law, or Gay-Lussac's law, as the case may be. *The ideal gas equation is therefore a good example of a single laconic equation that embodies several laws.* It is worth committing to memory as part of the naming/knowing stage of learning physics.

This principle of abstraction and condensation of facts—of encapsulating several ideas into one succinct equation—may be extended to any other tools you might use for studying, like octopus diagrams, or mind-maps, or flow charts, or mnemonics, such as those suggested in Chapters 7 and 19. This is especially pertinent to disciplines of study like history or medicine where equations are not commonplace, but where facts and their inter-relationships nevertheless need to be mastered.

45.9 Enumerate/List

The second, easy type of question begins with "Enumerate…" or "List…" where again the dots denote some subject. Here, you are asked to state a number of facts, often in a specific order.

★

To enumerate or list some facts, order them correctly, and write them down in that order, numbering or tabulating them if necessary.

Two example questions will help in understanding how to answer such questions.

45.9.1 Example: The halogens

This example question is taken from Chemistry. It requires some familiarity with the Periodic Table of elements [1]; I will assume that you have that familiarity. Here is the question:

QUESTION

List the halogens, along with their chemical symbols, in ascending order of atomic number.

PRE-COMMENTARY

This question merits discussion before an answer is proffered. First, you need to know *what* the halogens are. Second, you need to know their *order* by atomic number. Third, you need know their *chemical symbols.* Familiarity with the Periodic Table is essential on all three counts.

This kind of question will usually be asked only if your syllabus specifically states that you are expected to know this sort of information. Note, very importantly, that you are *not* asked to list the atomic numbers of these elements. It is as important to know what you *should not write down* as part of your answer, as it is to know what you should. Armed with this simple analysis, you might attempt the following table as your answer.

ANSWER

ELEMENT	CHEMICAL SYMBOL
Fluorine	F
Chlorine	Cl
Bromine	Br
Iodine	I
Astatine	At

COMMENTARY

If you do not know the order of the halogens by atomic number, you might attempt to order them by starting off with the gases, moving on to the liquids, and ending with the solids. This is where your knowledge of basic chemistry might assist you, when your memory falters.

Note the spelling for fluorine; it is all too easy to swap the letters u and o in the word fluorine. Note also that astatine has the symbol At rather than As, which stands for arsenic. Finally, note that the *chemical symbol* is different from the *molecular formula* for the elements. The first four halogens occur naturally as diatomic molecules. Therefore, while Br_2 is the molecular formula for naturally occurring bromine, its chemical symbol is merely Br.

Marks will be awarded for each of the following:

(a) whether you have listed all five halogens;

(b) whether you have spelt their names correctly;

(c) whether you have listed their chemical symbols correctly; and finally

(d) whether you have ordered the halogens correctly.

[6] Simple as it is, this question illustrates the composite, interleaved layers of facts and knowledge which need to be mastered to get full marks for the question.

45.9.2 Example: Kings and queens of Great Britain

Questions based on wars and dynasties are often a favourite for testing factual recall of historical material. Here is an example "List" question.

QUESTION

Name any five English or British monarchs, together with their royal houses, who reigned between Queen Elizabeth I and Queen Victoria, both inclusive.

PRE-COMMENTARY

This is an example of a *gift question.* You are already given the names of two queens and told that both may be included in the list. You could include them or leave them out, but it is much smarter to include them because you already know two names: you only need to associate the correct royal house with each. So, even if you knew nothing about this question, you would still gain some marks by naming Queens Elizabeth I and Victoria. Learn to capitalize on the information already available in gift questions.

Note that you are *not* asked to list the monarchs in *chronological order.* So, don't waste valuable time doing so. If you were asked to list them in chronological order, bear in mind that if you swap any two royals, you make *two* mistakes, not one. So, be very careful with such questions. If you know the names of the kings and queens, but not their royal houses, you might at most get half marks. This will help you decide whether or not to choose this question to answer, if it were optional.

[6] Once you know how marks are allocated, you will be able to correct your own answers when you attempt model examination papers as part of your revision or exam preparation.

ANSWER

MONARCH	ROYAL HOUSE
Elizabeth I	Tudor
Charles I	Stuart
Anne	Stuart
George III	Hanover
Victoria	Hanover

COMMENTARY

You could, of course, have named any other English or British monarchs meeting the criteria of the question. Incidentally, the current royal house of Britain was renamed from *Saxe-Coburg and Gotha* to *Windsor* during the First World War to deflect anti-German sentiments prevailing in Britain at that time.

45.10 Describe

Questions with the word "Describe" in them are next in ascending order of difficulty. When you are asked to describe something, do so clearly and completely. There is no need for an explanation or for a theoretical analysis. Often questions of this type are about what is heard, seen, or otherwise perceived.

★

Paint a clear and precise word picture of what is asked for.

45.10.1 Example: Wailing siren on an ambulance

QUESTION

Describe what you would hear as an ambulance vehicle with a wailing siren and flashing lights approaches you from a distance, passes by you, and recedes from you.

HURRIED ANSWER

The words *Doppler Effect* or *Doppler Shift* might pop up in your head instinctively and you might be tempted to answer the question with just those two words.

Commentary

The *Doppler Effect* is indeed the correct *name* for what takes place, but *it does not describe* what happens. Since you have been asked to describe what happens, you might not get any marks for your two-word answer and this might leave you a little mystified, not to say disappointed at your marks.

An ambulance vehicle announces its approach with its wailing siren and flashing lights. You are specifically asked about what you would *hear* as it pertains to the wailing siren; the flashing lights are outside the scope of the question.

Almost Correct Answer

Assuming that I am stationary, as the ambulance vehicle approaches me from afar, the pitch of its siren increases, until it passes me by. After that, as the vehicle recedes from me, its siren pitch decreases.

Post-Commentary

There is no need for any explanation. No need to mention the *Doppler Effect* or give any formulae for it. Subsequent parts of the question might indeed ask for the name of the effect or its mathematical elucidation; but we are here concerned only with the "Describe" part. Avoid unnecessary explanations because not only do they rob you of time for other questions, they also gain you no extra marks.

Additional Correct Answer

As the ambulance approaches me, the loudness of the siren increases, and reaches a maximum when it is directly in front of me. As the ambulance passes me by and disappears into the distance, the loudness of the siren falls again.

Final Commentary

The Almost Correct Answer identified the perceived *changes in frequency* but neglected to address the perceived *changes in loudness* mentioned in the Additional Correct Answer. If you neglected to mention the changes in either the frequency or the loudness (but not hopefully both!), you could lose some marks for the omission.

Incidentally, the change in loudness is governed by the *inverse square law* which is again not asked for.

The best students will also draw *sketch graphs* to indicate the variation of pitch and loudness with distance toward and away from the observer. Indeed, such a graph might be the fastest way to get the most marks if you are rushed for time and cannot write fast enough.

If you are interested in exploring more about this physical situation, take a look at the explanations at the HyperPhysics website [2] and at Wikipedia [3].

Finally, I think that the question itself is perhaps a little infelicitously phrased. *Relative motion* between observer and ambulance is what matters. The safest recourse here is to assume that you are "stationary" and that the ambulance is "moving". If you were in the ambulance yourself, or in another vehicle moving along with the ambulance, what you hear would be something quite different.

45.11 Recall versus Understanding

Before proceeding to the next type of question, we take a little breather to reflect on what exactly is being assessed in different types of examination question. As observed at the beginning of this chapter, examination questions may be divided into two broad classes: those which test *recall* and those which test *understanding*.

The foregoing types of questions mostly test recall. Recall relates primarily to the *naming* phase of learning. The subsequent types of questions test not only recall, but also understanding. They relate mostly to the *knowing* and *being* phases of learning. Problem solving questions and practical examinations relate to the *doing* phase of learning.

A typical examination question might start out as a *recall* question, and progress in its later parts into an *understanding* question. If the examiners are skilled, they will ease this passage from recall to understanding. You move from your base knowledge, through problem solving or practical abilities, into demanding intellectual analysis. In the next section, we consider questions that increasingly test understanding.

45.12 Explain

More demanding than the "Describe" questions are those with the word "Explain". Here you not only need to describe *what* happens, you also need to explain *why* it happens, or *how* it happens. You must be a full bottle not only on the facts but must also be able to marshal the underlying theory to give a coherent and convincing explanation of the observation.

> If necessary, describe what takes place. Then, explain clearly and coherently, what happens, or why it happens, or how it happens, as required. Sate assumptions and exceptions if any. Use jargon-free language.

45.12.1 Example: Mixing coloured paints

This example question with the word "Explain" is from Physics.

QUESTION

Explain what happens when two paints—one containing only yellow pigments, and the other containing only blue pigments—are mixed.

ANSWER WITH A DIAGRAM

Pigments exhibit the colours that they *reflect*. A blue pigment reflects predominantly light in the blue wavelengths; a yellow pigment reflects strongly in the yellow wavelengths. The *reflectance* or percentage of light reflected by the pigment, when plotted against wavelength typically appears as shown below. The two reflectance curves are generally not sharp spikes at some specific blue and yellow wavelengths, but rather resemble bell-shaped curves, centred in the blue and yellow wavebands, *reflecting light at other wavelengths as well.*

Recalling the rainbow colour mnemonic, VIBGYOR, it is apparent that yellow pigments reflect some green and orange wavelengths. Likewise, a blue pigment also reflects green and indigo. When both pigments are mixed, the overall reflected colours belong to wavelengths that are reflected by *both* pigments. This light happens to fall in the green-cyan-blue band of light. *So, the mixture of blue and yellow pigments will appear predominantly green,* given that human colour vision is most sensitive to green light.

ANSWER WITHOUT A DIAGRAM

When *lights* of different colours are mixed, as on a computer terminal or TV screen, the *additive colour model* applies. For example, a combination

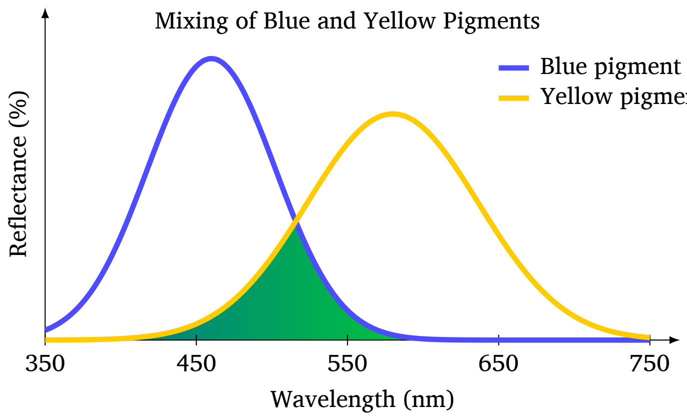

FIGURE 45.3: Typical reflectance curves of blue and yellow pigments plotted against wavelength in nanometres. Because pigments *reflect* light, the *subtractive colour model* applies. The colour of the mixed pigment is made up of the colours under the *intersection* of the two reflectance curves. These are green, cyan, and blue. Given that the human eye has a response to colour that peaks at the green wavelengths, *the mixed pigments appear mainly green.*

of equal parts of the primary colours—red, green, and blue—takes on a shade of grey, ranging from black to white.

On the other hand, when *pigments* are mixed, as in printing, the *subtractive colour model* applies. When the secondary colours—cyan, magenta, and yellow—are mixed, the result appears almost black. *A pigment takes on the colours it reflects.*

An ideal cyan pigment would therefore reflect the colours green and blue since cyan light is derived from mixing green and blue lights. Likewise, an ideal yellow pigment would reflect red and green. *When two pigments are mixed, the colour seen is that which is reflected by both pigments.* The common colour reflected by both cyan and yellow is green. Therefore, mixing cyan and yellow pigments would give a green coloured pigment. A blue pigment also reflects some light in the cyan and green regions. *So, a mixture of blue and yellow pigments appears green.*

COMMENTARY

To answer this question, you should know the following:

1. The visible spectrum is composed of light of an infinitude of colours—starting with red and ending with violet—conveniently remembered through the acronyms VIBGYOR or ROYGBIV.

2. When different coloured *lights* are mixed, their resultant colour is *additive*. This is something like a *set union* illustrated by a Venn diagram.
3. When two *pigments* are mixed; their resultant colour is *subtractive*. This is somewhat like a *set intersection* illustrated on a Venn diagram.
4. Paints derive their colour from pigments. The colour of a pigment is the colour perceived by the human visual system based on the *colours reflected by the pigment*.
5. The reflectance curve of a pigment—which quantifies the amount of light reflected against light wavelength—is centred around some dominant wavelength that is reflected the most, and gradually decays on either side of that wavelength. This dominant wavelength determines the colour of the pigment.
6. When two pigments are mixed, the colours reflected by the mixture consists of the colours reflected by *both* pigments.

How you choose to incorporate and express economically the above ideas is your call. Indeed, simply enumerating the ideas, as above, will constitute an adequate answer. The two model answers above are not definitive; they are merely illustrative.

A diagram is almost always more impressive and economical. Figure 45.3 has been prettied up for printing. A basic, black-and-white, but well-annotated sketch will suffice in an examination. A detailed caption is also unnecessary if its substance is included in your answer text.

The first answer adopts a more experimental approach to the explanation; the second has a more theoretical basis. The enumerated points above will suffice equally.

45.13 Analyze

Questions with the word "Analyze" are still more demanding. Simple recall of facts alone will not do. A mere description will not do. You need to add value to your answer by taking the issue apart, examining it carefully, and giving your assessment at the end. It is a bit like a jeweller appraising a gem. The opinion must be backed by facts or evidence. The later parts of almost all examination questions will require some form of analytical thinking and writing.

Pick the issue apart. Break the subject down to its causes and constituent parts. Give your verdict based on factual evidence. Be impartial, logical, complete, and convincing.

45.13.1 Example: Atomic Theory

QUESTION

Analyze the experimental evidence in support of the atomic theory of matter.

PRE-COMMENTARY

The atomic theory of matter itself arose out of an attempt to *analyze* or *break down* matter into its component parts. So, this question is a bit of a pun on the word "analyze". A sufficient answer is given below.

ANSWER

Our perceptions lead us to believe that matter is continuous rather than discrete. However, Kaṇāda in India and Democritus in Greece were among the ancient *philosophers* who propounded the then counter-intuitive theory that matter is composed of discrete indivisible entities. The word *atom* comes from the Greek *ατομος/atomos*, which means "uncuttable".[7]

The *scientific* view that matter could be discrete and atomic arose first in Chemistry from indirect evidence provided by the experimental analysis and synthesis of chemical compounds. It was found that when different samples of the same compound were analyzed, the elements composing it always occurred in the same proportion. This is called the Law of Constant Composition, or Proust's Law. Moreover, it was found that when elements combined, they always did so in the ratios of *small whole numbers*. Nature appeared to be digital or discrete in this regard. This is called the Law of Multiple Proportions, or Dalton's Law. While these laws lent credibility to the atomic view of matter, they were not definitive evidence for it.

[7] Although we now know that atoms themselves are composed of a hierarchy of still more elementary particles.

Because atoms cannot be seen, the atomic theory had to depend on a series of elegant and ingenious experiments that proved conclusively that matter was indeed composed of discrete indivisible particles. Physical effects that linked the measurable macroscopic properties with the unknown microscopic parameters provided the bridge across this perceptual chasm. Imagination, disciplined by mathematics, and supported by patient and careful experimentation were essential to this process.

The kinetic theory developed by James Clerk Maxwell, Ludwig Boltzmann, and others, carried the vivid mental picture of molecules as individual particles—perfectly elastic billiard balls if you will—that moved about quite freely in a gas and less freely in a liquid. Among its achievements, the theory related the temperature of a gas to the kinetic energy of its molecules. While this was a valuable theoretical insight, it was not sufficient evidence of atomicity.

Amadeo Avogadro's hypothesis—that samples of two different ideal gases at the same temperature, pressure, and volume contain the same number of molecules—allowed all gases to be studied generically and paved the way for the Ideal Gas Law. This gave a plausible explanation of the experimentally observed gas laws like Boyle's Law, Charles' Law and Gay-Lussac's Law, but stopped short of providing positive evidence for the existence of atoms.

It was the phenomenon of Brownian motion—named after the English botanist Robert Brown, who in 1827 saw pollen grains in a liquid performing an erratic and random dance under the microscope—that finally elevated the atomic hypothesis to a theory well founded on experimental evidence.

Almost eighty years were to pass before Brownian motion would be studied seriously and explained. As with almost all of twentieth century science, Albert Einstein had a hand in this. From 1905 onward, he published a series of theoretical papers [4] analyzing Brownian motion quantitatively as a manifestation of the atomicity of matter. One upshot of his work was the equation

$$\mathrm{D} = \frac{\mathrm{RT}}{\mathrm{N}}\left[\frac{1}{6\pi k\rho}\right]$$

where D is the coefficient of diffusion of the Brownian particle, R the universal gas constant, T the thermodynamic temperature, N the Avogadro number, k the coefficient of viscosity of the solvent, and ρ the radius of the Brownian particle, assumed to be a sphere. This equation related macroscopic and microscopic quantities and allowed one to estimate the size of the Brownian particle if N was assumed, and conversely, allowed N to be estimated if ρ was known.

Jean Baptiste Perrin devised a series of meticulous, ingenious, and elegant experiments [5] on Brownian motion to measure the value of N using Einstein's equation. The results he obtained were in accord with values of N obtained experimentally and independently, using other techniques like electrolysis, and Robert Andrews Millikan's famous oil drop experiments [6].

It was Einstein's theory and Perrin's experiments on Brownian motion that furnished conclusive evidence in support of the atomic theory. This congruence between theory and experiment established the atomic theory once and for all, and enabled the growth of modern scientific thought on its foundations.

Rutherford's gold foil experiment unexpectedly revealed an atom to be mostly empty space, with a charged, spatially localized nucleus, and lent further support to the atomic model while clarifying it. X-ray crystallography furnished evidence of periodic atomic lattices. Quantum Theory, and its consequent effects like photoelectricity, lasers, etc. have raised atomic theory to its presently unshakable pedestal by demonstrating not only the theoretical basis but also the practical utility of the atomic view.

The scanning tunnelling microscope invented in 1981 by Gerd Binnig and Heinrich Rohrer[8] finally enabled not only the convincing *visualization* of individual atoms but also their *manipulation,* leading to the science of nanotechnology. *We now have experimental evidence that allows individual atoms to be seen and moved.*

POST-COMMENTARY

There might be other pieces of evidence that I have left out. The framework for this answer was basically from memory, although I have consulted references because I did not answer the question under examination conditions. Moreover, this answer demonstrates how to weave relevant citations into your answer when you write a term paper or assignment.

[8]Einstein, Perrin, Millikan, Rutherford, and Binnig and Rohrer, all won Nobel Prizes in Physics for their work, underlining the fundamental importance of the atomic theory.

45.14 Discuss

A question with the word "Discuss" usually means that there is more than one accepted or valid viewpoint for the subject. This is more often the case with the humanities than with the sciences. Essay-type questions requiring anything from a single paragraph to several pages may also begin this way.

> State and analyze the issue from more than one viewpoint when you discuss it. You may, but need not, take sides, but you must explore and reason the issue impartially from different aspects.

If the current state of knowledge favours one or the other perspective, it might be appropriate to state it, time permitting.

If you are used to debating in school or university, you will recognize that this type of question requires you to consider both the pros and cons of the topic. In the process, you may give reasons why you think one or the other viewpoint is tenable, and therefore preferable.

In questions like these, it is often difficult to gauge how well you have done because there is considerable subjectivity in examiner satisfaction with your answer, and consequently in the way marks are awarded.

Because a "Discuss" question is most often an essay question, the question, answer and commentary are given in Chapter 37 devoted to a model essay. Please look therein for an illustration of the algorithmic method for writing an essay outlined in Chapter 36 as well as other remarks.

45.15 Compare and Contrast or Distinguish Between

"Compare and contrast" is a popular and time-tested way of wording examination questions. To *compare* two items is to list *both* their similarities *and* differences. To *contrast* two items is to highlight *only* their differences. Sometimes, the word "compare" by itself is used instead of the more verbose "compare and contrast" to convey the same meaning. The instruction "distinguish between" is used when two or more ideas, objects, etc., are to be discriminated, so that it is clear which is which and why. Think of a medical doctor making a differential diagnosis, based on your symptoms, to get to a definite cause and disease, and the idea will be clear.

When asked to "compare and contrast" or "distinguish between", state the similarities and differences between the items, ideas, subjects, events, or whatever is the subject of the question.

45.15.1 Example: Microeconomics and macroeconomics

QUESTION

Distinguish between microeconomics and macroeconomics.

ANSWER

The underlying discipline of economics applies to both microeconomics and macroeconomics. It is only the domain and scale of application and the economic parameters of interest that are different in the two cases.

Microeconomics is the "small scale" version of the discipline and applies to individuals or corporate entities like a company. Factors like resource allocation, provision of goods and services, tax and regulation regimes, etc., along with price regulation by supply and demand, are taken into account in order to achieve a specific goal, like tax minimization, production maximization, etc. The principal focus here is on the ups and downs, epitomized by supply and demand, and their influence on the economic goal to be attained.

Macroeconomics, on the other hand, applies to the economics of large scale systems like entire industries or countries. Its preoccupation is with a different set of "big picture" factors, like economic growth, unemployment, inflation, trade, balance of payments, and their effect on, say, GDP (gross domestic product). Macroeconomics is primarily concerned with economic growth and changes in national income, and methods to forecast and influence them.

It is not that there is one set of economic laws driving macroeconomics and another driving microeconomics, but rather that the factors that apply, and their domains and scales of application, are different, even though the theoretical economic basis on which both are founded is the same. Indeed, the two major sub-disciplines in economics as a whole are microeconomics and macroeconomics.

An analogy from physics might be apt at this juncture. The physics of the very small, sub-atomic world is governed by the laws of quantum theory. The physics of the very large, such as stellar systems, are governed by the laws of general relativity. It is not that the two theories are domain-specific and disjunct in their field of application, but rather that the effect of general relativity on the sub-atomic world, and of quantum theory on stellar systems is secondary, if not negligible in each case. But both theories are part of the one unified discipline called physics. So too with microeconomics and macroeconomics in economics.

COMMENTARY

This answer requires domain-specific knowledge in economics. If you have access to any authoritative economics text book, you will be able to assemble a good answer yourself, under non-examination conditions. The answer given here is merely indicative.

I have adopted the viewpoint of physics—with which I am more familiar—to provide a working metaphor for the difference between microeconomics and macroeconomics. The important point being conveyed is that in both physics and economics, the reality is governed by the same laws even though we sub-divide the discipline for *our* convenience into two sub-disciplines or two theories.

To answer questions such as these, you need basic knowledge of the discipline of economics buttressed by facts and arguments germane to your answer. The Web is a burgeoning resource for factual research because of its seemingly inexhaustible wealth and variety of information and knowledge. I encourage you to explore the Web and develop the knack of discovering sites offering good quality information and knowledge as part of your study and examination preparation.

As explained previously in Chapter 17, the only caveat is that the *quality* of Web-based information varies sitewise and topicwise, unlike books where publishers assure the quality of the published material.

With no attempt to recommend any particular site, because I have no control over the quality of what they offer, I must yet *inform* you that a large number of online resource sites do exist. A sampling of these is listed in this chapter [7–13], including a curious specialist site that specializes in the differences between two subjects, ideal for "compare and contrast" or "distinguish between" questions [14]. The Web is fast-changing and a website listed here might, over time, have ceased to be or reappeared in a new guise. Caveat videtor! Viewer beware!

45.16 Account for

When asked to "account for" something, you are asked to give the causative reasons for something or some situation. Often these reasons are not immediately obvious; hence such questions.

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Give the underlying reasons or causes that give rise to the observed effect, or event, or situation.

45.16.1 Example: Missing mass during nucleus formation

QUESTION

"The mass of a stable nucleus is always less than the sum of the individual masses of the constituent protons and neutrons." Account for this loss in mass.

ANSWER

A stable nucleus is energetically at a lower state than its constituent nucleons. The difference in mass between the stable nucleus and the sum of its component nucleons is given off as energy when the nucleus is formed, in accordance with $\Delta E = \Delta mc^2$ where ΔE is called the *nuclear binding energy* and Δm is the *mass defect*. The total mass-energy is indeed conserved and the missing mass has really been radiated away as photons or moved away as the kinetic energy of emitted particles during nucleus formation.

COMMENTARY

On the surface, this question seems to contravene the law of conservation of mass. That is why you are asked to account for the difference in mass, because you have to reconcile it with the law of conservation of mass. When viewed under the aegis of the more comprehensive law of conservation of mass-energy, whereby mass and energy are two aspects of the same phenomenon, it becomes clear that the missing mass "has simply moved away" either as the energy of photons or the kinetic energy of other particles. Once you have accounted for the missing mass, you are done.

45.17 Assumptions

Asking you to enumerate the assumptions that you used in order to arrive at your answer is now an increasingly popular part of examination questions. It is all too easy to commit facts to memory and to regurgitate them in an answer. When you are asked about assumptions, though, the integration of your recall and your understanding is tested at one go. Let us take an example from Mechanics which should be within the ambit of most people's everyday experience.

45.17.1 Example: Ball thrown in the air

QUESTION

It is known that a ball pitched at an acute, non-zero angle to the horizontal traces a parabolic path in the vertical plane before it falls down to the earth. What are the assumption(s) made in arriving at this deduction? If one or more of these assumptions do not hold, explain what you would expect to observe and why.

ANSWER

The parabolic trajectory is arrived at on the basis of the following assumptions:

1. The earth is so large compared to the ball that the gravitational field, and hence acceleration, experienced by the ball is *uniform and directed vertically downward* in the entire region traversed by the ball.

2. The velocity with which the ball is thrown up is much less than the *escape velocity* at and beyond which the ball would not return to the earth.

3. The ball is a *point mass,* rather than a physical solid with non-zero dimensions and a spherical shape. All the mass of the ball is therefore assumed to be concentrated at a single hypothetical point.

4. There is *no resistance from the air* to the motion of the ball.

The ball moves with *uniform velocity* in a straight line in the horizontal direction and with *uniform acceleration,* directed earthward, in the vertical direction. The combination of the resulting horizontal and vertical displacements results in the parabolic trajectory.

If the gravitational field were non-uniform, the vertical acceleration would not be uniform and the trajectory would not be parabolic. Its shape would vary with the field non-uniformity.

If the ball were thrown at a velocity greater[9] than the escape velocity, it would start orbiting the earth like a satellite, or might even escape the terrestrial gravitational pull altogether and make its way subject to the forces in space, such as the solar, lunar, and other gravitational fields.

The fact that the ball has dimensions and shape means that it could spin when pitched, as is well known to cricket and baseball players and fans. This spin is an additional type of motion. It could, for example, change the plane of the ball's motion from being in the vertical plane, to some other plane altogether, depending on the axis of rotation and angular momentum from the spin.

If there were air resistance, the horizontal motion would no longer be at uniform velocity; it too would suffer deceleration, like the vertical motion. Moreover, the vertical motion would not be under uniform acceleration, but rather experience deceleration that varies with velocity. Again, the trajectory would not be parabolic. Using energy arguments, one could guess that both maximum vertical and horizontal displacements would be less than in the ideal, frictionless case with the parabolic trajectory.

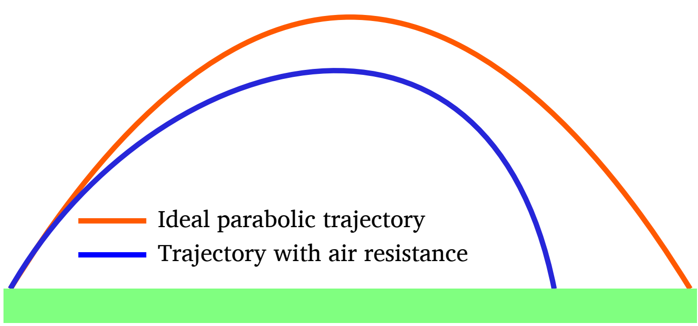

FIGURE 45.4: The ideal parabolic trajectory of a pitched ball is shown in orange. Air resistance retards motion both horizontally and vertically resulting in a real trajectory that looks like the one in blue above.

[9]Needless to say, this is a distinct impossibility when the ball is pitched by a human being!

COMMENTARY

This is a demanding question for which no amount of memorizing could prepare you. Despite there being no mathematics in the solution, the explanations squeeze out from you what knowledge you have integrated and made your own from your study of Mechanics.

The failure of each assumption is analyzed separately. In an examination context, it is neither meaningful nor practical to guess what would happen if two or more assumptions were to fail at the same time. That is what computer simulations are for.

From a quantitative angle, if the non-uniform acceleration vector $\boldsymbol{a}$ is a function of the velocity vector $\boldsymbol{v}$ given by $\boldsymbol{a}(\boldsymbol{v})$, as happens with air resistance, then we need to solve the vector differential equation

$$\dot{\boldsymbol{v}} = \boldsymbol{a}(\boldsymbol{v})$$

to proceed further. Note that this is not asked for in the question but illustrates the non-trivial nature of the problem once the simplifying and idealizing assumptions are removed.

Despite this, we can make some substantial statements about the trajectory using purely *qualitative* arguments, as shown above. It is often this "seat of the pants" reasoning that examiners are interested in testing in examinations rather than regurgitations of textbook solutions or the manipulations of mathematical gymnastics.

45.18 Prove

Questions that ask you to prove something arise invariably in mathematics and the mathematical sciences, like physics, engineering, etc. See Sections 42.7.2 and 42.7.3 in Chapter 42 for some guidelines on how to go about this.

45.19 Practical Examinations

If you are enrolled in subjects such as physics, chemistry, biology, or are doing engineering at university, you have most likely been required to attend practicals or laboratory sessions. Nowadays, even mathematics courses have computer-based mathematics laboratories that allow students to use software to explore ideas and conduct simulations that would be far too tedious with paper and pencil. Some students thrive in the laboratory while others dread it. Whatever your attitude, approach lab sessions as *relaxing* periods where "science is done," not only in your brain but also with your hands and senses, and you will slowly come to enjoy them.

If you are studying medicine, nursing, or some other praxis-based health-related discipline, it is imperative that you can translate book knowledge for the practical benefit of your patients. So, "clinical rounds" and practical examinations are an integral part of your education. *Lavish the greatest care during such sessions so that you get the most out of them.*[10]

> ★
>
> Do not stay away from practical sessions. They are an expensive, valuable, and vital part of your education.

Assuming that you have not been truant for lab sessions, you would have mastered the necessary techniques to do well in your practical examinations. And obviously, a practical examination tests your abilities in a laboratory. Specifically, practical examinations require you to:

1. read instructions, understand them, and carry them out exactly;
2. work independently on something practical that requires manual skill, observational acuity, careful documentation, logical analysis, and defensible inference;
3. not panic if things go wrong but to calmly and patiently backtrack and set things right so that you might go forward;
4. know safety in the laboratory and any necessary safety precautions applicable to your practical examinations;
5. appreciate accuracy, precision, and error and how the three are related, and the precautions you took to ensure precision and repeatability in your experiment.[11]

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[10] There are several iconic books, such as *Doctor in the House,* written by the English anaesthetist-author Richard Gordon about the practical aspects of doctors in training, that make for entertaining and educational reading; some have also been made into successful films.

[11] There is almost a century and a half's worth of knowledge on errors of observation and their treatment, starting from Sir George Airy's 1875 treatment [15], through Topping [16], Barford [17], Bevington and Robinson [18] to Drosg [19]. Make it your business to know something, if not most things, about this practical subject.

While most of us go through life without being tested for our practical knowledge on a daily basis, professionals like surgeons, physicians, teachers, airline pilots, astronauts, etc., need practical as well as theoretical ability to succeed in their chosen fields. Once again, I reiterate: do not neglect your practical sessions and you will do well in your practical examinations.

45.20 Oral Examinations

Oral examinations are often a prerequisite for the award of a higher degree by research, such as a Master's or PhD degree. Some universities even have an oral examination for Bachelor's degree students doing an honours programme. Preparing for an oral defence for a PhD, for example, is dealt with in Chapter 48. Please consult that chapter for useful hints. Specialist medical courses and many graduate courses also incorporate oral examinations as a way of testing students' knowledge at close range.

At an oral examination, you are not only being tested, but you are being tested *in person by other persons* who happen to be your examiners. This confrontation can be unsettling. Adopt the mental attitude that the examiners are your friends not your opponents. That will melt away apprehensions and negative emotions arising from a personal encounter.

At oral or *viva voce* examinations you cannot stall for time, mumble an excuse, or otherwise buy time while figuring out an answer to a question. You might need to concede that you do not know, or you might gracefully pose a question in reply, seeking clarification, while your mental gears whir energetically behind the scenes to formulate an answer. In this sense, oral examinations are probably the hardest test of your knowledge that you are likely to face in your academic life.

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The best preparation for an oral examination is mastery of your subject. Do not panic. Do not fear. Be calm. Be self-possessed. Answer clearly. Answer carefully. Answer to the best of your ability.

45.21 Attune yourself to the examiner's perspective

When you sit for an examination, you are a student hoping to score the most marks you can. But the person awarding those marks is an examiner who is usually neither present nor known. It will undoubtedly assist you if you can for an instant slip into the examiner's shoes and look at the question, and the expected answer, from the examiner's perspective.

That way, you will not omit stating an assumption. That way you will match your answers to the task words in your question. That way, you will state experimental precautions at a practical examination. That way you will track your oral examiner's thinking accurately and give the correct answer or concede gracefully.

Once you harmonize your answering with the examiner's expectations, your examiner will treat your answers as if they came from a "friend rather than a foe, an ally rather than an antagonist".

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Once you are in resonance with your examiner, your answers would be effortlessly correct, and in tune with the examiner's expectations. You would then have reached the "being" or *Zen* state in answering examination questions.

45.22 Real-world non-examination problems

Although, as a student you might think otherwise, the real world is actually much more difficult than an examination. In the artificial world of examinations, problems are carefully chosen so that they have neat, closed-form solutions that can be arrived at within a stipulated time. The knowledge you need to do well in an examination is tuned to achieving success in this artificial milieu. *You therefore only know how to solve problems that have neat solutions.*

The large, real-world problems of society—from renewable energy to global warming—are questions with no neat, readymade, closed form or unique solutions. Indeed, there are many problems for which there is no systematic methodology for finding a solution. What then do you do when you encounter such a problem? There is no set answer to this question, although there is a definite prescription.

> In the field of real-life experience, you must grow to maturity the knowledge you have gained from school and university, and apply what you know with wisdom, care, consideration, and humanity to benefit all humankind. That is a lesson that you have to earn and learn from life itself.

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[19] Manfred Drosg. *Dealing with Uncertainties: A Guide to Error Analysis*. Springer, 2007.

SUMMARY: ANSWERING EXAMINATION QUESTIONS

- Read the instructions carefully and follow them exactly.
- Read the question deliberately, understand it correctly, and then—and then alone—start answering it.
- Do not get bogged down by irrelevant details in multiple-choice questions.
- For multiple-choice questions, use common sense, estimation, elimination of wrong answers, and other time-saving techniques to home in on the correct answer.
- If you do not know the answer to a multiple-choice question, eliminate wrong answers first and intelligently best-guess the correct answer from the remaining choices.
- Recognize task words in questions and answer accordingly:

TASK WORD	MEANING
State	Write down
Enumerate	List one by one
Outline	Present the main points, in order, without the detail
Describe	Paint a word picture
Explain	Clearly give the meaning and/or reasons for something using jargon-free language
Account for	Give the causative reasons for something or some situation
Discuss	Look at both sides of an issue
Compare and contrast	Show the similarities and differences
Analyze	Break the subject down to its causes and constituent parts

- Answer questions pertinently.
- Calibrate detail and depth, explanation and exposition, to the task word.
- Include what is necessary.
- Exclude what is unnecessary.
- Sate assumptions, precautions, or other implicit conditions, in your answer.
- Strive for the *Zen* state in answering examination questions by harmonizing your answers with the examiner's expectations.
- Do not skip practical sessions or practical examinations.
- The best preparation for an oral examination is mastery of your subject.

PART I

UNIVERSITY STUDIES

48

So You Want to Do a PhD? A Guide for the Aspiring Doctoral Candidate

Synopsis

Why do you want to do a PhD? Proceed further only if you are doing it for yourself, out of your own interest, rather than because someone else wants you to.

Before you select a research topic, university, and supervisor, explore thoroughly, reflect and choose carefully, and having chosen, pursue your goal steadfastly until the end.

The only way you can fail to get a PhD is if you give it up yourself. Never lose motivation. Take breaks when necessary.

Master your subject. Read seminal papers. Formulate your hypothesis with help from your supervisors. Plan and execute your research competently and honestly. Manage your time frugally. Keep meticulous records. Write up your work in instalments. Prepare and present conference papers. Keep submitting and revising manuscripts to journals until they are accepted.

Rehearse repeatedly for your oral defence. Submit your thesis for examination. Make all corrections necessary for the final submission. Get your doctorate. Work to benefit humankind.

48.1 Introduction[1]

Those who are fortunate enough to go to university usually heave a sigh of relief when they graduate, often after fifteen to eighteen years of continuous education. They join the work force and are quite happy to put into practice what they have studied, serving society, and being rewarded in return with a good salary and social status.

With the rapid explosion and democratization of knowledge, however, merely completing an undergraduate degree does not always ensure a good job or a comfortable life. This spurs many to pursue graduate degree programmes that award a master's degree or a doctorate. Those who take the latter path may do so after joining the workforce in the "real" world, or they may remain solely in academia after their basic degree, to complete a doctorate, teach, and do research.

Whatever your purpose, and at whatever stage in your life you decide to do postgraduate work, there are certain issues that you must consider before you embark on a PhD.[2] This short guide is intended to assist the aspiring doctoral candidate in thinking things through before taking the PhD plunge. It will help you clarify for yourself, why you are doing what you are doing, what lies ahead, what pitfalls to avoid, and how to gain fulfilment in the process.

48.2 Brief history of the PhD as a degree

The title of the undergraduate degree usually has the word *bachelor* in it, and comes from the Latin *baccalaureus* meaning "apprentice" or "advanced student". Further study often leads to a degree with the word *master* in it, and this comes from the Latin *magister* meaning "master", "director", or "teacher", and denotes a higher qualification in that subject. Imagine an apprentice carpenter who has worked his way to become a master carpenter and the titles will fall into place.

The tradition of the doctoral degree originated in Germany. The PhD degree and its siblings all embody the Latin word *doctor* which means

[1]This book, and this chapter, including its title, was written in its entirety and independently, before I became aware of the book *The Unwritten Rules of PhD Research* [1] by Gordon Rugg and Marian Petre, whose first chapter has the same title as this chapter. Now that I am aware of their book, may I recommend it heartily to readers of this book, and especially of this chapter.

[2]Or other doctoral degree. The term *PhD* is used generically here and also applies to other earned research doctorates, such as the MD, LLD, JD, etc. Much of what is written here also applies to a Master's degree by research.

"teacher", *not* "physician"!

The PhD or Doctor of Philosophy degree comes from the Latin *Philosophiæ Doctor* meaning "teacher of philosophy". Historically, any discipline of study outside theology, medicine, and the law was considered to be philosophy. Later on, science was called natural philosophy. Today, the PhD is an advanced academic degree conferred by many universities in a variety of subjects. But bear in mind that theology, medicine, and the law, given their history, confer doctorates bearing other titles.

Some universities also confer a degree called a *licentiate,* from the Latin *licentia doctorandi* meaning "qualified to teach". This is sometimes equivalent to a master's degree. Some universities call the PhD the DPhil degree. In the United States and most Commonwealth countries, a PhD is sufficient for the holder to be accorded teaching rights at a university. Some European universities, however, require a PhD holder to submit a postdoctoral thesis through a process known as *habilitation* meaning "to make able to", before being accorded *venia legendi* or "permission for lecturing".

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In sum, a PhD in any discipline is simply a licence to teach that subject at university level.

48.3 Pre-requisites and processes

The aspiring doctoral candidate should have a very good undergraduate degree, or a master's degree, before she or he can enrol for a PhD. In many universities, the candidate is required to sit for *qualifying examinations* before doctoral enrolment is approved.

The requirements for the PhD degree vary with discipline, university, and country but they generally entail what is enumerated in List 48.1.

Some universities clearly stipulate it; others do not: But it is a universal expectation—even if not stated as an explicit requirement—that the PhD research should be of a high enough standard to merit publication in leading *peer-reviewed* journals.

Good research is fulfilled when published in high quality journals. For a PhD student, such a publication is a harbinger that the degree is almost in the bag. If a thesis is submitted, but with no journal publica-

REQUIREMENTS FOR THE PHD

1. Passing a number of qualifying examinations—usually at the final-year undergraduate level—in subjects relevant to the chosen field of research;
2. Enrolment in a number of advanced courses relevant to the field of research, and satisfactorily passing examinations in them;
3. Suitably framing a topic for research and getting approval from the university to undertake it;
4. Engaging in the actual research under the supervision of one or more qualified supervisors;
5. Documenting the research and its outcomes by writing a *dissertation* or *thesis* and getting it passed, with or without corrections, by a panel of expert examiners; and
6. Successfully defending the thesis orally before a panel of expert examiners and all-comers.

LIST 48.1: Requirements for the PhD degree common across many universities, disciplines, and countries.

tions yet, the thesis examiners will comment on whether the calibre of research is of sufficient quality to merit publication in reputable journals. Peer-reviewed research is the time-tested gold standard that has kept knowledge untainted by fraud and plagiarism: no new knowledge is admitted into the fold unless those held to be the experts deem it so worthy.

In praxis-oriented disciplines, e.g., music or drama, the thesis may form a small component and, say audio-visual documentation, a larger component of the research submission. In all cases, though, the requirement for the research to contribute significant, new knowledge remains unchanged.

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The primary requirement for a PhD is that the doctoral research should result in a substantial and original contribution to knowledge in the field.

48.4 Are you cut out for a PhD?

I firmly believe that you make your own destiny through the effort that you put into whatever you do. In that sense, as long as *you deeply desire* to do a PhD, you should be able to complete it successfully.

That said, the PhD is a marathon rather than a sprint. Those who do a PhD are crazy in one sense: they prefer plodding the dirt track of new knowledge to zooming comfortably along the autobahn of old knowledge. It demands sustained effort day after day, month after month. It is a single-minded quest best undertaken by those of monastic zeal. Like an intrepid explorer who braves uncharted seas and unforgiving terrain, you must be on fire to complete your research journey.

Some people are temperamentally unsuited to the PhD programme. By nature they do not relish doing difficult or demanding work over a long period. They have short attention spans and crave variety in what they do. Their minds would militate if asked to confront the same unsolved problem day after day. They are easily deterred by setbacks or failures. When faced with a challenge, they would rather walk away to something easier than brave the trial and fight the battle.

If you have these traits, the PhD would seem like drudgery or punishment rather than an exciting adventure. If you have no burning personal desire to do a PhD, you would be better off giving it a miss. Never let someone else force you into a doctoral programme if you yourself do not feel up to it.

48.5 Why do you want to do a PhD?

The PhD is, as we have seen, a licence to teach at tertiary level. If one asks oneself the question "Why do I want to do a PhD degree?" the answers could be one or more of the those shown in List 48.2.

Needless to say, the above list is not exhaustive, The reasons for doing a PhD are as numerous and varied as the human beings who enrol for the degree. But you must be clear why you want to do the degree before you embark upon it. There will be times during your candidature when you would doubt your own sanity for having enrolled for the degree. Or the sacrifice of time or family relationships might induce you to throw in the towel before you get the degree.

WHY DO I WANT TO DO A PHD DEGREE?

- I have won a scholarship to do a PhD.
- I want to teach at university level.
- I want to secure tenure at my university.
- I want to do research.
- I want to earn more money.
- I want to fulfil an ambition to do a PhD.
- I like the title Doctor before my name.
- I want to become a specialist working in a government agency or in industry.
- I want to start my own consulting firm, etc.

LIST 48.2: Reasons for doing a PhD degree.

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It is vital that you are clear in your own mind why you are enrolled for the PhD. Those reasons are the foundation upon which you build your work. If you lack that bedrock of personal conviction, and are doing a PhD simply to please someone else, you might want to re-consider your decision before it is too late.

48.6 Preparing for a PhD as an undergraduate

Knowing as an undergraduate that you want to do a PhD is a blessing. Knowing the area in which you want to do that research is a greater blessing. In such a case, is there anything that you can do while enrolled in your undergraduate programme to help fructify your ambition?

The best preparation is to lay an unshakeably sound foundation in the fundamental knowledge of your discipline. That demands excellent, consistent work throughout your undergraduate programme. Pay special attention to the later years of your bachelor's degree when the pace of knowledge "revs up".

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Never accumulate doubts about your subject.

If you are confused, or unclear, or have not mastered something, do not sweep it under the carpet. Your ignorance will return to haunt you later even if your examinations forgive you. Seek help until you finally understand what was once unclear.

Read widely even if your syllabus does not demand it. That way, you will acquaint yourself with current research areas and exciting new developments in your field. Cultivate casual familiarity with research material. Open some journals and browse them even if you do not understand much.

If you are inclined toward research in a particular area, choose elective units that will help lay a foundation in that area. Attending formal courses to build up knowledge is like constructing a pre-fabricated building: it is fast, easy, safe, and reliable. Use your undergraduate course units to help you later on in your PhD work.

Identify centres specializing in your proposed PhD area and visit them or explore them on the Web. If you have an internship programme as a pre-requisite for your bachelor's degree, spend that time at leading research laboratories or facilities. You will become familiar with actual work in the field and also strike up valuable acquaintances.

48.7 Selecting a research area

In many cases, selecting a research area is an obvious and uncomplicated decision. You *know* the field in which you earned your bachelor's and master's degrees. You wish to progress further in the same field by enlarging upon the research that you started in your earlier degrees, and that's that.

In other cases, it might be quite different. You might have started out as a pure mathematician and now want to do a PhD in physics. That is not too great a leap, and precisely one that many have made, sometimes in reverse!

To accommodate rare talent, some universities offer sandwich programmes in which a candidate can enrol, for example, for a PhD in engineering and also an MD in medicine. Or a combination of doctorates in law and engineering. There are some people who just love working in trans-disciplinary areas like these, and they thrive on the challenge, and excel because there are so few like them.

Other considerations could also come in. If you are contemplating a PhD in economics or finance and plan on making a killing at the stock exchange from your research, choosing your field would be like taking a gamble.

If you are intending to study marine biology, but live in a land-locked country, you must factor in living in another country during your PhD, and after you complete it. Likewise, if you wish to study the culture of the indigenous people of Australia, you will need to re-locate to Australia.

Location, availability of scholarships, prospective employers, the job market, and a host of factors all feed into the decision of choosing your field of doctoral research. Whatever your specific circumstances, if you do not love the area in which you wish to do research, you will not succeed, much less thrive.

Above all, choose to research something you love, for that will keep you going when all else tells you to stop.

48.8 Choosing a university

The choice of university is in many cases a foregone conclusion. It could be a university that is in your hometown, or the one which offered the most attractive scholarship, or the one that boasts of a world-famous researcher in the field, etc. Or the choice of university might simply be a Hobson's choice.

Universities the world over are becoming more corporatized, more homogeneous in their outlook, and more integrated with each other via the ubiquity of the Internet. Research at many universities is conducted at "Centres of Excellence" of one sort or another.

If the area in which you are contemplating research falls within the purview of such a centre at a particular university, and that centre has a good track record in research, that could help you make your choice of institution.

Bear in mind that your fellow PhD students contribute a great deal to the quality of your life at university. A good research centre will have a healthy diversity of PhD students, doing research on a variety of topics, who could go a long way toward enhancing your experience as a PhD candidate.

The only thing to avoid is shady "universities" which are not accredited, and which offer higher degrees within a matter of weeks, usually on payment of fees via the Internet. No serious doctoral candidate would countenance participating in these laughable scams, but I mention them to alert the unwary.

48.9 The PhD supervisor

The PhD supervisor is your friend, guide, and philosopher on your voyage toward a PhD. It is important that you search thoroughly, choose carefully, and stick with your choice steadfastly until you complete your PhD.

The choice of supervisor is entwined with the choice of research topic and, of course, choice of university. Take your time to make an informed choice that you are comfortable with, and that you do not regret later. Incidentally, the supervisor is also called the guide, the advisor, etc., depending on the university and its tradition.

Many universities now mandate that every doctoral student must have at least two supervisors, one of whom will act as principal supervisor. There are also often rules of engagement which dictate the minimum frequency of meetings, the documentation of meetings, the frequency of research progress reports, etc., during the candidature. While these are guidelines that prevent you from falling off the rails, they do not impact on the personal nature of interaction between supervisor and student.

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The choice of PhD supervisor can often make or break your PhD.

It is probably the most visceral aspect of your work: where your personality, work ethic, attitude to the topic, meticulousness or lack of it, etc., confront your supervisors' counterparts.

If there is a happy harmony, you will most likely sail smoothly. While a sympathetic supervisor may be a boon, an antagonistic supervisor could also be a blessing in disguise. He or she may present you with the very objections your examiners might, and it is better to encounter such opposition early in the day than to have your thesis dashed to bits at your oral defence, and have to return to the grinding wheel for another year or two, perhaps without financial support.

There are several archetypes for the supervisor. I shall enumerate three *extreme* cases: the autocrat, the laissez faire character, and the mentor-friend. Keep in mind that these are idealizations; real supervisors are likely to be mixtures of these extremes.

48.9.1 The autocrat

The autocrats wish you to work as a "glorified slave" doing their bidding. It is almost as if such supervisors would like to do the research themselves, but for the fact that they do not have enough pairs of hands and enough time. So, you the poor PhD candidate, function as an extension of themselves. You are not allowed to think for yourself or to offer an opposing viewpoint. You just do as you are told. Such a relationship is obviously unequal and does little to develop self-reliance and confidence in the research student.

Autocrats are often intolerant of errors. Remember that it is by making mistakes that the most enduring lessons are learned. Anyone who is unwilling to allow you to make mistakes is actually impeding your own educational development.

Some autocrats are very busy people wielding enormous influence among their peers, the university administration, and among research funding agencies. They may not have enough time to meet often or to allow a constructive two-way discussion. Moreover, their eminence might dissuade you from asking fundamental questions that are troubling you; *questions that you should never be afraid to ask.* Would you really like to work with someone like that?

If in the end, you decide for good reasons to do research under the supervision of an autocrat, make a pact with yourself that you will never give up your PhD because of problems with your supervisor.

48.9.2 The laissez faire or laid-back type

The laissez faire character is the antithesis of the autocrat and practises the gentle art of "non-interference," and often the even gentler art of laziness. Sometimes this might go to the extreme of meeting you perhaps only once during your entire candidature and of knowing your research topic only very superficially. If you are self-motivated, confident, and have mastery of your subject, this type of supervisor may be ideal for you. There are occasions, however, when it helps if you can bounce ideas off someone, be it an academic or a fellow student. If a laissez faire supervisor cannot perform this role, make sure that someone who can is on your panel of supervisors as a co-supervisor.

48.9.3 The mentor-friend

The supervisor who functions as friend and mentor is ideally suited to many students. This person is able to nurture the student without stifling his or her growth as a researcher. They have achieved sufficient eminence that they are not concerned that their students will steal the limelight away from them. Such supervisors are generally humble, approachable, affable, and disciplined enough to meet regularly, and to give truthful feedback, be it positive or negative. They do not punish you for making mistakes but rather emphasize the didactic nature of errors: what can you learn from them?

If you are fortunate enough in getting a mentor-friend type as your principal supervisor, you are generally in safe hands, but that does not absolve you from taking responsibility for your progress and timely completion.

48.9.4 Ask other PhD students

Other PhD students are often the most reliable source of information about how different academics stack up as PhD supervisors, especially from the student's viewpoint. Enquire and seek their counsel before you make your choice.

First, find out how many students have *completed* their PhD under a supervisor. If the supervisor has too few students, question why and think twice before going to him or her. Likewise, if the supervisor is chock-a-block with students, you should wonder how much time that person will have for you, and prudently stay away.

You should also ask other questions. For example, how soon does a supervisor grant a request for a meeting? How soon before a query relating to the research is answered? If it takes half a semester before a supervisor responds to an urgent question, perhaps you should look elsewhere. Frame your questions so that you can get objective answers untainted by opinion.

You should also ask subjective questions. For example, does the student feel that the supervisor is fair? How productive are meetings with the supervisor? Does the supervisor appear to be out of his or her depth in the particular research topic? Is there any aspect of the student-supervisor relationship that arouses unease in the student? Answers to such subjective questions reflect on both the student and the supervisor, and must be taken with a grain of salt. However, if all students under a particular supervisor give you adverse comments, that is a red flag which you should not ignore.

If you ask enough students—spread across different research areas and supervisors in the department—you will soon be able to form an overall picture from their answers that will help you select your supervisor, or least warn you about whom you should stay away from.

48.9.5 Grant applications

Research is critically dependant upon funding, and many PhD students are expected to pitch in when their supervisor writes a grant application. You might be pressed into service to provide material for these. But take care that you are not taken advantage of: for example, you should not be asked to shoulder the burden of writing the *whole* grant application. Your PhD scholarship could have come from a previous successful grant application. So, do your mite cheerfully if you are asked to contribute an explanatory paragraph or two, and perhaps a list of publications.

48.9.6 Detours

Beware of supervisors who constantly keep changing the goalposts of the research problem, perhaps to maintain continued research funding. The student is then a glorified servant, much like in the autocratic case, and the worst outcome is a hodge podge of unrelated research projects that cannot satisfy the university that it is a single integrated body of work, suitable for the award of a PhD.

Another danger from supervisors is that they could lead you down a track that takes you away from your main research topic and that could potentially be a dead end. It is critically important that you have a clearly defined research proposal, with well-crafted hypotheses, definite milestones, and a final goal. If not, you could be doing much and achieving little.

Because you are doing research whose very purpose is to uncover new knowledge, neither your supervisor nor you can be expected to know what your research outcomes would be, or where your research would ultimately lead. If you did, it would not be research in the first place. To avoid being trapped by dead ends, it is important that your supervisors have enough collective research supervision experience to identify potential cul de sacs into which you should not stray.

If you have a problem with your principal supervisor, you should be able to seek counsel, comfortably and confidentially, from your other supervisor(s) and resolve the matter productively and amicably with all of them.

48.9.7 Beware the rolling stone

Some students start out on a project with a supervisor who has inspired them, only to find out that the topic was not interesting enough, or that they could not work fruitfully with their supervisor. This leads to a change of topic, or of supervisor, or both. If this change were made without enough thought and analysis, but rather because of fear or anger or disappointment or some other emotion, it might lead to the same problems again. Avoid being caught in this trap of endless change of topic and supervisor. You will be wasting valuable time, and will achieve nothing in the end, unless you stick to your topic and supervisor.

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I reiterate: explore thoroughly, reflect and choose very carefully, and pursue your goal steadfastly until the end.

48.9.8 Respect for the supervisor

In some traditions, the teacher is accorded a veneration second to none: his or her words are sacrosanct. If you hail from such a background, you might unwittingly deify your PhD supervisor at university. Do not do so. Remember that he or she is another human being subject no less than you to the failings and foibles that assail the human state. Supervisors are not immune to the influence of funds and fame, politics and publications, factions and favours.

Your supervisor is a more knowledgeable person than you are. He or she is qualified to guide your research. But you are both engaged in a co-operative enterprise, jointly unearthing new knowledge. This requires a free and frank discussion of what needs to be done, etc. Do not be afraid to question your supervisor, whether to concur, clarify, correct, or contradict. If your relationship with your supervisor does not allow this freedom, think again before finalizing your choice.

By placing your supervisor on a pedestal and deifying him or her, you are doing a disservice both to your supervisor and yourself. It is unfair to superimpose divine infallibility on your supervisor. And you will be disappointed when you finally discover that your assumption was wrong, and expectation unrealistic.

> Respect your supervisor and in turn earn your supervisor's respect by your behaviour and work.

48.9.9 Honesty and integrity

As the last word on choosing supervisors, I would say that above all else, seek a principal supervisor who has honesty and integrity. If these qualities are lacking in an otherwise eminent person, my personal advice is to forgo the eminent supervisor and work under the guidance of a lesser achiever who has honesty and integrity.

48.9.10 Meeting your supervisor(s)

Once you have chosen your supervisor(s) and your research topic, you should discuss the frequency and purpose of your meetings with your supervisor(s): whom to meet, how often, when, and for what purpose. Do not put these meetings off but take care to schedule them regularly. You should typically meet your principal supervisor more often than your other supervisors. Also, there should be mutual respect for the clock on both your parts. You are both busy people, but your supervisor has more on his or her plate than you. Any time bequeathed to you by your supervisor is precious and should be treated respectfully as such.

Do not treat these meetings with your supervisor(s) perfunctorily. Make them count.

> Prioritize the issues you wish to discuss at each meeting with your supervisor(s). Your list should address the most pressing concerns and questions you have about your work, prior to each meeting.

Note and date each list of points discussed in a special notebook dedicated to such meetings. Record your supervisor's responses in the same notebook.

You will then have a running commentary on your progress or lack of progress viewed through both your own and your supervisor's lenses. Act upon any warnings your supervisor gives you regarding your pace of work. Your notebook would provide objective testimony one way or the other if, sadly, at some later stage you are pulled up for lack of progress and told by your supervisor that you were forewarned.

Equally, your notebook will help you to schedule your work, from reading the background literature, to planning your proposal, to writing up a conference paper, and finally, wrapping up your thesis. You will have dates and milestones for each stage of your work. You will also own responsibility for your progress. Do not depend on your supervisor to function as timekeeper for your work. Take charge of your schedule yourself.

By carefully preparing a list of questions or issues, and clearing them at fortnightly or monthly intervals with your supervisor, you establish a brisk and business-like schedule for your PhD work. You also indicate to your supervisor by your actions, that you are both serious about your work and also responsible enough to take charge of it. You will automatically command the respect of your supervisor as a good doctoral candidate. Likewise, if your supervisor gives you sage counsel at these meetings and solves or resolves problems, rather than gives you bureaucratic excuses, you will know that you have a supervisor whose experience and abilities will help steer you successfully toward your PhD.

48.10 The research proposal

Your research proposal is a vital document. It is a carefully worded statement of what you are setting out to research. It sets the tone and goal for anywhere between three years (full time) and eight years (part time) of your research life.

You do not wander willy nilly in search of inspiration on what research to do next. Instead, you follow a well-worn path to search out and scope a problem of sufficient interest and impact, but which has hitherto not been studied. The topic of research is then enunciated, so that regardless of research outcome, a degree may be awarded at the end. An example will help here, even if a little simplistic and contrived.

EXAMPLE: EXAMPLE

Suppose you wanted to do research to create a black-coloured rose. You could define your topic to be "The creation of a hybrid black-coloured rose." Suppose at the end of about four years of full time research, the closest you could get was a very deep purple rose, but not a strictly black one. Unfortunately, you may not have left yourself enough leeway to claim completion of research.

If instead, you had defined your proposal to be "An investigation into the production of a hybrid black-coloured rose," you could very well claim that a purple rose, of hitherto unseen depth of colour, is a significant advance. What is more, you could claim to have completed your research. You would also have laid a solid foundation for future research that could very well lead to a perfect black rose. ★

Universities vary in how seriously they approach the formulation of the research proposal. In some universities, after a period of preliminary study, completing examinable units, the candidate is asked to undertake a survey of the literature in the broad research area. She or he must master the basic knowledge in the field by self-study, and must identify the major unanswered questions, or existing contradictions or paradoxes, that are worthy of further study. In some cases, the research proposal would extend the envelope of existing knowledge, such as in the above hypothetical case of producing a hybrid black rose.

The candidate must carefully summarize the existing state of knowledge and formulate the research proposal carefully, so as not to prejudice the granting of a degree later on. This proposal must then be presented to a committee of academic faculty and perhaps, extra-mural experts, who will debate on its acceptability. They will ask questions like these:

The candidate will very likely be quizzed, and the proposal turned over several times like a pancake, and viewed from all angles, before the committee gives its blessing.

There are universities where there is no formal proposal process. Worse still, the student is given free rein and may not have any idea about where he or she is headed, until expiry of a scholarship or financial imperatives finally force the thesis to be written, and reality confronted.

It is then that the knotty problem of "what in the world have I worked on for four years?" suddenly strikes the student with alarming force. Panic often ensues before a plan is worked on for a patchwork quilt of a

Vetting a PhD Research Proposal

1. Is the proposal something that has already been done before?
2. Is it interesting enough to be worthwhile?
3. Is it neutral and unbiased?
4. Has the proposal been formulated carefully so that the student may be awarded the degree regardless of the research outcome, whether positive or negative?
5. Is it well-founded on the existing body of knowledge?
6. Is the scope feasible for the envisaged time frame?
7. What is the closest body of existing knowledge?
8. Who are the research leaders in the area?

List 48.3: Vetting a PhD Research Proposal.

thesis, in consultation with the panel of supervisors. Do not ever allow yourself to get into this mess. Even if your university is relaxed about a research proposal, make sure you are not, and take steps to formulate a rigorous and worthwhile proposal, in consultation with your panel of supervisors. It is time and effort well spent.

48.11 Visualizing the end: thesis writing and the oral defence

Successful people often strongly visualize the goal that they are striving to achieve. In your case, it is fulfilling the requirements for the PhD. This means writing the thesis and preparing for your oral defence. Neither of these should be taken lightly.

48.11.1 Thesis writing

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You should start work on your thesis from the very beginning.

Writing is a demanding discipline. Those who do it well make it seem effortless. Those who do not excel at it look upon it as an unpleasant chore. Those who dread it, put it off for as long as they can. Make sure you are not in the last category.

In Chapter 50, entitled "How to Write a Thesis", I have gone into the process of thesis writing in some depth. Read it carefully *before* you start your research work. You would then start off on the right footing. Do not think of writing up as an afterthought or something that you can do overnight.

> Believe me: it takes at least six months, and possibly a full year of sustained effort to produce a good quality thesis.

Your thesis is the first document by which your examining committee will encounter your work. Give it your best shot and do yourself a favour.

Thesis preparation can be aided by the judicious use of software. Use a document typesetting program and bibliographic reference management program that you are comfortable with. I personally use the freely available, open source, LaTeX [2] and XeTeX [3] document preparation systems based on TeX [4] and allied software for preparing illustrations and managing bibliographies.[3] The initial learning curve is steep, but once you have mastered that, the rewards are great. Especially valuable is the advice of hundreds, if not thousands, of academics and researchers from all fields, who are available for help through Internet newsgroups and mailing lists. I heartily recommend this path, if you have not made a choice yet.

Many universities now require the thesis to be available electronically, say in Portable Document Format (PDF). Ensure that whatever software you have chosen to prepare your thesis is capable of seamlessly exporting the output in the prescribed format, without any hassles.[4]

48.11.2 Oral defence

The oral defence[5] is perhaps the most dreaded part of the whole PhD process. There is something unnerving about a face-to-face encounter with experts who, however sympathetic they may be toward your work, put on an adversarial stance in order to ferret out any weaknesses that may lurk within it. The fact that they are not fellow students but experienced and learned academics might make this seem an uneven gladiatorial contest. Banish such thoughts and visualize yourself as the expert and your com-

[3]See the companion website for "how to" details. Also, take a look at the book *Using LaTeX to Write a PhD Thesis* and related texts [5].

[4]You do *not* want to spend time fighting your software at the eleventh hour to achieve this.

[5]Also called *viva voce* or simply *viva*, (literally "with living voice") at British and European universities.

mittee as a bunch of school kids whom you are going to enlighten. I do not mean that you should be arrogant, but please do not do yourself the disservice of becoming nervous or dumbstruck before your committee.

Prepare your talk using appropriate technology. Make sure that you have enough material but not too much. It is better to have too few slides than too many. Give yourself enough time. Speak clearly and slowly. Anticipate questions that you might be asked. Some experts may have only glancing acquaintance with your topic and could ask questions *around* the topic rather than on it. Be prepared for such questions by reading up thoroughly on related research. Of course you should be master of your own work. Practise as many as *twenty times,* if necessary, so that your oral defence is flawless. Realize that you get to do an oral defence of your PhD only once in your life. Let it be memorable and triumphant.

48.12 How not to “not get a PhD”[6]

The only way *not* to get a PhD is to give it up yourself.

There is no other way, once you have been enrolled as a doctoral candidate, and as long as you keep to your time limit and do not engage in deceit or other unacceptable behaviour. It is therefore instructive to look at the ways in which you might be persuaded to give up on your PhD and also how to avoid that. Some of the factors affecting your resolve to complete your PhD are shown in List 48.4. Each of these is considered in turn in the succeeding sections.

48.12.1 Time

Whether you do your PhD full-time or part-time, you need to complete it within a definite time frame. Many universities have a cut-off period—from the date of first enrolment—beyond which your PhD candidature will not be renewed. This varies by country and university, but you do not have unlimited time.

It is very easy to squander time during the early stages of your doctoral work. Your PhD might be your first taste of academic freedom away from the regimentation of undergraduate assignments and examinations.

[6]Inspired by chapter 4 of Phillips and Pugh’s *How to get a PhD: A handbook for students and their supervisors* [6].

FACTORS AFFECTING RESOLVE TO COMPLETE THE PHD

- Time
- Money
- Competing priorities
- Loss of motivation
- Scaling the next peak
- Academic politics
- Falling foul of bureaucracy

LIST 48.4: Factors affecting your resolve to complete the PhD.

But remember that this freedom comes with responsibility. You are responsible for your time; use it frugally. There is no immediate feedback when you lag behind or slack off. But retribution is swift and sudden when you run out of time.

If you need to work to fund your PhD, you have the added burden of apportioning time wisely to your various tasks. Whether you take on a graduate teaching assistantship running tutorials or laboratories, or you work as a paid research assistant for a professor, or you wait on tables during weekends at the local bistro, or you deliver pizzas every night for two hours, you must juggle your part-time work, your PhD research, your recreation, and your rest so that you do not short change yourself on any count.

When you are short of time, sleep schedules are often the first casualty. Do not accumulate a sleep debt. Thousands of years of evolution have tuned our bodies to synchronize activity with the hours of daylight and sleep with night time.

I have discussed the importance of sufficient sleep of good quality, and the pitfalls awaiting you if you forego it, in Section 2.4 Specifically, you might suffer burnout, become easily discouraged about your ability to complete the PhD, lose motivation, and give it up. Never let that happen.

Manage your time efficiently using the techniques given in Chapter 21. Juggle your time and your tasks intelligently. Most of all, overcome procrastination. *The hardest part of any task is to begin it.* Even if all you can manage is a few seconds or minutes, make a start doing whatever you are inclined to put off. Once you have begun, you will find it far easier to continue it to the end.

> Be brisk and businesslike from the very start of your programme.

Then you will not suffer the pressure of time, on top of other pressures. Do not procrastinate on researching your proposal, starting your experiments, or writing up. Start early. Revise your drafts several times. Submit a thesis that can be passed with few or no corrections.

48.12.2 Money

> Like time, money must be managed frugally. Do not squander either.

Since higher degree fees are prohibitive, you might be one of the many students who undertake their PhDs only if they can secure scholarships. These are usually for a fixed period—say three or four years—and are subject to satisfactory documented progress and could be renewed perhaps for an extra semester or two. But when the scholarship runs out, you may not have the luxury of working full-time on your research or thesis writing, free of worries about money.

If you can get an academic job—like a tutor or teaching assistant—at the university where you are enrolled, that would be ideal to fund you over the final, vulnerable lap when you complete your research and write up your thesis.

If not, you might be forced to seek employment outside academia. You then run the risk of drifting away from the immediacy of your research and settling into the workaday routine. You could also use work and its pressure as an excuse to get away from writing your thesis. If you fall into this trap, you risk not completing your thesis. Whatever the demands on your time and energy, discipline yourself and write up your thesis. Better still, complete your PhD and submit your thesis *before* your scholarship runs out. If you are disciplined from the beginning, you can do so easily.

If you are working to fund your PhD, you know the value of money and of consistent hard work. By enrolling for a PhD, you have already demonstrated great commitment to your research and your degree.

It is not so much money as time that you need to be attentive to. Manage your time effectively. Start all work early. Complete comfortably.

As human beings, we prefer variety to monotony. Use the variety imposed by your part-time work to refresh your mind and body to relieve any tedium triggered by your research work. When your paid work gets too taxing, seek refuge in your research work. Use every challenge to your advantage and tip the scales to your benefit.

48.12.3 Helplessness

About six months into your research proper, you will feel a sense of helplessness. You would have already done your literature search and perhaps written an interim report or two. But you would suddenly come to the realization that you are well and truly at sea and rudderless too! You would despair of ever being able to do research; the task seems hopelessly overwhelming.

You might be scared that someone else somewhere, working on the same topic, might pip you to the finish. Worse still, you might regret deciding to do a PhD in the first place. What seemed such a charming proposition at first, now seems like an endeavour undertaken in a fit of insanity. Like seasickness, a good many do not suffer from such feelings, but quite a few others do.

You need to pep talk yourself out of this mood of despair. This is where your reasons for doing your PhD come into play. They will keep you focused, on track, and on time.

48.12.4 Competing priorities

> Completing a PhD requires an almost monastic dedication to the task at hand.

If you are young and proceeding to your PhD immediately after finishing your first or second degrees, you might find your social life competing for attention. Romance and marriage can and often do claim your attention and time. You then have to balance your priorities so that you are fulfilled as a person without compromising on your goal of a PhD.

If you are a mature-age student with a spouse and family to look after, it is even more complicated. You need your spouse's unflinching support for your enterprise of getting a PhD. There could be role reversals, and even times when you are not available to your family for extended periods.

Each of these situations brings with it its own emotional stresses and strains. You need to be deft at juggling these competing demands and yet have your eye on the goal. If you are committed to your family and to your PhD, you will find a way to balance the two and succeed at both.

48.12.5 Loss of motivation

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> The worst thing that can happen is for you to lose motivation at some point in your PhD.

Perhaps some of the factors outlined previously have knocked a hole in your determination.

Perhaps you have just seen a newly published paper that is about the very thing you are working on, and you fear it may steal not only your thunder but also your chances of getting a PhD.[7] It is easy to get discouraged and view things as being worse than they really are. Perhaps the just published paper approaches your problem from another viewpoint and does not really pose a threat to your work. Indeed, it might highlight the importance of your research topic, as being of interest to other researchers, and thereby underscore the importance of your own research. So, your research could still be relevant, and new, and original.

Sometimes, you might develop a distaste for what you are doing because of over-exposure to the subject. It might be a monotonous laboratory routine, a software error you cannot localize, a theorem on which you are stuck, data that somehow seems inconsistent, or a performance that clearly does not gel. Whatever your field, and whatever your problem, this distaste is a sign that you need a break.

Contemplate a holiday, perhaps after presenting a conference paper overseas. Or do some writing up on your thesis. If nothing else appeals to you, do something mind-numbingly relaxing like typing up your biblio-

[7] This is one reason why you should publish your work as early as possible to establish priority. Do *not* wait until writing up to publish.

graphy. If you are in need of a fresh viewpoint, talk to some else: a fellow student, a faculty member who may not be on your panel, an academic from another discipline, or simply a family friend.

Do something sufficiently different from what you have been doing so far, and yet somehow linked to your work. You do not want to take a break that is so long or comfortable that you do not feel like returning to your PhD work. But you do need to do something definite and different to dispel the staleness or distaste that makes you lose motivation.

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Do not ever lose your motivation. Remember that even after your thesis has been examined, it is only after all corrections are in that you can relax and say, "Been there! Done that!"

48.12.6 Scaling the next research hill

The very opposite of loss of motivation is what I call the temptation to "scale the next research hill." It is all the more sinister because it is a weakness masquerading as a virtue. It is the temptation to keep going when you should be "downing tools" and finishing up.

You are all fired up and enthusiastic about your project. Your research work is humming along fine. You have surmounted some demanding problems and built up your confidence. In fact, things are going so swimmingly that you do not want to stop. *Having successfully completed the scope of your research project, you have just glimpsed another unsolved problem that beckons invitingly.* You are tempted to scale the next research hill. You are like the jogger who wants to keep running indefinitely. If the "high" from research is so wonderful, why would anyone want to stop?

But stop you must, at the right time, in order to wind up your work, write your thesis, and get your degree. You need to respect the ticking clock and changing calendar. You need to complete your degree in time, and on time.

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Resist the temptation to scale the next research hill. Stick strictly to your timetable and your scope of research.

Sometimes, your supervisor, especially if inexperienced, might join you in your enthusiasm and goad you to scale the next research hill. The prospect of more publications and perhaps even promotion all too often fuel such actions. In such cases, seek the counsel of a more experienced academic on your panel of supervisors to get a realistic assessment of what to do. If that person feels you should stop and write up, then so you must.

Once you have been awarded your degree, by all means pursue those loose research ends with gusto. The more you are in love with your research, the more irresistible it will become. Keep the prospect of doing this further research as a form of delayed gratification for all the sweat and toil of getting the PhD. But get your PhD first.

48.12.7 Academic politics

Be aware that politics exists in academia just as it does in other arenas of human activity. Do not get caught up in it. Develop the deftness to parry and dodge any inimical thrusts with minimal exertion.

Academic factionalism, old school tie networks, competitive rivalry, personal vendettas, and the like might tilt your research playing field so that it is no more level. The “sins” of the supervisor may be visited upon the student. It is not unheard of for graduate *students* to be thwarted in their research or funding efforts by academics who have an axe to grind with the student’s *supervisor*. However unfair this might be, remember that you are not at university to seek social justice, or to right perceived wrongs. You are at university to get a PhD.

Once you establish yourself as an apolitical, serious-minded scholar, you are likely to be unscathed by these bouts of political jousting.

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Be aware of academic politics. Avoid getting entangled in it. Keep focused on your research. Get your PhD.

48.12.8 Bureaucratic dereliction

Sometimes not filing a progress report to fulfil university or scholarship regulations can lead to your candidature being revoked. Avoid dereliction of bureaucratic requirements because it is such a trivial reason for which to jeopardize your PhD. Follow the rules and satisfy the powers that be, keeping your eyes ever on your goal.

48.13 Will power, guts, and gumption

There will be inevitable setbacks on your road to a PhD. You might have had a verbal tiff with your supervisor. You might have made an expensive mistake in an experiment. You might have received a rejection letter for a journal submission. The list is endless.

Do not be discouraged. You are not alone. Remember that the greatest scholars have had their books and papers rejected. Your first rejection letter betokens your membership into their distinguished company. Keep working away on your publications until they see the light of day. The stout of heart and firm of resolve will prevail in the end. For this you need will power, guts, and gumption.

48.14 The art of doing research

The PhD is ultimately a testament to the fact that you can conduct supervised research. The art of doing research is rapidly changing, especially in the interconnected world of knowledge available though the Internet and the World Wide Web. See Chapter 17 entitled "Harnessing the Power of the Web to Learn and Do Research" and the companion website for an up-to-date summary. Here we look briefly at the art of doing research per se.

48.14.1 Know your subject thoroughly

There is no substitute for knowledge. You are on the road to becoming a teacher (or doctor) in your subject at an advanced level. Needless to say, you should have already mastered the fundamentals of your subject by now. If you have not, you should resist the temptation to paper over the cracks in your knowledge.

If your knowledge of the foundations of your home discipline are shaky, the first thing to do is to acknowledge that fact. Then identify well-written undergraduate or graduate texts in the field and work through them. Find out if relevant final-year undergraduate or graduate units are taught at your university, and enrol in them if you can.

Surf the web. Many of the world's leading universities, like the University of California, Berkeley, Yale, MIT, the Indian Institutes of Technology, Stanford, Carnegie-Mellon University, Oxford, and The Open University, have all started putting some of their courses *online for free,*[8] [7–9], often complete with questions and assignments. You may use these for self-education and self-assessment. Employ whatever means you can to bring yourself up to speed in your discipline. Do not fret over the time spent on this: it is an investment in knowledge that you cannot do without.

Challenge yourself by attempting some past final-year examination papers in relevant units at your university. If you are able to breeze through them, you have an objective measure of your own mastery of the fundamentals of your subject. If you are in a praxis-based discipline, test yourself by asking an expert to gauge your performance in the area.

> ★ My unequivocal advice to you is to ensure that you have an unshakable foundation of knowledge in your chosen discipline(s) before you start on the specialized research that will earn you your PhD.

48.14.2 Read and internalize seminal papers

Once you have mastered the basics of your field, you are qualified to enter the inner sanctum of advanced knowledge. Identify the trailblazers in your area. Read their papers. This should give you a broad view of where the research is heading. This overview of the field is vital in formulating your research proposal.

This step in the process has an analogy in the way we see things. When something is far away we see it as the merest speck, un-identifiable as anything specific, but an object that is distinct from its background, nevertheless. As it approaches closer, we are able to make out a silhouette that allows us to *guess* what the object could be. As we get still closer, it is clear what the object is, but the details escape us. Finally when we are right in front of the object, we see it clearly for what it is, details and all.

This is exactly how your approach to your research field should pan out. The detail that you see close up cannot and should not obscure

[8] See the companion website for up to date offerings in the rapidly changing world of online education.

the global overview that you had when viewing the silhouette. Both are essential to demonstrate mastery of your chosen research area.

Once you have got the large scale picture, you should progress by reading and assimilating the classic seminal papers in the turf where you have chosen to pitch your research tent. Look at how leading researchers have surveyed the area. Do also read important and interesting papers by relatively unknown researchers because they might embody valuable insights. Do not gloss over a paper simply because of its age. The authors of yesteryear were often the crème de la crème of their generation and their papers could still yield rare nuggets of truth that might have been missed by others.

Most of all, you should *internalize* this newfound knowledge. Only then are you ready to ascend your chosen peak of research. Just as those attempting to conquer the Everest train hard, and methodically prepare themselves by scaling similar peaks, so should you use the papers written by others to prepare yourself to prosecute your own research.

48.14.3 Prepare a critical survey

By now you should have a solid feel for your research area. This is the time you should prepare a critical survey of the research literature in your field. Look at what has gone on before. Look at what is being done at present. Identify what needs to be done, but has not been. Single out interesting questions that have not been answered. Are there any contradictions or paradoxes or glaring omissions? Is there anything that you, as a relative newcomer, find strange or unsettling?

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This critical survey reflects your state-of-the-field summary of the research area, illuminated by your own insight, after your assimilation of the research literature.

Ideally, you now have the expert knowledge required to formulate your research proposal intelligently.

Julius Caesar is famed for his words “Veni, Vidi, Vici,” which translate to “I came, I saw, I conquered.” Let your critical survey be the “I saw” part of this process and let it lead to the “I conquered” stage of your PhD journey.

48.14.4 Repeat some previous work

You are now ready to commence your research proper. Sometimes this presents its own set of "starting problems." One way to ease your entry into research is to repeat something that someone else has already done successfully. It is a bit like cooking a dish for the first time, following a recipe written by a chef. Because someone else has already done it, there is no anxiety about whether what is proposed is feasible. This allows you to concentrate on technique and skills.

This will probably be your first taste of what it means to "do research." In all likelihood, you will love doing whatever you are doing and become totally absorbed in it. This mental absorption leads to great inner fulfilment. You get a "high" because you are creatively engaged in something you passionately believe in. There is also the added satisfaction that what you do could later benefit humankind in some way or another. Finally, when you successfully conclude your "repeat experiment" you would have gained the confidence that you too can do what others have done. Doing research now *does* seem to be within your grasp.

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This conviction or sense of "can do," earned through your own hard work, is the most valuable and durable research currency you will earn from your PhD.

48.14.5 Plan and execute your own original work

You are now ready to plan and execute your own original work. You should plan your work beforehand, whether it be laboratory-based experiments, a computer simulation, a field trip to an anthropological site, a psychology experiment needing many volunteers, an ethically-constrained study of a drug on humans, a performance of a forgotten dance, or analyzing the work habits of computer geeks. Take the advice of your supervisors for this.

You are likely to need time, money, hardware, software, some data, perhaps some laboratory animals or human volunteers, travel plans, etc. You should marshal all the necessary resources so that your research is not delayed by their lack. Make sure that all bureaucratic hurdles are surmounted, whether they be customs and immigration formalities, or getting the approval of university ethics committees.

There are specific questions that you need to answer if you are doing research related to human health. You will need controls for experiments, and double blind experimental protocols to exclude human and interpretational bias. If there are any such special issues impacting on your research, make sure you encounter them head on now, and seek professional statistical help if necessary, rather than wait to have your results trashed later because of poor protocols.

48.14.6 The hypothesis

Before you start on your original work, though, you need to flesh out something very important. This could happen over any period from the commencement of your candidature, through your course work, to the formulation of your research proposal, or a little later than that. But it must take place *before* you can start your original work. It is called *framing your hypothesis*.

If you hark back to your research proposal, discussed in Section 48.10, you would have enshrined somewhere in it your hypothesis, in some shape or form.

To understand what a hypothesis is, imagine you are a gold prospector, looking for the mother lode, based on some gold dust and perhaps a nugget or two, discovered by someone else. Colloquially speaking, your hypothesis is a *hopeful suspicion* that what you wish to find is really at the location where you will look for it. It is an educated guess—made on the basis of what is known, and your mastery of the subject—that you hope will help uncover the gold of new knowledge.

Depending on your field of study you might need one or more *sub-hypotheses* to buttress your main hypothesis and to guide the direction of your research every step of the way. To use another metaphor, they constitute a route map from your starting point to your destination. Each time you reach a town that is on the route, you can tick a sub-hypothesis as having been verified. Your main and sub-hypotheses will contribute to the "main story" and "subsidiary stories" you will tell when you write up your thesis later.

> One of the most important contributions your supervisors make to your research is to help you frame your main and sub-hypotheses.

Their experience and exposure to research in your field befits them eminently for this role. Make sure that you derive this benefit from your research association with them.

48.14.7 Devise your experiments

Only after tackling your hypothesis can you devise your experiments or engage in field work. Again, draw upon your supervisors' collective experience when you do this. Many a mistake, costing much in time and money, can be averted by taking advice from your supervisors and planning your experiments carefully.

If you are still a little foggy about "all this hypothesis mumbo-jumbo", I encourage you to scan Section 50.5.3 of Chapter 50. You would then see it in the context of your completed work as embodied in your thesis.

48.14.8 Keeping records

At this stage, you are giving birth to the original component of research that will go into your thesis. It is paramount that you should keep good records.

> Write results using ink, not pencil. Do not write on paper scraps but in a properly bound notebook. Record your thoughts, flashes of inspiration, the setbacks you faced, how you solved them, etc.

While all this material need not find its way into your thesis, some of it invariably will. Whatever will help a fellow researcher later in time, whether it be optimizing a protocol, or avoiding a pitfall, is valuable enough to be bequeathed, through your thesis, to a future generation of researchers.

Your original contribution will very likely come in many parts. There will be several experiments, or a number of projects, or a few performances, or a few of whatever, depending on your field of research. Each time you are finished with one chunk of work, make sure that you have written everything up clearly and left nothing unrecorded. Seek your supervisors' input on this. One of the most heart-breaking events you want to avoid is to repeat past work because some parameter was not recorded, or some drawing was missing, or some protocol was not properly observed, or because a multimedia record of a performance you directed was out of focus and noisy.

The first person who needs to understand your experiments or projects is yourself. If you cannot make head or tail of them when you look them over after, say six months, then you have done yourself in.

Be kind to yourself: do it well the first and final time.

At this stage, you should be somewhere between the half and three quarter mark of your PhD programme. Do not panic if you have consumed more time, but be aware that time is not on your side. Do your work systematically. Practise the self-discipline of recording everything permanently, neatly, legibly, understandably, and in one place.

48.14.9 If you get stuck

If you get stuck while conducting your original research, you can turn to many sources for help. Because the PhD is *supervised* research, you are not working alone. You have the benevolent oversight of your panel of supervisors. One of them will certainly have some word of wisdom that could guide you out of your current impasse. Then, there is the literature. There is the Web. And a Google search might be all you need to pick yourself up and get on track again. Moreover, there are fellow PhD students to whom you could turn for assistance.

The one thing you should not and cannot afford to do is to stagnate.

Imagine your research as being a vehicle, say a motorcar, and it suddenly gets stuck in the mud. You do not get out and wail your heart out in despair. You put your shoulder to the vehicle and try to get it out. Or you enlist the help of passersby. Or you call for help on your mobile phone. *You do what it takes to get it unstuck.* The same applies to your research work.

48.15 Writing conference and journal papers

Writing your first research paper is a new experience. But it need not be daunting. Because you have mastered your subject and critically surveyed the literature, you *know* your subject. And since you have read tens or hundreds of papers you know how a good paper is structured and written.

After you have completed some module of research, you could very well be in a position to prepare and present your first paper. You might experience some hesitation in doing this. Overcome that reluctance. You must make a start somewhere, and where better than at the beginning of your research?

Very likely, you think that what you have to say may not be significant enough to merit a conference or journal paper. Let the referees who review your paper decide that. Perhaps a fellow researcher or PhD candidate told you that your idea for a paper will not fly. Do not let that deter you. Let the naysayers have their nays; you need not accept their view.

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Do not deprecate your own work.

48.15.1 The conference paper

In one sense, the conference paper is really more difficult than the journal paper. For a start, you have to be very concise, because page limits are stricter. For another, you need to prepare a presentation of slides, and talk about your work before a live audience. Again, the time limits could be very strict, typically varying from five minutes to fifteen. In that time, a live audience of fellow researchers, who very likely have first-hand, expert knowledge of what you are saying, engage with you. They listen to what you have to say, they mentally compare your work with what they know, and then they ask you questions designed either to clarify or correct what you have said. You might have anticipated a question or two, but it does take some nerve to answer experts asking unrehearsed questions, while you are on your toes. The experience of presenting your first conference paper can also be reassuring, if it is well received.

Conference papers usually entail travel—sometimes overseas—and the associated expenditure. If you are not allocated an oral presentation, you would have to make a colourful poster of your work, neatly organized and well illustrated, to display at the conference. Making a high quality poster is an exercise in visual communication. You must put together at least one poster before you finish your PhD: it is a good learning experience. But posters too, are expensive. This is why I said that conference papers tend to demand more than journal papers: they consume more time, more effort, and more money.

If all this sounds a bit off-putting, start small. Present your first paper at a seminar at your university. Well-run research groups generally have scheduled research seminars sprinkled liberally throughout the year. Make sure that you book a slot and get to show and tell what you have done. You may then wish to graduate to a local symposium or a national meeting. From there, it is only a short hop to an international conference. Once you have fought and overcome any initial shyness or stage-fright, you will actually get to enjoy the opportunity for live dialogue and discussion.

One salutary side effect of conferences is that you get to meet others working in your area. They might be from a neighbouring city, or a continent away. If you meet someone with whom you feel comfortable enough to discuss research, you might have found a potential research collaborator with whom you could forge a productive working relationship. The Internet has nullified the tyranny of distance. Your collaborator might be five time zones away on the other side of the world, but you could be collaborating happily and publishing copiously.

48.15.2 The journal paper

The journal paper must be of a higher standard than the conference paper. It must be meticulously researched, clearly written up, adequately referenced, and revised several times after the first draft. You would generally want your paper to appear in a leading journal in your field. See Chapter 49 entitled "Getting Published in World-Class Journals: A Checklist for Researchers" for help on how to go about writing a journal paper, starting with why you want to do it, going through the motions of selecting a journal, and finally battening down the hatches and writing and revising it. Now, a word or two about what to avoid.

48.15.3 Plagiarism and cheating

If you are tempted to indulge in plagiarism or cheating or both, I suggest that you forget about your PhD and do something else instead. Knowledge thrives on honesty and integrity. Our civilization has advanced to where it is because each brick of knowledge on which it is built, is solid and of high standard. The worst disservice one can perform to knowledge as a whole is to corrupt the process by which it is gathered and guarded.[9]

> ★
>
> Do not plagiarize or cheat.

Plagiarism is using another's ideas or work and passing it off as your own. Whether you do this because you are slipshod in your work habits, or whether you do it with intentional malice, you are plagiarizing nevertheless. If you need to use someone else's ideas or work, give due credit by attribution in a citation—in a reference, or a footnote, or whatever is appropriate for your medium of publication. But never ever steal another's ideas or work and pass it off as your own.

Even worse than plagiarism is faking results. It is downright cheating. Mathematics, because of its infallible chain of logic, is thankfully, one subject in which cheating will always be uncovered eventually. A fake theorem that has been "proved" will eventually be unmasked for the falsehood that it is. But in most other disciplines, cheating may go undetected for a long time. Even the most reputable journals have fallen victim to fraudsters who have invaded the sacred niches of research. Your academic integrity must be guarded like a precious family heirloom: protect your own work and reputation by keeping meticulous records, and by documenting your work systematically.

48.16 What to do after you submit your thesis

While you are working on it, the PhD becomes a very major part of your life, or perhaps even your whole life. Once you have submitted your thesis, however, you are likely to feel a vacuum or even become depressed. The reason is that, what loomed ahead forever as a goal to be attained—namely writing up and submitting your thesis—has now un-

[9]See David L Goodstein's book *On Fact and Fraud: Cautionary Tales from the Front Lines of Science* to appreciate how onerous "policing" science can be.

believably been accomplished. The resulting void is uncomfortable and unsettling. Some hints on how to cope with this are in order.

Before you began your PhD voyage you would have had a mental picture of what you were going to do after that. Possible options include:

1. Returning to work with your previous employer if you had taken a leave of absence to pursue your PhD.
2. Seeking a position as a post-doctoral fellow at some university *other* than the one where you did your PhD.
3. Applying for a teaching position in academia, whether at the same or a different university.
4. Seeking placement in a national research agency, or the military, or government, or a Non-Governmental Organization (NGO), or a multi-national corporation.
5. Starting your own company or organization to do consulting, teaching, research, or whatever you feel is appropriate.

After thesis submission, you should seek to be gainfully employed in some such activity as the above. Try to be with an organization that allows you the freedom to work on your research publications for at least half a day each working week. That way, you maintain a link with your research while at work, and correcting your PhD thesis when the time comes, will be less of an ordeal.

48.17 The last lap

Depending on your university and its tradition, you would know the fate of your PhD thesis at the hands of the examiners, in a time frame ranging from a few minutes after your oral defence, to perhaps three months after submission. In most cases, it will be passed with minor corrections. You will then be given a deadline within which to make those corrections, and re-submit your thesis. The university can then declare your thesis "passed" and award you your PhD. Your task of getting a PhD is not complete until this final stage is reached.

> You need to keep alive the flame of academic enquiry—and the research and writing skills that you have carefully cultivated—until your PhD thesis has been corrected and finally graded as "passed."

If you are already working, there may be a tremendous reluctance to revert to "PhD candidate mode", and to correct the thesis carefully and make the final submission. Overcome this malaise and get the job over and done with. Running the last lap gives the greatest pain and demands the greatest effort and will. But it also gives the greatest joy. So do it just the same and emerge victorious.

48.18 Post-PhD blues

The PhD programme, while it lasts, is like a searing, all-consuming flame. Until the thesis is submitted, the fact that it has not been submitted will ceaselessly nag your waking hours. But what happens after it has been submitted and passed?

Some suffer post-PhD blues not unlike the post-natal depression that assails some new mothers. The journey has been so central to your life that its conclusion, instead of being a joyous celebration, becomes paradoxically a source of grief. What can you do if this happens to you?

Take preventive measures: they are always better than remedial ones. When one act of your life is over, move on to the next. Tarry no longer in the past. The PhD is now well and truly over.

Line up a job, or a holiday, or a business venture, or a post-doctoral fellowship: anything that can take your focus away from the PhD and let you move into the next phase of your life. Do this beforehand. Do not remain in vacant limbo until after your thesis has been passed. In this way, you will constructively engage your time and mind. You will make friends. You will take charge of your life. You might even make some money (or spend it on a holiday ☻).

Sometimes a PhD can make you "over-qualified" for the jobs that are available. If there are none where you live, harness the Web to find a prospective employer anywhere in the world. Apply well in advance for a visa if one is needed. The challenges and thrill of relocation will immunize you against post-PhD blues. Capitalize on the mobility that a

PhD confers on you.

Academia is generally kind to new PhDs. Postdoctoral positions abound. You can gain teaching or research experience or both.

Sometimes, the thought of doing unsupervised research might lead to anxiety or even fear. A postdoctoral position is often only marginally supervised and provides a comfortable transition from supervised to unsupervised research.

If your long-term goal is to remain in research, cultivate a network of researchers of your own vintage, and try collaborating with them. You will get feedback from your peers and you will grow together as a team. See Chapter 47 for more on unsupervised research. One of the best popular accounts of cutting edge doctoral research and collaboration is given almost tangentially in the book *SYNC: The Emerging Science of Spontaneous Order* by Steven Strogatz [11]. Read it and you will find that in real life, nestled among many failures and mistakes, are a few brilliant research successes. Your own expectations of research will then be grounded in a healthy reality.

If you are a mature age student, you might find jobs a little harder to come by. Ideally, in such a case, if you were working beforehand, you would have taken study leave from your employer to do your PhD. You can then just go back to your old job after your PhD, with no dramas.

If that does not apply, allocate a few hours each week to look for jobs even when you are writing your thesis. Diligence pays dividends. You can and will unearth that job which is uniquely waiting for you. But start early.

Even if all else fails, and you have to take a lower-paying job than before, what of it? You have fulfilled your dream of a PhD. You have succeeded! Your current job will pay you a steady income even if you consider yourself overqualified for it. Keep at it while seeking better prospects. You are bound to find something better if you keep looking. Dispel post-PhD blues by cultivating a sunny disposition.

48.19 Taking stock

The trials, tribulations, and travails on the road to a PhD are many. So, finishing your degree must carry some reward. Apart from the obvious improvement in your knowledge and employment prospects, what are the returns? How does completing a PhD transform you as a scholar and as a person?

Firstly, you learn discipline, not unlike in a monastery or the military. You respect the clock. You respect the work that you have to do. You respect schedules. You have become unafraid of hard work. You have learned to do what must be done, even if at first it appears uninviting, because you know *it has to be done*.

Alongside discipline, you develop a whole host of virtues: patience, perseverance, self-reliance, doggedness, optimism, endurance—the list goes on.

You learn to think for yourself.

This is perhaps the single greatest benefit of doing a PhD. You learn to think rationally and infer logically, always based on evidence. No one else will ever be able to pull the wool over your eyes again on matters scholastic.

You also learn to read articles critically and evaluate the work of others impartially based on merit alone. You realize that you exist within a society of professional peers with whom you must engage because you are all collectively enlarging the frontiers of knowledge as a cooperative enterprise.

Your PhD would have forced you to confront your own errors. By becoming acutely aware of your own failings, you will become more tolerant of the mistakes of others. You will thereby become a better-adjusted human being.

You develop the ability to resile—or bounce back—from failure and tackle the problem with renewed vigour. You become systematic in your approach, detailed in your planning, and thorough in your record keeping. You learn to spot anomalies quickly and extract what they can teach you.

You learn to organize your thoughts into a coherent story. You develop the ability to appreciate opposite viewpoints. You are unafraid of debate but unemotional about your standpoint. You can take criticism and learn from it. Bit by bit, you become an expert in the field you have chosen.

You master the art of preparing presentations for professional meetings. You learn to compose slides that let the story unfold at a comfortable pace, neither confusing your audience with too much haste nor boring them by too slow a pace. You learn to speak calmly, unhurriedly, and confidently. You value time and ask and answer questions courteously and to the point.

You have practised the art of writing clearly and concisely. You have learned how to write articles for learned journals in your field. You have mastered jargon but know your subject well enough to explain it to the layperson. A fledgeling scholar when you started, you now have eagle-wings that will allow you to soar the high skies of scholarly discourse.

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The PhD is a baptism by fire—no one comes out of it unchanged. It forever alters the way you think, the way you act, and the way you are: all for the better.

48.20 Suggestions for further reading

The book, *How to get a PhD: A handbook for students and their supervisors,* by Phillips and Pugh [6] is a classic worth consulting. Chapter 4 of that book, entitled "How not to get a PhD", inspired Section 48.12 here.

Mauch and Park have written the well-received *Guide to the Successful Thesis and Dissertation: A Handbook for Students and Faculty* [12]. Like Phillips and Pugh's book [6] this book is addressed to *both* students and faculty, allowing each party a (rare) glimpse of the other's viewpoint.

The PhD Application Handbook by Bentley [13] covers in greater detail many of the issues addressed in this chapter. Although it is directed principally to an audience contemplating doing their PhD in the United Kingdom, much of the advice given in the book is generic and more widely applicable.

There is one other book of which I became aware only after much of this book, and the whole of this chapter was completed. It is entitled *The Unwritten Rules of PhD Research* [1] by Gordon Rugg and Marian Petre. The book is an excellent reference which covers the contents of this and related chapters from a practical angle. I heartily recommend you to read it.

The early years of the twenty-first century ushered in a rash of new books devoted to helping master's and PhD students complete their programmes successfully. A representative list of these publications is available at the companion website.

48.21 Conclusion

The PhD should be undertaken only if you yourself feel the need to do so. It is a licence to teach at tertiary level. It is also the portal to a research career. Whatever the reason, start on your PhD only for non-frivolous, serious reasons, about which you yourself are convinced.

You should select your research area, supervisor, and university, only after searching thoroughly and deliberating carefully on all the available choices. Once you have chosen, stick with your choice.

Determine to succeed and never to give up. Once you have so resolved, do not let your gaze be deflected. Persevere until you are awarded the PhD degree. Hopefully, the process instils in you a lifelong love for acquiring and imparting new knowledge. Use your advanced knowledge to become a benefactor of humankind.

Best of Luck!

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[5] Nicola L. C. Talbot. *Dickimaw LaTeX Books.* 2012. URL: http://www.dickimaw-books.com/latex/index.html, visited on 2012-10-26.

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[12] James E Mauch and Namgi Park. *Guide to the Successful Thesis and Dissertation: A Handbook for Students and Faculty*. 5th ed. New York: Marcel Dekker, 2003.

[13] Peter J Bentley. *The PhD Application Handbook*. Maidenhead, Berkshire, England: Open University Press, 2006.

SUMMARY: SO YOU WANT TO DO A PHD?

- A PhD is a licence to teach at university.
- Be clear why you want to do a PhD. Proceed only if you have strong personal convictions.
- After committing yourself, never vacillate. Keep on at it until you complete your PhD successfully.
- Search carefully for a research field, supervisor, and university. Analyze your options and decide where you want to do your PhD, under whose supervision, and in what area.
- You cannot fail to get a PhD unless you give it up yourself.
- When you encounter difficulties, consult with your supervisors, your fellow students, and those with experience.
- Master your subject.
- Survey the field and draft a clear and feasible research proposal.

- Seek the help of your supervisors to formulate your hypothesis and any necessary sub-hypotheses.
- Plan your research and execute it with care.
- Write up your work, present papers at conferences, and publish in journals.
- Prepare carefully for your oral defence.
- Submit your thesis, make the necessary corrections, and fulfil your destiny armed with a PhD.

Publishing Papers in Peer-Reviewed Journals

Synopsis

Before starting work on a manuscript, ask the following W^7 questions about your proposed publication: what, why, where, how, when, which, and with whom?

Make sure that what you intend to say in your paper is worthwhile. Then write it in three drafts. Outline it in the first draft. Give it substance in the second. Polish it in the third.

Choose a suitable title to keep focused. Write the abstract, select keywords, and pen an outline to complete the first draft.

Keep the introduction and literature survey short and relevant. Partition your paper into methods, results, and discussion. Keep your conclusions complete, crisp, and accurate. Ensure that the intent of the paper as well as its major conclusions are in your abstract.

Proofread your manuscript. Check for logic, rigour, and continuity. Strengthen weak sections. Double check experimental results, graphs, and tables. Scrutinize references. Correct errors. Honour page length constraints. Use prescribed file formats for text and graphics. Submit the manuscript electronically.

Respond courteously, promptly, and thoroughly to the reviewers. Resubmit. Get published.

If your paper was rejected, review your work with your academic seniors. Repeat or extend your work. Redraft the manuscript. Submit once more to the same or a similar journal. Try unremittingly until your paper is published.

49.1 The need to publish

For graduate students and academics alike, the oracle of success drones on insistently and ceaselessly as the "publish or perish" mantra.

Graduate students, especially those doing a PhD, generally have to publish one or more research papers in internationally respected, peer-reviewed journals as part of the requirements for their research degree.

University academics who teach and do research have the dual burden of teaching well and also publishing highly cited papers in order to thrive in their chosen niches. Often their papers will be co-authored by their undergraduate or graduate students.

This chapter is intended to assist both graduate students and academics to get their research papers published in world-class journals.

> Your research is not completed until it has been published.

49.2 Preliminary questions

Notwithstanding the enormous pressures to publish, it is still proper to ask a few preliminary questions before plunging headlong into writing a paper. If you apply the W^7 framework of Section 19.5, you will come up with questions like those in List 49.1.

PRELIMINARY QUESTIONS

- What is the
  - subject of the paper?
  - purpose of the paper?
  - type of the paper?
- Why should the paper be published?
- Where should it be published?
- How should it be published?
- When should it be published?
- With whom should it be published?

LIST 49.1: Preliminary questions to ask before writing a paper.

Each of these questions is a good starting point in helping to clarify the process of writing and publishing academic papers. If you cannot answer these questions at the outset, you are not ready to write your paper yet. Discuss your difficulties with your academic supervisor. Talk it over with a more senior student. Walk away from writing the paper, take a fresh look at your work, think about your paper, and begin again.

> Never start a manuscript if you cannot answer these preliminary questions.

49.2.1 What is the subject of the paper?

This is the most important question. If there is no subject, then there is no paper. A research paper must have a clearly defined subject. It cannot be a "stream of consciousness" or "free association" exposition.

Imagine a dog tethered to a post. It can wander, but only within the radius defined by the post and the leash. The post is your subject. The dog is your paper. The leash is the discipline you display in writing your paper. If you ramble, or digress, or otherwise veer off course, your manuscript will not pack the punch that a research publication should, and is likely to be rejected.

> Identify the subject of your paper and stay true to it throughout.

49.2.2 What is the purpose of the paper?

Papers are published primarily to communicate research findings. They are the established mode for disseminating new knowledge. However, your precise reasons for publishing may often serve one or more specific ancillary purposes. List 49.2 shows some typical reasons for publishing papers.

The purpose will drive your *motivation* to get the paper published. Because enlightened self-interest is involved, you will be really fired up to publish it at all costs. The hurdles of correction and re-submission, or even rejection and re-submission, will not take the wind out of your sails. That eagerness is worth cultivating, especially as you weather through your first few manuscript submissions—and rejections—until you are published at last.

REASONS FOR PUBLISHING

- Dissemination of new research findings and knowledge
- Requirement for a master's or doctoral degree
- Academic career advancement through a greater number of publications
- Extension of previously published work of yourself or others
- Attempt to replicate another's experimental work
- Rebuttal or correction of another's previously published point of view
- Publication for academic merit after successful patent filing
- Tutorial to a subject area at research-level
- Invited survey paper for a plenary session at a conference or for a special issue of a journal

LIST 49.2: Reasons for publishing a paper.

The purpose behind your paper will also overtly or subtly affect the *scope* and *emphasis* you give to what you have written. The tone of a research-level tutorial, directed at a select, expert audience, will be different from that of an invited survey paper intended for a wider audience.

49.2.3 What type of paper is it?

Academic papers may be categorized in many ways. Here, I will adopt a simple, mutually exclusive binary classification.

49.2.3.1 Research papers

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A research paper disseminates new knowledge or extends existing knowledge.

You get to tell the story of your research simply, concisely, completely, clearly, and objectively. It is the sort of paper you need to write to notch up the requisite number of "brownie points", if you are a graduate student pursuing a research degree, or a junior academic seeking permanent appointment or tenure.

The perceived impact of the new knowledge in your paper will determine whether it finally gets published. So long as the new knowledge is neither too trivial nor too radical, there is a good chance of publication.

In this regard, experimental papers stand a better chance of acceptance than purely theoretical ones, if only because there is less to dispute when faced with experimental facts.

Because reviewers are only human, disruptive new ideas that upset the apple cart of established knowledge, will often be difficult to publish. Indeed, several Nobel prize winners have declared that they found it difficult to publish the very papers that later found their way into their prize citations.

Some research papers are written merely to *repeat previously reported work,* especially when a new method or technique is introduced. In these cases, your paper would be adding to or subtracting from the weight of scientific evidence needed to establish the efficacy and acceptability of the new technique.

49.2.3.2 Survey papers

★

A survey paper summarizes existing knowledge in a restricted discipline.

It offers an integrated, birds-eye view of a field, giving a coherent panoramic picture of the varied research landscape. It summarizes the state of the art and has the advantage of giving a clear global overview, even if lacking somewhat in detail. In some disciplines, there are *annual surveys* that collate in one place the significant new knowledge published in a particular year.

A survey paper will usually contain a very large number of references to the original literature. It is therefore an excellent starting point for novice researchers entering that particular field. To author a survey paper, you need to be widely read in the field, and also mature enough to give an overall assessment of it.

On occasion, leading researchers in a particular area are *invited* to write survey papers. Such papers usually carry the distinctive label *invited paper* and are often published in special issues of journals. At conferences, these surveys often take the form of *keynote addresses* at plenary sessions. An invited paper is a signal academic distinction accorded to eminent researchers.

If you are a beginning doctoral candidate in a field, who has done enough of a *critical survey* of the literature, you may wish to document your efforts as a survey paper. Such a survey, especially if there are none in the field, is a valuable addition to the state of knowledge, and merits publication.

In this type of paper, you summarize and critique the existing knowledge in the published literature. Special attention is paid to what is inconsistent, or what has been left undone, or what could be done better, etc. Your attention and emphasis would be directed toward the deficiencies that you hope to correct in your own research work. But bear in mind that such a survey paper might not qualify strictly as a research paper in the academic accounting schemes for research degrees employed by many universities.

A *tutorial paper* is a detailed, didactic survey meant to be accessible to a newcomer to the research field. In it, you weave together in one broad tapestry, the disparate strands of knowledge, using individual research papers as warp and woof. You should define terms clearly, and need to maintain a correct and consistent nomenclature and notation throughout, even if the different individual papers do not. It is a dicey juggling act because the papers you are weaving together will mostly be written by others, and there is no guarantee of notational consistency.[1]

49.2.4 Why? Is it worth publishing?

Although it sounds obvious, one of the very first questions you should ask—after determining the subject of a paper—is whether the intended material is worth publishing.

It *is* worth publishing if new knowledge is presented. Even if it is merely an extension or critical re-examination of old knowledge, it may still be worth disseminating. Publishing negative results can also be beneficial; it will prevent others from going down the same dead end and thereby save valuable research time, effort, and resources.

★

Attempt to publish a manuscript only if *you yourself* are convinced of its merit.

[1] When you perform the same weaving together—but of your own papers exclusively—to form a coherent picture, you would be writing your thesis. ☻

The reader of your paper does you the favour of investing time reading it. It is incumbent upon you as the author to return the favour by making your paper worth reading.

Several factors affect the publishability of a manuscript.[2] Here, I am looking at the criteria that will mark your manuscript as one deserving of publication. There are many abstract attributes that editors and reviewers of journals look for in a manuscript before they will let it past the "submission turnstile". If you do not get past this first barrier, your efforts at further publication will be frustrated, often without any constructive feedback on why your manuscript did not find favour in the first place.

A paper merits publication if it fulfils one or more of the criteria shown in List 49.3. Perhaps it is more important to recognize when the material is *not* worth publishing. Pretend that you are a reader of a journal, harried for time.

Factors Affecting Suitability for Publication

- Originality
- Contribution to new or existing knowledge
- Consistency, rigour, and quality
- Potential impact of the work
- Technical value of the advances made
- Compact survey of the field
- Entry level tutorial to a subject area

List 49.3: Factors affecting the suitability of a manuscript for publication.

> If you would not want to spend time reading your own paper, it is not worth publishing.

Trying to publish simply for the sake of chalking up one more publication in your list is not reason enough to publish.

There may also be institutional pressure to publish, even if it is only in a local journal of limited circulation, or a non-peer-reviewed journal.

[2]How to draft a paper and polish the final manuscript is discussed later in this chapter.

Under these circumstances, the individual academic must do his or her own soul-searching and act according to personal standards of probity and excellence.

If you are a research degree student, you should only publish in reputable peer-reviewed journals; publishing anywhere else will not help you get your degree.

49.2.5 Where: Conference paper or journal paper?

A journal paper endures. It is like a perennial plant. Conference proceedings, on the other hand, are more like seasonal wildflowers. Journals impose a higher standard of rigour and quality for the manuscripts they accept than most conferences.

For these reasons, universities traditionally award more weight and "brownie points" to journal papers than conference papers. The accompanying academic prestige is also greater for a journal paper.

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> You should always strive to publish your work in a reputable, peer-reviewed, international journal in your field.

That said, there is a legitimate place and purpose for conference papers, which we examine next.

Almost all new research will be presented first at a conference. This allows the new findings to be aired in a friendly, expert environment. One of the great advantages of a conference presentation is that you will be able to interact with others, at least some of whom would be experts in your chosen field.

The question time after the paper is presented is often a source for invaluable comments. Your audience would usually ask you questions—general or specific—about what you have done. They might point you to parallel developments of which you might not be aware. They could also challenge some of your assumptions or approaches. There may be a reference that was left out. Perhaps, there is another researcher doing the same work, who was not identified. There might be interesting new directions to explore. Or a comparison might be suggested of the new method against an older, established method. Such exchanges are part of intellectual Darwinism in action. It is generally civil and without personal rancour.

To benefit from such experiences, learn to listen to the other person, carefully and patiently. Only then should you respond. Remember that neither you nor your work is under judgement, and that your response need not always be a defence. This is not your thesis oral defence. It is rather an exposure of your work to the broader community, including specialists. They are there to help and guide. Their criticism is meant to improve your work. And you need not become defensive. If the criticism is valid, receive it gracefully. And follow it through.

There is no end of benefit from cross-pollination of ideas from the questions that a paper may evoke at a conference. Perhaps someone from a competing research group might ask a pointed question over lunch or dinner that could lead to better ideas or techniques. There is also an opportunity to share thoughts with kindred researchers that could very well lead to research collaboration in the future.

Another valid reason for submitting a conference paper is to allow the young undergraduate or graduate student to gain experience presenting new knowledge before an informed and expert audience. This will boost the confidence of the novice researcher. There may even be motivation for the student to try to win a "best student-paper award". Often, the student presenting the paper is the first author and assumes responsibility for everything, from paper content and formatting up to slide presentation, under the watchful eye of the supervisor. Such experience of hands-on preparation is invaluable to the young researcher and builds up confidence.

Conferences are planned one or more years in advance. The number of sessions and the durations of each dictate the time allocated to each orally presented paper. If too many papers are submitted, some will be relegated to poster sessions, which will involve greater expense and preparation. If there are too few papers, the standards of acceptance might be relaxed until there are enough papers to fill the time slots allocated to particular sessions. Be aware of these minutiae when submitting a conference paper. Aim for uncompromisingly high standards whenever you submit a paper regardless of these considerations.

It is customary for a conference paper to be suitably expanded and written up for publication in a journal. The standards for a journal paper are considerably more stringent than for a conference paper. While many conference papers undergo only a hurried peer review, if any at all, journal papers—especially those submitted to reputable journals—undergo at least two rounds of stringent peer review with at least two, and up to five, reviewers. This means that a manuscript that was "camera-ready" for a conference is nowhere near ready for a journal. Much extra effort must be lavished on the manuscript before it can be submitted to a major journal. Otherwise, the manuscript may be summarily rejected as not being worth reviewing.

When submitting to a journal, ensure that your manuscript follows the required format. If not, your manuscript might be rejected outright. There is intense competition to publish in a high class journal. Darwinian selection in the academic sphere will determine who publishes and who perishes. Ensure that you do not perish by default by not conforming to submission formats and rules. This applies especially to electronic submissions, which are displacing paper submissions as the norm in most quality journals. See Section 49.6 for details.

49.2.6 Where/Which journal should it be published in?

The choice of journal is a crucial step in the publication process. Usually, someone working in a research area is aware of the most highly reputed, peer-reviewed journals in the field. If you do not have that experience, discuss the choice of journal with a supervisor or senior colleague working in the same research field. Alternatively, you could discuss choice of journal with a librarian at your university library.

There are also published "league tables" of journals in each field. One example is the Journal Citation Reports (JCR) [1]. The *impact factor* [2] of different journals in a particular field, for example, may be used to prepare a list of candidate journals to which a manuscript may be submitted.

If you know that your paper is a major contribution, it is worth "going for gold" and submitting it to the flagship journal in the field. If you feel that your paper is significant, but is likely to be turned down by the top-notch journals, it might be prudent not to waste time but to submit it instead to a good, peer-reviewed journal of more modest reputation. You are the best judge, and will acquire this "gut feeling" through experience.

It is certainly *not* worth publishing in journals that are not peer-reviewed. Such publications, however meritorious, cannot lay claim to the prestige and authority conferred on a properly peer-reviewed publication in a journal of repute.

49.2.7 Where/Which online electronic journal?

Publication in a traditional journal incurs a cost for the author. Rather than being paid to publish, the author of a scholarly paper has to pay the publisher in order to publish in a journal. These charges might be levied on a per page or per paper basis. And the publisher could be either a professional society or a commercial publisher.

In addition, learned journals are usually not mass market publications. The cost of a single issue thus has to be defrayed across a limited number of academic institutional subscribers, including libraries. This leads to very high subscription rates for many journals.

Thus the author of a scholarly journal article has to *pay twice:* once to get published and again to subscribe to the journal to read what he or she has written.

The absurdity of the above situation has not escaped the notice of financially stretched academics. Together with the mindset spawned by the Internet and the World Wide Web, this has resulted in a growing tide of freely available *open-access* electronic journals managed largely by academics.

Many of these are *peer-reviewed* and *online,* and even though open-access, are not necessarily inferior to their expensive, paper-based cousins put out by traditional publishers. It is ultimately a matter of who publishes *in* them rather than who publishes them. If the leading lights in a field give cognizance to such journals, they cannot but grow in reputation and influence.

Such online journals are likely to be the norm in future as they promise a more rapid review cycle. There is also the added allure of open-access at a time when conventional journal subscriptions are skyrocketing, and university libraries are struggling to renew existing subscriptions, let alone enter into new ones.

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Become aware of reputable, peer-reviewed, online journals in your research area and submit manuscripts to them if you deem fit.

49.2.8 When: Special Issues of journals

From time to time, respected journals devote special issues to specific areas of research. Leading researchers in a field are usually invited to contribute surveys, and sometimes, even tutorials on the subject. Almost everyone who is someone in the field would have an article in such an issue.

If your paper is on a topic for which a special issue is planned in a good journal, it may be preferable to submit it to that journal, even if it is not quite the best in the field. Your paper is likely to be read by a wider audience if it appears in a special issue, and that increases its prestige and the chances of it being cited, replicated, and extended.

Of course, if you have attained the level of eminence where you are invited to present a paper—whether at a conference or to a special issue of a good journal—that is an even greater honour and recognition from your peers. It is unlikely though, that you would be reading this book then. ☻

49.2.9 With whom: Who is the first author?

The first author is generally the person who has done most of the research, and who has usually also done most of the writing for the manuscript. If the work being reported is the research conducted by a student enrolled as a candidate for a Masters or PhD degree, it would be sensible that she or he should be the first author. This is often necessary, because these higher degrees by research are earned on the basis of *individual, original* contributions. It would undermine the student's case if the first author of a research paper, publishing new experimental results, were the student's supervisor instead of the student.

It is an inevitable, if somewhat unpleasant fact of life, that politics plays a role in academia as in other human affairs. It is not unknown for supervisors—given their position of authority over the academic fate of the student—to nominate themselves as first authors. Indeed, there have even been cases where supervisors with surnames that come very early in the alphabet have insisted on being first authors in *all* papers simply on the basis of alphabetical ordering, incredible though this may sound. These are examples of abuse of power. Indeed, it is the supervisor's ethical responsibility to protect the student's interest, and one way of doing this is to ensure that the student is given due credit for original work by being listed as first author in a research paper.

Survey or review papers are somewhat different; they do not communicate new research. They are written because eminent researchers are asked to summarize the state of the art. They may also get written when a student accomplishes the same thing when doing a critical survey of the literature as part of a research thesis. In these circumstances, there can and should be flexibility in naming the first author. There are no hard and fast rules except that the ethical integrity of the parties involved should never be compromised.

It is often customary for the leader of the research team or the laboratory to be listed as *last author* in all papers emanating from that research group or laboratory. This may happen even if the leader, usually a senior academic, has done nothing except, perhaps get a successful research grant, to further the reported research. Indeed, there may be occasions where the senior academic may even be totally ignorant of what is being reported. This raises the issue of whether a non-contributing author should be listed at all, simply because of funding reasons, or simply to ensure repeated future funding for that project. This is where academic research meets politics and economics, and it must be left to individual institutions to determine their own ethical stands and standards.

49.2.10 Priority, patents, and such

The historical discoverer of an idea is not always the person credited with its discovery. Sometimes, the scientific atmosphere is ripe for a particular discovery to be made, and several people in different countries all make similar discoveries, which collectively usher in new knowledge and technology.

The discovery of radio waves and their use for human communication is one such case. Although Guglielmo Marconi is historically credited with the discovery of radio, several others, including Heinrich Hertz, Jagadish Bose, and Nikola Tesla could all lay claim to independent simultaneous discoveries of a similar nature.

Historically, Sir Isaac Newton and Gottfried Wilhelm Leibniz had a bitter feud over who first discovered (or invented?) the calculus [3]. Although Newton had understood, derived, and applied the methods of the calculus before Leibniz, he did not publish his work first. Leibniz on the other hand did not hesitate to publish. This classic case—of a feud over priority of discovery in a very fundamental area of mathematics—established once and for all that *you are not credited with a discovery unless you have published it first*.[3]

As a PhD student, you are required to discover new knowledge. If someone else publishes a paper on the discovery or invention you are trying to make, you have lost priority. That is a messy situation in which you do not want to find yourself. To resolve it would need proof that you have worked independently, and that your paper on your discovery had already been submitted to a journal on a date prior to the publication date of the competing paper. If your approach and method are different, your case becomes stronger.

Loss of priority involves not only you, the PhD student, but also your supervisor(s), and perhaps your faculty or university.

★

Priority, especially as a stepping stone to fulfilling the requirements of the PhD degree, or something more ambitious, is a good reason to want to publish, and publish soon.

When Wilhelm Roentgen discovered X-rays in 1895, he refused to patent his finding, intending instead that all mankind should benefit freely from his discovery. Pause for a moment to think about how you have personally benefited from Roentgen's altruistic decision, and thank him mentally for it.

It is a different story nowadays, though. Discoveries and inventions, especially in the scientific, technological, and medical fields hold the allure of commercial exploitation and monetary reward. These are extremely attractive because contemporary research is very expensive in terms of both the people and the machines involved. And how does a desire to patent impact on publication?

[3] There is an interesting account [4] of the undue haste that attended the publication in the journal *Nature* of the paper on the structure of DNA that was to win Francis Crick and James Watson their Nobel Prize.

Broadly speaking, a patent application is filed as a claim to priority for a method or technique of doing something. That something is disclosed for the first time in the patent with the express intent that anyone exploiting the method in the patent for commercial use will pay the owner of the patent a royalty for its use. If the patent is granted, this exclusive right to royalty expires, usually after 20 or so years.

Implicit in a patent application is a declaration that the subject of the patent has not been disclosed by anyone else elsewhere, and also that the applicant has himself or herself not disclosed it publicly prior to applying for the patent. What this means is that *you cannot publish in the open academic literature anything that you may later use as the basis for a patent application.*

But you may file a patent, and afterward, without prejudice to the patent, publish that information in the literature, usually without giving all the details for achieving whatever is being accomplished. In this case, someone intending to replicate the work will very likely have to refer to the patent itself, and very possibly pay royalty or licensing fees if the latter is used.

49.2.11 The final question: Is the material sufficient?

Bear in mind that the primary purpose of publication is the communication and dissemination of new knowledge. Your paper should contain enough information to allow someone else to repeat your work, irrespective of the field of research. If there is insufficient practical or technical detail to allow this, more material must be included before submission.

However, it is easy to err on the opposite side, and to smother the paper under too much detail. It would then most likely be rejected or returned for revision to reduce length. From repeated experience, it is possible to cultivate an inner judgement of the right degree of detail and the proper length for a paper.

Another purpose of publication is to showcase the new work in the context of existing knowledge in the field. Previous work should be described briefly with adequate citations to the relevant papers. It is also customary for any new methods or results to be compared to previously known ones. Such comparison may require running benchmarks to gauge single figures of merit, or statistical tests of efficacy, etc. It is obvious that the way in which the comparison is done varies with the field of research.

But comparison with earlier results there must be, before the paper can be considered seriously for publication.

Finally, quite separate from the comparison of experiment or methodology, it is necessary to have a balanced and substantially complete survey of previous work in the early parts of the manuscript. This again places the new work in context and allows the author(s) to air their subjective views on the strengths and weaknesses of the previous work later in the paper. Such critiques are part of the current scientific paradigm, and it is important not to be squeamish in stating one's considered opinions, but this should always be objective and without personal invective.

If your paper is not found lacking on all three counts—sufficient detail, comparison of results, and review of previous work—you can be reasonably confident that the material in it is sufficient. We have now completed the round of preliminary questions and are ready to tackle the actual writing of your paper.

49.3 The First Draft

I suggest here that you write your paper in *three drafts*. A draft is the name given to a series of activities, performed repeatedly, until the paper is developed enough to progress to the stage of the next draft. Obviously, we begin with the first draft.

Imagine you are painting a picture. You start off with a rough charcoal sketch of what you want to depict. It does not contain details. But one look at it will tell you whether you are planning the picture of a flower, or of a landscape, or of a personal portrait. It is *the big picture*. Without it, you have nothing to guide you as you work on the canvas. The first draft of your paper is that rough charcoal sketch that gives you the big picture.

49.3.1 Getting off the starting blocks

If you have asked and answered the preliminary questions, writing the first draft will be smooth and friction-free. If not, I suggest that you reread Section 49.2. It helps you to get moving at once.

You *outline* your paper in the first draft. Do not get bogged down by details. Resolutely resist tweaking things or polishing up what you write. In this phase of your paper, you are making an outline—a skeleton—that you can clothe later with substance.

There will be aspects of your work that you know so well and are so keen to record, that the words will rush and tumble out of your head as you apply your mind to writing your paper. Jot down these irrepressible inspirations for future use. Once the flood of thoughts has abated, get back to sketching your outline. Be disciplined.

This restraint is necessary for you to write a paper that is complete and balanced. Gushes of inspiration are welcome but they cannot totally dictate what you write. You need to give each part of the paper its due. Your first draft will encompass four items as shown in List 49.4.

The First Draft

1. Title
2. Abstract
3. Keywords
4. Outline

List 49.4: Components of the first draft

I find this order useful for keeping me on subject and on course. Rambling off subject or off course is detrimental to writing a paper. Your manuscript should be tightly focused on what you set out to say, and should never wander off, if it is to remain within specified page limits, and is to find favour with your referees. Choosing a good title helps to keep you on track, and this is what we consider next.

49.3.2 Start with the title

Choosing a *title* lets you start the first draft with the least pain and maximum return.

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Your title is a condensed description of what your paper is about.

When you choose your title, you get to reflect on and tell *yourself* what your paper is about. Because this is your first draft, your title could change between now and the final draft. But decide upon a title nevertheless. It will help you define the scope of your paper. The title will act like a railroad track that will keep you from wandering off into irrelevance.

If your paper is about a single subject, you could use the word or words describing it as a starting point. Toss words around for a title. Play with various words and word-combinations. Strike a balance by choosing something that is not too long and not too short; something that is neither too general nor too specific. Do not overload the title with jargon, but do not feel compelled to forswear jargon entirely.

Make each word count. Choose your words carefully. Most of all, try to make your title a *verbal snapshot* of what is most important in your paper. Once you have ruminated on your options, choose a draft title and use it to *scope* your paper.

49.3.3 The abstract

After the title, the most important part of your paper is the *abstract.* Write an abstract, whether of one line or one paragraph, summarizing the described work.

MINIMUM CONTENTS OF ABSTRACT

- What you did
- Why you did it
- How you did it
- What your results were
- How it advances knowledge

In Chapter 50, "How to Write a Thesis", I have recommended that when writing a *thesis,* you should write the abstract *after* everything else has been written. When writing a *paper,* this guideline should be relaxed.

Start writing the abstract from the very beginning. Your paper is far shorter than a thesis. So, your entire scope is always in your mind's eye. The close coupling between abstract and subject matter in a paper can and should be tracked as your paper evolves from first draft to submitted manuscript.

After you have written the whole paper in the second draft stage, read the abstract and the rest of the paper in one go. Revise each to be compatible with the other. Then iterate through as many first-second draft cycles as necessary.

World class journals are always indexed in databases of publications, which vary with discipline and specialization. Abstracts indexed in these databases have to conform to word limits. Honour these. Don't waffle. Parsimony helps; profligacy does not.

Polish up your abstract repeatedly so that it leaves out nothing of relevance and is yet self-contained and within the prescribed word limit. Writing a pithy abstract is one of those juggling acts of optimization that you need to master to keep publishing in good research journals.

49.3.4 The keywords

Along with your abstract, you need to furnish *keywords*. Practices differ across disciplines and journals. Your discipline might already have a *taxonomy* or *ontology* of keywords and your only task might be to select those that are the best fit.

For example, the Medical Subject Headings (MeSH) [5] are used for purposes of indexing books and articles on medicine. Mathematics has the Mathematics Subject Classification — MSC2020 [6] scheme. Similar taxonomies or ontologies will be in use in other disciplines.

To enable efficient indexing and retrieval of your paper, the journal or publisher would almost certainly ask for the appropriate subject headings applicable to your paper. These may also often be your keywords. Make it your business to know what these headings are and check with someone senior in your field that the headings you have chosen are indeed the most appropriate for your paper.

In cases where you are allowed free-text keywords, you again need to select the terms judiciously. *At least some words from your title should be included in your keywords.*

Re-visit your keywords and re-assess their aptness after you have written your full paper. Tweak them if necessary.

Just as you yourself, when you entered your field of research, queried databases to select the papers that you needed to read, so too would others entering your field. You would help them find your paper if you chose apt keywords. You would also be helping yourself in the bargain, because your paper would then be easily found, read, and possibly cited by others.

Choose keywords with the care they deserve.

49.3.5 Example: Putting it together thus far

To give you a flavour for choosing a title, assembling an abstract, and selecting keywords, let us consider some *contrived* examples from different disciplines.

EXAMPLE: SOME CONTRIVED SAMPLE PAPERS

Let us suppose that your research paper *compares two different methods for doing something.*[4] Let that something be called x where x could be:

1. solving ordinary differential equations in mathematics;
2. managing a patient suffering from a specific disease in medicine;
3. excavating an archaeological site;
4. implementing a Web-based software application; and
5. assessing the environmental health of a district.

The generic principles for comparing two methodologies do not vary with subject area; specialized considerations do. The approach below for homing in on the title, abstract, and keywords is generic rather than specialized. Let us start with the possible *titles,* which could respectively be:

1. A comparison of exact and numerical methods for solving ordinary differential equations;
2. Outcomes of managing patients suffering from X using two different treatment regimes;
3. Twin approaches to excavating a bronze-age archaeological site at Y: Preliminary results;
4. Implementing login/authentication using two different Web frameworks A and B: functionality, security, and maintainability; and

[4] Even though these might not qualify as *original* research.

5. Environmental health assessment of a district from the qualitative and quantitative standpoints.

Note that these are simply notional titles in which the variables like X, would in actuality, be given specific names such as, say, “typhoid”.

You might come up with shorter or better titles. Always strive for that which is short and snappy. Include keywords in the title where possible. The aim is to say the most with the fewest words. Avoid titles that read like paragraphs. If one method proves to be better than the other, phrase your title to reflect this fact: it makes your paper more attractive. The trick is to play around with the words until you get a title you are comfortable with.

Your *abstract* should concisely state, *using the jargon of the research field,* the two methods you are comparing, why you are doing so, your conclusion on whether one is to be preferred to the other, and for what reasons. State quantitative results and conclusions clearly. Highlight unexpected or interesting results concisely.

If a member of the intended audience of your paper were to read your abstract, he or she should be provoked to read your full paper. That is how you should write your abstract. Revise your abstract to keep it synchronized with your content and its emphasis as your paper evolves.

The *keywords* are next. If your subject has a taxonomy, use the correct categories and keywords from it. If free text is allowed, ask yourself what set of words or phrases best describes your work. They are usually nouns, and sometimes verbs. Use them as your keywords. It is a bit like a cartoonist looking at a face and exaggerating its salient features. *Become a cartoonist of words in your selection of keywords.* Let us have a go at plausible keywords for the five papers above:

1. ordinary differential equations, solution, exact, numerical;
2. treatment regime, (disease) X, patient outcome;
3. bronze-age, excavation;
4. login/authentication, (software) A, (software) B; and
5. qualitative, quantitative, environmental health assessment.

Notice that I have used single words and phrases, and drawn the keywords from the title in all cases. A good title has a ready supply of apt keywords. Try ranking keywords in descending order of importance. Then, truncate your list once you reach the allowed keyword limit. I have omitted the word "comparison" above. Include it if you have not exhausted your keyword quota; not otherwise.

If your subject area has a taxonomy or ontology, you might need to furnish the correct classification code along with your keywords.

For the first paper, which is on mathematics, you will need to type "ordinary differential equations" into a search box such as that shown on the 2020 Mathematics Subject Classification page of the American Mathematical Society (AMS) [7]. You would find that "ordinary differential equations" as a main heading is classified in the 2020 classification scheme as "34–XX". You are also presented with a host of other possibilities; choose appropriately.

For the second paper, in medicine, let us say that the disease X in question was "typhoid"; you would then type that word into the search box at the MeSH Browser of the National Library of Medicine (NLM) [8]. The online results will present you with a dizzying list of combinatorial options that you should whittle down until your indexing terms match the content of your paper.[5]

The above two forays into real life minutiae illustrate how complex knowledge has now become, and why careful classification is a necessary concomitant of the information explosion. It is something all manuscript authors must learn to navigate successfully. ★

49.3.6 The outline

With the title, abstract, and keywords under your belt, you are ready to plough into the hard work of writing the outline. This is the *big picture* phase of your work.

Write down the major headings of your paper. Generally, this will follow a standard schema such as that shown in List 49.5.

These headings represent an ordering of ideas that obeys one simple rule:

> ★
>
> Lead the reader from the known to the unknown.

[5] Medicine and its nomenclature is a rapidly evolving field and this contrived example is merely illustrative rather than definitive.

STANDARD SCHEMA OF A PAPER

1. Introduction
2. Previous work
3. Methods
4. Results
5. Discussion
6. Conclusions

LIST 49.5: Standard schema of a research paper.

The ordering of content in a paper is logical and functions like successive links in a chain. It mirrors the structure of a thesis, which is discussed in Section 50.5.1.

The actual headings you use might be more specific than these generic versions, but their function is the same.

★

Jot in point form what you want to say under each of these headings.

Whatever your research area, and whatever the contents of your research paper, make sure that your headings are logically connected and coherent. Once your headings are in place, you have the skeleton of your paper, ready to be fleshed out with facts and new knowledge. Writing your paper will then become so much easier.

It is better to start with a disciplined, properly-sized manuscript than to write your paper as if it were your thesis, only to later find yourself pruning your draft here, there, and everywhere.

Estimate the length of your paper against the prescribed page limits at this stage. Your *Results* and *Discussion* embody the new knowledge and will most likely take up the most space in your paper. Your *Introduction* and *Conclusions* are equally important, but might need to be condensed. Allocate to each part of your paper its legitimate page quota.

49.3.7 Introduction

The *Introduction* sets the scene for your work: what you did, why you did it, etc. Apply the W^7 framework of Section19.5 to ensure that your *Introduction* is complete. Do not repeat yourself; aim to be concise.

In brief conference or journal papers, your *Introduction* might also embody a critical review of previous work, sometimes called the literature review. For longer journal papers, you should dedicate a decent fraction of your paper to the literature review, as discussed next.

49.3.8 Previous Work

The *Previous Work* section sets the context for your work. It may also be called the *critical review of the literature.* If your work is the foreground or object in a picture, the *Previous Work* is the background against which it is viewed. It will *contrast* your work with what has been done previously, both by others and yourself.

If there is no contrast between what you are describing and what has already been done, there is little point in writing your paper. Only when contrast exists or improves can you justify your contribution as being new and useful. Use this criterion of contrast to highlight the merits of your work vis-a-vis previously reported work. Specific guidelines for assembling your literature review are given in Section 49.4.11 later in this chapter.

49.3.9 Methods

The *Methods* section is the meat of your paper. It explains how you did what you did to get what you were after. You are writing for the cognoscenti. Do not pitch your exposition at too basic a level.

Give enough detail to allow another qualified person to repeat your work. Conference papers' page lengths might preclude full descriptions, but you have the advantage of meeting your audience in person at conferences. If you are writing a journal paper, make sure that you describe what you did in full. If you are running short of space, cut out something else to allow a complete exposition of *Methods*.

Take care to state all the precautions you took. Anticipate criticism of your methodology during the review process, and state, in advance, all the measures you took to ensure the robustness of your methods.

49.3.10 Results

Condense your findings in the *Results* section. Use graphs and tables where applicable. State your results factually, without emotional embellishments or emphatic flourishes. Objectivity—not triumphalism—should be your hallmark. Do not discuss your results here. Simply lay out the facts, especially those that you wish to focus upon in the *Discussion* section.

49.3.11 Discussion

Your *Discussion* is where you add value to your work. Analyze your results here. It is also here that you compare your work with what has gone before, to highlight the merits of what you have done. It is the fulfilment of the contrast that was foreshadowed in the *Previous Work* section.

You might need to repeat others' previous work to establish a common baseline to allow comparison of your work with existing methods. Do this as part of your research work, well before you start writing your manuscript. If you fail to do this at the outset, you might find yourself having to do it very much later; see Section 49.9.2.

Do not make unsubstantiated claims for your work. At the same time, do not let modesty make you desist from rightfully claiming due merit. Be objective. Let your opinions rest solidly on facts. Ensure that everything factual that you rely upon has been made crystal clear in the preceding sections.

If there are drawbacks in your work, do not hide them. All work has both strengths and weaknesses. Your paper is being written only because you think the strengths greatly outnumber the weaknesses in your work. Make suggestions for further work to iron out any weaknesses in what you have reported. Your work should recommend itself because its manifest strengths so clearly overshadow any weaknesses.

49.3.12 Conclusions

The *Conclusions* should state concisely and clearly what you have accomplished through the reported work. Write it carefully because parts of it will find their way into the abstract. Make sure that your *Introduction* and *Conclusions* are consistent and tell the same coherent story.

Do not introduce fresh ideas here. Do not compare your work with others here. Finish all that in the *Discussion*.

> Summarize. Convince. Round off. Conclude.

49.4 The Second Draft

The title, abstract, keywords, and outline are all available from your first draft. Your ideas are now so ordered that your exposition will have both *thematic integrity* and *logical rigour*.

The second draft is where you really start writing to clothe the skeleton of the outline with the flesh of *substance* and bring your paper to life.

> The second draft is written and re-written as many times as necessary until it attains the shape and substance you desire to give your manuscript.

It is not a one-pass process. It is iterative. You hew and sculpt your words to give form to your thoughts. It is the heart of the writing process. The second draft is a forming—not a finishing—process.[6]

49.4.1 Thematic Integrity

Imagine watching a movie that keeps changing its complexion all the time. It starts off as a comedy, then suddenly turns into a science fiction thriller, and finally ends as a tragedy. Your expectations of what would come next would be dashed at each twist and turn, as one film genre morphed unpredictably into another. While this might be acceptable or even desirable in a movie, it is absolutely anathema to a research paper.

You are writing a story to communicate research findings. Fulfil your reader's expectations of what would come next, based on what has gone before. This is what is meant by *thematic integrity*. There is no scope for digression. There is no place for multiple foci of interest. It is not an epic that you are writing, with one tale within another tale, like a nested set of Russian matryoshka dolls.

> A linear flow is the simplest.

Start at the beginning. Move on briskly. Keep detours to a minimum, and explain any that you cannot avoid. Do not puzzle the reader. If you

[6] Not unlike making dough from flour.

depend on some result that was explained in full in a previous conference or journal publication of yours, provide a complete citation.

49.4.2 Logical Rigour

Regardless of your research discipline, your paper should fulfil the criteria for *logical rigour*.

> The story told in your paper must not flout the rules of logic.

It should also be consistent with the accepted rules for rigour in your discipline. Everything about your research, right from the design of the study, through your experimental technique, to the analysis of your results, interpretation of your findings, and your conclusions, should ring true.[7]

49.4.3 Substance

Be ever mindful of your audience and do not under- or over-assume its capabilities. Neither write up nor write down. Write to communicate without stress or distress.

Your reader is not privy to your knowledge. Neither is your reader an idiot. Do not tire the reader by mindless repetition. Nor infuriate the reader by taking rambling detours. Do not insult the reader by explaining that which may be safely assumed.

Define what needs to be defined. Explain where necessary. Give a reference citation to anything that you do not explain but later depend upon. Keep the reader on your side—neither confused nor bored—so that he or she will be persuaded by what you have to say.

Your paper should be as self-contained as possible, without straying into textbook territory. Keeping to the correct level of pedagogy is a delicate balancing act. Practise it until you get the hang of it.

The *jot*, *sort*, *write*, and *revise* sequence from Chapter 36 on "Essay Writing" is shown in List 49.6. Follow it. Your first draft should already have furnished you with a working outline of sorted, jotted points, which you can now build upon.

[7] Beware the glib generalization or unjustified and gratuitous comment that will consign your work to the ranks of snake oil papers!

WRITING SEQUENCE

1. Jot
2. Sort
3. Write
4. Revise

LIST 49.6: Writing sequence.

49.4.4 Illustrations, graphs, and tables

The substance of your paper will include a great many facts. Not all of these are best described by words or numbers alone.

An illustration or diagram can and often does convey a thousand words' worth of information. Massive amounts of data may be grasped faster and more completely from a graph or diagram. Graphs are best suited for showing trends and also how one thing varies with another. Diagrams are helpful in depicting spatial relationships, such as when describing an archaeological site or an experimental setup in a laboratory.

Tables are ideal for presenting information that has easily discerned patterns. Think of the Periodic Table of elements [9, 10] and the multiplication tables of your childhood. If some data is not arresting enough to be displayed as a graph, it is often relegated to a table, perhaps in an appendix to a paper.

Current standards for scientific truth require results to be replicable. If someone else wished to repeat your work to confirm or refute your findings, he or she needs enough data to permit comparison of results. Here again, tables are the preferred mode of presenting experimental data.

Your journal might stipulate a particular style—or file format in the case of electronic submissions—for graphs, diagrams and tables. Follow these stipulations strictly.

Producing helpful illustrations, graphs, and tables is both a technology-based skill that changes with time, and a timeless art that is cultivated with experience. Refer to Section 50.8.3, the companion website, and the relevant references in Section 49.14 for hints and details.

49.4.5 Read the Gopen and Swan paper

There is a landmark paper entitled "The Science of Scientific Writing" [11] by George D Gopen and Judith A Swan that is also available for free online [11]. If you have never heard of it or never read it yet, I strongly recommend that you do so. Although it was originally intended as a guide for writing scientific papers, the wisdom it embodies applies to writing papers in any academic discipline.

Even if you have read the paper before, it helps to review it every so often to ensure that you what you write is always easily absorbed by the reader.

> The cardinal rule is that your reader should not stumble on what you have written.

Do not lose you reader either to incomprehension or boredom. Keep your reader challenged but engaged by what you have written. Once you get the knack of it, "writing with the reader in mind" [11] will be quite a pleasant and enjoyable experience. It also helps if you have trained yourself to become your own observer auditor as explained in Chapter 3. You will then approach writing papers as a joy rather than as a chore.

49.4.6 Guideposts to writing well

Say first things first. Begin your sentence with the most important idea first. Do not waste the "premium real estate" at the beginning of the sentence with conditional or qualifying clauses. That will be like starting your sentence with an adjective or adverb, when you speak. It simply isn't done.

Use verbs liberally. They are words of action and add punch and life to your writing. There are many examples of how not to write in Chapter 50: refer to them if you have trouble with the way you write.

Vary sentence lengths and styles. A whole litany of short clipped sentences will read like a regimental sergeant major's drill commands. Equally, avoid too many lengthy and convoluted sentences. You are not writing to impress but to communicate. If your writing is obscure but erudite, you risk puzzling your reader. Aim for balance in your writing. Develop an inner barometer for what feels and sounds right and be guided by it. Practise until mastery is gained.

Because the substance of your paper will vary with discipline and subject, it would be pointless for me to give any detailed suggestions on how you should write the main body of your paper. You are the best judge of how to write it and what to put into it. Write the body of your paper under the different headings in consultation with your academic supervisors and colleagues.

49.4.7 Seek help to spot errors early

Seek help in areas where you could do with expert advice or knowledge. It is important that your manuscript does not contain errors of fact, technique, analysis, or interpretation. For this, a second or third pair of eyes is always welcome, ideally, those of your co-authors.

An unspotted error could mean re-writing your manuscript. So, do not wait to finish your paper before you ask for help. Seek help early. Get informed and expert comment *while* writing your second draft, and incorporate the suggested changes as you go along.

49.4.8 Seek specialist help if needed

Just because you are engaged in research does not mean that you have suddenly become omniscient. Au contraire, you might be knowing less than the layperson in some areas of general knowledge because of your narrow focus and specialization. In the course of writing your paper, you might stumble into one or more areas of your own ignorance. Seek expert help without delay.

EXAMPLE: SEEK HELP

Suppose that your field is in the humanities, but that you need to do some statistical analysis in your paper. If that is unfamiliar to you, or if you are unsure of the applicable assumptions, or allowable inferences, *seek help*.

Do your best to submit an error-free manuscript to your journal. Do not let a glaring mistake be spotted by the referees: such sloppiness will undo not only your paper, but perhaps even your research career!

Many universities provide assistance by qualified statisticians for authors in need. Use such help if available and needed. Seek the counsel of someone senior in your home department and follow it. Take whatever measures are necessary to make the substance of your paper as error-free as you can make it. ★

49.4.9 The literature

The work of others impacts on your paper in the *Previous Work* section where you *critically review the literature.* Your paper will end with a *list of references* of papers you have cited. And before you can either review or cite, you need to do a *literature search* for the papers. Since all three tasks are generic and not discipline-specific, they are considered in some detail below as part of writing the second draft.

49.4.10 Literature search

Before you can do a literature review, you must be able to *search* the databases to unearth the *relevant* papers that you will read, review, and cite in your paper. Learning how to do this is a central part of the craft of doing research, and is explained in Chapters 16 and 47. Review them if necessary. We recapitulate but briefly here.

In any search, there will be hits and misses. The hits are the papers you have found which you should read profitably; the misses are those of little or no relevance, or papers that are relevant, but which you have not found. The trick in formulating a good search is to use a combination of keywords and other database fields so that you get many hits and few misses. The results should not omit vital references while steering clear of what is outside your focus.[8]

Disciplines like medicine, mathematics, etc., have detailed classifications that permit comprehensive but focused searches. Familiarize yourself with the classification used in your area and formulate your searches judiciously. Seek the assistance of a reference librarian, especially if you are new to the game.

Marshalling good references that are relevant to your paper is not only a science but also an art. Besides mastering the technical aspects of doing a thorough literature search, you should also develop maturity and judgement to identify good references. It is assumed here that you already know how to do this, and that you have identified those core papers that you will read, cite, and review in your own paper.

49.4.11 Literature review

A *review* of the relevant literature is a necessary part of every research publication. It provides context for your own work, making it more ac-

[8]See also Section 49.4.13.6 later in this chapter.

cessible, by juxtaposing it with previously published material. The extent of your literature review and the way you present it depends upon the type and purpose of your publication.

49.4.12 Literature review for the survey paper

If you are writing a *survey paper,* your *sole* purpose is to summarize the state of knowledge and research in the chosen field accurately, clearly, concisely, comprehensively, and without repetition.

> Your whole survey paper is simply a global, thematically-linked review of the literature.

Because of this, lavish extra care on your literature review when preparing a survey paper. Your review should not merely be restricted to a single narrow focus but should rather capture in one broad sweep the landscape of the entire research area, segmented into sub-disciplines for easy reading and reference.

A survey does not include new research material but rather presents a succinct condensation of previous work. A survey paper can and often does include references not only to others' published work but also to the authors' own previous work in the field. Obviously, absence of bias is paramount.

Surveys should be current and comprehensive. They are convenient entry points into a research field and are therefore read for this purpose by postgraduate research students, postdoctoral fellows, and researchers entering a new field.

Landmark papers *must* be cited and summarized to allow a newcomer to grasp their import. Classic papers should be linked together—like flowers in a garland—to tell a coherent story of how research in the field has evolved over time.

If a survey is sloppily written, has glaring omissions, is biased, has erroneous citations, or is otherwise flawed, it fails to deliver. Therefore, take special care when writing the literature review for a survey paper: it is indeed the main game and course.

49.4.13 Literature review for the research paper

If you are writing a *research paper,* your primary purpose is to communicate *new* knowledge and findings. This new knowledge has to be presented in the context of old knowledge: this is the primary purpose of the literature review portion of your research paper.

> ★ Your review should be broad enough to provide a context for the new knowledge and yet narrow enough to permit detailed comparison with previous work.

First, work on a broad contextual overview that sets the scene for your work. Guard against an overly long review of prior work: you are not writing a survey paper here. A review that is too diffuse and general will blur the central message of your paper. So, do not attempt an encyclopaedic review, but rather tightly couple it to the subject matter at hand.

> ★ Aim to be concise, clear, and critical in that order.

Second, write a series of short and sharp assessments of previously published papers that bear relevance and significance to your work.

> ★ Cite and summarize.

Take care not to omit any important publications. The criterion to use for this phase of your review is to ask "Who else has done similar work that either directly addresses or obliquely impacts on my work?" Include your own previous work here, if relevant.

A journal paper demands a more careful literature survey than a conference paper. Restrictions on page length or duration of presentation often only allow a cursory survey of previous work in a conference publication. Be aware of this vital difference.

49.4.13.1 Zoomable digital map

To help you chalk out your literature review, think of it as a zoomable digital map capable of providing the broad picture at low resolution, but at the same time, allowing fine details in a *restricted region of interest* to be viewed at high resolution, when zoomed.

49.4.13.2 Critical review

A purely descriptive review adds no intellectual value from you. But a *critical review* does. The word *critical* should not be construed as a one-sided commentary pointing out the weaknesses alone, as in a debate.

> ★
>
> A critical review of a paper is an unbiased assessment of its strengths and weaknesses from your viewpoint.

There is room for fact-based objective analysis as well as for subjective opinions. Both should be defensible. You may critically review others' work to better highlight the merits of your own. Present an unbiased appraisal of their work—including strengths and weaknesses—sticking to published facts in preference to subjective opinion.

Rather than stating "their method was not very accurate" say instead that the error in their measurement was, for instance, ±50%, and let the facts speak for themselves. If in doubt, always rely on fact rather than on opinion.

If you have misunderstood a paper, your critical review of it would be a thorough pig's breakfast. You would be projecting your ignorance rather than your intellectual incisiveness, maturity, objectivity, and thoroughness. Do not so shoot yourself in the foot.

49.4.13.3 What to avoid

Do not compare others' work with your own *present work* in your literature review. There is another place for it later in your paper where you discuss your results and compare them with others' work, hopefully claiming superiority for your method!

> ★
>
> The literature survey is strictly about previous relevant work done by others or by you to provide a context and benchmark for your current work.

A frequently cited paper by renowned authors often attains classic status in the field. It *must* be cited and summarized in a survey paper.

But many authors feel obliged to cite such a paper in their research papers as well, even if it is only *incidental* to the work being described. Avoid this temptation.

> ★ Cite and review only what is relevant to your work.

Bear in mind that the literature review is something that will be read not only by the reviewers refereeing your paper but also by your readers, after publication. Your reviewer will be critical of omissions of landmark papers, relevant to your work.[9] Your reader will be irate if a paper is cited erroneously and has cost him or her a futile trip to the library. Do not earn the displeasure of either reviewer or reader.[10]

49.4.13.4 Comparable work

Never neglect to cite comparable work in your literature review. This is especially important if that work is in direct competition with yours. You cannot ignore or run away from your competition on the sports field. Neither can you do so in the academic arena.

If you have omitted a paper comparable to yours—published in a journal or conference of equivalent standing—you betray a lack of mastery of your subject. This will not go down well with the reviewers of your paper. There are enough resources like online databases for you to do a thorough search for comparable publications. Seek the assistance of librarians to frame a comprehensive search so that no paper of relevance escapes your search net.

Citing comparable work enables the reader to decide whether or not to continue reading your paper. If you have cited a paper that the reader has avidly read and followed, then she or he is likely to follow up by reading your paper as well.

49.4.13.5 Attribution

When you refer to another's work in passing, or compare it with your own, or use results from it, you must *cite* it. Using another's work unattributed is tantamount to the capital sin of plagiarism. This applies

[9] There is also the slight danger that a reviewer who finds his or her work omitted from the literature review will feel a little miffed and would therefore be less inclined to be generous in recommending publication. Reviewers are human after all!

[10] It will help you publish more papers in the future. ☻

to your own work as well. If you use results from something you published earlier, cite that work. As time goes on, you will develop your own sense of what is new enough to require a citation, and what is established enough to be accepted as part of the prevailing paradigm.

49.4.13.6 Which papers to cite?

Which papers should you cite when preparing a research manuscript for publication? You alone can answer this question, and you acquire that skill as you write more and more papers.

If your field is a mature one, there are likely to be tens, hundreds, or even thousands of papers. You then have the mammoth task of sifting through that exuberant pile to select those you should include in your review. Unless you are writing a survey paper, do not cite so many papers that your own work gets buried among them.

Choose the most *significant* papers and read them. Next, choose the most *relevant* papers and read them. Once you have read both sets, pare down the number of publications that you will include in your review.

As you gain proficiency in your field of research, you will recognize a good, relevant paper, regardless of its provenance, simply on its own merits. Once you feel that you have acquired that skill, rejoice that you are maturing as a researcher in your field.

49.4.13.7 Reading cited papers

Only cite papers that you have read. If a paper is merely tangentially relevant to your work, you might get off with having read only the abstract. But never cite a paper that you have not sighted (pun intended).

You will not have time to go through each paper with a fine-toothed comb, but you should read each publication that you cite. Divide the time available to you for reading papers by following the method suggested in Reading cited papersList 49.7.

Hark back to Chapter 46 to refresh your mind on how to read a research paper.

49.4.14 List of references

The list of references appears at the end of your paper and should be in the format required by your journal and its publisher. Include in the list of references every paper that you cite.

Reading Cited Papers

1. Rank the papers in some order of descending importance to your work, such as a combination of relevance and significance.
2. Divide the ranked papers into three piles: A, B, and C.
3. Read the A-pile papers in full, and carefully.
4. Read the abstracts and browse the full papers in the B-pile.
5. Read the abstracts alone for the C-pile.

List 49.7: Reading cited papers.

Unfortunately, there is no single citation standard. The conventions in use vary with discipline and publisher. If the software used to generate your manuscript supports it, you would be able to automatically generate from a bibliographic database your sorted list of references, in the required format. I recommend that you do this because it takes your mind off one needless chore. Refer to this book's companion website on the software to do this.

So long as all mandatory fields are filled in correctly in the database, and there are no spelling errors, your citations should come out perfectly. Take care to ensure that the authors' names are spelled correctly.

When I review a manuscript for a journal, one of the first things I do is to look at the list of references. If it is shoddily done, I already know that the authors lack thoroughness.

The references, being listed *last,* are often given the *least* attention. They may also be processed by programs other than the main one used to typeset the document. Therefore, spelling errors might have crept in undetected. *Moreover, and worse, the references might be erroneous, duplicated, or incomplete.* There is nothing as unhelpful and irritating to a reader—intending to replicate your work—as a reference that does not exist. Errors in your list of references are easily spotted and betray a lack of attention to detail that will unfavourably predispose your referees toward your manuscript. So, make your list of references letter perfect and do yourself a favour.

49.4.15 Taking stock

Run through the checklist of List 49.8 to take stock of what you have accomplished in the first two drafts. You should be able to answer all questions in the affirmative. If not, you know what to attend to and where.

CHECKLIST

- ✔ Is the paper worth publishing?
- ✔ Is it being published in the right journal or conference?
- ✔ Is the title apt?
- ✔ Is the paper faithful to the title?
- ✔ Are the conclusions crisp and accurate?
- ✔ Does the abstract embody the purpose of the paper?
- ✔ Are the keywords appropriate?
- ✔ Are the conclusions clearly stated in the summary?

LIST 49.8: Checklist on completion of second draft.

Your second draft should have yielded a complete, working manuscript. It might be rough around the edges but there will be no glaring omissions or inclusion of gratuitous detail. Every section has been filled out. There are no gaps. All the substance of your paper is there. Your manuscript is structurally sound and there are absolutely no glaring errors of fact, logic, or methodology.

You have now finished the most gruelling part of writing a paper: fleshing it out with facts and making your case to your community of expert peers. It is always gratifying and liberating to know that the hardest part is over. You are now ready for the third draft, which should, by and large, be less arduous.

49.5 The Third Draft

If the first draft is the rough sketch, and the second draft is the painting of your picture, the third draft is where you correct little blemishes and add finishing touches to your masterpiece. During this phase, you *refine* your manuscript according to List 49.9.

THE THIRD DRAFT

Logical continuity: there should be no missing links in your logic or argument.

Coherence: your story must unfold without any gaps in the telling.

Flow: your story should flow from start to finish and not stutter and sputter at various points.

Style: your reader should not stumble on any passage while reading your paper.

Language: no errors in spelling, grammar, typography, usage, citation, etc.

LIST 49.9: Guidelines for third draft revision.

49.5.1 Logical continuity and coherence

Each time you finish a spot of work on the manuscript, allow a few days to lapse. Then read what you have written, either as one integral whole, or in three sittings at most. Then, ask yourself the questions shown in List 49.10.

PROOFREADING THE MANUSCRIPT

- Does the paper read and feel like a single whole?
- Are there any lapses in logic?
- Are there any missing links that obfuscate what is being said?
- Are there any missing, mis-labelled, or wrongly referenced diagrams, graphs, or tables?
- Are there any duplicate references or wrong citations?

LIST 49.10: Questions to ask when proofreading the manuscript.

Delete repetitive passages. Correct grammatical errors. Rewrite sentences to improve readability. Correct spelling mistakes.

Get someone not connected with your paper to look at it critically. It could be an academic colleague or postgraduate student. You and your co-authors are likely to be too close to the paper to see flaws and errors that would be obvious to an outsider to your work.

49.5.2 Flow, style, and language

Like art or music appreciation, the fluency of your paper and the style of language it embodies is a subjective assessment. Ensure that not only your content but also your language flows well. Adopt a style suited to an academic paper: avoid slang, spell out acronyms in full when they are first introduced, use a neutral, objective tone, etc. Do not hide behind the thicket of jargon or the boulder of verbiage.

★

The best way to polish up your manuscript is to read it yourself many times with intervals between successive readings.

The easy errors to catch are those of wrong spelling. Use a computer program for this but beware that when you write *form* instead of *from* the spellchecker will not alert you to it.

I tend to repeat some words, which I would invariably gloss over during proofreading. This became disconcerting enough that I wrote a small Perl script to explicitly alert me to repeated words or phrases. Get to know your own peculiar idiosyncratic errors. Identify your own blind spots when proofreading, and come up with sensible solutions of your own.

A quick pair of eyes that sees what is *actually* there rather than what *should* be there is very helpful. Get someone else, like a co-author, to help you if you feel you are too close to your manuscript to trap errors like these.

If, in several readings of your manuscript, you find yourself stumbling over the *same* passage again and again, especially because the expression is infelicitous, or because the meaning is unclear, *know for sure* that you need to revise that passage. Read and follow the guidelines of Gopen and Swan [11]. Rewrite sentences. Rephrase ideas. Swap words around. Track the source of the ambiguity or confusion and nip it out. Revise what you have written until *you* can read it without stumbling.

SOURCE FILE FORMATS

1. Microsoft Word document format (.doc) [12]
2. LibreOffice open document format (.odt) [13]
3. LaTeX source document format (.tex) [14, 15]
4. Plain text format (.txt) in ASCII [16] or Unicode encoding [17]

LIST 49.11:

This read-rewrite cycle can get quite taxing. After concerted work on your manuscript, let it rest awhile like a yeasted bread dough. Return to it after a day or two and re-read it with a fresh outlook and an unjaded pair of eyes. Does it feel smooth and polished? If not, go through another round of polishing up.

If it passes muster, give it to a co-author, supervisor, or academic colleague, for them to cast a critical mind and eye on it. There should be few or no stylistic errors by virtue of repeated scrutiny and careful proofreading. If all of these criteria have been fulfilled, your manuscript is ready for submission, which is what we look at next.

49.6 Electronic manuscript submission

Electronic submission of manuscripts is now the norm rather than the exception. Almost no journal asks for photographs on photographic paper any more: digital images suffice. The same applies to text and illustrations like diagrams and graphs, as well as for tables. Until the late 1990s, each of these would have been laboriously hand-drawn or manually assembled from the experimental data. In the twenty-first century, though, they are generated by computer programs.

One consequence of this change is that, whereas paper-based submissions were standard, except perhaps for paper sizes, electronic manuscripts may be submitted digitally in several *formats*. In most fields, this is generally pared down to the four *source file* formats shown in List 49.11

Publishers usually provide *document templates*. These help cast the content of a manuscript into the chosen mould or *style* of the journal. If your publisher furnishes a template, you are in luck. If not, use something sensible for your discipline.

Text should usually be typeset in *double spacing* to allow referees to annotate the manuscript during review. Citation styles should strictly follow house rules.

Photographic images should be saved in Portable Network Graphics (PNG) [18] or Tagged Image File Format (TIFF) [19] formats. The image resolution should be what your journal stipulates.

Graphics should preferably be submitted in a *scalable graphic format* like Portable Document Format (PDF) [20] or Scalable Vector Graphics (SVG) [21]. There could also be rules for the layout of tables.

Some journals require that graphic and tabular material should not be integrated with the text but must be submitted separately, although this requirement seems increasingly artificial and out of tune with the natural flow of electronic submission.

The submitted manuscript is almost always requested to be a *single file* in the chosen format[11] or a *single archive of individual files* which may be in different formats. Upload this onto the manuscript submission site using a Web browser or other program. You will get an acknowledgement email or other communication on successful upload. You might even be told when the review of your manuscript would be completed. You then sit tight, and wait, and hope for the best! ☻

A simple tasklist for electronic manuscript submission is shown in Algorithm 49.1.

49.7 The peer-review process

A world-class journal is always peer-reviewed. So, if you want to publish in the best journals, you need to have some inkling of the peer-review process. It is at present the "gold standard" for assessing new knowledge. And what better way to understand it than by means of an analogy involving gold?

Suppose you have inherited a piece of jewellery. You do not know its value but would like to find out how much it is worth. What would you do? You would get it assayed by *expert* jewellers so that you might know its true worth or lack of it.

[11] Technological changes will certainly usher in new formats as time goes on; refer to the companion website for current links and details.

Electronic Manuscript Submission

- ✔ Typeset text using publisher-supplied document template and file format.
- ✔ Save all graphs and diagrams in the preferred format such as PDF or SVG.
- ✔ Save all images in lossless image formats such as PNG or TIFF.
- ✔ Package the lot into a single file or archive.
- ✔ Upload the file/archive onto the manuscript submission website using a Web browser or other program.

Algorithm 49.1: Tasklist for electronic manuscript submission.

The peer review of your manuscript is analogous to getting your jewellery valued by an expert jeweller. And just as you would get second or third expert opinions on the value of your jewellery, so also, most journals, in fairness to contributing authors, appoint two to five experts to review the suitability of your manuscript for publication. The purpose is to ensure that only original, new knowledge that is accurate and useful is granted the privilege and prestige of publication.

Peer reviews are generally *transparent* in that the rationale for the assessment is usually made known to the authors, but practices can and do vary across disciplines and journals.

There are journals which insist on *double blind reviews* where neither the author(s) nor the reviewer(s) are aware of one another's identities or affiliations. The hope is that persons and personalities are factored out from the review process. A reviewer then can neither favour a friend nor show disfavour to an adversary. Human emotions and frailties are thereby avoided and the review is kept purely professional and objective, and therefore unbiased.

Other journals practise *blind reviews* in which the authors' names, and sometimes, even their institutional affiliations, are made known to the reviewers, but the reverse is not true. The authors are kept in the dark about the identity or affiliations of their reviewers. Human and emotional considerations can creep into such a review process, but c'est la vie!

49.8 Publication status: accept, revise, or reject

A few weeks or months after submitting your manuscript, you might receive that much anticipated or dreaded postal letter or email from the journal editor notifying you of the status of your paper—which could be one of three possibilities: accepted, revise and re-submit, or rejected.

Moreover, you will usually be given the *logical reasons* for the status. Very often the comments of each reviewer will be made known verbatim. As a consequence, you know exactly *why* the referees have accepted or rejected your manuscript, or suggested resubmission after certain changes are made. So, not only are *experts* reviewing your manuscript, they do so *transparently*.

Figure 49.1 illustrates the fate of a manuscript that is submitted for peer-review.

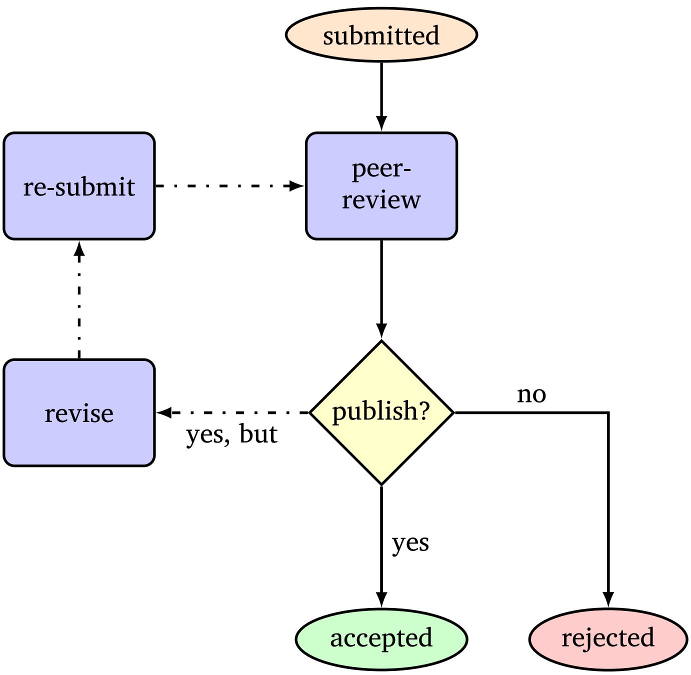

FIGURE 49.1: The fate of a manuscript submitted for publication can be one of three: accepted, revise and re-submit, or rejected. The revise and re-submit process might run for one or more cycles, as indicated by the dash-dotted lines.

49.8.1 Accepted

Although highly unlikely, the most favourable outcome is when your manuscript is accepted with no changes. It means that, apart from possible cosmetic changes to your manuscript, you need not revise it any further. You have succeeded in your quest to publish your paper. You have won! Rejoice!

49.8.2 Revise and re-submit

This is conditional success. Your manuscript will be published subject to satisfactory completion of revisions required by the reviewers and the editor. This in itself is a great outcome if it is a "'gold standard" journal in your field with an extremely low rate of acceptance.

But it often means *more work*. There is also a *time limit* attached to resubmission. Sometimes, the revision involves merely re-writing or editing parts of your manuscript. But other times, it could translate into several months of further work. This might be to gather new data to repeat your own work, to improve reliability of your results. Or it might require you to run new experiments to benchmark your work against established methods. The tasks and their reasons vary. If you wish to see your work in print, you need to punctiliously fulfil whatever is required.[12]

★

Accept this outcome gratefully. Do what needs to be done conscientiously, completely, cheerfully, and on time.

49.8.3 Rejected

This last is like receiving an unhappy diagnosis. But unlike a terminal illness, a manuscript rejection leaves the door open for hope: you might be able to improve your paper and publish it elsewhere. See Section 49.12 for what you can do further.

49.9 Making the most of your reviews

Every manuscript and every review is unique. Reviews are meant to be rebutted. Even with a flat rejection, you should voice your response. If nothing else, it teaches you how to better play the paper publishing game.

[12]Even if it means working severely overtime. ☻

I like to classify reviews into three broad categories: laconic, detailed, and wrong. Knowing how to cope with each category would help you get the most out of your reviews. Bear in mind, though, that the archetypes given below are just that—archetypes.

49.9.1 Laconic

The laconic reviewer says little either because of unfamiliarity with the precise subject matter of the paper, or because he or she is habitually terse. Such a review can be favourable or dismissive. If favourable, it might lavish praise on your work, but tell you little about what you need to do to improve the paper. If dismissive, it might again not give any hints for improvement.

With a favourable review, you need do little more. But a brief, unfavourable review, lacking detail, sheds little or no light on how to improve your manuscript. Or perhaps even why it was rejected in the first place.

In such a case, you should try to read between the lines to guess if the reviewer really understood your work, or is trying to hide his or her ignorance behind the screen of brevity. If you are convinced that the reviewer has misunderstood your work, the best you can do is to clarify what you have done and rebut the reviewer as explained in Section 49.11.

49.9.2 Detailed

The detailed review is the most helpful of the three archetypes discussed here. The reviewer has taken his or her task seriously and given your paper a detailed and thoughtful going over. If there are blatant errors of fact, they will be pointed out. Sometimes, such errors will completely upset the conclusion of your paper and make it academically non-viable.

The detailed review ideally tells you what you have got right, what you have got wrong, what your paper lacks, and what further work could fruitfully be done. Such a manuscript review—whether it recommends acceptance or rejection—is worth its weight in gold. Rarely would you come across a peer reviewer with enough time and dedication to help you along with such care and academic rigour. Cherish and learn from such a review.

One common suggestion from many detailed reviews would be that your method should be compared, usually quantitatively, with other existing methods. This is simply to place your proposed new method in the context of current practice.

Indeed, it is often a sign of poor manuscript preparation if you launch directly into your work without reference to the prevailing "gold standard" methods. You will never fall into this trap if you follow the suggestions in Section 49.3.8. If results from existing methods cannot be *directly* compared with yours, you should go back and repeat others' work on your data, with similar nomenclature, notation, variables, units, etc., to permit easy comparison.

49.9.3 Wrong

A wrong review is one where the reviewer has clearly and totally misunderstood the import of your paper. Whether it is in basic knowledge, aim of paper, experimental techniques, field work, hypotheses, conclusions, or analysis, a wrong review would betray misapprehension in one or all areas. Such a review is manifestly unhelpful. It may be infrequent, but it happens, and is especially frustrating when there is only a single reviewer. You have two courses of action here.

First, you could write to the editor of the journal co-coordinating the review to request a new set of referees to review your paper because you have evidence of total or partial misunderstanding of what your paper is about. Mark that what you are saying might not sit well with the editor, who might have chosen the original referees in the first place! That is a risk that you must take because responding is better than suffering in silence.

The second recourse is to look for an equally good, high quality journal in which known experts in your field have published. There is a convention that those who have published in a subject area in a journal are often called upon to review papers relating to their field. In such a case, you have a fresh opportunity to get your paper looked at by another team of experts in the hope that *they* do not misunderstand the import of your work.

If several journal submissions all result in wrong reviews, it is very likely that your paper has been written poorly, unwittingly converting it into something it is not. This might be because you have hidden the purpose of your paper behind a thicket of jargon, or of mathematical abstruseness, or of a mass of data, etc.

It is time for you to review the preliminary questions in Section 49.2. Can you clearly summarize the purpose of the paper in one word or one sentence? Revise and recast your manuscript to say simply and clearly what your paper is about. The "Mark II" version of your paper should be significantly better than the first. Then, and then alone, should you re-submit to a journal. Anything else would not be fair to yourself, or your prospective referees, or both.

49.10 Revising your paper

There are two important things to remember when revising your paper for re-submission. First, thoroughly fulfil all revision requirements. There is nothing more annoying to an editor or reviewer than to get a "second round" manuscript that has missed out on some important aspects of the called-for revision. Do not play "ping-pong" with your reviewers. Try to publish after the first revision. Make the review process work in your favour.

Second, revise and re-submit within the time frame that you have been given. You are chasing a moving target when you do research. Deadlines serve a purpose. Your work could become stale, or be overtaken by others' work, or worse, eclipsed by new developments. It is paramount that you revise and re-submit within the allowed grace period to remain relevant and original.

49.11 Responding to your reviewers

Your paper might need to go through several submit-review-revise-re-submit cycles before it gets published. Each round of revision and resubmission should be accompanied by a response to the reviewer's comments as well as the revised manuscript. Because your reviewers are experts, it is more likely that the changes they have suggested are reasonable. Therefore, your best response would be to revise and thoroughly fulfil all their recommendations. Your final published paper will be all the stronger for it.

Disagree with the reviewers' observations if you feel they have not understood what you meant. In that case rephrase what you have written. But do not engage in a war of words with your reviewers: it is not an equal game.

> Always be courteous when responding to reviewers.

49.12 Coping with rejection

The most valuable outcome from a manuscript rejection is that your paper has been evaluated by experts. What is more, you also have their detailed comments on your work. And all this free of charge (not counting the effort and time that both you and your reviewers have put in). So, however cruel you might think your reviewers have been, realize that they have acted altruistically toward you.

Rejection of your paper by one journal is not the end of everything. Several Nobel prize-winners have had their prize-winning papers initially rejected, and sometimes more than once! You are therefore in good company if your paper has been rejected.

Moreover, your first rejection letter is a rite of passage, indicating that you have come of age as a researcher. So, take heart: rejection of your paper is not the end of the world, although it might seem like that at first.

The rejection should lead to a "post-mortem" examination of your manuscript and the work reported therein. Determine whether the problem lies with the subject itself, or merely with its exposition, or both. Seek the counsel of more experienced researchers on both manuscript and reviews. Decide on the future course of work.

Do further work if called for. Or rewrite until you get your ideas across clearly. Or do both if justified. Resubmit to the same or another journal. Most of all, do not give up. Keep trying until you get published. Recall and follow the guidelines in Chapter 10 on "Learning from Failure".

49.13 Time frame

Finish a paper within six weeks. Putting it off diminishes the probability of it ever being finished and seeing the light of day. When you receive your reviewers' reports, do what needs to be done without delay. The sooner you act, the earlier you get published. Conversely, if you delay working on a manuscript that is to be revised, you might miss your deadline—often not more than six months—and risk losing the chance to publish. Do not snatch defeat from the jaws of victory.

49.14 To Explore Further

There is a wealth of books on how to prepare manuscripts for academic publications. Some deal with the process generically, as we have done here. Others are discipline-specific.

First among the recommended references—and one that I depend upon constantly—is the paper by Gopen and Swan [11] that I have previously alluded to in Section 49.4.5. Since there is a freely available online version of this paper, I strongly recommend you to read it. I have also benefited greatly from reading and applying the principles in David Lindsay's book *A Guide to Scientific Writing* [22].

A more complete treatment of much of what is in this chapter may be found in the eminently practical and very readable book by Björn Gustavii entitled *How to Write and Illustrate Scientific Papers* [23]. It has dedicated chapters on graphs, drawings, and tables, and even one on the PhD thesis. *Scientific Papers and Presentations* by Martha Davis [24] is broad in scope, covering the spectrum from papers, through theses, to presentations. It contains much valuable advice garnered from experience. Still on scientific publishing, another useful reference is *Guide to Publishing a Scientific Paper* by Ann M. Körner [25]. This book addresses writing a scientific paper per se in detail and has a wealth of practical advice ranging from setting out figures and legends, to writing the paper, to responding to reviewers.

How to Write a Paper edited by George M Hall [26] is another comprehensive book that not only covers papers but also presentations. The book has a distinct medical flavour but contains enough generic practical advice to benefit authors of academic papers from other disciplines as well. Another subject-specific book covering writing, papers, theses, and presentations is Nicholas J Higham's *Handbook of Writing for the Mathematical Sciences* [27]. *Writing Philosophy Papers* by Zachary Seech [28] is again subject-specific and addresses papers that are written as part of course work in philosophy rather than for learned journals. Another book, in the same vein, but aimed at students of psychology is Rosnow and Rosnow's *Writing Papers in Psychology* [29].

An almost encyclopaedic monograph on academic writing, with a bias towards the sciences, is James Hartley's *Academic Writing and Publishing: A Practical Handbook* [30]. I will wrap up with one last book that is likely to be useful to many: *English for Writing Research Papers* by Adrian

Wallwork [31]. It is aimed at researchers from *any* discipline who write research papers and is designed to especially benefit those for whom English is not their first language.

There are also numerous blogs and articles on the Web devoted to writing academic papers. For example, the eminent mathematician, Terence Tao, has written a blog specifically on writing mathematics papers [32] which embodies valuable advice, and also features numerous links to related articles. Surf and search the Web until you find something authoritative and helpful written specifically for your discipline.

Before I wind up this section, I would like to recommend you to view and absorb this hour-long presentation by the British computer scientist, Simon Peyton Jones, entitled *How to write a great research paper* [33] on on YouTube. If you would prefer a PDF version of the talk, it is available as a delightful, to-the-point slide presentation[34] as well. In a tongue-in-cheek, cheery, picturesque, and memorable fashion, he explains why and how you should write a paper. Although he uses examples from computer science, his recommendations and sound advice are universally applicable to all academic disciplines. I strongly recommend that you read and follow the guidelines given in this presentation. It is a little gem.

49.15 Wrapping up

Publishing your first paper is a rite of passage in your academic life. It marks you out as a person who is serious about the acquisition and dissemination of knowledge. It signals your entry into the world of scholarship as a bona fide researcher.

The reasons to publish are various. Whatever they are for you, do not publish unless you yourself are convinced of the worth of your manuscript. Then, clarify in your own mind the subject of your paper and the story you intend to tell. Stick to it. Tell it clearly and well.

Write your manuscript in three drafts. Outline it in the first draft. Flesh it out with substance in the second. Polish it in the third. Get your co-authors, and academic colleagues uninvolved in your paper, to critique it. Your submitted manuscript should embody your best efforts.

Respond to your reviewers with confidence and enthusiasm. Do not dilly dally. Fulfil any called for revision promptly and thoroughly. Resubmit and rejoice in being published.

If your reviewers rejected you paper, try to piece together from their remarks an action plan to revive your paper for another attempt at publication. Go through the reviews *carefully and unemotionally* with your co-authors and academic benefactors. Never give up. Try again. If *you* are convinced your paper should see light of day, it certainly will.

49.A Appendix: The need for peer-review

Peer-review is held to be the best method at present for evaluating new knowledge and ensuring its correctness. A word about the need for peer-review is therefore appropriate here.

One could argue, that in these days of the Internet, the World Wide Web (WWW), and the free flow of information, there is no need for peer-review. Simply by posting your results on the WWW, you are allowing the whole world to view them. If there are any errors, surely those who view your website will comment on and correct what you have written. It sounds seductively convincing. Indeed, this is the paradigm for open-source software development, which has proven to be very robust [35].

However, this argument misses one central point. Participants in the open-source software development process are almost always experts who know how to write software. But the viewers of your results on the Web are unlikely to be domain experts. Indeed, they could be students seeking knowledge, rather than experts critiquing the work of others that they chance upon when surfing the Web.

When your paper is peer-reviewed, *experts* evaluate the worth of your work and its suitability for publication. This screening process is rigorous and cannot be supplanted by the laissez-faire, market-oriented behaviour of Web users who might not be experts themselves. Moreover peer-review assures quality *before* publication.

In my humble opinion, whatever its shortcomings, the peer-review process is currently the best method we have in place for evaluating and admitting new knowledge. Peer review has its faults and it has failed on occasion to detect or unearth fraud [36]. Nevertheless, it is superior to the open-ended system of publishing on the Web and letting the readers decide on the basis of *caveat lector* or "reader beware" because the content might not be entirely correct.

49.B Appendix: Resurrecting a stalled or dormant paper

When faced with the rampant *publish or perish* paradigm in academia—especially when you are short of fresh lines of research and are looking to extend some previous work—it is logical to cast a wistful eye on a stalled or dormant paper from your past. We now look at whether and how to resurrect such a paper.

49.B.1 Origin of stalled or dormant papers

A *stalled* manuscript is one that is abandoned before completion. It could be due to lack of time, competing interests, changed priorities—e.g., the work for which you are paid takes precedence over unpaid research—or plain lack of willpower to push through to completion, also known less euphemistically as procrastination. ☺

A *dormant* manuscript is one that has been completed but not published. For example, a manuscript submitted to a journal might get rejected or returned for revision. But other pressing tasks might preclude you from revising and re-submitting it on time. Sometimes, the changes called for by your referees might, in your opinion, be misconceived, and therefore not worth the effort of revision.

Most likely, you have a stalled or dormant paper buried in your personal graveyard of unpublished manuscripts. You might at some later date—because you need to increase your publication-count, or because you have time to spare or a rekindled interest in the field—wish to work again on the manuscript to finish it with enough effort and care to ensure that it sees the light of day as a published paper.

49.B.2 Resuming work on a stalled paper

Resuming work on a stalled paper requires motivation, discipline, and continuous effort until completion. If you lack this triad, you are unlikely to succeed.

I like to think that Newton's First Law of Motion—basically that starting, stopping, and changing direction are not without effort—applies not only to physical motion but also to activities of the mind. You will face difficult odds when you resume work on a stalled paper and you must be prepared to counter them with resolute effort.

49.B.3 Does your manuscript still merit publication?

The passage of time robs stalled initiatives of their relevance. Your stalled manuscript could count among the many victims of Father Time. So, first, you need to convince yourself that your stalled or dormant manuscript still merits publication, and that it will benefit the research community in your field.

It might happen that you have lost priority in the subject of the manuscript. What was once a little-researched and unpublished field is now a mature area of investigation with several landmark papers. If that happens, you must *dispassionately* analyze whether resurrecting your lost manuscript in its original form is indeed worthwhile.

If the prime focus of the manuscript is no more an unresolved question, you need to see if there are other aspects of it that still merit publication. If so, you need to change the *focus* of your paper to centralize this new contribution, and possibly expand upon it.

49.B.4 Avoid regret and self-pity

Old manuscripts evoke nostalgia. Resist the temptation to indulge in a generous bout of regret and self-pity, wallowing in memories of what could have been but was not. *Do not waste more time on lost time.* It will take you nowhere. Rather, overcome such moods with detachment and determination. View your manuscript not as a relic from your past but as a fresh and fortuitous foundling.

49.B.5 Is experimental work still involved?

If fresh experimental work is involved before you can complete your manuscript, then ensure that you attend to it as well. Rewriting and revising might be a breeze compared to re-running experiments. They could take a lot more time and effort, not to mention resources that need to be acquired or booked ahead of time. Plan to re-do your experiments well in advance—along with your rewriting efforts—so that your progress is smooth and not punctuated by delays in acquiring new experimental data.

49.B.6 Be focused

Now that you have convinced yourself that your paper should be completed, be focused in your efforts to propel it to publication. Do not meander or wander hither and yon just because your curiosity is piqued in one or another new direction. Make notes for future work if you stumble across new insights, but remain steady on your course to get the stalled or dormant paper published.

Set a specific time frame for completion and stick to it unswervingly.

If there are newer citations that you need to include, do so after a renewed and meticulous literature search. Do not skimp on this vital step because the research world moves on relentlessly and does not forgive Rip Van Winkles who have slept through contemporary research.

49.B.7 Avoid repeating past errors

Take care not to repeat whatever caused your paper to stall or be rejected in the first place. If it was shoddy experiments, revise them first. If it was a major methodological, logical, or derivational error, correct it. If it was poor citations, pay attention to your literature search. If it was lack of interest on your part, promise yourself—*before* you resume work on it—that you *will finish* the paper this time. It is senseless to pour more time and effort on a paper that you are still half-hearted about.

Before you resume work on the paper resolve unwaveringly to complete and publish it.

49.B.8 Get it done

How to overcome procrastination, how to write in general, how to write a paper, and how to write a thesis, etc., are all covered *in extenso* in this book. As you finally settle down to writing the paper, follow the guidelines therein. Set a schedule. Stick to it. Write it in three stages as exhorted. Get a suitable person to review it. Dispatch it for publication. And pat yourself on the back. ☻

49.C Appendix: Ethical afterword

There was a time when academic research and its findings were above reproach and suspicion. Alas, times have changed and the documented instances of plagiarism, falsification, and re-publication of already published material are growing with time. Papers have been withdrawn after being published, even in prestigious journals, when it was discovered that results had been falsified [36].

Accordingly, I have taken the liberty of a little advocacy in the appendix of this chapter to exhort researchers, academics, and students to adopt a strict, zero-tolerance approach to these unethical practices. Stringently forswear wrong actions even if they promise dizzying academic heights because your fall, when discovered, will be just as precipitous.

49.D Appendix: Plagiarism, Falsification, Recycling—The three plagues

Ethics should pervade the whole of our lives, not merely the educational or research aspects. The time-tested exhortations are:

THREE DON'TS

- ✖ Don't steal.
- ✖ Don't lie.
- ✖ Don't cheat.

These principles do not change with time and clime. Follow them and you will not regret it. In the context of academic publishing, failure to follow these precepts results in plagiarism, falsification, and recycling of previously published work, or "double dipping".

49.D.1 Do not steal: Do not plagiarize

It is a sad reflection on our times that in many universities, students and researchers are advised formally not to plagiarize. In former times, it would have been assumed that those without the character that ruled out stealing would not venture into a life of learning. The competition for finite funding, dwindling resources, and scarce opportunities, by an ever increasing cohort of the educationally aspirational means that we have to be digitally clear about practices like plagiarism. Failure to attribute or acknowledge the work of others, and passing it up as your own work, is plagiarism, and it is a form of stealing. I repeat:

Plagiarism is stealing.

Whether or not you get caught, plagiarism is wrong. Do not do it in any phase of your educational life. Plagiarism is especially virulent when it infects research. The clean and clear thread of responsibility and priority for new knowledge is lost. It becomes a tangled skein instead. The already taxed peer-review process has to assume the added burden of policing the ethical and moral rectitude of authors, in addition to assessing the technical merit of their contributions.

49.D.2 Do not lie: Do not falsify

Falsification of data—leading to false conclusions—is another, equally if not more pernicious, infection threatening all research. This practice poisons the whole lake of knowledge. All knowledge becomes tainted by the suspicion that it too could be false. The sciences especially are quests for truth. The basis for experimental research is that a hypothesis, or guess of what might be true, is submitted to the test of experimental investigation. If the results bear out the original guess, the hypothesis is verified. Otherwise, the hypothesis must be modified or discarded.

When someone hopelessly in love with research funding or personal advancement decides to sacrifice objective, impartial reporting of truth, and substitutes convenient and expedient falsehood in their place, all of science is undermined.

Falsification of results is treachery against truth.

If I had my way, someone who is inclined to falsify should be banished from doing research. It is that serious.

If you wish to falsify data or evidence, for whatever reason, please do *not* do research.

There is another side to this issue. You might have been careless when acquiring your data. Perhaps it was an overseas field trip and you failed to record some parameter or other, due to inexperience, forgetfulness, or some other reason. Do not fake the missing data. Interpolation, however cleverly done or justified, is simply interpolation. It is not data. If you made a genuine mistake or omission, record the omission as such. If at all possible, try to repeat that measurement or try re-acquiring the data. But do not substitute bogus "data" for the missing data: that is lying and cheating.

49.D.3 Do not cheat: Do not recycle your papers

Fierce competition for limited research funding in many countries and universities, means that those who are viewed as successful researchers often get priority allocation of scarce resources. Sadly, and all too often, "successful research" is measured by the *quantity* of research output, usually the *number* of published papers, rather than by their *quality* and *impact*.

Many academics and some students fall prey to the temptation of tweaking previously published papers a little bit, and presenting the results as new material to another conference or journal. This means a longer and more impressive publication list. But it also corrodes the researcher's sense of what is ethical. Fight this temptation. It is better to have fewer publications of high quality, which embody original research and are frequently cited, than a plague of papers that are recycled after a few tweaks, simply to swell publication lists on curriculum vitae and research grant applications.

★

Do not recycle old papers.

I recommend that early research findings be first presented at a conference as a poster or oral paper. Later, when all the results are in and have been analyzed, a full research paper in a respectable journal is merited. Once this has been done, fight the urge to add "salad dressing" or "garnishing" and re-publishing essentially the same work as one or more new papers. Instead, direct your efforts on follow-up research.

Keep on discovering new knowledge and disseminating it through new publications. If you adhere to this principle, you will have a productive and fulfilling research career, and one that is above reproach.

49.D.4 Rigour, Respect, and Responsibility

The Government Office for Science of the British Government proposed in 2007 that scientists engaged in research should adopt three principles:

UNIVERSAL ETHICAL CODE

1. Rigour.
2. Respect.
3. Responsibility.

These three attributes make a good starting point for the qualities of character needed in those who commit themselves to scientific research. Underlying all these qualities, however, is the assumed virtue of *honesty*. It is implicit in the term *rigour*. You may read more about this online at the relevant website [37].

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SUMMARY: PUBLISHING PAPERS IN PEER-REVIEWED JOURNALS

- Ask the W^7 questions—introduced in Chapter 19—about your paper before starting work on it.
- Write the paper only if *you* think it worthwhile in your own eyes.
- The first draft comprises the title, abstract, keywords and outline.
- The substance of your paper is added in the second draft: introduction, previous work, methods, results, discussion, conclusion, references.
- Check for logic, rigour and continuity in your third draft. Correct grammar and spelling. Rewrite for clarity. Proofread the manuscript. Ask others to critique your paper.
- Satisfy yourself that your paper is as good as you can get it to be.
- Prepare and submit the manuscript electronically.
- Respond to reviewers' comments with confidence and enthusiasm. Correct any misconceptions expressed in the reviews. Carry out all suggested changes promptly and thoroughly.
- Re-submit the paper and cycle through the submit-review-revise process until the manuscript gets published.
- On rejection, refuse to give up. Analyze carefully the reasons for rejection. Enlist help from your academic superiors. Plan new experiments. Rewrite your paper. Do whatever needs doing. Re-submit to the same or a similar journal until the paper gets published.

50

How to Write a Thesis: A Working Guide

Synopsis

This is a short guide on how to write a thesis at both the undergraduate and postgraduate levels. A thesis may be analyzed into three S's: *structure, substance* and *style*. Structure confers logical coherence; substance, significance and depth; and style, elegance and appeal.

State your hypothesis clearly, ensuring that it is both reasonable and testable. Keep meticulous records and write up rough drafts of your work as you go along. Begin writing your thesis proper with the experimental chapters. Progress to the literature review, introduction, and conclusions. Write the summary or abstract last, *after* writing the conclusions.

Write clearly and directly, with the reader's expectations always in mind. Lead the reader from the known to the unknown. Write clearly, precisely, and briefly. Think, plan, write, and revise. Follow layout guidelines and check spelling and grammar. Re-read, seek criticism, and revise. Submit your best effort as your completed thesis.

50.1 Thesis writing in context

Sometime during your university education, you might be required to write a thesis. It could be an honours thesis at undergraduate level or a master's or doctoral thesis at postgraduate level. The trouble is that universities usually provide no formal instruction on how to write a thesis; you are simply expected to "know" how to do so.

This chapter is written to help you out of the above predicament. It is a guide on writing a thesis, both at the undergraduate and postgraduate levels. I have tried to make the chapter discipline-neutral. Even if some of the material seems tailored to students of engineering and science, this chapter may be read with benefit by students from all disciplines.

Thesis writing is part of a chain of academic activities that starts with enrolment in a programme of study, and ends with the award of a degree. Earlier phases, such as selecting a supervisor and research topic, doing a literature search etc., are covered in Chapter 48. The art of doing research—whether for an assignment or for a research degree—is covered in Chapter 47.

50.2 The three components of a thesis

Every thesis may be abstracted into three components:

1. Structure;
2. Substance; and
3. Style.

The *structure* of a thesis is governed by logic and is invariant with respect to subject. The *substance* varies with subject, and its quality is determined by the specialist technical knowledge and mastery of essentials exhibited by the author. *Style* has two components: *language* and *layout*. The former deals with the usage of English as a medium of sound technical communication; the latter with the physical presentation of the thesis on paper, according to the requirements laid out by the university.

All three components—structure, substance and style—influence one another. A good thesis will not be found wanting in any of these three.

50.3 What is a thesis and why write one?

thesis /ˈθiːsɪs/ *n* **1** a proposition to be maintained or proved. **2** a dissertation especially by a candidate for a degree. [Middle English via Late Latin from Greek = putting, placing, a proposition, etc.] [1]

hypothesis /hʌɪˈpɒθɪsɪs/ *n***1** a proposition made as a basis for reasoning without the assumption of its truth. **2** a supposition made as a starting point for further investigation from known facts. [Late Latin from Greek *ὑπόθεσις/hypothesis* ‘foundation’; Greek *ὑπό/hypo* ‘under’] [1]

One might infer from the etymology above that a thesis is an (obligatory) offering *placed* at the desk of the examiner by a candidate who wishes to get a degree. This is the most common—and often only—reason why a thesis is written. But there *are* other reasons for writing a thesis.

A thesis is a written record of the work that has been undertaken by a candidate. It constitutes objective *evidence* of the author’s *knowledge* and *capabilities* in the field of interest and is therefore a fair means to *gauge* them. Although thesis writing may be viewed as an unpleasant obligation on the road to a degree, the *discipline* it induces may have lifelong benefits.

Most of all, a thesis is an attempt to *communicate*. All knowledge, and especially science, begins with curiosity, follows on with experiment and analysis, and leads to findings which are then shared with the larger community of academics and perhaps even the public.

A thesis is therefore not merely a record of technical work, but is also an attempt to communicate it to a larger audience.

50.4 The undergraduate and postgraduate theses

The differences between the undergraduate and postgraduate theses is one of *degree*[1] rather than *kind*. They share a common structure and need for logical rigour. It is only in the substance and the emphasis placed on it that the differences arise. For example, the university might require that:

> A PhD thesis shall be a substantial and original contribution to scholarship, for example, through the discovery of knowledge, the formulation of theories or the innovative re-interpretation of known data and established ideas [2].

An undergraduate thesis is usually graded on the *quality of research*, the *significance of the contributions* and the *style of presentation*.

Thus, the undergraduate thesis is judged on a similar basis to the postgraduate one. Indeed, the three most commonly cited qualities that earn an undergraduate thesis the first class grade at one university are *originality*, *independence*, and *mastery* [3].

Candidates writing a higher degree thesis—and the PhD thesis in particular—are required to present their research in the context of existing knowledge. This means a *thorough* and *critical review* of the literature, not necessarily limited to the narrow topic of research, but covering the general area. The PhD candidate should also show clearly what *original contributions* she or he has made [2]. Although neither of these requirements applies strictly to undergraduate work, the candidate should demonstrate familiarity with previous relevant work in his or her thesis.

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In short, a thesis, whether undergraduate or postgraduate, is evidence of the candidate's capacity to carry out independent research under the guidance of a supervisor, and to analyze and communicate the significant results of that work.

The candidate for higher degrees must demonstrate, in addition, mastery of the literature and indicate clearly which is his or her original work, and why it is significant.[2]

[1] Pun unintended!

[2] In addition to thesis submission, many universities also require magisterial and doctoral candidates to conduct an oral defence of their work before a committee of examiners.

50.5 Structure

50.5.1 Thesis structure

The regulations governing the PhD degree at any university usually stipulate the structure of the PhD thesis. These regulations vary with university and across time. Typical PhD regulations governing the format for the doctoral thesis might read so [4]:

1. Title page: gives the title of the thesis in full, the candidate's names and degrees, a statement of presentation in the form "This thesis is presented for the degree of Doctor of Philosophy of the University of …", the department and year of submission.
2. Summary or Abstract—of approximately 300 words. (It should not exceed 700 words.) The Abstract or summary should summarize the appropriate headings, aims, scope and conclusion of the thesis.
3. Table of Contents
4. Acknowledgements
5. Main Text
6. Bibliography or References
7. Appendices

The format of the undergraduate thesis is similar.

The thesis proper consists of the Main Text, numbered Item 5 above. If we zoomed in on the Main Text, we should see something like this [5, p 110]:

(a) Chapter 1: Introduction

(b) Chapter 2: Review of the Literature

(c) Chapter 3: Materials and Methods

(d) Chapters 4 to n: Experimental Chapters

(e) Chapter $(n+1)$: General Discussion or Conclusions

If we now zoomed in on any Experimental Chapter (labelled (Item (d)) above), we should expect to see [5]:

i. A brief introduction

ii. Experimental procedure (methods and materials)

iii. Results

iv. Discussion

This structure reflects the time-honoured format of science experiments:

SCIENCE EXPERIMENT FORMAT

1. Aim
2. Materials and Methods
3. Observations
4. Results
5. Discussion
6. Conclusions

LIST 50.1: Format for recording of science experiments.

We have just dissected the structure of a (scientific or engineering) thesis but have we obtained any insights in return?

50.5.2 Rationale for structure

The rationale for the structure in List 50.1 is simply that *a thesis must tell a story clearly and convincingly*. The components of the structure impart logical continuity to the thesis in much the same way that links in a chain confer on it integrity and strength. There is a flow in the logic, as shown in Table 50.1, which is adapted from Barrass [6, p 131]:

Any flaw in the reasoning or gap in the logic will be easily spotted if this structure is strictly followed.

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Thus, the structure of the thesis is designed to enforce logical and scientific rigour and make it easy to read.

Follow the structure and you can be sure that you are telling your story in the right order. But what exactly *is* your story?

50.5.3 The hypothesis underpins the thesis

The *hypothesis* is all important. *It is the foundation of your thesis.* It gives coherence and purpose to your thesis. Go back to Section 50.3 to review

Structure	Logic
Introduction/Aim	*What did you do and why?*
Materials and Methods	*How did you do it?*
Observations/Results	*What did you find?*
Discussion	*What do your results mean to you and why?*
Conclusions	*What new knowledge have you extracted from your experiment?*

Table 50.1: The relationship between the structure of an experimental chapter in a thesis, and its underlying logic. It applies equally well to the entire thesis.

the meaning and etymology of this word. If it is hard to grasp what hypothesis means, these explanations might help:

- The hypothesis defines the aim or objective of an experiment, that if some likely but unproven proposition were indeed true, we would expect to make certain observations or measurements.
- A hypothesis is an imaginative preconception of *what might be true* in the form of a declaration with verifiable deductive consequences [7, p 18].
- Hypotheses are the larval forms of theories [7, p 20].
- ‘In every useful experiment, there must be some point in view, some anticipation of a principle to be established or rejected’; *such anticipations are hypotheses* [7, John Gregory quoted by Medawar, p 22].

Indeed, the great French physiologist, Claude Bernard, has written:

> A hypothesis is …the obligatory starting point of all experimental reasoning. Without it, no investigation would be possible, and one would learn nothing: one could only pile up barren observations. To experiment without preconceived ideas is to wander aimlessly. [7, p 30]

Your *hypothesis* must *fit the known facts*[3] and *be testable*. To comply with the first, you must have read the literature. To comply with the second, you must do the experiment. This is why the hypothesis is central to scientific investigation [5].

If you find time, read an account of the famous Michelson-Morley experiment [8] to understand that if hypothesis and experiment are in conflict, it is experiment that prevails and hypothesis that falls. If an experiment shows that a hypothesis is incorrect, then that hypothesis must be erroneous, no matter how attractive. Moreover, failure of a hypothesis may lead to a re-examination of assumptions, refutation of shaky theories, and ultimately to new knowledge, as happened in this case.

50.5.4 Does an engineering thesis need a hypothesis?

Hypotheses may be relevant to science theses, but are they relevant to engineering theses?[4] Because engineers *invent* rather than *discover*, does an engineering thesis need a hypothesis?

Yes, all the more so, because invention is a more tightly *directed* activity than discovery; and the two are not mutually exclusive anyway! I prefer the word hypothesis: *that which underlies a thesis*; you may be more familiar or comfortable with *aims* or *objectives*. The hypothesis is the *electromotive force* or EMF for your thesis.

Suppose your project involves using Artificial Neural Networks (ANNs), in conjunction with appropriate hardware, to sort good apples from bad. The hypothesis for this project may be, 'It is possible to sort good apples from bad using ANNs and suitable hardware'. Note that implicit in your hypothesis is a definition of acceptable levels of accuracy (how do you *quantify* the words 'possible', 'good', and 'bad'?).

Suppose that on completing your project, you discovered that the system you had devised works well with green apples, but not with red ones. You would have *discovered new knowledge* and would be able to suggest a *revised hypothesis* as the starting point for further investigation. Your own project would have demonstrated[5] the correctness of a hypothesis

[3]But you should not be afraid to explore the unknown. If the "known fact" that "atoms are indivisible" had not been challenged, we would not have known of electrons, let alone quarks.

[4]Or to theses in other areas?

[5]Philosophers of science contend [7] that a hypothesis cannot be proved conclusively, but only falsified. We will steer clear of this controversy here.

like ‘It is possible to sort good green apples from bad green apples, with an accuracy of better than 90%, using ANNs and suitable hardware’.

Never forget that underlying every thesis, there must be a hypothesis. It is what your story is all about. If you keep your hypothesis in view, you will never stray into irrelevance when writing your thesis, which is what we look at next.

> A thesis is not merely a collection of pearls; it is a necklace elegantly strung together.

50.6 Substance

50.6.1 Begin at the beginning[6]: keep records

> The content of your thesis is being continuously gathered throughout the period of your project or research. Remember this and keep clear, well-annotated records in your own research notebook.

You can afford to be wordy and repetitive here, because you do not want to be lost when you refer to it later on. Because it is a running record of experiment and observation, its only requirement is fidelity; not subsequent correctness.

Michael Faraday was an experimental scientist par excellence. His diary of his researches can serve very well as a model of how your own research notebook should be like. For example, in one volume of his diaries [10], he has recorded the following:

- Freehand drawings of experimental setups [pp 248–9]. You should do the same; your diagrams in your record book need not be works of art: save that for the thesis!

- His accurate description of what he *believed* he was *perceiving*:“It still *smelt strongly of Electricity*” [p 200]. The italics are his. Today we may hide a smirk if anyone talks about smelling electricity; but

[6]“Begin at the beginning,” the King said gravely, “and go on till you come to the end: then stop.” *Alice’s Adventures in Wonderland* by Lewis Carroll [9, p 158]

remember that these are the observations of a scientific pioneer. Do not be afraid to record your *perceptions accurately*.

- His own questions to himself: "Can induction through air take place in curves or round a corner?" [p 420]. Such questions serve to clarify your own thoughts and to steer further work.

In summary, your record book is where you *record your thoughts, perceptions and measurements, using words, numbers and pictures, as and when they are still fresh in your mind.*

Plan your experiments so that one experiment has only one hypothesis. Many experiments may together shed light on a larger, unifying hypothesis.

Assuming that your experimental work is going well, the spectre of writing it up, so that it looks like a thesis, still looms ahead. How do you do that?

In the following sections, we take a look at some guidelines on how to write well. This is followed by advice from some experienced professors on how to write a good thesis. The material that follows is the core of this working guide: so pay attention to it and try to understand it thoroughly.

50.6.2 Write with the reader in mind

All communication involves two parties: the sender of the message and the receiver; in written communication, they are the writer and the reader.

> If you write with the reader in mind you are more likely to communicate successfully.

To fix this concept in your mind, I will introduce two analogies from electrical engineering which should be familiar to members of that profession:[7]:

1. The maximum power transfer theorem: [11, p 432] The transfer of power from a source to a load is maximum if the load impedance is the complex conjugate of the source impedance (see Figure 50.1). *The matching of source and load impedances for maximum*

[7] Please ignore if you are not an electrical engineer.

power transfer to occur is analogous to matching the writer's technique to the reader's expectations for maximum communication to occur.

2. There are *no reflections* on an ideal, lossless transmission line if it is *matched* or terminated with a load that is equal to the *characteristic impedance* of the transmission line [12, p 355]. The *reflections* at the end of a transmission line are like the reader's *confusion* at what the writer intended to convey; such confusion is minimized again by *matching what the reader expects with what the writer provides*.

Gopen and Swan [13][8] have written an excellent article introducing scientific method into scientific writing. They claim that readers have certain implicit expectations about *what to encounter* and *when*, each time they read a sentence. If the writer matches these expectations, communication takes place easily; otherwise confusion or misinterpretation results. They exhort the writer *to write so as to match the reader's expectations*. The reader should not waste the effort that would go into *understanding the substance* of the writing, in trying to *guess* what the writer *intended to mean*. Although they warn that "there can be no fixed algorithm for good writing", they give seven sound generic guidelines that are worth re-stating here [13]:

1. Follow a grammatical subject with its verb, as soon as possible.

2. Place in the position of importance (stress position) the "new information" you want the reader to emphasize in his or her mind.

3. Place the person or thing whose story is being told at the beginning of a sentence in the topic position.

4. Place appropriate "old information" (material discussed earlier) in the topic position to provide *linkage* with what has gone before and *context* for what is to come later.

5. Make clear the action of every clause or sentence in its verb.

6. Provide context for your reader before asking him or her to consider anything new.

[8] I am indebted to Emeritus Professor David R Lindsay for introducing me to this article. An online version is available at `http://www.americanscientist.org/issues/feature/the-science-of-scientific-writing/1`

7. Match the emphasis conveyed by the *substance* with the emphasis anticipated by the reader from the *structure*.

In summary, match the reader's expectations by constructing sentences skilfully.

> Lead the reader from the known to the unknown.

Write with the reader in mind: this is usually the examiner, but do not forget the poor student who gets to continue your project the next year. If your thesis is not clear enough, she or he may be condemned to repeat your work before making further progress, losing valuable time in the process.

50.6.3 Think, plan, write, revise

> Think. Plan. Write. Revise.

This is the cycle advocated by Barrass [6] in his short but very useful book on scientific writing. Messy thinking leads to messy writing: cluttered, obscure and uninviting. Think and plan before you write and revise.

Writing is not a linear process but a cyclic one. What appears first may be written last, with the benefit of hindsight and a unified perspective. But, where does one start; how does one revise, and how many times? As an entrée, let us listen to those with experience.

50.6.4 Attikiouzel's aphorisms [14]

1. *Start writing early.* Do not delay writing until you have finished your project or research. Write complete and concise "Progress Reports" *as and when* you finish *each nugget* of work. This way, you will remember everything you did and document it accurately, when the work is still fresh in your mind. This is especially so if your work involves programming or field investigation.

2. *Spot errors early.* A well-written "Technical Report" will force you to think about what you have done, *before* you move on to something else. If anything is amiss, you will detect it at once and can easily

correct it, rather than have to re-visit the work later, when you may be pressured for time and have lost touch with it.

3. *Write your thesis from the inside out.* Begin with the chapters on your own experimental work. You will develop confidence in writing them because you know your own work better than anyone else. Once you have overcome the initial inertia, move on to the other chapters.

4. *End with a bang, not a whimper.* First things first, and save the best for last. First and last impressions persist. Arrange your chapters so that your first and last experimental chapters are sound and solid.

5. *Write the Introduction after writing the Conclusions.* The examiner will read the Introduction first, and then the Conclusions, to see if the promises made in the former are indeed fulfilled in the latter. Ensure that your Introduction and Conclusions match 100%.

6. *"No man is an Island".*[9] The critical review of the literature places your work in context. Usually, one third of the PhD thesis is about others' work; two thirds, what you have done yourself. After a thorough and critical literature review, the PhD candidate must be able to identify the major researchers in the field and make a sound proposal for doctoral research.

7. *Estimate the time to write your thesis and then multiply it by three to get the correct estimate.* Writing at one stretch is very demanding and it is all too easy to underestimate the time required for it; inflating your first estimate by a factor of three is more realistic.

50.6.5 Lindsay's laws [5, 15]

1. Research is finished only after it is written up. What you write must *communicate* and *persuade*.

2. The hallmarks of scientific writing are *precision, clarity* and *brevity, in that order*.

[9] No man is an Island, entire of itself; every man is a piece of the Continent, a part of the main; if a clod be washed away by the sea, Europe is the less, as well as if a promontory were, as well as if a manor of thy friends or of thine own were; any man's death diminishes me, because I am involved in Mankind; And therefore never send to know for whom the bell tolls; It tolls for thee.—John Donne (1571–1631), *Meditation XVII*

3. Try to write as if you were speaking to someone: “see a face”. This way you get to say it directly and clearly.

4. Write (your chapters) in *four drafts*:

   a) first: putting the facts together

   b) second: checking for coherence and fluency of ideas

   c) third: readability

   d) fourth: editing

   Full details are given in Lindsay’s book [5, chapters 1 to 4].

5. The Introduction should embody the (unified) hypothesis. The reader finds in a clearly expressed hypothesis the skeleton of the thesis on which hangs all of the skin and meat that will be presented later.

6. The scope and emphasis of the Literature Review must be directly relevant to the subject of the thesis.

7. Include a common chapter that presents in one place all the experimental details common to all your experimental chapters. This avoids boring repetition and clears the way for a more fluent presentation of experimental results in different chapters without the intervening distraction of tedious methodology.

8. Experiments and results must be set out in careful detail in individual chapters. See Item i. to Item iv. on page 701 for the structure of each experimental chapter. Where several related experiments are grouped into a single chapter, it is preferable to present this sequence individually for each experiment but to conclude with *one* Discussion. This will meld the experiments together and unify the chapter.

9. The General Discussion or Conclusions integrate the whole thesis and present its main points at one place. This should be done in the context of the unifying hypothesis of the thesis. The Introduction and this chapter along with the Summary or Abstract are the most important parts of the thesis.

50.6.6 Hartmann's hints [16]

Listed below are hints on writing the *PhD thesis*, gleaned from a seminar I once attended[16], with points made largely by the first speaker during split group discussion, and subsequently by all three speakers at a panel discussion. Undergraduate students may optionally skip this section.

1. *Title.* The title should be succinct, focused and objective, giving, if possible, the scope of the thesis.

2. *Abstract or Summary.* Examiners will look here to find out whether it is new knowledge; and if so what.

3. *Introduction.* Remember that the introductory pages are important because they create the first, and perhaps lasting, impression on the examiner. Use flow diagrams, headings, sub-headings etc., to create and sustain interest.

4. *Literature Review.* This should be a *critical synthesis* of the state of the knowledge. Especially important are the areas needing further investigation: what has not been done, as well as what has been done, but for which there is a conflict in the literature. The examiner finds out *how the candidate thinks* from reading this section.

5. *Hypothesis Testing.* The hypothesis must be framed carefully and experiments designed thoughtfully to test it.

6. *Materials and Methods.* Ensure proper quality control and statistical planning and analysis. Retain enough details to allow repetition of experiments for up to seven (7) years, as legally required.

7. *General Discussion or Conclusions.* You may afford to be speculative here.

8. Examiners ask the following questions when reading a thesis:

   - Has the student read all the references?
   - What questions does this thesis raise?
   - What richness does it contain that can spawn other work?
   - What is the quality of flow of ideas?

9. Keep in mind that examiners read a thesis in instalments and display a *natural benevolence,* i.e., they do not set out to read a thesis with the aim of failing the student.

10. Read the *whole* thesis to pick up repetition.

11. Read your thesis for *ideas* and read it again for *editing* (see Item 4 in Section 50.6.5).

50.6.7 Cobbling together your first draft

According to Newton's first law, *starting something new* is difficult because inertia must be overcome.[10] Writing a thesis from scratch is no exception. This is why I suggested that you start writing your thesis *before* you know you are writing it: by keeping complete notes in your research notebook and by writing "Progress Reports" as and when you complete each module of work.

Use whatever writing techniques you are familiar and comfortable with. If, for example, you like to jot down *bullet points* before you formally commit your thoughts to writing, do so by all means. If you have used *mind-maps* [17] in your study technique, you may wish to apply them to write your thesis too. Marshall whatever resource or technique that has worked for you, and use it to help you write your thesis.

You are now familiar with the structure of the entire thesis and also with that of each experimental chapter (see Section 50.5.1). You have also benefited from the counsel of several experienced academics. Let us now tackle the nitty-gritty of actually writing the thesis, more or less in the order you should go about it.

50.6.8 The Experimental Chapters

Each of these should preferably be self-contained and clearly focused. Think of the story you want to tell.

★

Choose and present only those results that are relevant to your hypothesis.

[10] I have taken pedagogic licence here by extrapolating Newton's laws from the physical to the mental.

A morass of experimental results un-illuminated by a hypothesis and unembellished by a discussion is insulting and confusing to your reader.

The sections in your chapter should follow the experimental schema set out in Figure 50.2. State your hypothesis clearly. Indicate all assumptions. Include enough information about materials and methods to enable another suitably qualified person to repeat your experiments. Relegate tedious but necessary details to an Appendix, so that there are no breaks in the flow of ideas in your presentation.

If you chose some "magic numbers" for your programs, or some specific conditions for your experiment that may not be readily apparent to your reader, *explain* the reasons for your choice here.

Do not mix Materials and Methods with Results [18]; they are quite distinct as shown in Figure 50.2. It is customary to describe your Methods before the Materials. For example, you would describe your algorithm before giving details about the dataset on which you developed and tested it. Use informative headings. If you are using a method that has already been documented in the literature, do not describe it in full; describe it briefly or not at all, and give a reference citation [15].

When to present your results in a table and when to show them in a graph is discussed in Section 50.8.3.

If your results convey no sense of the new or the unexpected, you must ask yourself whether they are the right results to present, and also whether your hypothesis was well-framed in the first place. If your results are insipid, if they say nothing new, shed no light on what was unknown, and generally convey no sense of excitement or new knowledge, you should sit down and think carefully about everything you have done. A discussion with your supervisor may also be in order.

★

Do not present results chronologically; present them logically.

Adopt a standard nomenclature for all your chapters and introduce this in one place, preferably in a chapter preceding your experimental work, and entitled "Common Materials and Methods", or "Experimental Framework and Notation", or something similar. Do not change your symbols and their meanings as you go along: this will irritate your reader no end.

Check all facts and results at least once, twice if possible.

Use SI units and the preferred abbreviations. It is unprofessional to write 75 mhz when you mean 75 MHz. Leave a blank space between the number and the SI unit and do not put a full stop after the abbreviation, unless it is at the end of the sentence.

I repeat: try to present your Results separately from your Discussion. There is a temptation to commingle fact and opinion, but resist it. Your work will be easier to understand if your results (measurements, observations, perceptions) are separated from your discussion (inferences, opinions, even conjectures).

The Discussion section of your experimental chapter is where you add value to your work.

This is where you comment on your results. Why are they what they are? What meaning can you wrest from them? Are they in accord with accepted theory? What do they mean with respect to your hypothesis? Do your results uphold your assumptions? How do you treat unexpected or inconsistent results? Can you account for them? Do your results suggest that you need to revise your experiments or repeat them? Do they indicate a revised hypothesis? What are the limitations in your methodology? How do your results fit in with the work of others in the field? What additional work can you suggest?

An A^+ student distinguishes himself or herself by the quality, depth, knowledge and subject mastery that is apparent from the discussion. Even if the hypothesis fell as a result of the experiment, an excellent discussion of results alone can earn you an A^+.

Throughout your thesis, and especially in your experimental chapters, there should be *no gaps in the flow of logic.* Keep the links of a chain in mind. Each link is connected to two other links: one before and one after. Absence of any one link is a weakness. Absence of both means there is no chain!

To sum up, your overall purpose is to tell a good story: interesting, coherent, and plausible. Use your results to serve this purpose, keeping the hypothesis in mind.

50.6.9 The Literature Review

The literature review is the backdrop against which you present your work. It must be selective, but substantial enough for the merits of your work to be judged in relation to what is known. It is especially critical for a PhD thesis where the claim of originality should be defended with a thorough and critical review of the literature, especially in your specific area of research. You should capture the essence of current knowledge and comment critically on where the interesting questions and inconsistencies lie. The literature review is vital to justify your hypothesis, which must be consistent with what is known. If you present your literature review objectively but selectively, so that it does not stick out as an extraneous chapter, but merges into the larger story of your thesis, you would have done well.

50.6.10 The Introduction and Conclusions

The Introduction is where you "soft launch" your reader on the work described in your thesis. *Lead the reader from the known to the unknown.* State the hypothesis clearly. Give a preview of your thesis, globally, and chapter by chapter. Your Introduction has done its work if you have captured the reader's curiosity and interest in this first chapter.

The Conclusions record the power of your scientific thinking. You have to unite all that has gone before with a "thread of unified perspective". This is where you say why you think your story is a good one and present evidence from your work to support your claim. The fate of your hypothesis is revealed here: did it stand, fall, or require modification? You may briefly compare your work with that of others, present whatever new knowledge has been gained from your work, and suggest what may be done to further new knowledge. The Conclusions should give a sense of fulfilment and finality to your thesis, and give the reader some satisfaction that the time spent on reading it has not been in vain.

Write the Introduction *after* you have written the Conclusions and make sure the two match each other (see Section 50.6.4).

50.6.11 Linking your chapters

While you are writing your thesis, you might suddenly remember that an idea in Chapter 3 needs to be linked to an idea in Chapter 5, etc. This is a healthy sign because it means that you are integrating your work and seeing your thesis as one whole in your mind. These forward and backward linkages give continuity to your thesis. Keep a stack of pages, one for each chapter, where you can write down these aides-mémoire, *as and when they occur to you.* As you finish writing each chapter, check the "linklist" for that chapter and ensure that you have not forgotten anything.

50.6.12 The Summary or Abstract

The Summary or Abstract is perhaps the most difficult part to write. Do not make the mistake of trying to write it first: you will waste time and get discouraged.

★

The Summary or Abstract should be written last.

You will then have a feel for the story being told by your thesis: a bird's eye view so to speak, that was lacking when you had your nose to the grindstone, writing the Experimental Chapters or the Literature Review. *This unified perspective is vital to writing the Summary.*

I have found the following exercise very helpful in trying to focus the mind on what the point of a thesis (or paper or article) is. Try condensing your thesis in:

- one word;
- one line;
- one sentence;
- one paragraph;
- one page; and
- one chapter.

This method is somewhat like asking a dying man for a message: he will tell you only the most important thing(s). You begin at the most "compressed" level of describing your thesis and successively relax the constraint on the number of words to achieve increasing levels of detail. Somewhere along the way, you should have written your one- to two-page abstract, summarizing your thesis adequately. This is a disciplined way to distill what is important from what you have written.[11] If you have not gone through this process yourself, it is unfair (and risky) to expect the examiner to do it for you.

50.6.13 Writing other parts of your thesis

The Title should be neither too long nor too short. It should be focused and interesting. It should include the keywords you might use to describe your work in a scientific paper or thesis-abstracting system. Try to use some verbs rather than a long list of nouns.

The Acknowledgements should include sources of financial support and all those whose help you have sought and got, and all those whose work you have directly built upon.

The Bibliography should only contain references you have actually read. To quote an unread paper is misleading and dangerous. In engineering theses, references are usually cited by number, in order of citation. Follow the bibliographic convention applicable to your discipline.

Sometimes, it may be necessary to digress from your main story to explain something, especially for completeness. For example, it may be some experimental details, an analytical method, a program listing, etc., that is not central to your story, but whose exclusion would make your thesis incomplete. Include such material in an Appendix. Moreover, do not parrot textbook material in an Appendix just to give your thesis length or to impress your examiners. In all likelihood, they would ignore such material and could take marks off for gratuitous length.

50.6.14 Polishing up your thesis

As and when each chapter is written, read it for understanding, paying attention to the flow of logic and sense of continuity. Then read it again, paying attention this time to how comprehensible it is. Finally, read it once more paying attention to spelling, grammar, typography, placement

[11] The Abstract is *not* a summary of the *entire* contents of your thesis, but only of its *salient* points, including the *major findings and conclusions*.

of illustrations, etc. In these three stages, you are evaluating the chapter for its structure, substance and style (see also Section 50.7.1).

At each reading, revise your thesis as you feel appropriate.

When all the chapters are in place, read the thesis again, paying attention this time to *overall* understanding, coherence, comprehensibility and presentation.

> ★
>
> Get your supervisor, and anyone else whom you can approach, to read and criticize the early drafts of your thesis.

The more you polish up your thesis, the better your chances of getting high marks for it. A well-written thesis is like a piece of highly polished fine furniture: its elegance bespeaks its worth.

50.6.15 The time element

It is very easy to underestimate the time needed to plan, write and revise your thesis.

> ★
>
> As a general guideline, allow one to three months for writing up an undergraduate thesis and at least six months for a PhD thesis.

As another rule of thumb, triple your initial estimate to arrive at a more realistic time frame.

The task of writing up will not loom large at the end of your project if you have written your thesis in instalments as suggested in this guide.

Do not procrastinate, however much you dislike writing. Remember that writing up is also an integral part of your project or research work. Schroeder gives an interesting analysis, using a self-similar model, of how "…the longer one works on such a project without actually concluding it, the more remote the expected completion date becomes" [19, p 157].

This paragraph is addressed especially to PhD students. The period when you are writing up is the period when you are most vulnerable: the excitement of the research is now behind you, your scholarship would be running out or might already have, financial pressures will intensify, and there may be an obligation to work part-time and write up part-time. There may also be attractive job offers vying for your attention. Do not lose motivation during this difficult period. Loss of motivation is one of the principal ways in which you can deprive yourself of your PhD [20]. Write up your thesis and get on with the rest of your life.

50.6.16 Do's and Don'ts in Science and Engineering

- Do keep records as you go along and date them.
- Do systematic work.
- Don't claim precision where it is not justified.
- Don't present a conjecture as a fact.
- Don't plagiarize.
- Don't falsify records or cook up data.

50.7 Style: Language

50.7.1 The craft of writing good English

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Writing good English is a craft. It has to be learned by careful reading and even more careful writing.

You must develop your own *style:* no one can teach or bequeath it to you. It helps to read books devoted to the subject [21–27], but it helps even more to read exemplars of good writing. I particularly like and recommend the books of the chemist Peter Atkins [28, 29] and the biophysicist Harold Morowitz [30] which popularize science. These authors have demonstrated how it is possible to present science simply, correctly and engagingly.

As you progress in developing your own style, you will develop an internal feedback mechanism that will tell you just when the rhythm, length, and structure of a sentence is right, and when it needs revising.

★

Read what you have written, slowly and carefully. If you find yourself backtracking for any reason, revise what you have written.

This may be because of bad sentence structure, poor punctuation, excessive sentence length, poorly expressed ideas, or an unfortunate choice of words. Whatever the cause, take the trouble to revise it: if you yourself stumble on your own writing, your reader is bound to stumble too. The least courtesy you can do to your reader is to revise your writing.

Verbs are words of action. They infuse life and meaning to your writing. A long catalogue of nouns is lifeless; throw in a verb to add some sparkle!

Style and substance are intertwined. Say *clearly* why the busy reader should give *you* her time and attention, when so many others are clamouring for it, and say this *early*. Think of your writing as a tense wire connecting your reader to you. If everything you say is old hat to the reader, the wire is slack and you have lost your reader to boredom or even sleep. If everything you say is new and not linked to something the reader already knows, the wire is too taut and will break at some point. You will again lose your reader, but this time to incomprehension.

> Monotony leads to boredom; unpredictability to confusion. You have a duty to keep the reader challenged but not frustrated, engaged but not confused, comfortable but not bored.

The sections that follow are devoted to clarifying what good scientific writing is and should be.

50.7.2 Ambiguity and clarity

Ambiguity has its place. The novel *Finnegans Wake* by the great Irish author James Joyce [31], was first published in 1939. Starting with its title, the novel was open to several interpretations. Indeed, Joyce had claimed that this book "…would keep the professors busy for centuries" [32] and that is indeed one of its merits. It has proven to be such a rich source of layered meanings that there is at least one interpretive book with a *scientific* flavour, that has a chapter entitled "*Finnegans Wake*: The Complexity of Artificial Life" [33].

Scientific writing, however, must be unambiguous and the scientific thesis is no exception. It must communicate clearly,[12] precisely, and briefly.

[12]Those for whom English is a second language sometimes mistakenly think that good English should be convoluted. This is not true. Good English is clear and easy to read and understand. The cardinal rule is to *keep it simple.*

Say what was done; how it was done; why it was done etc., following the guidelines of Gopen and Swan in Section 50.6.2, to minimize the possibility of ambiguity and misinterpretation.

50.7.3 Precision

Precision distinguishes science as a field of intellectual endeavour. It is vital in quantitative work. Precision allows your work to be repeated by others for verification and extension. Vagueness hides in expressions like "quite small", "a considerable length" etc. Avoid them. They will besmirch your writing and your work.

★

Precision, accuracy, and experimental error are an inseparable triad. You should know how they differ and why they are related.

If not, read a good text on the subject, for example, Barford [34], or Topping [35]. Precision is related to *resolution* of measurements; accuracy, to *fidelity* with truth; and error with *departure* from truth. All measurements embody errors, limited by technique, instrumentation and other factors.

Do not record a measured voltage, for example, to five decimal places simply because a digital multimeter displays it to that many decimal places. Generally, if a measured voltage is quoted as 5 V, it means the value could be in error by *half the least significant digit*, i.e., the true value lies within the interval 5 ± 0.5 V. Two other popular conventions used in stating experimental results are: $\langle v \rangle \pm \sigma_v$ and $\langle v \rangle \pm 3\sigma_v$ where $\langle v \rangle$ is the mean of a series of measurements of voltage, v, and σ_v is the standard deviation. State the convention you have used in your thesis and stick to it throughout.

50.7.4 Brevity

Each of us is faced with more information than we can cope, let alone digest. The reader of your thesis is no exception. As a courtesy to your reader, be brief. Repetition frustrates the able reader. However, brevity must not be at the expense of clarity or precision.

Avoid saying the same thing twice except by choice.

Eschew expressions like "in order to", "as a result of", etc. When revising your thesis, try deleting phrases and expressions that are "fillers"; in most cases, what remains would be clearer and read better.

The use of acronyms is convenient and often unavoidable in specialist writing. Some acronyms like "laser",[13] have become entrenched in the common vocabulary. However, acronyms hold other, darker attractions, especially for students: they may be used to advertise the writer's erudition[14] or to separate the cognoscenti from the "ignoscenti". Such use of acronyms is best avoided, or it could lead to their proliferation, and the disease, *acronymosis*,[15] which destroys readability and sacrifices clarity for brevity.

50.7.5 Examples of what to avoid

Lindsay [5] gives ten categories of cumbersome expressions that should be avoided in writing a scientific paper or thesis. These are summarized below (using his examples, mostly):

1. *Clusters of nouns.* When clustered together, all nouns, except the last, function as adjectives. Avoid expressions like "chemical healing suppression" and say instead, "suppression of healing *by* chemicals", or "suppression *of* chemical healing", or whatever else you intended to mean. Use prepositions to make your meaning clear.

2. *Adjectival clauses.* Instead of "an innovation based return on investment culture", say "a culture *of* innovation based on return-on-investment" or whatever you actually meant to say. Again, use prepositions to make your meaning clear, even if this construction is longer.

3. *Subordinate clauses at the beginning.* This style puts the unimportant bits first and the important ones later. It may be good electronics to

[13] *L*ight *A*mplification by *S*timulated *E*mission of *R*adiation.

[14] SMTP sounds so much more learned (and complicated) than "*S*imple *M*ail *T*ransfer *P*rotocol" when used in the context of e-mail.

[15] I am indebted to Emeritus Professor David R Lindsay for introducing me to this priceless word.

do so (LSB[16] first), but it is bad English. Avoid beginning sentences with constructions like "Despite the fact that ...", "Notwithstanding the fact that ...", etc. Compare these two versions:

> Thus, although there were too few plots[17] to show all of the interactions which we sought [subordinate clause, apologetic], under the conditions of the experiment [subordinate phrase, conditional], copper and zinc acted additively [5, p 47].
>
> Thus, copper and zinc acted additively under the conditions of our experiment, although there were ...[5, p 47]

The second sentence certainly reads better. It is also a good example of putting the important information in the *topic position*, which is at the beginning (see Section 50.6.2).

4. *Nouns instead of the verbs from which they are derived.* Avoid writing "Recording of pulse rates was made"; instead write, "Pulse rates were recorded". We have improved the original sentence in three ways by doing this. We have:

    a) replaced the original *dummy* verb "made" with the *genuine* verb "recorded";

    b) shortened the sentence; and

    c) sharpened the impact.

5. *Use of filler verbs.* Do not write "We conducted a study of group III-V compounds"; instead say, "We studied group III-V compounds". The second sentence has five words; the first, eight. Again, a dummy verb has been replaced with a genuine verb and the sentence has been shortened and strengthened. Examples of dummy-verb constructions to be avoided are "to be present", "to occur", "to perform", "to obtain", etc.

6. *Use of passive voice rather than active voice.* Passive voice is appropriate when the doer of an action is unknown or is irrelevant. Otherwise, passive voice lengthens and weakens the sentence, whereas

[16] Least Significant Bit
[17] Plot of ground, presumably. My footnote.

active voice is direct, succinct and more forceful. Compare “Patients were observed by two people for signs of abnormal behaviour” [5, p 49] with “Two people observed the patients ...” [5, p 49].

7. *Use of imprecise words.* Do not use words like “quite”, “some”, “considerable”, “a great deal”, etc. in scientific writing. It is imprecise and unhelpful to the reader. Be quantitative: you are writing an engineering thesis. Sometimes, you may wish to avoid numerical precision for some compelling reason. If you want to avoid writing “Fifty-two percent of the images were correctly classified”, do not say “The majority of the images were correctly classified”, but rather “Slightly over half the images were correctly classified”.

8. *Use of compound prepositions.* Debaters and politicians use expressions like “in the case of”, “in respect of”, etc., usually to gain time to think of a proper answer during a debate or a press conference. Such expressions dilute the force of the simple, direct statement: they have no place in your thesis.

9. *Multiple negatives.* A double negative, when used carefully, has impact or conveys just the right shade of meaning. Multiple negatives do not. They serve only to confuse and should be avoided. What does “not unreasonably inefficient” really mean? Anytime you cause your reader to backtrack or pause for mental breath to take in meaning, you have done yourself and your reader a disservice. (Remember the reflections on the transmission line in Section 50.6.2.)

10. *Unfamiliar abbreviations and symbols.* Stick to SI units and prefixes. If you *have to* introduce a new unit called a *flip* make sure that you define it somewhere, introduce an abbreviation consistent with the SI system, use SI prefixes, and stick with your nomenclature all through.

50.7.6 Punctuation

Good punctuation makes reading easy.

The simplest way to find out where to punctuate is to read aloud what you have written. Each time you pause, you should add a punctuation symbol.

There are four major pause symbols, arranged below in ascending order of "degree of pause":

- *Comma.* Use the comma to indicate a short pause or to separate items in a list. A pair of commas may delimit the beginning and end of a subordinate clause or phrase. Sometimes, this is also done with a pair of "em dashes" which are printed like this: —.

- *Semi-colon.* The semi-colon signifies a longer pause than the comma. It separates segments of a sentence that are "further apart" in position, or meaning, but which are nevertheless related. If the ideas were "closer together", a comma would have been used. It is also used to separate two clauses that may stand on their own but which are too closely related for a colon or full stop to intervene between them.

- *Colon.* The colon is used before one or more examples of a concept, and whenever items are to be listed in a visually separate fashion. The sentence that introduced the itemized list you are now reading ended in a colon. It may also be used to separate two fairly—but not totally—independent clauses in a sentence.

- *Full stop or period.* The full stop ends a sentence. If the sentence embodies a question or an exclamation, then, of course, it is ended with a question mark or exclamation mark, respectively. The full stop is also used to terminate abbreviations like etc., (for et cetera), e.g., (for exempli gratia), et al., (for et alia) etc., but not with abbreviations for SI units.

The readability of your writing will improve greatly if you take the trouble to learn the basic rules of punctuation given above. For further guidance on punctuation, I recommend the books by Carey [36], Gowers [26], and Vallins [21, 22].

50.7.7 The I/We Active/Passive controversy

There is a pervasive belief that because scientific writing should be objective, one should avoid the first person singular pronoun “I”.[18] This belief is embedded in another deeper conviction: scientific writing must be in the passive voice, again in the interests of objectivity, because the subject “I” is thereby avoided. Some of those who hold these views are passionate about them. Others, are less dogmatic (see for example, Lindsay [5] and item Item 6 of Section 50.7.5). So what is acceptable and what is not? Is there any “right way”?

I read the writings of Faraday, Maxwell, and Rayleigh to get some light on the matter, and discovered the following:

1. The first person singular pronoun, “I”, is used by them liberally when they describe experiments they have themselves performed, or where they introduce new nomenclature, or when they refer to their personal conjectures or beliefs. I suspect that this practice springs from the times when papers were literally *read* at meetings of learned societies before they appeared in journals. The use of “I” was both natural and authoritative in that context. Examples of the use of ‘I’ are given below:

    > Many bodies are decomposed directly by the electric current, their elements being set free; these I propose to call *electrolytes*. Water, therefore, is an electrolyte.—*Michael Faraday* in [37, p 113]

    > I have recently been engaged in describing and defining the lines of magnetic force …*i.e.* those lines which are indicated in a general manner by the disposition of iron filings or small magnetic needles, around or between magnets; …—*Michael Faraday* in [38, p 407]

    > I first observed this peculiarity of my eyes when observing the spectrum formed by a very long vertical slit. I saw an elongated dark spot running up and down in the blue, as if confined in a groove, and following the motion of the eye as it moved up or down the spectrum, but refusing to pass out

[18] The plural, “we” somehow seems more acceptable, perhaps because it has royal connections!

> of the blue into other colours.—*James Clerk Maxwell* [39, p 435]

> It is now, I believe, generally admitted that the light which we receive from the clear sky is due in one way or another to small suspended particles which divert the light from its regular course.—*Lord Rayleigh* [40, p 87]

2. The first person plural pronoun, "we" is used when stating facts, assumptions or previously derived results; in (mathematical) proofs; and especially in textbooks where a didactic tone is normal. The use of "we" conveys the impression of a dialogue between writer and reader: something that is lacking with "I". Here are some examples:

   > When we turn to radiation phænomena, then we obtain the highest proof, that though nothing ponderable passes, yet the lines of force have a physical existence independent, in a manner, of the body radiating, or of the body receiving the rays.—*Michael Faraday* [38, p 409]

   > We have used the phrase Lines of Force because it has been used by Faraday and others. In strictness, however, these lines should be called Lines of Electric Induction.—*James Clerk Maxwell* [41, p 98]

   > We have seen that the electrical charge on the surface of the glass is attracted by the rubber.—*James Clerk Maxwell* [41, p 318]

   > The symmetry also requires that the intensity of the scattered light should vanish for the ray which would be propagated along the axis; for there is nothing to distinguish one direction transverse to the ray from another. We have now got what we want.—*Lord Rayleigh* [40, p 89]

3. Passive voice is used in textbooks and in describing facts, and experiments done by others, or where it does not matter who did the experiments:

   > There was also another effect produced, especially by the use of large electrodes, which was both a consequence and a proof

> of the solution of part of the gas evolved there. The collected gas, when examined, was found to contain small portions of nitrogen. This I attribute to the presence of air dissolved in the acid used for decomposition.—*Michael Faraday* [37, p 127]

> In each cell the copper plate is placed horizontally at the bottom and a saturated solution of sulphate of zinc is poured over it.—*James Clerk Maxwell* [41, p 397]

> There are two methods by which the pitch of a resonator may be determined without the use of a stream of air. The simplest, and in many cases the most accurate, method consists merely in tapping the resonator with the finger or other hammer of suitable hardness, and estimating with the aid of a monochord the pitch of sound so produced….The other method is one of which I have had a good deal of experience, and which I can rely upon to give results of moderate accuracy. It consists in putting the ear into communication with the interior of a resonator, and determining to what note of the scale the resonance is loudest.—*Lord Rayleigh* [40, p 320]

It is clear that some very eminent scientists had no hesitation in using the first person singular pronoun "I" to describe what they did, perceived or inferred. This usage is direct and is preferable to the passive voice, especially when used to describe what *you yourself* did. If, for modesty or other reasons, you are uncomfortable with using the pronoun "I", use the passive voice instead, but *not* the first person plural pronoun "we", which is inappropriate for two related reasons:

1. You are describing work that you have *individually* done rather than some *collective* effort for which the plural number would be apt.

2. Most university regulations are clear, especially for the PhD thesis, that *your original* work and contributions must be clearly distinguished from that of others [2]; again the plural number would be incorrect when describing this work.

50.7.8 Examples of good writing

I now present two examples of good scientific writing with some commentary:

> An atom is a body which cannot be cut in two. A molecule is the smallest portion of a particular substance. No one has ever seen or handled a single molecule. Molecular science, therefore, is one of those branches of study which deal with things invisible and imperceptible by our senses, and which cannot be subjected to direct experiment.—*James Clerk Maxwell* [39, p 361]

This is one of the founding fathers of the kinetic theory of gases holding forth on his home ground. These are the opening lines of a paper entitled *Molecules*, originally delivered before the British Association and published in *Nature*, Vol. VIII.

Maxwell uses the etymology of the word atom—from the Greek *ατομος/atomos*, which means "not cuttable"—to define it clearly and directly. The expression "cannot be cut in two" is more picturesque and powerful than the usual textbook definition, "smallest indivisible particle", that we have been brought up on. He then progresses to molecules and provokes our *interest* in what these mysterious, invisible, imperceptible entities might be. We are left anticipating what ingenious experiments he might have devised to demonstrate the existence and properties of molecules. If you can draw your reader into your work like this, you have written a good thesis.

Now for the second example:

> A structure is an arrangement of particles, such as atoms, molecules, or ions. For example, a crystal is a definite structure. It is distinct from a gas, a liquid, or even a splodge[19] of butter, because in these the arrangements of particles are indefinite. Whereas in a crystal we can be sure to find a particle at some definite location relative to another, …in the "structureless" states of gases, liquids, and amorphous solids, the relative locations of particles are indefinite …
>
> We can summarize these remarks (and sow the seed for the generalization) by saying that the particles of crystalline solids are arranged *coherently*: the locations are *correlated*. In contrast, in gases (and to a smaller extent in liquids) the locations are uncorrelated. The idea that *structure signifies coherence,* with orderly regiments of particles, whereas *lack of structure signifies incoherence,* with a hodge-podge of

[19] The use of a colloquial expression like "splodge" is permissible because the extract is from a book written to popularize science.

> locations, neatly captures solids as structures but allows gases to escape as structureless.—*Peter Atkins* [29, pp 179–180]

This is a fine example of leading the reader from the known to the unknown, progressively increasing the complexity of ideas. Atkins paints a picture in words, first relating structure to regularity in position. Then he moves on to the more mathematical and subtler concept of coherence and relates it to structure. The last sentence summarizes and binds together the three ideas: structure, positional regularity, and coherence. The parenthetical statement "sow the seed for the generalization" again keeps the reader anxiously waiting for the rabbit out of the hat.

50.7.9 Spelling and grammar

Check the spelling of all words in your thesis, *including those in your bibliography*, using a good spelling-checker. Use British or American spelling consistently throughout. There is some confusion about which spelling is correct: for example, is it "organize" or "organise"? British usage *allows both* [26, p 239] where appropriate, as in this case. The only exceptions are words like "surmise" which are never spelt with a "z" at all. Be consistent, once you have made your choice and do not mix "organize" with "organise" in your thesis. Note also that most spelling-checker programs are unaware of the advice of authorities like Gowers [26]. If your spelling-checker does not pick up repeated words like "the the", write a simple program to warn you of them. Check also that you have not written "and" where you meant "an" and vice versa. Such errors will not be trapped by a spelling-checker. Be careful with grammar-checkers: I do not trust them.

If there are glaring spelling errors in your thesis, examiners will get the impression that it is "poorly finished" and will not rate it highly for presentation.

> ★ Time spent in checking spelling and grammar is therefore time well spent.

50.8 Style: Layout

The layout is the packaging for your thesis. A pleasing font and adequate margins make your thesis visually attractive. The convention is to choose a font with serifs (e.g., Times Roman) for the main text and a sans serif font (e.g., Helvetica) for text *inside* diagrams. All figure captions should be in the same font as the main text, preferably at one size smaller. The details of thesis layout are considered next.

50.8.1 Format

The regulations governing size of paper, size of margins, etc., vary with department and university. The following criteria are typical:

> Theses are not restricted to one volume. They should be double or one and a half space typed on A4 paper with a left hand margin of 4 cm. There should be a 2 cm margin on all other edges. Typing on both sides is encouraged, and margins should be mirrored accordingly. [2]

The actual regulations may vary with time and the interested student is referred to the website pertinent to his or her department and university.

50.8.2 Word Processor vs Markup Language

Until the 1980s, theses were typed on typewriters and diagrams drawn by hand. This has changed with the advent of personal computers. You now type your own thesis at a computer terminal and use a word- or document-processing program to produce letter-perfect output. Microsoft Word and Corel WordPerfect are examples of WYSIWYG[20] word-processing programs, while LaTeX and XeLaTeX are examples of customizable document processing systems based on the extremely powerful TeX typesetting engine. If you do not know the difference between these two options, you should browse the web and find out, before making an informed choice about which to use. I shall refrain from advising here because I have a marked preference.

[20] *W*hat *Y*ou *S*ee *I*s *W*hat *Y*ou *G*et

50.8.3 Diagrams, Graphs and Tables

By its very nature, scientific writing includes the *judicious* use of diagrams, graphs and tables. When do you present your results using a graph and when do you tabulate them?

A table invokes an expectation of regularity. So present in a table dull, unremarkable data that must nevertheless be presented. Make sure that your variables are in different columns. Your rows for any given column should represent different observations of a given variable.

A graph should be reserved for exciting findings or interesting, but unexpected results. Trends, departures from trends, dramatic behaviours of variables, etc., are good candidates for graphs.

Caption all diagrams, graphs, and tables so that they may be read by themselves, independently of the main text, by a reader who wants only to skim your work. It is discourteous to embed the explanation or commentary for a diagram or graph somewhere in the text and let the skimmer hunt for it.

Refer in your text to every diagram, graph, and table, especially in the sections where you present and discuss your results.

The ready availability of graphical software should not entice you into presenting everything indiscriminately in pictorial form. Emphasis is rightly gained with sparing and selective use; and this applies to the use of diagrams, graphs, *italics* and **bold** typeface.

Three books that give helpful guidance on presenting diagrams, graphs and tables are: [18, 42, 43].

50.8.4 Table of Contents, Bibliography and Index

There are facilities to generate the table of contents, bibliography and index automatically using word- or document-processing programs. Learn how to use them and unburden yourself from the bookkeeping that goes with manually numbering references, figures, etc. You have more than enough on your plate, writing a good thesis, to fritter away your attention on such minutiae.

50.9 Suggestions for further reading

Language and usage change with time. So do university thesis guidelines. Bear this in mind as you read books devoted to helping you write your thesis.

This chapter is peppered with many references; so my suggested reading list is short and selective. The classic on British usage and style—originally written by Fowler—has been revised by Burchfield and is now called *The New Fowler's Modern English Usage* [44]. Its transatlantic counterpart is the legendary *The Elements of Style* by Strunk and White [45]. Other valuable references include Michael Harvey's *The Nuts & Bolts of College Writing* [46], Zinsser's *On Writing Well* [47] and La LaRocque's *The Book on Writing* [48]. Take from these books what appeals to you and develop your *own consistent* style from that.

50.10 Conclusions

Writing a thesis well is simple, if you know how. There are three aspects: structure, substance and style, but all three are entwined.

Start at the beginning by keeping good records. Understand what it is you are doing and why. Be clear what story you are going to tell. Keep the hypothesis to the fore always. Stick to the thesis structure you have been given.

Start writing your Experimental Chapters first. If you have done a Literature Review, write it next. Then complete the rest: Conclusions, Introduction, and Summary, in that order. The other bits and pieces like the Appendices may be written as you go along.

Think, plan, write, revise. Think clearly and write carefully. Clarity, precision and brevity are the three watchwords. Leave no gaps in the chain of logic or ideas you express. Avoid verbiage. Avoid clutter. Develop your own writing style by careful reading and even more careful writing. Polish what you have written by repeated reading and revision. Ask your supervisor to critique your thesis draft and amend it accordingly.

Enjoy writing your thesis and good luck!

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SUMMARY: HOW TO WRITE A THESIS

- Writing a thesis well is simple if you know how.
- There are three aspects:
  - structure
  - substance
  - style

  but all three are entwined.
- Start at the beginning by keeping good records.
- Understand what it is you are doing and why.
- Be clear what story you are going to tell.
- Keep the hypothesis to the fore always.
- Stick to the thesis structure you have been given.
- Start writing your Experimental Chapters first.
- If you have done a Literature Review, write it next.

- Then complete the rest:
  - Conclusions
  - Introduction
  - Summary

  in that order.
- The other bits and pieces like the Appendices may be written as you go along.
- Think, plan, write, revise.
- Think clearly.
- Write carefully.
- The three watchwords are:
  - Clarity
  - Precision
  - Brevity

  in that order.
- Leave no gaps in the chain of logic or ideas you express.
- Avoid verbiage.
- Avoid clutter.
- Develop your own writing style by
  - careful reading; and
  - even more careful writing.
- Polish what you have written by repeated reading and revision.
- Ask your supervisor to critique your thesis draft and amend it accordingly.
- Submit your best effort as your completed thesis.

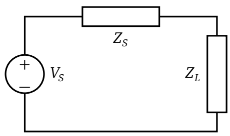

FIGURE 50.1: Maximum power is transferred from the source V_S if the load impedance Z_L is the complex conjugate of the source impedance Z_S, i.e., if $Z_L = Z_S^*$ [11, p 432].

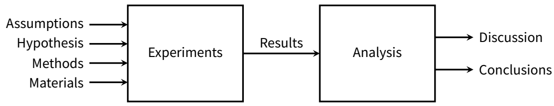

FIGURE 50.2: This diagram illustrates the relationship between the different stages in the experimental process. Do not intersperse your Results with Materials and/or Methods. Resist the temptation to pepper your Results section with a Discussion.

Part J

Working Life

54

The Last Word

Synopsis

Learn to think.
Unleash your imagination.
Follow your dreams.
Stick with it.
Never give up.
Succeed!

Colophon

This book has been typeset using the TeX program, the Unicode-aware XeTeX typesetting engine, and the LaTeX document preparation system.

The serif font used for the body text is Charis SIL The sans-serif font is Source Sans Pro, and the `monospaced` font is `Fira Mono`. Text body has been typeset in 12/14 pt.